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Sumudu Iterative Method for solving Nonlinear Partial Differential Equations

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Abstract

In this paper, we apply a new technique combined by a Sumudu transform and iterative method called the Sumudu iterative method for resolving non-linear partial differential equations to compute analytic solutions. The aim of this paper is to construct the efficacious frequent relation to resolve these problems. The suggested technique is tested on four problems. So the results of this study are debated to show how useful this method is in terms of being a powerful, accurate and fast tool with a little effort compared to other iterative methods.

Keywords: Sumudu Transform, Iterative Method, Nonlinear Partial Differential Equations.

1.Introduction

In the previous few decades, the non-linear equations Ordinary Differential Equations(ODEs) and Partial Differential Equations(PDEs) represented the most important mathematical formulations occurring in the physical phenomena and engineering fields. It can be described via ODEs, PDEs and integral equations. PDEs have be a good gadget for characterizing these natural phenomena of science and engineering models, like wave propagation, Korteweg-de Vries equation and heat flow [1]. With this great expansion of differential equations in applied mathematics and physics, especially the non-linear PDEs which are the subject of our study. Mathematicians faced some problems in solving some of these equations which required the development of new methods to find an exact or approximate solution to them. In recent years, most researchers fundamentally have studied solutions of nonlinear PDEs and ODEs via utilizing different methods, like the Variational

Iteration Method[1], Adomian Decomposition Method [2-3], Homotopy Perturbation Method [4], Laplace transform and Modified Variational Iteration [5], Natural Decomposition Method [6], the reduced differential transform method as in [7-8],the Sumudu transform [9-10] and Sumudu Decomposition Method [11]. We introduce a reliable method of an integral transform which is named the Sumudu Iterative Method (SIM) which we implement to get exact solutions to nonlinear PDEs. Temimi and Ansari have proposed an Iterative Method (IM) to resolve linear and non-linear functional equations, [12-15].The IM has been successfully applied in many researches to solve some linear and non-linear PDEs and ODEs, non-linear delay differential equations, higher order integro-differential equations and Korteweg-de Vries equations [16-18]. However one of the most important achievements and applications of integral transform methods is solving non-linear PDEs by the relationship between the Sumudu transform and iterative method. For this purpose, we present a new technique. We think this method has not been implemented yet to solve non-linear PDEs; the results of the examples show that this method is an accurate and powerful technique and it does not need to impose any additional restrictions to get the analytical solution of these problems. It is a qualified method for reducing the number of calculations while keeping the solution is more accurate and efficient. In this work, the examples of non-linear PDEs which are used in [1] will be solved by using the SIM.
The basic concept of this work is included in section 2 and 3 as we introduce the definition and properties of the Sumudu transform. Section 4, we clarify the methodology of SIM. Section 5 is dedicated to illustrate the SIM to four problems, the results show that the method is easy and accurate to implement. Section 6 is devoted to debate and conclusion of this paper.

2. Sumudu Transform

Sumudu transform is one of the integral transformations that emerges after Laplace transform where it is derived from the Laplace transform by setting $\mathrm{p} = \frac{1}{\mathrm{s}}$ to resolve differential equations subject to the initial conditions. This transform is relatively new but has many good properties for resolving problems in computational science and control engineering. It is interesting that in Sumudu transform variable u and transformed function $\mathrm{F(u)}$ in u-domain are transacted as replicas of t and $\mathrm{f(t)}$ in the t-domain so thus the consistency of unit in the differential equations describing a physical process can be preserved even after transformations, as for, in Laplace transform variable s and transformed function $\mathrm{F(s)}$ in s-domain are transacted as toys in this operation [19]. We will present here the definition of Sumudu transform and some of its properties. We start definition of Sumudu transform of a function $\mathrm{u(x,t)}$ where $\mathrm{u(x,t)}$ is continuous.
Let $\mathrm{u(x,t)}$ be continuous and of exponential order. Then Sumudu transform $\mathrm{u\,(x,t)}$ is written as [20]
$\mathrm{U(x,s)} = \mathbb{S}[\mathrm{u(x,t\,)}] = \frac{1}{\mathrm{s}}\int_0^{\infty} \mathrm{u(x,t)}e^{\frac{-\mathrm{t}}{\mathrm{s}}}\mathrm{dt}, -\tau_1 < u < \tau_2$,where $-\tau_1$ and $\tau_2 > 0$, (2.1)

3.Properties of Sumudu Transform [20]:-

Let $\mathrm{u\,(x,t)}$ be a piecewise continuous function having exponential order. If $\mathrm{U(x,s)}$ is Sumudu transform of $\mathrm{u\,(x,t)}$, then Sumudu transforms of partial derivatives that function as follows,

(i) $\mathbb{S}\left[\frac{\partial \mathrm{u(x,t)}}{\partial \mathrm{t}}\right] = \frac{1}{\mathrm{s}}\,[\mathrm{U(x,s)} - \mathrm{u(x,t)}].$ (3.1)

(ii) $\mathbb{S}\left[\frac{\partial \mathrm{u(x,t)}}{\partial \mathrm{x}}\right] = \frac{\mathrm{d[U(x,s)]}}{\mathrm{dx}}$. (3.2)

(iii) $\mathbb{S}\left[\frac{\partial^2 \mathrm{u(x,t)}}{\partial \mathrm{t}^2}\right] = \frac{1}{s^2}\,\mathrm{U(x,s)} - \frac{1}{s^2}\mathrm{u(x,0)} - \frac{1}{\mathrm{s}}\,\frac{\partial \mathrm{u(x,0)}}{\partial \mathrm{t}}$. (3.3)

(iv) $\mathbb{S}\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right] = \frac{d^2[U(x,s)]}{dx^2}$. (3.4)

Table 1. S-Transform of some functions.

f(t)	$G(s) = \mathbb{S}[f(t)]$
1	1
t	s
$\frac{t^{n-1}}{(n-1)!}, n = 1,2, \ldots$	s^{n-1}

Table 2. S-Transform properties.

Formula	Comment
$\mathbb{S}\left[\frac{\partial u(x,t)}{\partial t}\right] = \frac{G(x,s) - u(x,0)}{s}$	Sumudu transforms of function derivatives
$\mathbb{S}\left[\frac{\partial^2 u(x,t)}{\partial t^2}\right] = \frac{G(x,s) - u(x,0)}{s^2} - \frac{1}{s}\frac{\partial u(x,0)}{\partial t}$	
$\mathbb{S}\left[\frac{\partial^n u(x,t)}{\partial t^n}\right] = \frac{G(x,s) - u(x,0)}{s^n} - \frac{1}{s^{n-1}}\frac{\partial u(x,0)}{\partial t} - \cdots - \frac{1}{s}\frac{\partial^{n-1} u(x,0)}{\partial t^{n-1}}$	
$\mathbb{S}[au_1(x,t) + bu_2(x,t) = a\mathbb{S}[u_1(x,t)] + b[u_2(x,t)]$ where a and b are constants	linearity property
$\mathbb{S}[\int_0^t u(x,\tau)d\tau] = sG(x,s)$	Sumudu transform of an integral function
$\mathbb{S}[u(x,at)] = G(x,as)$ where a is a constant	First scale preserving theorem

4. Fundamental Idea of the Iterative Method

The bacis steps of IM. It is rewritten that any PDE can be written as**:**

$$L(u(x,t)) + N(u(x,t)) + h(x,t)) = 0 . \quad (4.1)$$

Subject to the conditions $C(u, \frac{\partial u}{\partial t}) = 0$. (4.2)

So, thus $u(x,t)$ the unknown function, x and t indicates the independent variables, whereas L, N represent linear and non-linear operators, respectively, $h(x,t)$ represents inhomogeneous term which is a renowned function and C the conditions operator for a problem. The essential idea of the iterative method to solve Eq. (4.1) with initial approximation for Eq.(4.2) is a primary step, by assuming that the initial guess $u_0(x,t)$ is a solution of the problem $u(x,t)$ and solution of the equation

$$L(u_0(x,t)) + h(x,t) = 0, C(u_0, \frac{\partial u_0}{\partial t}) = 0 . \quad (4.3)$$

To generate next iterative of solution, we put Eq.(4.1) as follows

$$L(u_1(x,t)) + h(x,t) + N(u_0(x,t)) = 0, C(u_1, \frac{\partial u_1}{\partial t}) = 0 . \quad (4.4)$$

After several simple iterative steps of the solution, the generic form of this equation is

$$L(u_{n+1}(x,t)) + h(x,t) + N(u_n(x,t)) = 0 , C(u_{n+1}, \frac{\partial u_{n+1}}{\partial t}) = 0 \quad (4.5)$$

Evidently, each iteration of the function $u_n(x,t)$ represents effectively the only solution for Eq. (4.1).

5. Sumudu Iterative Method

We clarify the Sumudu Iterative Method (SIM) algorithm via considering general nonlinear inhomogeneous PDEs of the form:

$$L(u(x,t)) + R(u(x,t)) + N(u(x,t)) = h(x,t). \quad (5.1)$$

Submit to ICs

$$u(x,0) = f_1(x)\ ,\ 0 \le x \le l \ \text{ and } \ u_t(x,0) = f_2(x),\ t > 0. \quad (5.2)$$

So, the second order linear differential operator is L about $L = \frac{\partial^2}{\partial t^2}$, linear operator for less order than L is R, N represents the non-linear differential operator and the source term is h(x,t).

We stratify S-Transform of Eq.(5.1) to get:

$$\mathbb{S}[L(u(x,t))] + \mathbb{S}[R(u(x,t))] + \mathbb{S}[N(u(x,t))] = \mathbb{S}[h(x,t)]. \quad (5.3)$$

Utilizing properties in Table 1,2 respectively above in section 3 with the given initial conditions and for equation of order 2, we get:

$$\frac{U(x,s)}{s^2} - \frac{u(x,0)}{s^2} - \frac{u_t(x,0)}{s} + \mathbb{S}[R(u(x,t))] + \mathbb{S}[N(u(x,t))] = \mathbb{S}[h(x,t)]. \quad (5.4)$$

Replace Eq.(5.2) into Eq.(5.4) to get:

$$\frac{U(x,s)}{s^2} - \frac{f_1(x)}{s^2} - \frac{f_2(x)}{s} + \mathbb{S}[R(u(x,t))] + \mathbb{S}[N(u(x,t))] = \mathbb{S}[h(x,t)]. \quad (5.5)$$

From Eq.(5.5), we obtain:

$$\frac{U(x,s)}{s^2} = \frac{f_1(x)}{s^2} + \frac{f_2(x)}{s} + \mathbb{S}[h(x,t)] - \mathbb{S}[R(u(x,t))] - \mathbb{S}[N(u(x,t))]. \quad (5.6)$$

Eq.(5.6) becomes

$$U(x,s) = f_1(x) + sf_2(x) + s^2\mathbb{S}[h(x,t)] - s^2\mathbb{S}[R(u(x,t))] - s^2\mathbb{S}[N(u(x,t))]. \quad (5.7)$$

Subsequently, taking inverse Sumudu transform of Eq.(5.7) implies that:

$$\mathbb{S}^{-1}\{U(x,s)\} = \mathbb{S}^{-1}\{f_1(x) + sf_2(x) + s^2\mathbb{S}[h(x,t)]\} - \mathbb{S}^{-1}\{s^2\mathbb{S}[R(u(x,t))]\} - \mathbb{S}^{-1}\{s^2\mathbb{S}[N(u(x,t))]\}. \quad (5.8)$$

From Eq.(5.8), we can get

$$u(x,t) = H(x,t) - \mathbb{S}^{-1}\{s^2\mathbb{S}[R(u(x,t))]\} - \mathbb{S}^{-1}\{s^2\mathbb{S}[N(u(x,t))]\}. \quad (5.9)$$

Note that $H(x,t)$ represents a term engendering from source term and specified IC.

Hence to transact with the new correction function of the Sumudu iterative method

$$u_{n+1}(x,t) = H(x,t) - \mathbb{S}^{-1}\{s^2\mathbb{S}[R(u_n(x,t))]\} - \mathbb{S}^{-1}\{s^2\mathbb{S}[N(u_n(x,t))]\}, n \ge 0. \quad (5.10)$$

To get out how this method works, we follow next steps as follows:

Step 1:

$u_0(x,t) = H(x,t)$

Step 2: The next iteration is

$$u_1(x,t) = H(x,t) - \mathbb{S}^{-1}\{s^2\mathbb{S}[R(u_0(x,t))]\} - \mathbb{S}^{-1}\{s^2\mathbb{S}[N(u_0(x,t))]\}. \quad (5.11)$$

Step (3) : After several simple Sumudu iterative steps of the solution, the general form of this equation which is

$$u_{n+1}(x,t) = H(x,t) - \mathbb{S}^{-1}\{s^2\mathbb{S}[R(u_n(x,t))]\} - \mathbb{S}^{-1}\{s^2\mathbb{S}[N(u_n(x,t))]\}, n \geq 0. \quad (5.12)$$

Evidently, each iteration of the function $(u_{n+1}(x,t))$ represents effectively the only solution for Eq. (5.12).
In the above steps, this method has the merit that solution can be found in the first steps manually and easily.

6.Illustrative Examples

In this part, we implement four examples and then compare our solutions to the existent exact solutions.

Example 6.1

Solve the inhomogeneous advection problem:

$$u_t + \frac{1}{2}(u^2)_x = x . \quad (6.1)$$

Submit to the IC $u(x,0) = 2.$ (6.2)

Taking Sumudu transform of Eq.(6.1),we have:

$\mathbb{S}[u_t] + \mathbb{S}[\frac{1}{2}(u^2)_x] = \mathbb{S}[x].$

Utilizing the properties in Table 1,2 and the properties mentioned above, we get:

$$\frac{U(x,s)}{s} - \frac{u(x,0)}{s} = x - \frac{1}{2}\,\mathbb{S}[(u^2)_x] . \quad (6.3)$$

Replace Eq.(6.2) into Eq.(6.3), we get:

$$U(x,s) = 2 + xs - \frac{s}{2}\,\mathbb{S}[(u^2)_x] . \quad (6.4)$$

Subsequently, taking inverse Sumudu transform of Eq.(6.4) implies that:

$$\mathbb{S}^{-1}[U(x,s)] = 2 + x\mathbb{S}^{-1}[s] - \mathbb{S}^{-1}\left\{\frac{s}{2}\mathbb{S}[(u^2)_x]\right\}. \quad (6.5)$$

Table 1and 2, Eq.(6.5) becomes

$$u(x,t) = 2 + xt - \mathbb{S}^{-1}\left\{\frac{s}{2}\,\mathbb{S}[(u^2)_x]\right\}. \quad (6.6)$$

By the new correction function from Eq.(6.6), we can get:

$$u_{n+1}(x,t) = 2 + xt - \mathbb{S}^{-1}\left\{\frac{s}{2}\,\mathbb{S}[(u_n^2)_x]\right\}, \ n \geq 0. \quad (6.7)$$

Now, we apply the new Sumudu iterative method.

So from Eq.(6.7), we conclude

$u_0(x,t) = 2 + xt.$

$u_1(x,t) = 2 + xt \ -\mathbb{S}^{-1}\left\{\frac{s}{2}\ \mathbb{S}[(u_0^2)_x]\right\}.$

$u_1(x,t) = 2 + xt - \ t^2 - \frac{1}{3}xt^3.$

And

$u_2(x,t) = 2 + xt - \mathbb{S}^{-1}\left\{\frac{s}{2}\ \mathbb{S}[(u_1^2)_x]\right\}, n \geq 1\ .$

$u_2(x,t) = 2 + xt - \ t^2 - \frac{1}{3}xt^3 + \frac{5}{12}t^4 + \frac{2}{15}xt^5 - \frac{1}{63}xt^7).$

Therefore, we persist in this way to obtain a generic recursive relation via canceling a noise terms and we get:

$$u_{n+1}(x,t) = 2\left(1 - \frac{1}{2!}t^2 + \frac{5}{4!}t^4 \ldots\right) + x\left(t - \frac{1}{3}t^3 + \frac{2}{15}t^5 - \frac{1}{63}t^7 + \cdots \ldots\right)$$

Hence, we can conclude the exact solution as follows $u(x,t) = 2\text{secht} + x\,\text{tanht}$.

Example 6.2.

Solve the Burgers equation of the form:

$u_t + uu_x = u_{xx}$, with the IC $u(x,0) = 2\tan x$. (6.8)

Taking Sumudu transform of Eq.(6.8), we have:

$\mathbb{S}[u_t] + \mathbb{S}[uu_x] = \ \mathbb{S}[u_{xx}].$

Using the properties in Table 1, 2 and submitting to the IC above to obtain:

$\frac{U(x,s)}{s} - \frac{2\tan x}{s} = \mathbb{S}[u_{xx} - uu_x]\ .$ (6.9)

Subsequently, taking inverse Sumudu transform of Eq.(6.9) to get:

$\mathbb{S}^{-1}[U(x,s)] = 2\tan x + \mathbb{S}^{-1}\{s\ \mathbb{S}[u_{xx} - uu_x]\}.$ (6.10)

Table 1and 2, Eq.(6.10) becomes

$u(x,t) = 2\tan x + \mathbb{S}^{-1}\{s\ \mathbb{S}[u_{xx} - uu_x]\}.$ (6.11)

Thus, we persist in this manner to obtain a general recursive relation from Eq.(6.11):

$u_{n+1}(x,t) = 2\tan x + \mathbb{S}^{-1}\{s\ \mathbb{S}[u_{nxx} - u_n u_{nx}]\}, n \geq 0.$ (6.12)

Now, we apply the new Sumudu iterative method.

So from Eq.(6.12), we conclude

$u_0(x,t) = 2\tan x$

Note that

$u_1(x,t) = 2\tan x + \mathbb{S}^{-1}\{s\ \mathbb{S}[u_{0xx} - u_0 u_{0x}]\}.$

$u_1(x,t) = 2tanx + \mathbb{S}^{-1}\{s\ \mathbb{S}[4\tan(x)sec^2(x) - 4\tan(x)sec^2(x)]\} = 0.$

So, we can conclude $u_{n+1}(x,t) = 0, \forall n \geq 0$. Hence the exact solution as

$\mathrm{u(x,t) = 2tanx}$.

Example 6.3.

Consider the non-linear PDE

$$\mathrm{u_t = 2u(u_x)^2 + u^2u_{xx}}, \text{ with IC } \mathrm{u(x,0) = \frac{x+1}{2}}. \tag{6.13}$$

Taking Sumudu transform of Eq.(6.13), we have:

$\mathbb{S}[\mathrm{u_t}] = \mathbb{S}[\mathrm{2u(u_x)^2 + u^2u_{xx}}]$

Using the properties in Table 1,2 and submit to the IC above we obtain:

$$\frac{\mathrm{U(x,s)}}{\mathrm{s}} - \frac{\frac{\mathrm{x+1}}{2}}{\mathrm{s}} = \mathbb{S}[\mathrm{2u(u_x)^2 + u^2u_{xx}}]\ . \tag{6.14}$$

Subsequently, taking inverse Sumudu transform of Eq. (6.14) to get:

$$\mathbb{S}^{-1}[\mathrm{U(x,s)}] = \frac{\mathrm{x+1}}{2} + \mathbb{S}^{-1}\{\mathrm{s}\ \mathbb{S}[\mathrm{2u(u_x)^2 + u^2u_{xx}}]\}. \tag{6.15}$$

Table 1and 2 Eq.(6.15) becomes

$$\mathrm{u(x,t)} = \frac{\mathrm{x+1}}{2} + \mathbb{S}^{-1}\{\mathrm{s}\ \mathbb{S}[\mathrm{2u(u_x)^2 + u^2u_{xx}}]\}. \tag{6.16}$$

Thus, we persist in this manner to obtain a general recursive relation from Eq.(6.16) :

$$u_{n+1}(x,t) = \frac{x+1}{2} + \mathbb{S}^{-1}\{s\ \mathbb{S}[2u_n(u_{nx})^2 + (u_n)^2u_{nxx}]\}, n \geq 0. \tag{6.17}$$

Now, we apply the new Sumudu iterative method.

So from Eq.(6.17), we conclude

$\mathrm{u_0(x,t)} = \frac{\mathrm{x+1}}{2}$,

$\mathrm{u_1(x,t)} = \frac{\mathrm{x+1}}{2} + \mathbb{S}^{-1}\{\mathrm{s}\ \mathbb{S}[\mathrm{2u_0(u_{0x})^2 + (u_0)^2u_{0xx}}]\}.$

$u_1(x,t) = \frac{x+1}{2}(1 + \frac{1}{2}t).$

And

$$\mathrm{u_2(x,t)} = \frac{\mathrm{x+1}}{2} + \mathbb{S}^{-1}\{\mathrm{s}\ \mathbb{S}[\mathrm{2u_1(u_{1x})^2 + (u_1)^2u_{1xx}}]\}, \forall \mathrm{n} \geq 1.$$

$\mathrm{u_2(x,t)} = \frac{\mathrm{x+1}}{2}(1 + \frac{1}{2}\mathrm{t} + \frac{3}{8}\mathrm{t}^2 + \frac{1}{8}\mathrm{t}^3 + \frac{1}{64}\mathrm{t}^4),$

We persist in this technique to obtain::

$\mathrm{u(x,t)} = \frac{\mathrm{x+1}}{2}(1 + \frac{1}{2}\mathrm{t} + \frac{3}{8}\mathrm{t}^2 + \frac{1}{8}\mathrm{t}^3 + \frac{1}{64}\mathrm{t}^4 + \cdots).$

Which is the exact solution of Eq.(6.13).

Example 6.4.

The non-linear inhomogeneous Klein Gordon equation of the form:

$$u_{tt} - u_{xx} + u^2 = x^2t^2, \text{ with ICs } u(x,0) = 0, u_t(x,0) = x. \quad (6.18)$$

Taking Sumudu transform of Eq.(6.18), we have:

$$\mathbb{S}[u_{tt}] - \mathbb{S}[u_{xx}] + \mathbb{S}[u^2] = \mathbb{S}[x^2t^2]$$

Utilizing properties in Table 1,2 and the properties mentioned above, we get:

$$\frac{U(x,s)}{s^2} - \frac{u(x,0)}{s^2} - \frac{u_t(x,0)}{s} = 2x^2s^2 + \mathbb{S}[u_{xx} - u^2]. \quad (6.19)$$

Substitute the initial condition in Eq.(6.19) to obtain:

$$U(x,s) = xs + 2x^2s^4 + s^2\mathbb{S}[u_{xx} - u^2]. \quad (6.20)$$

Subsequently, taking inverse Sumudu transform of Eq.(6.20), we get:

$$\mathbb{S}^{-1}[U(x,s)] = x\mathbb{S}^{-1}[s] + 2x^2\mathbb{S}^{-1}[s^4] + \mathbb{S}^{-1}\{s^2\mathbb{S}[u_{xx} - u^2]\}. \quad (6.21)$$

Table 1and 2 Eq.(6.21) becomes

$$u(x,t) = xt + \frac{x^2t^4}{12} + \mathbb{S}^{-1}\{s^2\mathbb{S}[u_{xx} - u^2]\}. \quad (6.22)$$

By the new correction function from Eq.(6.22), we can get:

$$u_n(x,t) = xt + \frac{x^2t^4}{12} + \mathbb{S}^{-1}\{s^2\mathbb{S}[u_{nxx} - (u_n)^2]\}, n \geq 0 . \quad (6.23)$$

Now, we apply the new Sumudu iterative method.

So from Eq.(6.23) , we conclude

$$u_0(x,t) = xt + \frac{x^2t^4}{12},$$

$$u_1(x,t) = xt + \frac{x^2t^4}{12} + \mathbb{S}^{-1}\{s^2\mathbb{S}[u_{0xx} - (u_0)^2]\}.$$

$$u_1(x,t) = xt + \frac{t^6}{180} - \frac{x^3t^7}{252} - \frac{x^4t^{10}}{12960}.$$

We would like to mention here, as we go forward with iterations the terms after xt will begin to decrease and get close to zero, hence we can cancel this noise term and get the exact solution $u(x,t) = xt$.

7.Conclusion

In this work we introduce a new efficient method for resolving non-linear partial differential equations that is the Sumudu iterative method, which represents a new addition to the previous iterative methods such as the Adomain decomposition method, Variation Iteration Method, Homotopy perpetration Method, etc. Our method is characterized by its speed and ease of use in addition to the fact that it often gives the exact solution after a few iterations. Our aim of this work is a step towards using applications of the Sumudu iterative method to resolve nonlinear problems arising in various field scopes of science mathematical physics and engineering, which may be debated in further work.

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