On Reliability Estimation for the Exponential Distribution Based on Monte Carlo Simulation

Abbas Najim Salman
abbasnajim66@yahoo.com
Dept. of Mathematics / College of Education for Pure Science/ Ibn Al – Haitham- University of Baghdad
Taha AnwarTaha
tahaanwar36@gmail.com
Directorate- General for Education of Anbar/ Ministry of Education

Abstract

This Research deals with estimation the reliability function for two-parameters Exponential distribution, using different estimation methods; Maximum likelihood, Median-First Order Statistics, Ridge Regression, Modified Thompson-Type Shrinkage and Single Stage Shrinkage methods. Comparisons among the estimators were made using Monte Carlo Simulation based on statistical indicter mean squared error (MSE) conclude that the shrinkage method perform better than the other methods

.

Keywords: The Exponential distribution, Median-First Order Statistics, Ridge regression, Modified Thompson, Shrinkage methods, Mean squared error.

1. Introduction

The Exponential distribution is generally use for unit failure model that have an constant failure rate, it be able to be with one or two parameters. "Maguire, Pearson and Wynn (1952) studied mine accidents and showed that time intervals between industrial accidents follow exponential distribution";[6]. "Cohen and Helm (1973) used (BLUE), (MLE), (ME), (MVUE) and MME to estimate the parameters of the exponential distribution"; [3]. "Peter (1974) used robust M-estimation method for the scale parameter, with application to the exponential distribution"[8]. " Lawless (1977) studied a confidence interval for the scale parameter and obtained a prediction interval for a future observation"; [5] . "Afify (2004) and Muhammad and Ahmed (2011) reviewed and compared several methods for estimating the two-parameter exponential distribution";[1 and 7]. "Two new classes of confidence interval for the scale parameter proposed by Petropoulos (2011)"; [9] and "Lai and Augustine (2012) obtained interval estimates for the threshold (location) parameter and derived a predictive function for the two parameter";[4].

The aim of this research is to estimate the reliability function for two-parameters Exponential distribution, using different estimation methods ; Maximum likelihood, Median-First Order Statistics, Ridge Regression, Modified Thompson-Type Shrinkage and Single Stage Shrinkage methods where a location parameter (α) is known.

The probability density function of two parameters Exponential distribution is given by

$$f(t_i,\alpha,\beta)=\begin{cases}\frac{1}{\beta}exp-\left(\frac{(t_i-\alpha)}{\beta}\right) & \alpha<t_i<\infty\\ 0 & o.w\end{cases} \quad (1)$$

$$\Omega=\{(\alpha,\beta);\alpha>0,\beta>0\}$$

Where α is a location parameter and β is a scale parameter.

The mean and the variance of Exponential distribution are:

$$M_t=E(t)=\beta+\alpha \quad (2)$$

$$\sigma^2{}_t=var(t)=\beta^2 \quad (3)$$

$$E(t^2)=2\beta^2+2\,\alpha\beta+\alpha^2 \quad (4)$$

The distribution, reliability and hazard functions of Exponential distribution respectively as below:

$$F(t)=1-exp\left(-\frac{(t-\alpha)}{\beta}\right) \quad (5)$$

$$R(t)=exp\left(-\frac{(t-\alpha)}{\beta}\right) \quad (6)$$

$$h(t)=\frac{1}{\beta} \quad (7)$$

 https://doi.org/ 10.30526/2017.IHSCICONF.1814

2. Experimental

Estimation Method (Theoretical Part)

In this section, we discussed five estimation methods for the parameter and the reliability function of two-parameter Exponential distribution

Maximum likelihood method

The likelihood function L(t_i , α , β) is defined as below, [3]

$$\mathrm{L} = \prod_{i=1}^{n} f(t_i\,,\alpha\,,\beta) \tag{8}$$

$$\mathrm{L} = \prod_{i=1}^{n} \frac{1}{\beta} exp\left(-\left(\frac{(t_i-\alpha)}{\beta^2}\right)\right) \tag{9}$$

Taking the Natural logarithm equation (9) , so we get the following:

$$\mathrm{Ln\,L} = -\mathrm{n\,Ln}\,\beta + \sum_{i=1}^{\mathrm{n}} \left(\frac{(t_i-\alpha)}{\beta}\right) \tag{10}$$

The partial derivative for equation (10) with respect to unknown parameter β, is:

$$\frac{\partial \mathrm{Ln\,L}}{\partial \beta} = \frac{-n}{\beta} + \sum_{i=1}^{\mathrm{n}} \left(\frac{(t_i-\alpha)}{\beta^2}\right) \tag{11}$$

Equating equation (11) to zero to solve this equation:

$$\frac{-n}{\beta} + \sum_{i=1}^{\mathrm{n}} \frac{(t_i-\alpha)}{\beta^2} = 0$$

$$\frac{-n\,\hat{\beta} + \sum_{i=1}^{n}(t_i-\alpha)}{\hat{\beta}^2} = 0$$

$$-n\,\hat{\beta} + \sum_{i=1}^{n}(t_i-\alpha) = 0$$

$$n\,\hat{\beta} = \sum_{i=1}^{n}(t_i-\alpha)$$

$$\therefore\ \hat{\beta}_{ML} = \frac{1}{n}\sum_{i=1}^{n}(t_i-\alpha) \tag{12}$$

Then the estimation of Reliability function for the two-parameters Exponential distribution using ML technique will be

For more information about the Conference please visit the websites:
http://www.ihsciconf.org/conf/
www.ihsciconf.org

$$\hat{R}_{ML} = \exp(-\frac{(t_i - \alpha)}{\hat{\beta}_{ML}}) \quad (13)$$

Median-First Order Statistics Method

"In this modification the second moment is replaced by $Me_t = t_{me}$, where Me_t is the population median and t_{me} is the sample median";[7]. The Exponential distribution median can be found by

$$0.5 = \int_0^A f(x)\ dt \quad (14)$$

$$\frac{1}{\beta}\int_0^A \exp[-\frac{(t_i - \alpha)}{\beta}]\ dt = \frac{1}{2}$$

$$A = \alpha + \beta\ (ln2)$$

The C.D.F. of $t_{(1)}$ defined below

$$F(t_{(1)}) = pr[t_{(1)} < t] = 1 - pr[t_{(1)} > t)]$$

$$= 1 - pr[t_{(1)} > t\ , t_{(2)} > t, \dots, t_{(n)} > t]$$

$$= 1 - [pr(T > t)]^n = 1 - [R(t)]^n$$

So, the P.D.F. of $t_{(1)}$ became

$$f(t_{(1)}) = \frac{dF(t_{(1)})}{dt_{(1)}}$$

$$= \frac{n}{\beta}\exp\left[-n\left(\frac{t_{(1)}-\alpha)}{\beta}\right)\right] \qquad ; \alpha < t_{(1)} < \infty \quad , \beta > 0$$

Then, the mathematical expectation of a random variable $t_{(1)}$ is

$$E[t_{(1)}] = \int_\alpha^\infty \frac{tn}{\beta}\exp\left[-\left(\frac{n(t_{(1)} - \alpha)}{\beta}\right)\right] dt$$
$$= \alpha + \frac{\beta}{n}$$

Where $t_{(1)}$ *refer* to the first order statistic which is represents the smallest values of t.
If $(\hat{\alpha}\ ,\ \hat{\beta})$ is unbiased estimator to $(\alpha\ , \beta)$ respectively then the equationrealized

$$\alpha = t_{(1)} - \frac{\beta}{n} \quad (15)$$

Thus, we have

$$\alpha + \beta\ (ln2) = t_{me} \quad \rightarrow \quad \alpha = t_{me} - \beta\ (ln2)$$
(16)

By equating equations (17) and (18), we get

$$t_{me} - \beta(ln2) = t_{(1)} - \frac{\beta}{n} \quad \rightarrow \quad t_{me} - t_{(1)} = \beta(ln2) - \frac{\beta}{n}$$

$$t_{me} - t_{(1)} = \beta\left((\ln 2) - \frac{1}{n}\right)$$

$$\hat{\beta}_{MD} = \frac{(t_{me} - t_{(1)})}{\left((\ln 2) - \frac{1}{n}\right)} \qquad (17)$$

Then the approximate estimation of Reliability function for the two-parameter Exponential distribution using Median-First Order Statistics Method (MD) will be

$$\hat{R}_{MD} = exp\left(-\frac{(t_i - \alpha)}{\hat{\beta}_{Md}}\right) \qquad (18)$$

Ridge Regression method (RR)

"The ridge regression (RR) estimates of A and B can be obtained by minimizing the error sum of squares for the model Y_i = a+bX_i Subject to the single constraint that $a^2+b^2 = \rho$ where ρ is a finite positive constant "; [1].
i.e.; $L = \sum_{i=1}^{n}(Y_i - a - bX_i)^2 + \lambda(a^2 + b^2 - \rho)$

w.r.t a and b. when these derivatives are equated to zero, we obtain the following two equations

$$\sum_{i=1}^{n} Y_i = (n + \lambda)a + b\sum_{i=1}^{n} X_i$$

$$\sum_{i=1}^{n} X_i Y_i = a\sum_{i=1}^{n} X_i + b(\lambda + \sum_{i=1}^{n} X_i^2)$$

Solving above two equations for a and b we get

$$\hat{a} = \frac{(\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} X_i Y_i) - \sum_{i=1}^{n} Y_i (\lambda + \sum_{i=1}^{n} X_i^2)}{(\sum_{i=1}^{n} X_i)^2 - (n + \lambda)(\lambda + \sum_{i=1}^{n} X_i^2)}$$

$$\hat{b} = \frac{(\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i) - (n + \lambda)\sum_{i=1}^{n} X_i Y_i}{(\sum_{i=1}^{n} X_i)^2 - (n + \lambda)(\lambda + \sum_{i=1}^{n} X_i^2)}$$

For two parameter Exponential distribution when α is known we recognize that $Y_i = t_i$, b = β

X_i= [-ln (1-F (t_i))] i = 1, 2,…, n .

$$\hat{\beta} = \frac{\sum t_i \sum(-\ln(1-F(t_i))) - (n+\lambda)\sum t_i(-\ln(1-F(t_i)))}{[\sum(-\ln(1-F(t_i)))]^2 - (n+\lambda)(\lambda + \sum[-\ln(1-F(t_i))]^2)} \quad (19)$$

Then the approximate estimation of Reliability function for the two-parameter Exponential distribution using ridge regression technique (RR) will be

$$\hat{R}_{RR} = exp\left(-\frac{(t_i-\alpha)}{\hat{\beta}_{RR}}\right) \quad (20)$$

Where 0<λ<1 is the ridge coefficient. The present paper suggested λ= exp((-n+1) /(n^2)) and 0< exp((-n+1) /(n^2)) < 1 .

Modified Thompson-Type Shrinkage Estimator

" The shrinkage estimation method is the Bayesian approach depending on prior information regarding the value of the specific parameter θ from past experiences or previous studies. However, in certain situations, prior information is available only from of an initial guess value (natural origin) $\theta_\circ$ of θ";[10]. In such a situation, it is natural to begin with $\hat{\theta}$ (e.g., MLE) and adapt it by touching $\theta_\circ$.Thompson has suggested the problem of shrink an unbiased estimator $\hat{\theta}$ of the parameter θ toward prior information (a natural origin) $\theta_\circ$by shrinkage estimator $\psi(\hat{\theta})\hat{\theta} + (1-\psi(\hat{\theta}))\theta_\circ$, 0≤ $\psi(\hat{\theta}) \leq 1$, which is more efficient than $\hat{\theta}$ if $\theta_\circ$ is close to θ and less efficient than $\hat{\theta}$ otherwise; [10].

"the prior information $\theta_\circ$ is a natural origin and, as such, may arise for any one of a number of reasons e.g., we are estimating θ and (a) we believe $\theta_\circ$ is closed to the true value of θ, or (b) we fear that $\theta_\circ$ may be near the true value of θ, that is, something bad happens if $\theta_\circ = \theta$, and we do not know about it (that is, something bad happens if $\theta_\circ \approx \theta$ and we do not use $\theta_\circ$)";[2],[10].

Where, $\psi(\hat{\theta})$ is so called shrinkage weight factor; $0 \leq \psi(\hat{\theta}) \leq 1$ which represent the belief of $\hat{\theta}$,and (1- $\psi(\hat{\theta})$) represent the belief of $\theta_\circ$.Thompson noting that the shrinkage weight factor may be a function of $\hat{\theta}$ or may be constant and the chosen of the shrinkage weight factor is(ad hoc basis).

The shrinkage weight function $\psi(\hat{\theta})$ can be found by minimizing the mean square error of $\tilde{\theta}$:

$$MSE(\tilde{\theta}_{TH}) = E(\tilde{\theta}_{TH} - \theta)^2 \quad (21)$$

$$= E(\psi(\hat{\theta})\,\hat{\theta}_{ML} + (1-\psi(\hat{\theta}))\,\theta_\circ - \theta)^2$$

The partial derivative for above equation w .r. t. to $\psi(\hat{\theta})$ is

$$\frac{\partial MSE(\tilde{\theta})}{\partial \psi(\hat{\theta})} = 2\psi(\hat{\theta})E\left(\hat{\theta}_{ML} - \theta_\circ\right)^2 - 2\left(1 - \psi(\hat{\theta})\right)\left(\hat{\theta}_{ML} - \theta_\circ\right)^2 \qquad (22\ )$$

Equating equation (24) to zero to solve this equation:

$$\frac{\partial MSE(\tilde{\theta})}{\partial \psi(\hat{\theta})} = 0$$

$$2\psi(\hat{\theta})MSE(\hat{\theta}_{ML}) - 2\left(1 - \psi(\hat{\theta})\right)\left(\hat{\theta}_{ML} - \theta_\circ\right)^2 = 0$$

$$\psi(\hat{\theta}) = \frac{\left(\hat{\theta}_{ML} - \theta_\circ\right)^2}{\left[MSE(\hat{\theta}_{ML}) + \left(\hat{\theta}_{ML} - \theta_\circ\right)^2\right]}$$

The modified shrinkage weight factor will be considered as below

$$\psi(\hat{\theta}) = \frac{\left(\hat{\theta}_{ML} - \theta_\circ\right)^2}{\left[MSE(\hat{\theta}_{ML}) + \left(\hat{\theta}_{ML} - \theta_\circ\right)^2\right]}\ (0.001) \qquad (23)$$

Therefore, the modified shrinkage estimator of β will be as below:

$$\hat{\beta}_{MT} = \beta_\circ + \frac{\left(\hat{\beta}_{ML} - \beta_\circ\right)^3}{\left[MSE(\hat{\beta}_{ML}) + \left(\hat{\beta}_{ML} - \beta_\circ\right)^2\right]} * (0.001) \qquad (24)$$

Where $\beta_\circ$, refer to prior estimate of β.

Then the approximate estimation of Reliability function for the two-parameter Exponential distribution using modified Thompson-type shrinkage estimator (MT) will be

$$\hat{R}_{MT} = exp\left(-\frac{(t_i - \alpha)}{\hat{\beta}_{MT}}\right) \qquad (25)$$

Single Stage Shrinkage Estimator

"Single stage shrinkage estimation method is the same as the method of Thompson-Type shrinkage estimator $\psi(\hat{\theta})\hat{\theta} + (1 - \psi(\hat{\theta}))\theta_\circ$, $0 \leq \psi(\hat{\theta}) \leq 1$, which is define in section 2.4 above but we consider the shrinkage weight factor $\psi(\hat{\theta})$ as a function of $var(\hat{\theta})$; "[2] .

In this paper we consider , $\psi(\hat{\beta}_{ML}) = e^{-\left(\frac{1}{var(\hat{\beta}_{ML})}\right)}$ where the $var(\hat{\beta}_{ML}) = \frac{\beta^2}{n}$

$$\hat{\beta}_{ST} = e^{-\left(\frac{1}{var(\hat{\beta}_{ML})}\right)}\hat{\beta}_{ML} + \left(1 - e^{-\left(\frac{1}{var(\hat{\beta}_{ML})}\right)}\right)\beta_\circ \qquad (26)$$

Then the approximate estimation of Reliability function for the two-parameters Exponential distribution using single stage shrinkage estimator (ST)will be

$$\hat{R}_{ST} = exp\left(-\frac{(t_i-\alpha)}{\hat{\beta}_{ST}}\right) \tag{27}$$

3. Results & Discussion

Simulation Study

We carried out Monte Carlo simulation in order to compare the performance of all the estimators proposed in the preceding section. The programs were written in Matlab (2013b). The results depend on 1000 simulation iteration. Generated random samples of different sizes by observing that if U is uniform (0, 1), then $t = \alpha + \beta\,[-\log(1-U)]$ is Exponential of (α , β). The sample sizes considered were n=25, 50, 75, 100 and the shape parameter was taken as α = 0.2, 0.5. In all cases, we set the scale parameter β =3, 3.5, 4. We used 1000 replications to estimate by using the ML, MD, RR, MT and ST methods. The process of simulation strategy is explained the numerical results in the Table (1) and Table (2). And comparison among all propose estimators where made on MSE which is defined as follow

$$MSE\left(\hat{R}(t)\right) = \frac{\sum_{i=1}^{L}\left(\hat{R}_i(t) - R_i(t)\right)^2}{L}$$

Where $\hat{R}_i(t)$ is the specific estimated reliability, $R_i(t)$ refer to specific real reliability and L=1000 refer to the number of replications.

3.2 Discussion and Numerical Analysis

1. When n=25, the estimator minimum MSE vibration between $\hat{R}_{ST}$ and $\hat{R}_{MT}$ and they performed good than the others estimators in the sense of MSE, then follow by $\hat{R}_{MT}$, $\hat{R}_{MD}$, $\hat{R}_{ML}$ and $\hat{R}_{RR}$ respectively for all α and β.
2. When n=50,75 and 100 , we can see from the Table (2) , the MSE of estimator $\hat{R}_{ST}$ less than of the MSE of the other estimators , thus it will be the best in the sense of MSE and follow by the estimators by $\hat{R}_{MT}$, $\hat{R}_{MD}$, $\hat{R}_{ML}$ and $\hat{R}_{RR}$ respectively.
3. For all n and for all β, the MSE of the proposed estimators are vibrate **with respect to** α.

The results of the simulation study are reported in the tables (1) and (2):

Table (1): Estimate Reliability of Exponential distribution based on simulations

n	β	α	R_{RL}	$\hat{R}_{ML1}$	$\hat{R}_{MD}$	$\hat{R}_{RR}$	$\hat{R}_{MT}$	$\hat{R}_{ST}$
25	2	0.2	0.0341	0.0311	0.0354	0.7692	0.0341	0.0341
		0.5	0.0140	0.0245	0.0171	0.7171	0.0140	0.0140
	2.5	0.2	0.0626	0.0530	0.0820	0.7641	0.0626	0.0626
		0.5	0.1110	0.0386	0.1279	0.8081	0.1109	0.1110
	3	0.2	0.0991	0.0395	0.0635	0.7639	0.0991	0.0991
		0.5	0.0893	0.0624	0.0502	0.7544	0.0892	0.0893
50	2	0.2	0.0236	0.0142	0.0138	0.8631	0.0236	0.0236
		0.5	0.0237	0.0186	0.0202	0.8632	0.0237	0.0237
	2.5	0.2	0.0026	0.0055	0.0070	0.7459	0.0026	0.0026
		0.5	0.0086	0.0163	0.0160	0.7912	0.0086	0.0086
	3	0.2	0.0088	0.0106	0.0201	0.7557	0.0088	0.0088
		0.5	0.0238	0.0244	0.0259	0.8019	0.0238	0.0238
75	2	0.2	0.0017	0.0049	0.0078	0.8450	0.0017	0.0017
		0.5	0.0073	0.0114	0.0154	0.8784	0.0073	0.0073
	2.5	0.2	0.0139	0.0167	0.0092	0.8686	0.0139	0.0139
		0.5	0.0097	0.0129	0.0268	0.8582	0.0097	0.0097
	3	0.2	0.0117	0.0206	0.0290	0.8388	0.0118	0.0118
		0.5	0.0092	0.0179	0.0189	0.8308	0.0092	0.0092
100	2	0.2	0.0094	0.0066	0.0027	0.9116	0.0094	0.0094
		0.5	0.0020	0.0020	0.0025	0.8841	0.0020	0.0020
	2.5	0.2	0.0075	0.0058	0.0078	0.8857	0.0075	0.0075
		0.5	0.0011	0.0011	0.0015	0.8452	0.0011	0.0011
	3	0.2	0.0228	0.0118	0.0106	0.8937	0.0228	0.0228
		0.5	0.0043	0.0044	0.0082	0.8503	0.0043	0.0043

Table (2): Mean Squared Error (MSE) of Reliability estimates based on simulations

n	β	α	$\hat{R}_{ML}$	$\hat{R}_{MD}$	$\hat{R}_{RR}$	$\hat{R}_{MT}$	$\hat{R}_{ST}$
25	2	0.2	8.8661e-09	1.6801e-09	5.4040e-04	4.2742e-17	1.1463e-31
		0.5	1.0933e-07	9.5551e-09	4.9431e-04	3.2088e-14	3.1178e-24
	2.5	0.2	9.1547e-08	3.7569e-07	4.9207e-04	6.6791e-15	2.3506e-23
		0.5	5.2429e-06	2.8634e-07	4.8594e-04	5.7627e-12	1.7333e-36
	3	0.2	3.5488e-06	1.2662e-06	4.4190e-04	3.7563e-12	6.0057e-25
		0.5	7.1999e-07	1.5285e-06	4.4240e-04	3.6421e-13	1.4778e-19
50	2	0.2	8.8349e-08	9.7428e-08	7.0465e-04	6.8744e-14	0
		0.5	2.6716e-08	1.2540e-08	7.0470e-04	6.4173e-15	0
	2.5	0.2	4.8101e-10	9.3650e-09	5.8745e-04	6.7782e-19	2.4385e-17
		0.5	6.0132e-08	5.4459e-08	6.1243e-04	9.3136e-15	2.5984e-13
	3	0.2	3.3587e-09	1.2837e-07	5.5796e-04	1.6550e-17	1.0752e-13
		0.5	3.0973e-10	4.2670e-09	6.0538e-04	1.1146e-21	5.2519e-15
75	2	0.2	1.0020e-08	3.7668e-08	7.1120e-04	2.0645e-15	2.0825e-20
		0.5	1.6789e-08	6.5521e-08	7.5878e-04	1.8533e-15	4.3237e-22
	2.5	0.2	7.8791e-09	2.2677e-08	7.3051e-04	1.0563e-16	2.0281e-18
		0.5	1.0242e-08	2.9311e-07	7.2002e-04	4.2731e-16	5.7699e-18
	3	0.2	7.8496e-08	2.9791e-07	6.8397e-04	1.6989e-14	1.7126e-13
		0.5	7.5074e-08	9.3980e-08	6.7492e-04	1.8214e-14	2.3336e-13
100	2	0.2	7.5865e-09	4.4484e-08	8.1402e-04	1.1724e-15	7.7964e-34
		0.5	8.7731e-15	2.5500e-10	7.7805e-04	2.8446e-31	1.6974e-36
	2.5	0.2	2.9358e-09	1.2003e-10	7.7132e-04	1.6660e-16	1.3086e-24
		0.5	1.1759e-12	1.4244e-10	7.1239e-04	5.0870e-24	1.1373e-26
	3	0.2	1.2099e-07	1.4817e-07	7.5841e-04	9.2427e-14	8.1256e-21
		0.5	1.2835e-11	1.5244e-08	7.1580e-04	6.5160e-23	3.4843e-21

4. Conclusions

From the Table (2), one can be noticed that, the shrinkage method perform better than the other methods in the sense of MSE, and we recommend to use this type of estimation which is depend on prior information from the past experiences or previous studies.

Acknowledgment

The authors wanted to provide thanks to the referees and to the Editor for constructive suggestions and valuable comments which resulted in the improvement of this article

Reference

[1] E. E. Afify, Linear and Nonlinear regression of exponential distribution. Inter. Statist., online Statistics Journal. 2004

[2] A.N. Al-Joboori, el al "Single and Double Stage Shrinkage Estimators for the Normal Mean with the Variance Cases", International Journal of statistic, 38, 2, 1127-1134. 2014

[3] A. C.Cohen& F. R.Helm, Estimation in Exponential Distribution. Technometrics, 15, 415-418. http://dx.doi.org/10.1080/00401706.1973.104890541973

For more information about the Conference please visit the websites:
http://www.ihsciconf.org/conf/
www.ihsciconf.org

[4] J. Lai, , & C. W. Augustine, Interval estimation of the two-parameter exponential distribution. Journal of Probability and Statistics, Article ID 734575. http://dx.doi.org/10.1155/2012/734575. 2012
[5] J. F. Lawless, Prediction intervals for the two parameter exponential distribution. Technometrics, 19(4), 469-472. http://dx.doi.org/10.1080/00401706.1977.10489587,1977.
[6] B. A. Maguire ; E.S. Pearson, and A. H. A. Wynn, The time intervals between industrial accidents. *Biometrika* 39, 168-180. 1952
[7] Z. R. Muhammad & S. A. Ahmed, Estimation accuracy of exponential distribution parameters. Pakistan Journal of Statistics and Operation Research, 7(2), 217-232. 2011
[8] F. T. Peter, Huber-Sense Robust M-Estimation of a Scale Parameter, with Application to the Exponential Distribution. *JASA*, *74*, 147-152. 1974
[9] C. Petropoulos, New classes of improved confidence intervals for the scale parameter of a two parameter exponential distribution. Statistical Methodology, 8(4), 401-410. 2011. http://dx.doi.org/10.1016/j.stamet.2011.03.002.
[10] J.R. Thompson, Some Shrinkage Techniques for Estimating the Mean. J. Amer. Statist. Assoc. 63.113-122. 1968

For more information about the Conference please visit the websites:
http://www.ihsciconf.org/conf/
www.ihsciconf.org