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The New Complex Integral Transform "Complex Sadik Transform" and It's Applications

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Abstract

In this work, we present a novel complex transform namely the "Complex Sadik Transform". The propositions of this transformation are investigated. The complex transform is used to convert the core problem to a simple algebraic equation. Then, the answer to this primary problem can be obtained to find the solution to this equation and apply the inverse of the complex Sadik transform. As well, the complex Sadik transform is applied and used to find the solution of linear higher order ordinary differential equations. As well, we present and discuss, some important real life problems such as pharmacokinetics problems, nuclear physics problems, and Beam problems.

Keywords: Complex integral transformation, the inverse of complex transform, Sadik transform, ordinary differential equations.

1. Introduction

In (2018), researcher Sadik L. sheikh [11] presented a new integral transformation defined as follows :

The Sadik integral transform of $g(t)$ is defined as :

$$\mathbf{S}_a[g(t)] = \mathbf{F}(v^\alpha, \beta) = \frac{1}{v^\beta}\int_0^\infty g(t)e^{-v^\alpha t}dt$$

where $v \in \mathbb{C}$, $\alpha \in \mathbb{R}^*$, and $\beta \in \mathbb{R}$.

The following properties of Sadik integral transform [11] :

1 If $g(t) = t^n$, then $\mathbf{S}_a[t^n] = \frac{n!}{v^{n\alpha+(\alpha+\beta)}}$, $n \geq 0$.

2 If $g(t) = e^{at}$, then $\mathbf{S}_a[e^{at}] = \frac{v^{-\beta}}{v^\alpha - a}$, where a is a constant.

3 If $g(t) = \sin at$, then $\mathbf{S}_a[\sin at] = \frac{av^{-\beta}}{v^{2\alpha}+a^2}$.

4 If $g(t) = \cos at$, then $\mathbf{S}_a[\cos at] = \frac{v^{\alpha-\beta}}{v^{2\alpha}+a^2}$.

5 If $g(t) = \sinh at$, then $\mathbf{S}_a[\sinh at] = \frac{av^{-\beta}}{v^{2\alpha}-a^2}$

6 If $g(t) = \cosh at$, then $\mathbf{S}_a[\cosh at] = \frac{v^{\alpha-\beta}}{v^{2\alpha}-a^2}$.

7 $\mathbf{S}_a\left[g^{(n)}(t)\right] = v^{n\alpha}\mathbf{F}(v) - \sum_{k=0}^{n-1} v^{k\alpha-\beta} g^{((n-1)-k)}(0)$.

Now, the complex Sadik transform is a new complex transform and it is applied and used to find the solution of ordinary differential equation and has applications in domains such as engineering, applied physics, and signed processing [3,4,7].

We analyze functions in the set **C** defined by a novel complex transform defined for functions of exponential order:

$\mathbf{C} = \{g(t)$: there exists M, L_1 and L_2 are greater than zero such that

$$|g(t)| < Me^{-iL_j|t|}, \text{ if } t \in (-1)^j \times [0,\infty), j = 1,2\}$$

where i is a complex number.

The constant M must be a finite number for a particular function $g(t)$ in the set **C**, while L_1 and L_2 are may be finite or infinite.

The Complex Sadik Transform (CST) denoted by the operator $\mathbf{S}_a\{.\}$, the transform as follows:

$$\mathbf{S}_a^c[g(t)] = \mathbf{F}^c(s^\alpha, \beta) = \frac{1}{s^\beta}\int_0^\infty g(t)e^{-is^\alpha t}dt$$

where $s \in \mathbb{C}$, $\alpha \in \mathbb{R}^*$, and $\beta \in \mathbb{R}$.

The aim of this work (complex transform) to find the solution of higher order ordinary differential equations.

Many researchers have proposed new integral transformations for the purpose of solving ordinary and partial differential equations and their applications [2,8,9,12].

2. A Novel Complex Transform "Complex Sadik Transform" of Important Functions

In this section, we present the complex Sadik transform of famous functions:

1 If $f(t) = t^n, n \in \mathbb{N}$, then $\mathbf{S}_a^c\{t^n\} = (-i)^{n+1}\frac{n!}{s^{n\alpha+(\alpha+\beta)}}, s > 0$.

Proof. Since $\mathbf{S}_a^c[t^n] = \frac{1}{s^\beta}\int_0^\infty t^n e^{-is^\alpha t}dt$

Let $u = it \to du = idt$ or $\frac{du}{i} = dt$ or $-idu = dt$ and we know $u = it \to -iu = t$, when $t \to 0$ then $u \to 0$ and when $t \to \infty$ then $u \to \infty$, that is:

$$\begin{aligned}
\mathbf{S}_a^c[t^n] &= \frac{1}{s^\beta}\int_0^\infty t^n e^{-is^\alpha t}dt \\
&= \frac{1}{s^\beta}\int_0^\infty (-iu)^n e^{-s^\alpha u}(-i)du \\
&= \frac{1}{s^\beta}\int_0^\infty (-i)^n u^n e^{-s^\alpha} u(-i)du \\
&= (-i)^{n+1}\frac{1}{s^\beta}\int_0^\infty u^n e^{-s^\alpha u}du = (-i)^{n+1}\mathbf{S}_a[t^n] \\
&= (-i)^{n+1}\frac{n!}{s^{n\alpha+(\alpha+\beta)}} \cdot s > 0
\end{aligned}$$

2 If $f(t) = e^{at}, a$ is a constant number, then :

$$\mathbf{S}_a^c[e^{at}] = \frac{-1}{s^\beta}\left[\frac{a}{(s^{2\alpha}+a^2)} + i\frac{s^\alpha}{(s^{2\alpha}+a^2)}\right],\ s > a$$

Proof. Since

$$\begin{aligned}\mathbf{S}_a^c[e^{at}] &= \frac{1}{s^\beta}\int_0^\infty e^{at}e^{-is^\alpha t}dt\\ &= \frac{1}{s^\beta}\int_0^\infty e^{-t(is^\alpha - a)}dt\\ &= \frac{1}{s^\beta}\frac{1}{is^\alpha - a},\\ &= \frac{1}{s^\beta}\frac{1}{(is^\alpha - a)}\frac{(-is^\alpha - a)}{(-is^\alpha - a)},\\ &= \frac{1}{s^\beta}\left[\frac{(-1)(a+is^\alpha)}{(s^{2\alpha}+a^2)}\right],\\ &= \frac{-1}{s^\beta}\left[\frac{a}{(s^{2\alpha}+a^2)} + i\frac{s^\alpha}{(s^{2\alpha}+a^2)}\right],\ s>a\end{aligned}$$

3 Let $f(t) = \sin(at), a$ is a constant number, then

$$\mathbf{S}_a^c[\sin(at)] = \frac{-a}{s^\beta(s^{2\alpha}-a^2)}, s>|a|$$

Proof. Since

$$\begin{aligned}\mathbf{S}_a^c[\sin(at)] &= \frac{1}{s^\beta}\int_0^\infty \sin(at)e^{-is^\alpha t}dt\\ &= \frac{1}{s^\beta}\int_0^\infty \frac{e^{iat}-e^{-iat}}{2i}e^{-is^\alpha t}dt\end{aligned}$$

after simple computations, we get:

$$\mathbf{S}_a^c[\sin(at)] = \frac{-a}{s^\beta(s^{2\alpha}-a^2)}, s>|a|$$

The result will be benefit in determining the complicated transform of:

4 $\mathbf{S}_a^c[\cos(at)] = \frac{-is^\alpha}{s^\beta(s^{2\alpha}-a^2)}, s>|a|.$

5 $\mathbf{S}_a^c[\sinh(at)] = \frac{-a}{s^\beta(s^{2\alpha}+a^2)}, s>0.$

6 $\mathbf{S}_a^c[\cosh(at)] = \frac{-is^\alpha}{s^\beta(s^{2\alpha}+a^2)}, s>0.$

2.1. The Sadik and Complex Sadik Integral Transforms for Some Basic Functions

In this section, we will present the Sadik transform and the novel complex transform for some basic functions in the following **Table 1**:

Table 1: Sadik transform and the complex Sadik integral transform for some basic functions

Functions $g(t)$	$\mathbf{S}_a[g(t)] = \mathbf{F}(s)$ **"Sadik Transform"**	$\mathbf{S}_a^c[g(t)] = \mathbf{F}^c(s)$ **"Complex Sadik Transform"**
$t^n, n\in\mathbb{N}$	$\frac{n!}{s^{n\alpha+(\alpha+\beta)}}$	$(-i)^{n+1}\frac{n!}{s^{n\alpha+(\alpha+\beta)}}$
e^{at}, a constant	$\frac{1}{s^\beta(s^\alpha-a)}$	$\frac{-1}{s^\beta}\left[\frac{a}{(s^{2\alpha}+a^2)}+i\frac{s^\alpha}{(s^{2\alpha}+a^2)}\right]$
$\sin(at)$	$\frac{a}{s^\beta(s^{2\alpha}+a^2)}$	$\frac{-a}{s^\beta(s^{2\alpha}-a^2)}$
$\cos(at)$	$\frac{s^\alpha}{s^\beta(s^{2\alpha}+a^2)}$	$\frac{-is^\alpha}{s^\beta(s^{2\alpha}-a^2)}$
$\sinh(at)$	$\frac{a}{s^\beta(s^{2\alpha}-a^2)}$	$\frac{-a}{s^\beta(s^{2\alpha}+a^2)}$

$\cosh(at)$	$\frac{s^{\alpha}}{s^{\beta}(s^{2\alpha}-a^2)}$	$\frac{-is^{\alpha}}{s^{\beta}(s^{2\alpha}+a^2)}$

2.2. The inverse of Sadik Complex Integral Transform :

If $\mathbf{S}_a^c\{g(t)\} = \mathbf{F}^c(s)$ is the Sadik complex transform, then $g(t) = (\mathbf{S}_a^c)^{-1}[\mathbf{F}^c(s)]$ is said to be an inverse of the Sadik complex transform.

In this section, we present the inverse of Sadik complex integral transform of simple functions:

1 $(\mathbf{S}_a^c)^{-1}\left[(-i)^{n+1}\frac{n!}{s^{n\alpha+(\alpha+\beta)}}\right] = t^n.$

2 $(\mathbf{S}_a^c)^{-1}\left[\frac{-1}{s^{\beta}}\left[\frac{a}{(s^{2\alpha}+a^2)} + i\,\frac{s^{\alpha}}{(s^{2\alpha}+a^2)}\right]\right] = e^{at}.$

3 $(\mathbf{S}_a^c)^{-1}\left[\frac{-a}{s^{\beta}(s^{2\alpha}-a^2)}\right] = \sin(at).$

4 $(\mathbf{S}_a^c)^{-1}\left[\frac{-is^{\alpha}}{s^{\beta}(s^{2\alpha}-a^2)}\right] = \cos(at).$

5 $(\mathbf{S}_a^c)^{-1}\left[\frac{-a}{s^{\beta}(s^{2\alpha}+a^2)}\right] = \sinh(at)$

6 $(\mathbf{S}_a^c)^{-1}\left[\frac{-is^{\alpha}}{s^{\beta}(s^{2\alpha}+a^2)}\right] = \cosh(at)$

3. Complex Sadik Integral Transform of Derivatives:

Let $f(t)$ be a continuous function and piecewise continuous on any interval, then the complex Sadik transform of first derivative of $f(t)$ is given by:

$$\mathbf{S}_a^c[f'(t)] = \frac{1}{s^{\beta}}\int_0^{\infty} f'(t)e^{-is^{\alpha}t}dt, \text{ integrating by parts.}$$

$$\text{Let } u = e^{-is^{\alpha}t},\ dv = f'(t)dt$$

$$du = -is^{\alpha}e^{-is^{\alpha}t}dt,\ v = f(t)$$

$$\frac{1}{s^{\beta}}\left[-f(0) + is^{\alpha}\int_0^{\infty} e^{-is^{\alpha}t}f(t)dt\right]$$

$$\frac{-f(0)}{s^{\beta}} + is^{\alpha}\frac{1}{s^{\beta}}\int_0^{\infty} e^{-is^{\alpha}t}f(t)dt$$

$$\frac{-f(0)}{s^{\beta}} + is^{\alpha}\mathbf{S}_a^c[f(t)]$$

$$\mathbf{S}_a^c[f'(t)] = \frac{1}{s^{\beta}}\left[-f(0) + is^{\alpha}\int_0^{\infty} e^{-is^{\alpha}t}f(t)dt\right],$$

or

$$\mathbf{S}_a^c[f'(t)] = is^{\alpha}\mathbf{F}^c(s) - \frac{f(0)}{s^{\beta}}$$

Therefore, when substituted $f(t)$ by $f'(t)$ and $f'(t)$ by $f''(t)$, we get

$$\mathbf{S}_a^c[f''(t)] = (is^{\alpha})^2\mathbf{F}^c(s) - \frac{f'(0)}{s^{\beta}} - \frac{is^{\alpha}f(0)}{s^{\beta}}$$

Similarly,

$$\mathbf{S}_a^c[f'''(t)] = (is^{\alpha})^3\mathbf{F}^c(s) - \frac{1}{s^{\beta}}[f''(0) + is^{\alpha}f'(0) + (is^{\alpha})^2 f(0)].$$

In general:

$$\mathbf{S}_a^c\left[f^{(n)}(t)\right] = (is^{\alpha})^n\mathbf{F}^c(s) - \frac{1}{s^{\beta}}[f^{(n-1)}(0) + is^{\alpha}f^{(n-2)}(0) + (is^{\alpha})^2 f^{(n-3)}(0) + \cdots + (is^{\alpha})^{n-2}f'(0) + (is^{\alpha})^{n-1}f(0)]$$

or $\mathbf{S}_a^c\left[f^{(n}(t)\right] = (is^\alpha)^n\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\sum_{k=1}^{n}(is^\alpha)^{k-1}f^{(n-k)}(0)\right]$.

Theorem 3.1. Let $\mathbf{F}^c(s)$ be the complex Sadik integral transform of $f(t)(\mathbf{F}^c(s) = \mathbf{S}_a^c[f(t)])$, then :

$$\mathbf{S}_a^c\left[f^{(n)}(t)\right] = (is^\alpha)^n\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\sum_{k=1}^{n} (is^\alpha)^{k-1}f^{(n-k)}(0)\right].$$

Proof. By Mathematical Induction

1 For $n = 1$,

$$\mathbf{S}_a^c[f'(t)] = is^\alpha\mathbf{F}^c(s) - \frac{f(0)}{s^\beta}$$

Thus true for $n = 1$.

2. Assume that, true for $n = m$ that means:

$$\mathbf{S}_a^c\left[f^{(m)}(t)\right] = (is^\alpha)^m\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\sum_{k=1}^{m} (is^\alpha)^{k-1}f^{(m-k)}(0)\right]$$

3 we want to prove for $n = m + 1$

$$\begin{aligned}
\mathbf{S}_a^c\left[f^{(m+1)}(t)\right] = \mathbf{S}_a^c\left[\left(f^{(m)}(t)\right)'\right] \quad &= is^\alpha\mathbf{S}_a^c\left[f^{(m)}(t)\right] - \frac{1}{s^\beta}\left[f^{(m)}(0)\right],\\
&= is^\alpha\mathbf{S}_a^c\left[f^{(m)}(t)\right] - \frac{f^{(m)}(0)}{s^\beta},\\
&= is^\alpha\left[\left(is^\alpha\right)^m\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\sum_{k=1}^{m} (is^\alpha)^{k-1}f^{(m-k)}(0)\right]\right] - \frac{f^{(m)}}{s}(0)\Bigg]\\
&= (is^\alpha)^{m+1}\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\sum_{k=1}^{m} (is^\alpha)^{k}f^{(m-k)}(0)\right]\Bigg] - \frac{f^{(m)}(0)}{s^\beta}\\
&= (is^\alpha)^{m+1}\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\left[\sum_{k=1}^{m} (is^\alpha)^{k}f^{(m-k)}(0)\right] + f^{(m)}(0)\right],\\
&= (is^\alpha)^{m+1}\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\sum_{k=0}^{m} (is^\alpha)^{k}f^{(m-k)}(0)\right]\Bigg],\\
&= (is^\alpha)^{m+1}\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\sum_{k=1}^{m+1} (is^\alpha)^{k-1}f^{(m-(k-1))}(0)\right]\Bigg]\\
&= (is^\alpha)^{m+1}\mathbf{F}^c(s) - \frac{1}{s^\beta}\left[\sum_{k=1}^{m+1} (is^\alpha)^{k-1}f^{(m+1-k))}(0)\right]\Bigg],
\end{aligned}$$

So theorem is true for $n \in \mathbb{N}$.

4. Applications of Complex Sadik Integral Transform:

In this section, we introduce three real life problems: pharmacokinetics problem, nuclear physics and Beam problems.

Example 4.1. For a physical explanation of the present scheme, we consider a problem from the field of "pharmacokinetics" for solving the concentration of the drug at any given time " t " in the blood during continuous intravenous injection of drug and find its solution in this application. This application can be written in terms of 1^{st} order linear ordinary differential equation with constant coefficients as [1,5,6].

$$\frac{dg(t)}{dt}+\lambda g(t)=\frac{\gamma}{\text{volume}}, \text{ where } t>0 \qquad (1)$$

with initial conditions

$$g(0)=0 \qquad (2)$$

Here:

$g(t)$: is the drug concentration in the blood at any time " t ".

λ: is the constant velocity of elimination.

γ : the rate of infusion (in mg/min.)

Volume: volume in which drug is distributed.

Complex Sadik transform of both sides of equation (1) gives:

$$\mathbf{S}_a^c\left\{\frac{dg(t)}{dt}\right\}+\lambda\mathbf{S}_a^c\{g(t)\}=\frac{\gamma}{\text{volume}}\mathbf{S}_a^c\{1\} \qquad (3)$$

Applying Theorem 3.1, we get:

$$is^{\alpha}\mathbf{F}^c(s)-\frac{g(0)}{s^{\beta}}+\lambda\mathbf{F}^c(s)=\frac{\gamma}{\text{volume}}\frac{-i}{s^{\alpha+\beta}} \qquad (4)$$

The use of the initial condition equation (2) in (4) gives:

$$is^{\alpha}\mathbf{F}^c(s)+\lambda\mathbf{F}^c(s)=\frac{-i\gamma}{\text{volume } s^{\alpha+\beta}}.$$

$$\mathbf{F}^c(s)=\frac{\gamma}{\text{volume}}\frac{-is^{-\beta}}{s^{\alpha}(\lambda+is^{\alpha})} \qquad (5)$$

Applying inverse complex Sadik transform in equation (5), we get:

$$g(t)=\frac{\gamma}{\text{volume}}(\mathbf{S}_a^c)^{-1}\left[\frac{-is^{-\beta}}{s^{\alpha}(\lambda+is^{\alpha})}\right]$$

By a fractional fraction, after simple computations, we get:

$$g(t)=\frac{\gamma}{\lambda\text{ volume}}\left[1-e^{-\lambda t}\right].$$

Which is the required concentration of drug at any given time " t" in the blood during continuous intravenous injection of a drug.

Example 4.2. "Complex Sadik Transform in Nuclear physics":

Consider the first order linear differential equation:

$$\frac{dg(t)}{dt}=-\lambda g(t)$$

This differential equation is the fundamental relationship describing radioactive decay, where $g(t)$ represents the number of un decayed atoms remaining in a sample of radioactive isotope at the time " t " and λ is the decay constant, [7,10].

We can apply the complex Sadik transform to find the solution to this differential equation. Rearranging the above differential equation, we obtain:

$$\frac{dg(t)}{dt}+\lambda g(t)=0$$

Taking complex Sadik transform on both sides, we have:

$$\mathbf{S}_a^c\left\{\frac{dg(t)}{dt}\right\}+\lambda\mathbf{S}_a^c\{g(t)\}=0$$

then:

$$is^{\alpha}\mathbf{S}_a^c\{g(x)\} - \frac{g(0)}{s^{\beta}} + \lambda\mathbf{S}_a^c\{g(t)\} = 0$$

$$(is^{\alpha} + \lambda)\mathbf{S}_a^c\{g(t)\} = \frac{g(0)}{s^{\beta}}$$

$$\mathbf{S}_a^c\{g(t)\} = \frac{g(0)}{s^{\beta}(is^{\alpha} + \lambda)}, \text{ here } g(0) = g_0$$

Then

$$\mathbf{S}_a^c\{g(t)\} = \frac{g_0}{s^{\beta}(is^{\alpha} + \lambda)}$$

Now, we take the inverse complex Sadik transform on both sides, we obtain:

$$\begin{aligned} g(t) &= g_0(\mathbf{S}_a^c)^{-1}\left\{\frac{1}{s^{\beta}(is^{\alpha} + \lambda)}\right\} \\ &= g_0(\mathbf{S}_a^c)^{-1}\left\{\frac{1}{s^{\beta}(is^{\alpha} + \lambda)}\frac{\lambda - is^{\alpha}}{\lambda - is^{\alpha}}\right\} \\ &= g_0(\mathbf{S}_a^c)^{-1}\left\{\frac{1}{s^{\beta}}\left[\frac{\lambda}{s^{2\alpha} + \lambda^2} - i\frac{s^{\alpha}}{s^{2\alpha} + \lambda^2}\right]\right\} \\ &= g_0(\mathbf{S}_a^c)^{-1}\left\{\frac{-1}{s^{\beta}}\left[\frac{-\lambda}{s^{2\alpha} + \lambda^2} + i\frac{s^{\alpha}}{s^{2\alpha} + \lambda^2}\right]\right\} \\ &= g_0 e^{-\lambda t} \end{aligned}$$

Which is indeed the correct formula for radioactive decay.

Example 4.3. Problem to Beams:

A beam that is hinged at its ends, $x = 0$ and $x = L$ carries a uniform loud w_0 per unit length. Find the deflection at any point P.

Solutions:

The ordinary differential equation and boundary conditions are:

$$\frac{d^4y}{dx^4} = \frac{w_0}{E_1}, \ 0 < x < L \tag{6}$$

$$y(0) = y''(0) = 0, \ y(L) = y''(L) = 0 \tag{7}$$

where E is young's modulus, I is the moment of inertia of the cross section about an axis normal to the plane of bending and EI is said to be the flexural rigidity of the beam.

Some physical quantities associated with the application are:

$$y'(x), M(x) = EIy''(x) \text{ and } S(x) = M'(x)EIy''(x)$$

which respectively represent the "Slope", bending moment, and shear at a point P.

Taking complex Sadik transform of both sides of equation (6) , we get, if $\mathbf{F}^c(s) = \mathbf{S}_a^c\{y(x)\}$,

$$(is^{\alpha})^4\mathbf{F}^c(s) - \frac{1}{s^{\beta}}[y'''(0) + is^{\alpha}y''(0) + (is^{\alpha})^2y'(0) + (is^{\alpha})^3y(0)] = \frac{w_0}{EI}\left(\frac{-i}{s^{\alpha+\beta}}\right)$$

$$(is^{\alpha})^4\mathbf{F}^c(s) - \frac{1}{s^{\beta}}\left[C_2 + (is^{\alpha})^2C_1)\right] = \frac{-w_0 i}{EIs^{\alpha+\beta}},$$

$$s^{4\alpha}\mathbf{F}^c(s) = \frac{-w_0 i}{EIs^{\alpha+\beta}} + \frac{1}{s^{\beta}}\left[C_2 - s^{2\alpha}C_1)\right],$$

$$\mathbf{S}_a^c\{y(x)\} = \mathbf{F}^c(s) = \frac{-w_0 i}{EIs^{5\alpha+\beta}} + \frac{C_2}{s^{4\alpha+\beta}} - \frac{C_1}{s^{2\alpha+\beta}}$$

Inverting to find the solution:

$$y(x) = C_1x + C_2\frac{x^3}{3!} + \frac{w_0}{EI}\frac{x^4}{4!},$$

or

$$y(x) = C_1 x + C_2 \frac{x^3}{6} + \frac{w_0}{EI}\frac{x^4}{24}.$$

From the last two conditions in Equation (7) , we find:

$$C_1 = \frac{w_0 L^3}{24EI},\ C_2 = \frac{w_0 L}{2EI}.$$

Thus, the required deflection is :

$$y(x) = \frac{w_0}{24EI} x(L-x)(L^2 - Lx - x^2).$$

It is possible to calculate the bending moment and shear at any point P of the beam, and in particular, at the ends.

5. Conclusions

The definition and applications of the novel complex Sadik transform to solve ordinary differential equations have been demonstrated.

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