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Some Games in ỉ- PRE- g- separation axioms

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Abstract

The primary purpose of this subject is to define new games in ideal spaces via ỉ- pre - g- open set. The relationships between games that provided and the winning and losing strategy for any player were elucidated.

Keywords. ỉ- pre- g- open set, ỉ- pre- g- open function, ỉ- pre- g- cotinuous function, ỉ- pre- g- separation axioms and game.

1.Introduction

Kuratowski [1] presented in 1933. A collection ỉ ⊂ Ꝑ(X) is claims an ideal on a nonempty set X when the following two conditions are satisfied; (i) Ƀ ∈ ỉ whenever Ƀ ⊂ Ⱥ and Ⱥ ∈ ỉ (ii) Ⱥ ∪ Ƀ ∈ ỉ whenever Ⱥ and Ƀ ∈ ỉ. Vaidyanathaswamy [2]. Provides the concept of ideal spaces by giving the set operator $(\,)^*$: Ꝑ(X) ⟶ Ꝑ(X). Which is local function, so the topological spaces were circulated, claims ideal space and symbolize by (X, Ṭ , ỉ), [3-5].
Mashhour, Abd El- Monsef and El- Deeb, present the concept of "pre- open set", a set Ⱥ in (X, Ṭ) is a pre-open when Ⱥ ⊆ cl(int(Ⱥ)) [6]. Many researchers at that time used this concept in their studies [7-9].

Also, Ahmed and Esmaeel [10], use this concept to provide an ỉ - pre - g - closed set (symbolizes it, ỉpg - closed). If Ⱥ- Ụ ∈ ỉ and Ụ is a pre-open set, implies to cl(A) - Ụ ∈ ỉ , so a set Ⱥ in (X, Ṭ , ỉ) is ỉpg - closed. And the set Ⱥ in X claims ỉ - pre - g - open set (symbolizes it, ỉpg - open), if X - Ⱥ is ỉpg - closed. The collection of all ỉpg - closed sets (respectively, ỉpg - open sets) in (X, Ṭ , ỉ) symbolizes it ỉpg - C(X) (respectively, ỉpg - O(X)). And ỉpg - O(X) is finer than Ṭ.

A space (X, Ṭ, ỉ) is namely ỉpg -$Ṭ_0$-space (respectively ỉpg -$Ṭ_1$-space, ỉpg -$Ṭ_2$-space), if for each element $r_1 \neq r_2$, there is an ỉpg-open set containing only one of them (respectively there is an

İpg-open sets U_1 and U_2, satisfies $r_1 \in (U_1 - U_2)$ and $r_2 \in (U_2 - U_1)$, there is an İpg-open sets U_1 and U_2, satisfies $r_1 \in U_1$ and $r_2 \in U_2$ such that $U_1 \cap U_2 = \emptyset$)[11].
The main point of this article is to provide new types of games in ideal spaces by using the concept of İpg - open set.

2. İ - Pre- g- openness on Game.

This portion is to provide new types of game by using the concept of İpg - openness, where the relationships between them are discussed. In the theory of game , there is always at least two participants called players P_1 and P_2. Denoted for player one by P_1 and symbolizes for player two by P_2 and G be a game between two players P_1 and P_2. The set of choices $I_1, I_2, I_3, \ldots\ldots, I_m$ for each player. These choices are claims round, steps or options. In this research with games of type "Two- Zero- Sum Games". The games will be defined between two players and the payoff for any one of them equals to the loose of other player [11-13]

A function S is a strategy for P_1 as follows $S = \{S_m : A_{m-1} \times B_{m-1} \to A_m$, such that $(A_1, B_1, \text{------}, A_{m-1}, B_{m-1}) = A_m\}$ similarly a function T is a strategy for P_2 as follows $T = \{T_m ; A_m \times B_{m-1} \to B_m$, such that $(A_1, B_1, \text{------}, A_{m-1}, B_{m-1}, A_m) = B_m\}$. [15].
In this work, we provide the winning and losing strategy for any player P in the game G, if P has a winning strategy in G which symbolizes ($P \hookrightarrow G$), if P does not has a winning strategy symbolizes ($P \not\hookrightarrow G$), if P has a losing strategy symbolizes ($P \hookleftarrow G$) and if P does not has a losing strategy symbolizes ($P \not\hookleftarrow G$).

Definition 2.1. Let (X, T) be a topological space, define a game $G(\dot{T}_0, X)$ (respectively, $G(\dot{T}_0, \dot{I})$)as follows: The two players P_1 and P_2 play an inning for each natural numbers, in the m- th inning, the first round, P_1 will choose $x_m \neq \varsigma_m$. Next, P_2 choose $U_m \in T$ (respectively $U_m \in$ İpg- $O(X)$) such that $x_m \in U_m$ and $\varsigma_m \notin U_m$, P_2 wins in the game where $B = \{U_1, U_2, U_3, \ldots, U_m, \ldots\}$ satisfies that for all $x_m \neq \varsigma_m$ in X there exist $U_m \in B$ such that $x_m \in U_m$ and $\varsigma_m \notin U_m$. Other hand P_1 wins.

Remark 2.2. For any ideal topological space$(X, T, \dot{I})$:
1.if ($P_2 \hookrightarrow G(\dot{T}_0, X)$) then ($P_2 \hookrightarrow G(\dot{T}_0, \dot{I})$).
2.if ($P_2 \hookleftarrow G(\dot{T}_0, X)$) then ($P_2 \hookleftarrow G(\dot{T}_0, \dot{I})$).
3.if ($P_1 \hookrightarrow G(\dot{T}_0, \dot{I})$) then ($P_1 \hookrightarrow G(\dot{T}_0, X)$).

Proposition 2.3. If $(X, T, \dot{I})$) is $\dot{T}_0$-space (respectively, İpg- $\dot{T}_0$- space) $\longleftrightarrow$ ($P_2 \hookrightarrow G(\dot{T}_0, X)$). (respectively, ($P_2 \hookrightarrow G(\dot{T}_0, \dot{I})$).
Proof: since $(X, T, \dot{I})$ is $\dot{T}_0$- space (respectively, İpg- $\dot{T}_0$- space), then, in the m- th inning, any choice for the first player P_1, $x_m \neq \varsigma_m$, the second player P_2 can be found $U_m \in T$(respectively, $U_m \in$ İpg- $O(X)$) $U_2 \in T$ (respectively $U_m \in$ İpg- $O(X)$). So $B = \{U_1, U_2, U_3, \ldots, U_m, \ldots\}$ is the winning strategy for P_2.
($\Leftarrow$) Clear.

Corollary 2.4. ($P_2 \hookrightarrow G(\dot{T}_0, X)$) (respectively, ($P_2 \hookrightarrow G(\dot{T}_0, \dot{I})$)$\longleftrightarrow \forall x_1 \neq x_2$ in X , $\exists \dot{F} \in F$ (respectively$\exists \dot{F} \in$ İpgC(X)) such that, $x_1 \in \dot{F}$ and $x_2 \notin \dot{F}$.

Corollary 2.5. If $(X, T, \dot{I})$ is $\dot{T}_0$-space (respectively, İpg- $\dot{T}_0$-space)$\longleftrightarrow$ ($P_1 \not\hookrightarrow G(\dot{T}_0, X)$). (respectively ($P_1 \not\hookrightarrow G(\dot{T}_0, \dot{I})$).

Proposition 2.6. If (X, T, I) is not T_0-space (respectively, not Ipg- T_0-space)$\longleftrightarrow$$(P_1 \hookrightarrow G(T_0, X))$ (respectively, $(P_1 \hookrightarrow G(T_0, I))$.

Corollary 2.7. If (X, T, I) is not T_0-space (respectively not Ipg- T_0-space) $\longleftrightarrow$$(P_2 \not\hookrightarrow G(T_0, X))$ (respectively$(P_2 \not\hookrightarrow G(T_0, I))$**.**

Definition 2.8. Let (X, T, I) be a topological space, define a game $G(T_1, X)$ (respectively $G(T_1, I)$)as follows: The two players $P_1$and P_2 play an inning for each natural numbers, in the m-th inning, the first round, P_1 will choose $x_m \neq y_m$ where $x_m, y_m \in X$. Next, P_2 choose U_m, $V_m \in T$(respectively, U_m, $V_m \in I$pg-$O(X)$) such that $x_m \in (U_m$-$V_m)$ and $y_m \in (V_m$-$U_m)$, P_2 wins in the game where $B = \left\{\{U_1, V_1\}, \{U_2, V_2\} \dots, \{U_m, V_m\} \dots\right\}$ satisfies that for all $x_m \neq y_m$ in X there exist $\{U_m, V_m\} \in B$ such that $x_m \in (U_m$-$V_m)$ and $y_m \in (V_m$-$U_m)$. Other hand P_1 wins.

Remark 2.9. For any ideal topological space(X, T, I):
1. if $(P_2 \hookrightarrow G(T_1, X))$ then $(P_2 \hookrightarrow G(T_1, I))$.
2. if $(P_2 \hookleftarrow G(T_1, X))$ then $(P_2 \hookleftarrow G(T_1, I))$.
3. if $(P_1 \hookrightarrow G(T_1, I))$ then $(P_1 \hookrightarrow G(T_1, X))$.

Proposition 2.10. If (X, T, I) is T_1-space (respectively Ipg-T_1-space $\longleftrightarrow$$(P_2 \hookrightarrow G(T_1, X))$. (respectively, $(P_2 \hookrightarrow G(T_1, I))$.

Proof: ($\Longrightarrow$) Let (X, T, I) be a topological space, in the first round, P_1 will choose $x_1 \neq y_1$. Next, since (X, T, I) is T_1- space (respectively Ipg-T_1- space) P_2 can be found $U_1, V_1 \in T$ (respectively U_1, $V_1 \in I$pg-$O(X)$) such that $x_1 \in (U_1$-$V_1)$and $y_1 \in (V_1$-$U_1)$, in the second round, P_1 will choose $x_2 \neq y_2$. Next, P_2 can be found U_2, $V_2 \in T$ (respectively U_2, $V_2 \in I$pg-$O(X)$) such that $x_2 \in U_2$-V_2 and $y_2 \in (V_2$-$U_2)$, in the m-th round P_1 will choose $x_m \neq y_m$, Next, P_2 can be found U_m, $V_m \in T$(respectively, U_m, $V_m \in I$pg-$O(X)$) such that $x_m \in (U_m$-$V_m)$ and $y_m \in (V_m$-$U_m)$. So $B = \left\{\{U_1, V_1\}, \{U_2, V_2\}, \dots, \{U_m, V_m\}, \dots\right\}$ is the winning strategy for P_2.

($\Longleftarrow$) Clear.

Corollary 2.11. $(P_2 \hookrightarrow G(T_1, X))$ (respectively, $(P_2 \hookrightarrow G(T_1, I))$$\longleftrightarrow$ $\forall\, x_1 \neq x_2$ in X $\exists\, F_1, F_2 \in F$ (respectively $\exists\, F_1, F_2 \in I$pg-$C(X)$) such that, $x_1 \in F_1$ and $x_2 \notin F_1$ and $x_1 \notin F_2$ and $x_2 \in F_2$.

Corollary 2.12. (X, T, I) is T_1-space (respectively, Ipg-T_1-space) $\longleftrightarrow$$(P_1 \not\hookrightarrow G(T_1, X))$. (respectively $(P_1 \not\hookrightarrow G(T_1, I))$.

Proposition 2.13. (X, T, I) is not T_1-space (respectively, not Ipg-T_1-space$\longleftrightarrow$ $(P_1 \hookrightarrow G(T_1, X))$ (respectively$(P_1 \hookrightarrow G(T_1, I))$.

Corollary 2.14. (X, T, I) is not T_1-space (respectively, not Ipg-T_1-space) $\longleftrightarrow$ $(P_2 \not\hookrightarrow G(T_1, X))$ (respectively$(P_2 \not\hookrightarrow G(T_1, I))$**.**

Definition 2.15. [10], [13] Let (X, T) be topological space, define a game $G(T_2, X)$(respectively $G(T_2, I)$) as follows: The two players $P_1$and P_2 play an inning for each natural numbers, in the m-th inning, the first round, P_1will choose $x_m \neq y_m$. Next, P_2 choose U_m, V_m are disjoint, U_m,

$V_m \in T$(respectively, U_m, $V_m \in$ Ípg-O(X)) such that $x_m \in U_m$ and $\varsigma_m \in V_m$. P_2 wins in the game where $B = \{\{U_1, V_1\}, \{U_2, V_2\}, \ldots, \{U_m, V_m\}, \ldots\}$ satisfies that for all $x_m \neq \varsigma_m$ in X there exist $\{U_m, V_m\} \in B$, such that $x_m \in U_m$ and $\varsigma_m \in V_m$. Other hand P_1 wins.

Remark 2.16. For any ideal topological space(X, T, İ):

1. if ($P_2 \hookrightarrow G(\dot{T}_2, X)$) then ($P_2 \hookrightarrow G(\dot{T}_2, \dot{I})$).
2. if ($P_2 \hookleftarrow G(\dot{T}_2, X)$) then ($P_2 \hookleftarrow G(\dot{T}_2, \dot{I})$).
3. if ($P_1 \hookrightarrow G(\dot{T}_2, \dot{I})$) then ($P_1 \hookrightarrow G(\dot{T}_2, X)$).

Proposition 2.17. If (X, T, İ) is $\dot{T}_2$-space (respectively, İpg-$\dot{T}_2$-space) $\longleftrightarrow$ ($P_2 \hookrightarrow G(\dot{T}_2, X)$). (respectively, ($P_2 \hookrightarrow G(\dot{T}_2, \dot{I})$).

Proof: (⇒) Let (X, T, İ) be a topological space, in the first round, P_1 will choose $x_1 \neq \varsigma_1$. Next, since (X, T, İ) is $\dot{T}_2$- space (respectively, İpg-$\dot{T}_2$- space), P_2 can be found $U_1$and $V_1 \in T$ (respectively $U_1$and $V_1 \in$ İpg-O(X)) such that $x_1 \in U_1$ and $\varsigma_1 \in V_1$, $U_1 \cap V_1 = \emptyset$, in the second round, P_1 will choose $x_2 \neq \varsigma_2$. Next, P_2 choose $U_2$and $V_2 \in T$(respectively $U_1$and $V_2 \in$ İpg-O(X)) such that $x_2 \in U_2$ and $\varsigma_2 \in V_2$, $U_2 \cap V_2 = \emptyset$, in the m-th round, P_1 will choose $x_m \neq \varsigma_m$. Next, P_2 choose U_m and $V_m \in T$(respectively, $U_m$and $V_m \in$ İpg-O(X)) such that $x_m \in U_m$ and $\varsigma_m \in V_m$, $U_m \cap V_m = \emptyset$.

So $B = \{\{U_1, V_1\}, \{U_2, V_2\}, \ldots, \{U_m, V_m\} \ldots\}$ is the winning strategy for P_2.

(⇐) Clear.

Corollary 2.18. If (X, T, İ) is $\dot{T}_2$-space (respectively, İpg-$\dot{T}_2$-space) $\longleftrightarrow$ ($P_1 \nrightarrow G(\dot{T}_2, X)$). (respectively, ($P_1 \nrightarrow G(\dot{T}_2, \dot{I})$).

Proposition 2.19. (X, T, İ) is not $\dot{T}_2$-space(respectively not İpg-$\dot{T}_2$-space) $\longleftrightarrow$ ($P_1 \hookrightarrow G(\dot{T}_2, X)$) (respectively ($P_1 \hookrightarrow G(\dot{T}_2, \dot{I})$).

Corollary 2.20. (X, T, İ) is not $\dot{T}_2$-space (respectively not İpg-$\dot{T}_0$-space) $\longleftrightarrow$($P_2 \nrightarrow G(\dot{T}_2, X)$) (respectively($P_2 \nrightarrow G(\dot{T}_2, \dot{I})$)**.**

Remark 2.21. For any space (T, X, İ)

1. ($P_2 \hookrightarrow G(\dot{T}_{i+1}, X)$)(respectively $G(\dot{T}_{i+1}, \dot{I})$); i = {0,1} then ($P_2 \hookrightarrow G(\dot{T}_i, X)$)(respectively $G(\dot{T}_i, \dot{I})$).
2. ($P_2 \nrightarrow G(\dot{T}_{i+1}, X)$)(respectively $G(\dot{T}_{i+1}, \dot{I})$); i = {0,1} then ($P_2 \nrightarrow G(\dot{T}_i, X)$)(respectively $G(\dot{T}_i, \dot{I})$).

The following (fig) illustrates the relationships given in Remark 2.2

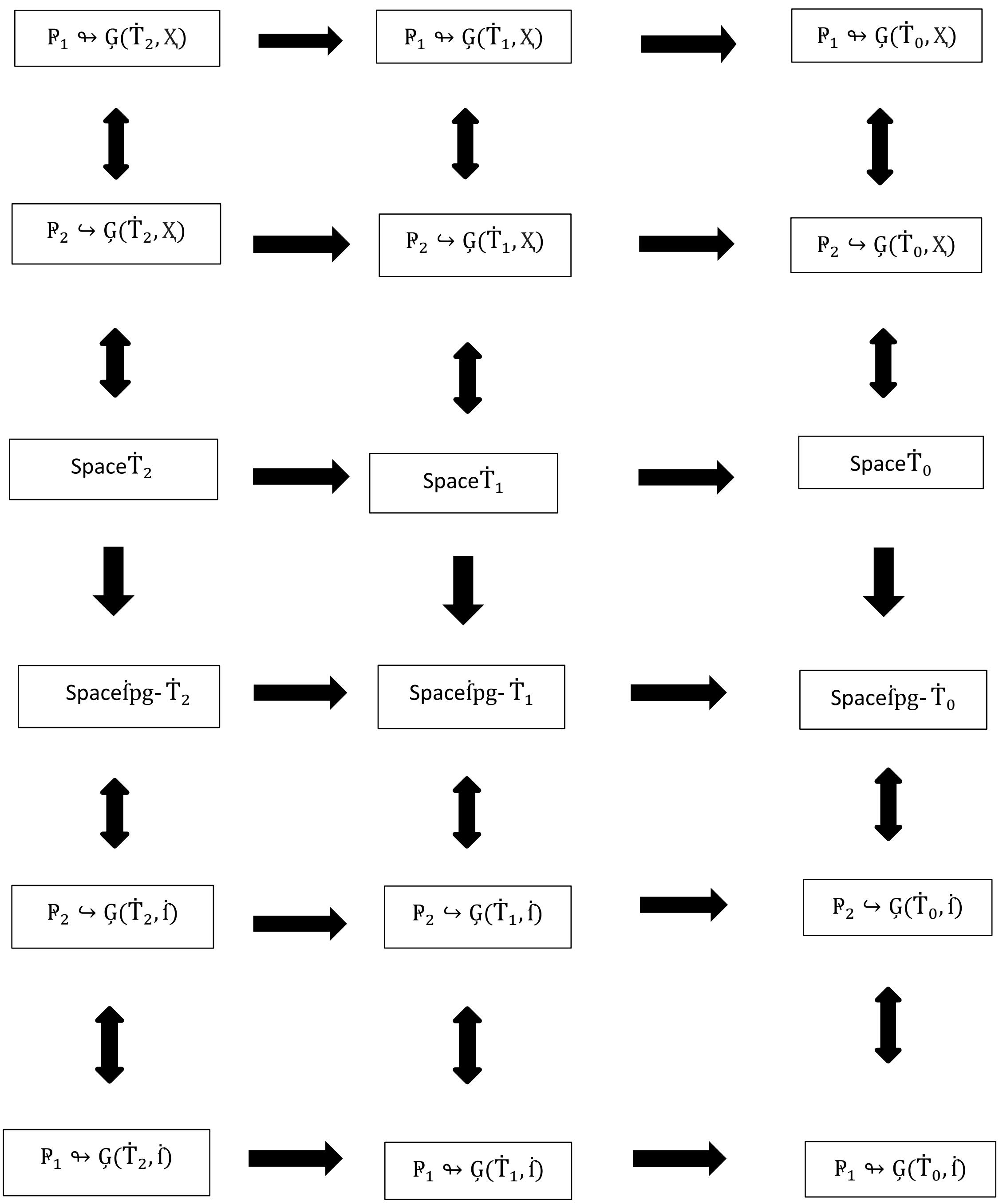

Figure 1. The winning strategy for Ᵽ_2 in $\text{Ģ}(\dot{T}_i, \text{X̧})$, i= {0, 1, 2}

Remark 2.22. For any space $(\text{Ț}, \text{X̧}, \dot{f})$

1. $(\text{Ᵽ}_1 \hookrightarrow \text{Ģ}(\dot{T}_i, \text{X̧}))$(respectively $\text{Ģ}(\dot{T}_i, \dot{f})$); i = {0,1} then $(\text{Ᵽ}_1 \hookrightarrow \text{Ģ}(\dot{T}_{i+1}, \text{X̧}))$(respectively $\text{Ģ}(\dot{T}_{i+1}, \dot{f})$).
2. $(\text{Ᵽ}_2 \looparrowright \text{Ģ}(\dot{T}_i, \text{X̧}))$(respectively $\text{Ģ}(\dot{T}_i, \dot{f})$); i = {0,1} then $(\text{Ᵽ}_2 \looparrowright \text{Ģ}(\dot{T}_{i+1}, \text{X̧}))$(respectively $\text{Ģ}(\dot{T}_{i+1}, \dot{f})$).

The following **Figure** illustrates the relationships given in Remark 2.22:

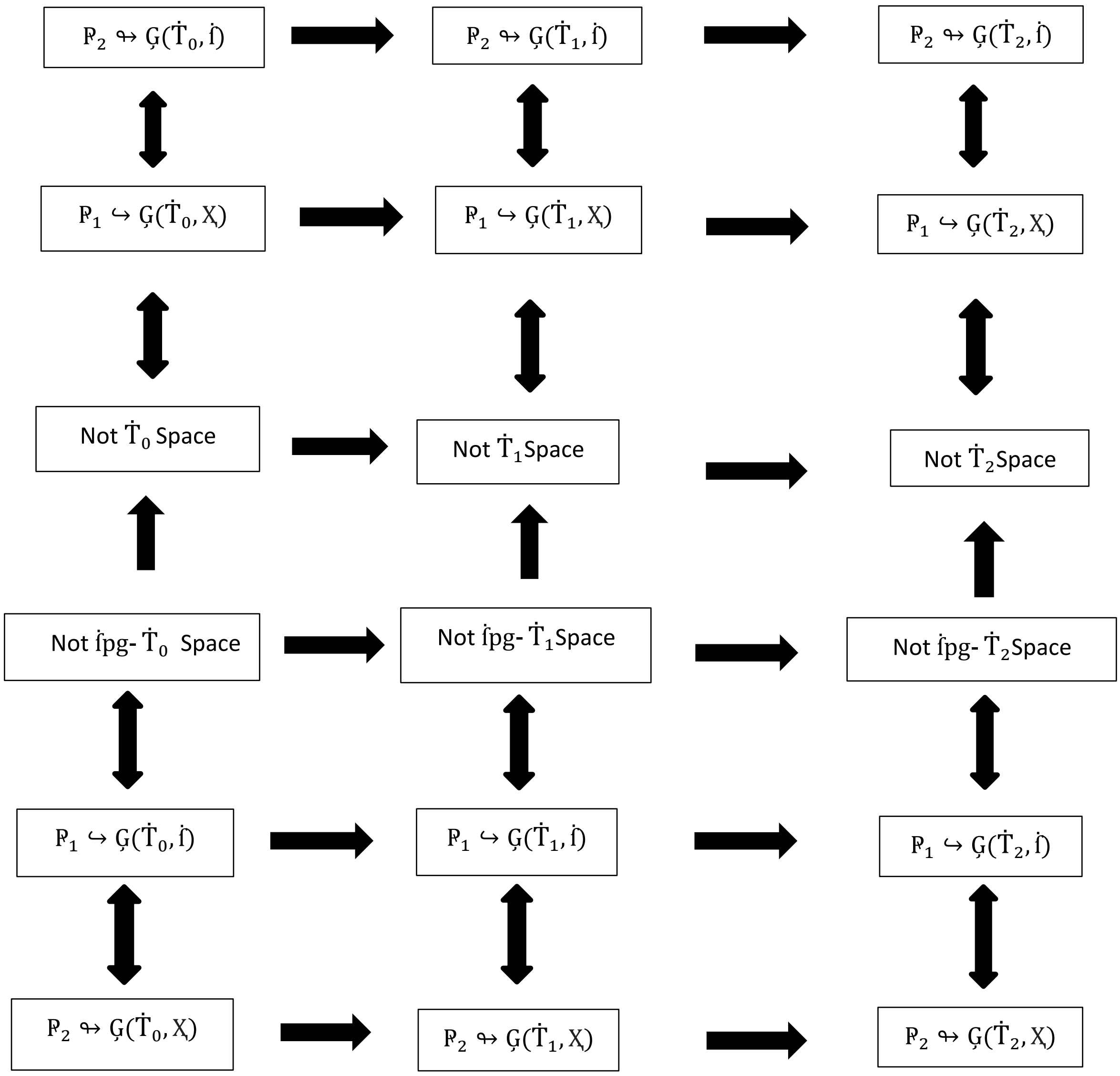

Figure 2. The winning strategy for $Ᵽ_1$ in $Ģ(Ṫ_i, Ẋ)$, i= {0, 1, 2}

3. The games with open functions via ḟpg-open sets.

By using open function via ḟpg-open sets; you can determine the winning strategy for any players in $Ģ(Ṫ_i, Ẋ)$); and $Ģ(Ṫ_i, ḟ)$) where i={0,1,2}.

Definition 3.1. (1) A function $ʄ : (Ẋ, Ţ, ḟ) \rightarrow (Ύ, Ƭ, ɉ)$ is

1. ḟ-pre-g-open function, symbolizes ḟpgo-function if ʄ(Ụ) is a ɉpg-open set in Ύ whenever Ụ is an ḟpg-open set in Ẋ.
2. $ḟ^*$-pre-g-open function, symbolizes $ḟ^*$pgo-function if ʄ (Ụ) is a ɉpg-open set in Ύ whenever Ụ is an open set in Ẋ.
3. $ḟ^{**}$-pre-g-open function, symbolizes $ḟ^{**}$pgo-function if ʄ (Ụ) is an open in Ύ whenever Ụ is an ḟpg-open set in Ẋ.

Proposition 3.1. If the function ƒ : (Ẋ, Ṭ, İ) → (ϓ, Ŧ, ɉ) is surjective open (respectively İ-pre-g-open function) and ($Ꝑ_2$ ↪ Ģ($\dot{T}_i$, Ẋ)) (respectively($Ꝑ_2$ ↪ Ģ($\dot{T}_i$, İ)) then ($Ꝑ_2$ ↪ Ģ($\dot{T}_i$, ϓ) (respectively ($Ꝑ_2$ ↪ Ģ($\dot{T}_i$, ɉ)), where (i=0,1and 2 respectively).

Proof **(1).** In the game Ģ($\dot{T}_i$, ϓ) (respectively, Ģ($\dot{T}_i$, ɉ)) where(i=0), in the first round, $Ꝑ_1$will choose $ς_1 \neq z_1$ such that $ς_1, z_1$ ∈ ϓ. Next, $Ꝑ_2$ in Ģ($\dot{T}_0$, ϓ) (respectively $Ꝑ_2$ in Ģ($\dot{T}_0$,ɉ) will hold account $ƒ^{-1}(ς_1)$, $ƒ^{-1}(z_1)$ ∈ Ẋ, $ƒ^{-1}(ς_1) \neq ƒ^{-1}(z_1)$, but ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, Ẋ)) (respectively ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, İ)), ∃$Ụ_1$ ∈ Ṭ (respectively ∃$Ụ_1$ ∈ İpg-O(Ẋ)), $ƒ^{-1}(ς_1)$ ∈ $Ụ_1$ and $ƒ^{-1}(z_1)$ ∉ $Ụ_1$ since ƒ is an open respectively İ-pre-g-open function then $ς_1$ ∈ ƒ($Ụ_1$) and z_1 ∉ ƒ($Ụ_1$) this implies $Ꝑ_2$ in Ģ($\dot{T}_0$, ϓ) (respectively $Ꝑ_2$ in Ģ($\dot{T}_0$,ɉ)) choose ƒ($Ụ_1$) is open (respectively ɉpg-open sets), in the second round, $Ꝑ_1$ in Ģ($\dot{T}_0$, ϓ) (respectively $Ꝑ_1$ in Ģ($\dot{T}_0$,ɉ) choose $ς_2 \neq ẓ_2$ such that $ς_2$, $ẓ_2$ ∈ ϓ. Next, $Ꝑ_2$ in Ģ($\dot{T}_0$, ϓ) (respectively $Ꝑ_2$ in Ģ($\dot{T}_0$,ɉ)) will hold account $ƒ^{-1}(ς_2)$, $ƒ^{-1}(ẓ_2)$ ∈ Ẋ, $ƒ^{-1}(ς_2) \neq ƒ^{-1}(ẓ_2)$, but ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, Ẋ)), (respectively ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, İ)), ∃$Ụ_2$ ∈ Ṭ(respectively ∃$Ụ_2$ ∈ İpg-O(Ẋ)), $ƒ^{-1}(ς_{ṃ})$ ∈ $Ụ_2$ and $ƒ^{-1}(ẓ_2)$ ∉ $Ụ_2$, then $ς_2$ ∈ ƒ($Ụ_2$) and z_2 ∉ ƒ($Ụ_2$) this implies $Ꝑ_2$ in Ģ($\dot{T}_0$, ϓ) (respectively $Ꝑ_2$in Ģ($\dot{T}_0$, ϓ) will choose ƒ($Ụ_2$) is open (respectively ɉpg-open sets) and in the ṃ-th round, $Ꝑ_1$ in Ģ($\dot{T}_0$, ϓ) (respectively $Ꝑ_1$ in Ģ($\dot{T}_0$,ɉ) choose $ς_{ṃ} \neq ẓ_{ṃ}$ such that $ς_{ṃ}, ẓ_{ṃ}$ ∈ ϓ. Next, $Ꝑ_2$ in Ģ($\dot{T}_0$, ϓ) (respectively $Ꝑ_1$ in Ģ($\dot{T}_0$,ɉ) will hold account $ƒ^{-1}(ς_{ṃ})$, $ƒ^{-1}(ẓ_{ṃ})$ ∈ Ẋ, $ƒ^{-1}(ς_{ṃ}) \neq ƒ^{-1}(ẓ_{ṃ})$, but ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, Ẋ)), (respectively ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, İ)), so,∃$Ụ_{ṃ}$ ∈ Ṭ(respectively ∃$Ụ_{ṃ}$ ∈ İpg-O(Ẋ)); $ƒ^{-1}(ς_{ṃ})$ ∈ $Ụ_{ṃ}$ and $ƒ^{-1}(ẓ_{ṃ})$ ∉ $Ụ_{ṃ}$, then $ς_{ṃ}$ ∈ ƒ($Ụ_{ṃ}$) and $z_{ṃ}$ ∉ ƒ($Ụ_{ṃ}$); this implies $Ꝑ_2$ in Ģ($\dot{T}_0$, ϓ) (respectively $Ꝑ_2$ in Ģ($\dot{T}_0$,ɉ)will choose ƒ($Ụ_{ṃ}$) is open (respectively ɉpg-open sets); thus Ƀ = {ƒ{ $Ụ_1$}, ƒ{$Ụ_2$}, ... , ƒ{$Ụ_{ṃ}$}...} is the winning strategy for $Ꝑ_2$ in Ģ($\dot{T}_0$, ϓ) (respectively, $Ꝑ_2$ in Ģ($\dot{T}_0$,ɉ)).

(2). In the game Ģ($\dot{T}_i$, ϓ) (respectively, Ģ($\dot{T}_i$, ɉ)) where (i=1), in the ṃ-th inning, $Ꝑ_1$will choose $ς_{ṃ} \neq z_{ṃ}$ such that $ς_{ṃ}, z_{ṃ}$ ∈ ϓ. Next, $Ꝑ_2$ in Ģ($Ŧ_1$, ϓ) (respectively, $Ꝑ_2$ in Ģ($\dot{T}_1$,ɉ) will hold account $ƒ^{-1}(ς_{ṃ})$, $ƒ^{-1}(z_{ṃ})$ ∈ Ẋ, $ƒ^{-1}(ς_{ṃ}) \neq ƒ^{-1}(z_{ṃ})$, but ($Ꝑ_2$ ↪ Ģ($\dot{T}_1$, Ẋ)) (respectively, ($Ꝑ_2$ ↪ Ģ($\dot{T}_1$, İ)), ∃$Ụ_{ṃ}$, $Ṿ_{ṃ}$ ∈ Ṭ (respectively ∃$Ụ_{ṃ}$, $Ṿ_{ṃ}$ ∈ İpg-O(Ẋ)), $ƒ^{-1}(ς_1)$ ∈ ($Ụ_{ṃ}$-$Ṿ_{ṃ}$) and $ƒ^{-1}(z_{ṃ})$ ∈ ($Ṿ_{ṃ}$ -$Ụ_{ṃ}$) and since ƒ is an open ,respectively İ-pre-g-open function; this implies $Ꝑ_2$ in Ģ($\dot{T}_1$, ϓ) (respectively $Ꝑ_2$ in Ģ($\dot{T}_1$,ɉ)) choose ƒ($Ụ_{ṃ}$) , ƒ($Ṿ_{ṃ}$) are open (respectively ɉpg-open sets), thus Ƀ = {{ƒ($Ụ_1$), ƒ($Ṿ_1$)}, {ƒ($Ụ_2$), ƒ($Ṿ_2$)}, ... , {ƒ($Ụ_{ṃ}$), ƒ($Ṿ_{ṃ}$)}...} is the winning strategy for $Ꝑ_2$ in Ģ($\dot{T}_1$, ϓ)) (respectively $Ꝑ_2$ in Ģ($\dot{T}_1$,ɉ)).In the same way, we can proof ($Ꝑ_2$ ↪ Ģ($\dot{T}_2$, ϓ) (respectively($Ꝑ_2$ ↪ Ģ($\dot{T}_1$,ɉ)) but ƒ($Ụ_{ṃ}$) ∩ ƒ($Ṿ_{ṃ}$) = ∅.Thus, Ƀ = {{ƒ($Ụ_1$), ƒ($Ṿ_1$)}, {ƒ($Ụ_2$), ƒ($Ṿ_2$)}, ... , { ƒ($Ụ_{ṃ}$), ƒ($Ṿ_{ṃ}$)}...} is the winning strategy for $Ꝑ_2$ in Ģ($\dot{T}_2$, ϓ)(respectively $Ꝑ_2$ in Ģ($\dot{T}_2$,ɉ)).

Proposition 3.3. If the function ƒ : (Ẋ, Ṭ, İ) → (ϓ, Ŧ, ɉ) is surjective İ*pgo-function and ($Ꝑ_2$ ↪ Ģ($\dot{T}_i$, Ẋ)), then, ($Ꝑ_2$ ↪ Ģ($\dot{T}_i$,ɉ)) , where (i=0,1and 2 respectively).

Proof **(1).** In the game Ģ($\dot{T}_i$,ɉ), where (i=0), in the first round, $Ꝑ_1$ will choose $ς_1 \neq ẓ_1$ such that $ς_1, ẓ_1$ ∈ ϓ. Next, $Ꝑ_2$ in Ģ($Ṭ_0$,ɉ) will hold account $ƒ^{-1}(ς_1)$, $ƒ^{-1}(ẓ_1)$ ∈ Ẋ, $ƒ^{-1}(ς_1) \neq ƒ^{-1}(z_1)$, but ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, Ẋ)), ∃$Ụ_1$ ∈ Ṭ, $ƒ^{-1}(ς_1)$ ∈ $Ụ_1$ and $ƒ^{-1}(z_1)$ ∉ $Ụ_1$, and since ƒ is İ*pgo-function this implies $Ꝑ_2$ in Ģ($\dot{T}_0$, Ẋ) will choose ƒ($Ụ_1$) is a ɉpg-open set, in the second round, $Ꝑ_1$ in Ģ($\dot{T}_0$,ɉ) choose $ς_2 \neq ẓ_2$, $ς_2$, $ẓ_2$ ∈ ϓ. Next, $Ꝑ_2$ in Ģ($\dot{T}_0$,ɉ) will hold account $ƒ^{-1}(ς_2)$, $ƒ^{-1}(ẓ_2)$ ∈ Ẋ, $ƒ^{-1}(ς_2) \neq ƒ^{-1}(ẓ_2)$, but ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, Ẋ)), ∃$Ụ_2$ ∈ Ṭ, $ƒ^{-1}(ς_2)$ ∈ $Ụ_2$ and $ƒ^{-1}(ẓ_2)$ ∉ $Ụ_2$, this implies $Ꝑ_2$ in Ģ($\dot{T}_0$, Ẋ) will choose ƒ($Ụ_2$) is a ɉpg-open set and in ṃ-th round $Ꝑ_1$ in Ģ($\dot{T}_0$,ɉ) choose $ς_{ṃ} \neq ẓ_{ṃ}$, $ς_{ṃ}, ẓ_{ṃ}$ ∈ ϓ, Next, $Ꝑ_2$ in Ģ($\dot{T}_0$,ɉ) will hold account $ƒ^{-1}(ς_{ṃ})$, $ƒ^{-1}(ẓ_{ṃ})$ ∈ Ẋ, $ƒ^{-1}(ς_{ṃ}) \neq ƒ^{-1}(ẓ_{ṃ})$, but ($Ꝑ_2$ ↪ Ģ($\dot{T}_0$, Ẋ)), ∃$Ụ_{ṃ}$ ∈ Ṭ, $ƒ^{-1}(ς_{ṃ})$ ∈ $Ụ_{ṃ}$ and $ƒ^{-1}(ẓ_{ṃ})$ ∉ $Ụ_{ṃ}$, this implies $Ꝑ_2$ in Ģ($\dot{T}_0$, Ẋ) will choose ƒ($Ụ_{ṃ}$) is a ɉpg-open set, thus Ƀ{ƒ{ $Ụ_1$}, ƒ{$Ụ_2$} ..., ƒ{$Ụ_{ṃ}$}...} is the winning strategy for $Ꝑ_2$ in Ģ($\dot{T}_0$, Ẋ)).

(2). In the game Ģ $(\dot{Ṭ}_i, ɉ)$ where$(i = 1)$, in the ṃ-th round $Ᵽ_1$ in Ģ$(\dot{Ṭ}_1, ɉ)$ choose $ς_{ṃ} \neq ẓ_{ṃ}$, $ς_{ṃ}, ẓ_{ṃ} \in ϒ$. Next, $Ᵽ_2$ in Ģ$(\dot{Ṭ}_1, ɉ)$ will hold account $ʄ^{-1}(ς_{ṃ}), ʄ^{-1}(ẓ_{ṃ}) \in Ꭓ$, $ʄ^{-1}(ς_{ṃ}) \neq ʄ^{-1}(ẓ_{ṃ})$, but ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_1, Ꭓ)$), $\exists Ụ_{ṃ}, ṿ_{ṃ} \in Ṭ$, $ʄ^{-1}(ς_{ṃ}) \in (Ụ_{ṃ}$-$ṿ_{ṃ})$ and $ʄ^{-1}(z_{ṃ}) \in (ṿ_{ṃ}$ -$Ụ_{ṃ})$, this implies $Ᵽ_2$ in Ģ$(\dot{Ṭ}_1, Ꭓ)$ will choose $ʄ(Ụ_{ṃ})$ and $ʄ(ṿ_{ṃ})$ are ɉpg-open sets, thus $Ƀ = \{\{ʄ(Ụ_1), ʄ(ṿ_1)\}, \{ʄ(Ụ_2), ʄ(ṿ_2)\}, \dots, \{ʄ(Ụ_{ṃ}), ʄ(ṿ_{ṃ})\}\dots\}$ is the winning strategy for $Ᵽ_2$ in Ģ$(\dot{Ṭ}_1, Ꭓ)$). By the same way we can proof($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_2, Ꭓ)$) but, $ʄ(Ụ_{ṃ}) \cap ʄ(ṿ_{ṃ}) = \emptyset$. Thus $Ƀ = ʄ(Ụ_{ṃ}) \cap ʄ(ṿ_{ṃ}) = \emptyset$ is the winning strategy for $Ᵽ_2$ in Ģ$(\dot{Ṭ}_2, Ꭓ)$).

Corollary. If the function $ʄ : (Ꭓ, Ṭ) \to (ϒ, Ƃ)$ is a surjective open function and $Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_i, Ꭓ)$, then $Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_i, ɉ)$, where i= {0, 1, 2}.

Proposition 3.4. If the function $ʄ : (Ꭓ, Ṭ, \dot{ſ}) \to (ϒ, Ƃ, ɉ)$ is a surjective $\dot{ſ}^{**}$pgo-function and ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_0, \dot{ſ})$ then, ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_0, ϒ)$, where (i = 0,1and 2 respectively).

Proof(**1).** In the game Ģ$(\dot{Ṭ}_i, ϒ)$ where$(i = 0)$, in the first round, $Ᵽ_1$in Ģ$(\dot{Ṭ}_0, ϒ)$ will choose $ς_1 \neq ẓ_1$ such that $ς_1, ẓ_1 \in ϒ$. Next, $Ᵽ_2$ in Ģ$(\dot{Ṭ}_0, ϒ)$ will hold account $ʄ^{-1}(ς_1), ʄ^{-1}(ẓ_1) \in Ꭓ$, $ʄ^{-1}(ς_1) \neq ʄ^{-1}(ẓ_1)$, but ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_0, \dot{ſ})$), $\exists Ụ_1 \in \dot{ſ}pgO(Ꭓ)$, $ʄ^{-1}(ς_1) \in Ụ_1$and $ʄ^{-1}(ẓ_1) \notin Ụ_1$, $ς_1 \in ʄ(Ụ_1)$ and $z_1 \notin ʄ(ṿ_1)$ and since ʄ is $\dot{ſ}^{**}$pgo-function this implies $Ᵽ_2$ in Ģ$(\dot{Ṭ}_0, \dot{ſ})$ will choose $ʄ(Ụ_1)$ such that $ς_1 \in ʄ(Ụ_1)$, $ẓ_1 \notin ʄ(Ụ_1)$ open, in the second round, $Ᵽ_1$ in Ģ$(Ṭ_0, ϒ)$ choose $ς_2 \neq ẓ_2$, $ς_2, ẓ_2 \in ϒ$. Next, $Ᵽ_2$ in Ģ$(\dot{Ṭ}_0, ϒ)$ will hold account $ʄ^{-1}(ς_2), ʄ^{-1}(ẓ_2) \in Ꭓ$, $ʄ^{-1}(ς_2) \neq ʄ^{-1}(ẓ_2)$, but ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_0, \dot{ſ})$), $\exists Ụ_2 \in \dot{ſ}pgO(Ꭓ)$, $ʄ^{-1}(ς_2) \in Ụ_2$ and$ʄ^{-1}(ẓ_2) \notin Ụ_2$, $ς_2 \in ʄ(Ụ_2)$ and $z_2 \notin ʄ(ṿ_2)$ this implies $Ᵽ_2$ in Ģ$(\dot{Ṭ}_0, \dot{ſ})$ will choose $ʄ(Ụ_2)$ and in ṃ-th round $Ᵽ_1$ choose $ς_{ṃ} \neq ẓ_{ṃ}$, $ς_{ṃ}, ẓ_{ṃ} \in ϒ$. Next, $Ᵽ_2$ in Ģ$(\dot{Ṭ}_0, ϒ)$ will hold account $ʄ^{-1}(ς_{ṃ}), ʄ^{-1}(ẓ_{ṃ}) \in Ꭓ$, $ʄ^{-1}(ς_{ṃ}) \neq ʄ^{-1}(ẓ_{ṃ})$, but ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_0, \dot{ſ})$), $\exists Ụ_{ṃ} \in \dot{ſ}pgO(Ꭓ)$, $ʄ^{-1}(ς_{ṃ}) \in Ụ_{ṃ}$ and $ʄ^{-1}(ẓ_{ṃ}) \notin Ụ_{ṃ}$, $ς_{ṃ} \in ʄ(Ụ_{ṃ})$ and $z_{ṃ} \notin ʄ(ṿ_{ṃ})$ this implies $Ᵽ_2$ in Ģ$(\dot{Ṭ}_0, \dot{ſ})$ will choose $ʄ(Ụ_{ṃ})$; thus $Ƀ = \{ʄ(Ụ_1), ʄ(Ụ_2), \dots, ʄ(Ụ_{ṃ})\dots\}$ is the winning strategy for $Ᵽ_2$ in Ģ$(Ƃ_0, ϒ)$).

(2). In the game Ģ$(\dot{Ṭ}_i, ϒ)$where$(i = 1)$, in the ṃ-th round $Ᵽ_1$choose $ς_{ṃ} \neq ẓ_{ṃ}$, $ς_{ṃ}, ẓ_{ṃ} \in ϒ$. Next, $Ᵽ_2$ in Ģ$(\dot{Ṭ}_1, ϒ)$will hold account $ʄ^{-1}(ς_{ṃ}), ʄ^{-1}(ẓ_{ṃ}) \in Ꭓ$, $ʄ^{-1}(ς_{ṃ}) \neq ʄ^{-1}(ẓ_{ṃ})$, but ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_1, \dot{ſ})$), $\exists Ụ_{ṃ}, ṿ_{ṃ} \in \dot{ſ}pg$-$O(Ꭓ)$,$ʄ^{-1}(ς_{ṃ}) \in (Ụ_{ṃ}$-$ṿ_{ṃ})$ and $ʄ^{-1}(ẓ_{ṃ}) \in (ṿ_{ṃ}$ -$Ụ_{ṃ})$, so $Ᵽ_2$ in Ģ$(Ṭ_1, \dot{ſ})$ will choose $ʄ(Ụ_{ṃ})$, $ʄ(ṿ_{ṃ})$;thus $Ƀ = \{\{ʄ(Ụ_1), ʄ(ṿ_1)\}, \{ʄ(Ụ_2), ʄ(ṿ_2)\}, \dots, \{ʄ(Ụ_{ṃ}), ʄ(ṿ_{ṃ})\}\dots\}$ is the winning strategy for $Ᵽ_2$ in Ģ$(Ṭ_1, ϒ)$).

In the same way, we can proof ($Ᵽ_2 \hookrightarrow (\dot{Ṭ}_2, ϒ)$), but $ʄ(Ụ_{ṃ}) \cap ʄ(ṿ_{ṃ}) = \emptyset$.

Thus $Ƀ = \{\{ʄ(Ụ_1), ʄ(ṿ_1)\}, \{ʄ(Ụ_2), ʄ(ṿ_2)\}, \dots, \{ʄ(Ụ_{ṃ}), ʄ(ṿ_{ṃ})\}\dots\}$ is the winning strategy for $Ᵽ_2$ in Ģ$(\dot{Ṭ}_2, ϒ)$).

4. The games with a continuous function via $\dot{ſ}$pg-open sets.

In this part, we will using *continuous* function via $\dot{ſ}$pg- open set to explain a winning strategy for $Ᵽ_1$ and $Ᵽ_2$ in Ģ$(\dot{Ṭ}_i, Ꭓ)$ and Ģ$(\dot{Ṭ}_i, \dot{ſ})$ where I ={0,1,2}.

Definition 3.6. (1) A function $ʄ : (Ꭓ, Ṭ, \dot{ſ}) \to (ϒ, Ƃ, ɉ)$ is;

1. $\dot{ſ}$-pre-g-continuous function, symbolizes $\dot{ſ}$pg-continuous, if $ʄ^{-1}(ṿ) \in \dot{ſ}pgO(Ꭓ)$ for all $ṿ \in Ƃ$.
2. Strongly-$\dot{ſ}$-pre-g-continuous function, Symbolizes strongly-$\dot{ſ}$pg-continuous, if $ʄ^{-1}(ṿ) \in Ṭ$, for all $ṿ \in ɉpgO(ϒ)$.
3. $\dot{ſ}$-pre-g-irresolute function, symbolizes $\dot{ſ}$pg-irresolute, if $ʄ^{-1}(ṿ) \in \dot{ſ}pgO(Ꭓ)$ for all $ṿ \in ɉpgO(ϒ)$.

Proposition 4.6. If the function $ʄ : (Ꭓ, Ṭ, \dot{ſ}) \to (ϒ, Ƃ, ɉ)$ is an injective $\dot{ſ}$-pre-g-continuous function and ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_i, ϒ)$ then ($Ᵽ_2 \hookrightarrow$ Ģ$(\dot{Ṭ}_i, \dot{ſ})$), where (i=0,1and 2 respectively).

Proof **(1).** In the game $\c{G}(\dot{T}_i,\dot{f})$ where $(i = 0)$, in the first round, P_1 will choose $x_1 \neq r_1$ such that, x_1, $r_1 \in X$. Next, P_2 in $\c{G}(\dot{T}_0,\dot{f})$ will hold account $f(x_1)$, $f(r_1) \in Y$, $f(x_1) \neq f(r_1)$, but ($P_2 \hookrightarrow \c{G}(\dot{T}_0,Y)$, $\exists v_1 \in \mathcal{T}$, $f(x_1) \in v_1$ and $f(r_1) \notin v_1$, but f is $\dot{f}$pg-continuous function, so $f^{-1}(v) \in \dot{f}pgO(X)$, this implies P_2 in $\c{G}(\dot{T}_0,\dot{f})$ choose $f^{-1}(v_1)$ is an $\dot{f}pgO(X)$, in the second round, P_1 in $\c{G}(\dot{T}_0,\dot{f})$ will choose $x_2 \neq r_2$ such that x_2, $r_2 \in X$. Next, P_2 in $\c{G}(\dot{T}_0,X)$ will hold account $f(x_2)$, $f(r_2) \in Y$, $f(x_2) \neq f(r_2)$, but ($P_2 \hookrightarrow \c{G}(\dot{T}_0,Y)$, $\exists v_2 \in \mathcal{T}$, $f(x_2) \in v_2$ and $f(r_2) \notin v_2$, this implies P_2 in $\c{G}(\dot{T}_0,\dot{f})$ choose $f^{-1}(v_2)$ is an $\dot{f}pgO(X)$ and in m-th round P_1 in $\c{G}(\dot{T}_0,\dot{f})$ will choose $x_m \neq r_m$ such that x_m, $r_m \in X$. Next, P_2 in $\c{G}(\dot{T}_0,X)$ choose $f(x_m)$, $f(r_m) \in Y$, $f(x_m) \neq f(r_m)$, but ($P_2 \hookrightarrow \c{G}(\dot{T}_0,Y)$, $\exists v_m \in \mathcal{T}$, $f(x_m) \in v_m$ and $f(r_m) \notin v_m$, this implies P_2 in $\c{G}(\dot{T}_0,\dot{f})$ choose $f^{-1}(v_m)$ is an $\dot{f}pgO(X)$ thus $B = \{ f^{-1}(v_1), f^{-1}(v_2), \dots, f^{-1}(v_m)\}\dots\}$ is winning strategy for P_2 in $\c{G}(\dot{T}_0,\dot{f})$).

(2) In the game $\c{G}(\dot{T}_i,\dot{f})$ where $(i = 1)$, in m-th round P_1 in $\c{G}(\dot{T}_1,\dot{f})$ will choose $x_m \neq r_m$ such that x_m, $r_m \in X$. Next, P_2 in $\c{G}(\dot{T}_1,X)$ will hold account $f(x_m)$, $f(r_m) \in Y$, $f(x_m) \neq f(r_m)$, but ($P_2 \hookrightarrow \c{G}(\dot{T}_1,Y)$, $\exists U_m, v_m \in \mathcal{T}$, $f(x_m) \in (U_m - v_m)$ and $f(r_m) \in (v_m - U_m)$, this implies P_2 in $\c{G}(\dot{T}_1,\dot{f})$choose $f^{-1}(U_m)$, $f^{-1}(v_m)$, are $\dot{f}pgO(X)$, thus $B = \left\{\{f^{-1}(U_1), f^{-1}(v_1)\}, \{f^{-1}(U_2), f^{-1}(v_2)\}, \dots, \{ f^{-1}(U_m), f^{-1}(v_m)\}\dots\right\}$ is winning strategy for P_2 in $\c{G}(\dot{T}_1,\dot{f})$). By the same way we can prove $P_2 \hookrightarrow \c{G}(\dot{T}_2,\dot{f})$.but $f^{-1}(U_m) \cap f^{-1}(v_m) = \emptyset$, thus $B = \left\{\{f^{-1}(U_1), f^{-1}(v_1)\}, \{f^{-1}(U_2), f^{-1}(v_2)\}, \dots, \{ f^{-1}(U_m), f^{-1}(v_m)\}\dots\right\}$ is winning strategy for P_2 in $\c{G}(\dot{T}_2,\dot{f})$).

Proposition 4.7. If the function $f : (X, T, \dot{f}) \to (Y, \mathcal{T}, \dot{j})$ is an injective strongly-$\dot{f}$pg- continuous and ($P_2 \hookrightarrow \c{G}(\dot{T}_i,\dot{j})$) then ($P_2 \hookrightarrow \c{G}(\dot{T}_i,X)$) where (i=0,1and 2 respectively).

Proof **(1).** In the game $\c{G}(\dot{T}_i,X)$ where $(i = 0)$, in the first round, P_1 will choose $x_1 \neq r_1$ such that $x_1, r_1 \in X$. Next, P_2 in $\c{G}(\dot{T}_0,X)$) will hold account $f(x_1)$, $f(r_1) \in Y$, $f(x_1) \neq f(r_1)$, but ($P_2 \hookrightarrow \c{G}(\dot{T}_0,\dot{j})$, so $\exists v_1 \in \dot{j}pgO(Y)$), $f(x_1) \in v_1$ and $f(r_1) \notin v_1$ but f is strongly-$\dot{f}$pg- continuous then, $f^{-1}(v_1) \in T$ this implies P_2 in $\c{G}(\dot{T}_0,X)$ choose $f^{-1}(v_1)$, in the second round, P_1 in $\c{G}(\dot{T}_0,X)$ choose $x_2 \neq r_2$ such that $x_2, r_2 \in X$. Next, P_2 in $\c{G}(\dot{T}_0,X)$ will hold account $f(x_2)$, $f(r_2) \in Y$, $f(x_2) \neq f(r_2)$, but ($P_2 \hookrightarrow \c{G}(\dot{T}_0,\dot{j})$, $\exists v_2 \in \dot{j}pgO(Y)$), $f(x_2) \in v_2$ and $f(r_2) \notin v_2$, this implies P_2 in $\c{G}(\dot{T}_0,X)$ choose $f^{-1}(v_2)$ and in m - th round, P_1 in $\c{G}(\dot{T}_0,X)$ choose $x_m \neq r_m$, $x_m, r_m \in X$. Next, P_2 in $\c{G}(\dot{T}_0,X)$ will hold account $f(x_m)$, $f(r_m) \in Y$, $f(x_m) \neq f(r_m)$, but ($P_2 \hookrightarrow \c{G}(\dot{T}_0,\dot{j})$, $\exists v_m \in \dot{j}pgO(Y)$, $f(x_m) \in v_m$ and $f(r_m) \notin v_m$, this implies P_2 in $\c{G}(\dot{T}_0,X)$ choose $f^{-1}(v_m) \in T$,thus $B = \{f^{-1}\{ v_1\}, f^{-1}\{v_2\} \dots, f^{-1}\{v_m\}..\}$ is winning strategy for P_2 in $\c{G}(\dot{T}_0,X)$.

(2). In the game $\c{G}(\dot{T}_i,X)$, where $(i = 1)$, in the m- th round P_1 in $\c{G}(\dot{T}_1,X)$ choose $x_m \neq r_m$ such that $x_m, r_m \in X$, P_2 in $\c{G}(\dot{T}_1,X)$ will hold account $f(x_m)$, $f(r_m) \in Y$, $f(x_m) \neq f(r_m)$, but ($P_2 \hookrightarrow \c{G}(\dot{T}_1,\dot{j})$, $\exists U_m, v_m \in \dot{j}pgO(Y)$, $f(x_m) \in (U_m - v_m)$ and $f(r_m) \in (v_m - U_m)$, this implies P_2 in $\c{G}(\dot{T}_1,X)$ choose $f^{-1}(U_m)$, $f^{-1}(v_m) \in T$. Thus $B = \left\{\{f^{-1}(U_1), f^{-1}(v_1)\}, \{f^{-1}(U_2), f^{-1}(v_2)\}, \dots, \{ f^{-1}(U_m), f^{-1}(v_m)\}\dots\right\}$ is winning strategy for P_2 in $\c{G}(\dot{T}_1,X)$. In the same way, we can prove $P_2 \hookrightarrow \c{G}(\dot{T}_2,X)$, but $f^{-1}(U_m) \cap f^{-1}(v_m) = \emptyset$. Thus $B = \left\{\{f^{-1}(U_1), f^{-1}(v_1)\}, \{f^{-1}(U_2), f^{-1}(v_2)\}, \dots, \{ f^{-1}(U_m), f^{-1}(v_m)\}\dots\right\}$ is winning strategy for P_2 in $\c{G}(\dot{T}_2,\dot{f})$).

Corollary 4.8. Let $ʄ : (\text{X}, \text{Ț}, \dot{\text{Ĩ}}) \to (\Upsilon, \text{Ꚍ}, \text{ɉ})$ is injective Strongly-Ĩpg-continuous function and $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_i, \text{ɉ})$, then$(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_i, \dot{\text{Ĩ}})$, where (i = 0,1and2 respectively).

Proposition 4.9. If the function $ʄ : (\text{X}, \text{Ț}, \dot{\text{Ĩ}}) \to (\Upsilon, \text{Ꚍ}, \text{ɉ})$ is an injective open continuous (respectively Ĩ-pre-g-irresolute function) and $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \Upsilon)$ respectively $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \text{ɉ}))$ then $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \text{X})$ (respectively $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}}))$.

Proof(1): In the game $\text{Ģ}(\dot{\text{T}}_0, \text{X})$(respectively in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$), in the first round, Ᵽ_1 will choose $ӿ_1 \neq r_1$, $ӿ_1, r_1 \in \text{X}$, Next Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \text{X})$(respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$)choose $ʄ(ӿ_1)$, $ʄ(r_1) \in \Upsilon$, $ʄ(ӿ_1) \neq ʄ(r_1)$, but $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \Upsilon)$(respectively$(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \text{ɉ}))$, $\exists \text{ṿ}_1$ Ꚍ (respectively $\exists \text{ṿ}_1$ ɉpgO(Υ)), $ʄ(ӿ_1) \in \text{ṿ}_1$ and $ʄ(r_1) \notin \text{ṿ}_1$ and since ʄ is open continuous (respectively Ĩ-pre-g-irresolute function)this implies Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \text{X})$ (respectively in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$) choose $ʄ^{-1}(\text{ṿ}_1)$, in the second round, Ᵽ_1 in $\text{Ģ}(\dot{\text{T}}_0, \text{X})$ (respectively in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$) choose $ӿ_2 \neq r_2$ such that $ӿ_2$, $r_2 \in \text{X}$. Next, Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \text{X})$ (respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$) choose $ʄ(ӿ_2)$, $ʄ(r_2) \in \Upsilon$, $ʄ(ӿ_2) \neq ʄ(r_2)$, but $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \Upsilon)$(respectively$(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \text{ɉ}))$, $\exists \text{ṿ}_2 \in$ Ꚍ (respectively $\exists \text{ṿ}_2 \in$ ɉpgO(Υ)), $ʄ(ӿ_2) \in \text{ṿ}_2$ and $ʄ(r_2) \notin \text{ṿ}_2$, this implies Ᵽ_2 in $\text{Ģ}(\text{Ț}_0, \text{X})$ (respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$) choose $ʄ^{-1}(\text{ṿ}_2)$and in ṃ-th step Ᵽ_1 in $\text{Ģ}(\dot{\text{T}}_0, \text{X})$ (respectively in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$) choose $ӿ_ṃ \neq r_ṃ$, $ӿ_ṃ$, $r_ṃ \in \text{X}$. Next, Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \text{X})$ (respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$) choose $ʄ(ӿ_ṃ)$, $ʄ(r_ṃ) \in \Upsilon)$, $ʄ(ӿ_ṃ) \neq ʄ(r_ṃ)$, but$(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \Upsilon))$(respectively $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_0, \text{ɉ}))$,$\exists \text{ṿ}_ṃ \in$ Ꚍ(respectively $\exists \text{ṿ}_ṃ \in$ ɉpgO(Υ), $ʄ(ӿ_ṃ) \in \text{ṿ}_ṃ$ and $ʄ(r_ṃ) \notin \text{ṿ}_ṃ$ this implies Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \text{X})$ respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$) choose $ʄ^{-1}(\text{ṿ}_ṃ)$, thus $\text{Ƀ} = \{ʄ^{-1}\{\text{ṿ}_1\}, ʄ^{-1}\{\text{ṿ}_2\} \ldots, ʄ^{-1}\{\text{ṿ}_ṃ\} \ldots\}$ is winning strategy for Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \text{X})$) (respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_0, \dot{\text{Ĩ}})$).

(2). In the game $\text{Ģ}(\dot{\text{T}}_1, \text{X})$, (respectively $\text{Ģ}(\dot{\text{T}}_1, \dot{\text{Ĩ}})$) , in the ṃ-th round, Ᵽ_1 in $\text{Ģ}(\dot{\text{T}}_1, \text{X})$ (respectively in $\text{Ģ}(\dot{\text{T}}_1, \dot{\text{Ĩ}})$) choose $ӿ_ṃ \neq r_ṃ$ such that $ӿ_ṃ$, $r_ṃ \in \text{X}$. Next, Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_1, \text{X})$ (respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_1, \dot{\text{Ĩ}})$) choose $ʄ(ӿ_ṃ)$, $ʄ(r_ṃ) \in \Upsilon)$, $ʄ(ӿ_ṃ) \neq ʄ(r_ṃ)$, but $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_1, \Upsilon))$, $\exists \text{Ụ}_ṃ, \text{ṿ}_ṃ \in$ Ꚍ (respectively $\exists \text{Ụ}_ṃ, \text{ṿ}_ṃ \in$ ɉpgO(Υ)); $ʄ(ӿ_ṃ) \in (\text{Ụ}_ṃ - \text{ṿ}_ṃ)$ and $ʄ(r_ṃ) \in (\text{ṿ}_ṃ - \text{Ụ}_ṃ)$, this implies Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_1, \text{X})$(respectively $\text{Ᵽ}_2$in $\text{Ģ}(\dot{\text{T}}_1, \dot{\text{Ĩ}})$) choose $ʄ^{-1}(\text{Ụ}_ṃ), ʄ^{-1}(\text{ṿ}_ṃ)$ thus $\text{Ƀ} = \{\{ʄ^{-1}(\text{Ụ}_1), ʄ^{-1}(\text{ṿ}_1)\}, \{ʄ^{-1}(\text{Ụ}_2), ʄ^{-1}(\text{ṿ}_2)\}, \ldots, \{ʄ^{-1}(\text{Ụ}_ṃ), ʄ^{-1}(\text{ṿ}_ṃ)\} \ldots\}$ is winning strategy for Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_1, \text{X})$(respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_1, \dot{\text{Ĩ}})$). By the same way we can prove $\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_2, \text{X})$ respectively, $\text{Ᵽ}_2$in $\text{Ģ}(\dot{\text{T}}_2, \dot{\text{Ĩ}})$, but $ʄ^{-1}(\text{Ụ}_ṃ) \cap ʄ^{-1}(\text{ṿ}_ṃ) = \emptyset$ thus $\text{Ƀ} = \{\{ʄ^{-1}(\text{Ụ}_1), ʄ^{-1}(\text{ṿ}_1)\}, \{ʄ^{-1}(\text{Ụ}_2), ʄ^{-1}(\text{ṿ}_2)\}, \ldots, \{ʄ^{-1}(\text{Ụ}_ṃ), ʄ^{-1}(\text{ṿ}_ṃ)\} \ldots\}$ is winning strategy for Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_2, \text{X})$)(respectively Ᵽ_2 in $\text{Ģ}(\dot{\text{T}}_2, \dot{\text{Ĩ}})$).

Corollary 4.10. If $ʄ : (\text{X}, \text{Ț}) \to (\Upsilon, \text{Ꚍ})$ is homeo then $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_i, \text{X}))$ $\longleftrightarrow$ $(\text{Ᵽ}_2 \hookrightarrow \text{Ģ}(\dot{\text{T}}_i, \Upsilon))$ such that (i=0,1and 2 respectively).

5.Conclusion

The main aim of this work is to submit new near open sets which are called Ĩ-pre-g-closed sets and it is complement Ĩ-pre-g-open set, and interested also in studying new species of the games by application separation axioms via Ĩ-pre-g-open sets and gives the strategy of winning and losing to any one of the two players in $\text{Ģ}(\dot{\text{T}}_i, \text{X})$, i= {0, 1, 2} .

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