On generalized b*-Closed Sets In Topological Spaces

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Abstract

In this paper, we introduce and study the concept of a new class of generalized closed set which is called generalized b*-closed set in topological spaces (briefly .g b*-closed) we study also. some of its basic properties and investigate the relations between the associated topology.

Keywords: gb* -closed set, gb -closed set,g-closed set.

1. Introduction

Levine[9] introduced the concept of generalized closed sets (briefly ,g-closed) and studied their most fundamental properties in topological spaces . Arya and Nour[6], Bhattacharya and Lahiri[7], Levine[10], Mashhour[11], Njastad[13]and Andrijevic[3,4] introduced and investigated generalized semi-open sets,semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α - open sets, semi pre-open sets and b-open sets which are some of the weak forms of open sets and the complements of these sets are called the same types of closed sets. A.A.Omari and M.S.M.Noorani[14] introduced and studied the concept of generalized b-closed sets(briey gb-closed) in topological spaces. Recently Sundaram and Sheik John [15] introduced and studied w-closed sets. S.Muthuvel and R.Parimelazhagan [12] introduced and studied b*closed sets , A.Poongothai and R.Parimelazhagan [5] introduced and studied strongly b*-closed set in topological spaces.
In this paper, we introduce a new class of sets, namely gb*- closed sets for topological spaces. this class lies between the class b*-closed set and strongly b*-closed set.

2.Preliminaries

Let (X,T) be topological spaces and A be a subset of X .The closure of A and interior of A are denoted by cl(A) and int(A) respectively, union of all b-open (semi-open , pre-open , α – open) sets X contained in A is called b- interior (semi- interior, pre-interior, α – interior ,respectively) of A, it is denoted by b-int (A)(s-int(A),p-int(A), α-int(A), respectively),The intersection of all b-closed (semi- closed , pre- closed , α – closed) sets X containing A is called b- closure (semi- closure, pre- closure , α – closure ,respectively) of A and it is denoted by bcl(A) (scl(A) ,pcl(A) , αcl(A) ,respectively).In this section , we recall some definitions of open sets in topological spaces.

Definition 2-1[15]*:* A subset A of a topological space (X,T) is called a pre –open set if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.

Definition 2-2[10]*:*A subset A of a topological space (X,T) is called a semi –open set if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.

Definition 2-3[3]: A subset A of a topological space (X,T) is called a α –open set if $A \subseteq int(cl(int(A))$ and α -closed set if $cl(int(cl(A)) \subseteq A$.

Definition 2-4[8]*:*A subset A of a topological space (X,T) is called a β–open set if $A \subseteq cl(int(cl(A))$ and β-closed set if $int(cl(int(A)) \subseteq A$.

Definition 2-5[1]*:*A subset A of a topological space (X,T) is called a b –open set if $A \subseteq cl(int(A)) \cup int(cl(A))$ and b-closed set if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

Definition 2-6[9]:A subset A of a topological space (X,T) is called a generalized –closed set (briefly, g-closed) if $cl(A) \subseteq U$,whenever $A \subseteq U$ and U is open set .

Definition 2-7[7]*:*A subset A of a topological space (X,T) is called a semi generalized closed set (briefly, sg-closed) if $scl(A) \subseteq U$,whenever $A \subseteq U$ and U is semi- open set .

Definition 2-8[8]:A subset A of a topological space (X,T) is called a generalized α-closed set (briefly gα-closed) if αcl(A) ⊆ U ,whenever A ⊆ U and U is α- open set .

Definition 2-9[2]:A subset A of a topological space (X,T) is called a generalized b-closed set (briefly gb -closed) if bcl(A) ⊆ U ,whenever A ⊆ U and U is open set .

Definition 2-10[8]:A subset A of a topological space (X,T) is called a generalized β-closed set (briefly gβ-closed) if βcl(A) ⊆ U ,whenever A ⊆ U and U is open set .

Definition 2-11[5]:A subset A of a topological space (X,T) is called weakly generalized closed set (briefly wg-closed) if cl(int(A)) ⊆ U ,whenever A ⊆ U and U is open set .

Definition 2-12[15]:A subset A of a topological space (X,T) is called wekly-closed set (briefly w-closed) if cl(A) ⊆ U ,whenever A ⊆ U and U is semi- open set .

Definition 2-13[12]:A subset A of a topological space (X,T) is called b*-closed set if int(cl(A)) ⊆ U ,whenever A ⊆ U and U is b- open set .

Definition 2-14[5]:A subset A of a topological space (X,T) is called g* -closed set if cl(A) ⊆ U ,whenever A ⊆ U and U is g- open set .

Definition 2-15[5]:A subset A of a topological space (X,T) is called a g*b -closed set if bcl(A) ⊆ U ,whenever A ⊆ U and U is g- open set.

Definition 2-16[5] :A subset A of a topological space (X,T) is called strongly b*-closed set (briefly, sb*-closed) if cl(int(A)) ⊆ U ,whenever A ⊆ U and U is b- open set .

Definition 2-17[5] :A subset A of a topological space (X,T) is called b**-open set if A⊆int(cl(int(A)))∪cl(int(cl(A))) and b**-closed set if cl(int(cl(A))∩int(cl(int(A)) ⊆ A).

3. Generalized b*-closed sets.

In this section , we introduce and study the concept of generalized b*-closed set in topological spaces . Also we study the relationship between this set and the other types of sets.

Definition 3-1*:* A subset A of a topological space (X,T) is called generalized b*-closed set (briefly, gb*-closed) if int(cl(A)) ⊆ U ,whenever A ⊆ U and U is gb- open set .

Theorem 3-2:Every closed set is gb* -closed set.

Proof : Assume that A is a closed set in X then cl (A)=A ,and U be any gb-open set where A ⊆ U . Since int(A) ⊆ A . implies that int(cl(A) ⊆ U .Hence A is gb* -closed set in X.

Remark 3-3: The converse of the Theorem [3- 2] need not be true as seen by the following example.

Example3-4: let X={ a,b,c } with T={X , ∅ , {a} }.In this topological space , the sub set A={b} is gb*- closed set but not closed set .

Theorem 3-5: A set A is gb*-closed set iff int cl(A)-A contains no non-empty gb-closed set.

Proof : Necessity: Suppose that F is a non-empty gb-closed subset of int(cl(A)) such that $F \subseteq int(cl(A)) - A$. then $F \subseteq int(cl(A)) \cap A^c$.Therefore $F \subseteq int(cl(A))$ and $F \subseteq A^c$. Since F^c is gb-closed set and A is gb*-closed set , $int(cl(A)) \subseteq F^c$. thus $F \subseteq (int(cl(A)))^c$. Therefore $F \subseteq \left(int(cl(A))\right) \cap (int(cl(A)))^c = \emptyset$ .Therefore F= ∅ and this implies that int(cl(A))-A contains no non-empty gb-closed set .

Sufficiency : Assume that int(cl(A))-A contains no non-empty gb-closed. Let $A \subseteq U$,U is gb-open set .Suppose that int(cl(A)) is not contained in U , then $int(cl(A)) \cap U^c$ is a non-empty gb-closed set of int(cl(A))-A which is a contradiction. Therefore $int(cl(A))U \subseteq$ and hence A is gb*-closed set .

Theorem 3-6: Let $B \subseteq Y \subseteq X$,if B is gb^*-closed set relative to Y and that Y is both gb-open *and gb*-closed set in (X,T) then B is gb*-closed set in (X,T).*

Proof: Let $U \subseteq B$ and U be a gb-open set in (X,T).But Given that $B \subseteq Y \subseteq X$. Therefore $B \subseteq Y$ and $U \subseteq B$. This implies that $Y \cap U \subseteq B$. Since B is gb*-closed set relative to Y, Then $Y \cap U \subseteq int(cl(Y))$.(i.e) $Y \cap U \subseteq Y \cap int(cl(Y))$.implies that $U \subseteq Y \cap int(cl(Y))$.

thus $U \cup [int(cl(B))]^c \subseteq [Y \cap int(cl(B))] \cup [int(cl(B))]^c$.

This implies that $U \cup [int(cl(B))]^c \subseteq int(cl(Y)) \subseteq int(cl(B))$.

Therefore $U \subseteq int(cl(B))$. Since int(cl(B)) is not contained in$[int(cl(B))]^c$.

Thus B is gb*-closed set relative to X.

Theorem 3-7: Let $A \subseteq Y \subseteq X$ and suppose that A is gb^* -closed set in X then A is gb^* -closed set relative to Y.

Proof : Assume that $A \subseteq Y \subseteq X$ and A is gb* -closed set in X . To show that A is gb* -closed set relative to Y , let $A \subseteq Y \cap U$ where U is gb-open in X . Since A is gb* -closed set in X , $A \subseteq U$ implies that $int(cl(A)) \subseteq U$, (i.e) $Y \cap int(cl(A)) \subseteq Y \cap U$. where $Y \cap int(cl(A))$ is interior of closure of A in Y . Thus A is gb* -closed set relative to Y.

Theorem 3-8: If A is a gb* -closed set and $A \subseteq B \subseteq int(cl(A))$ then B is a gb* -closed set.

Proof : Let U be a gb -open set of X , such that $B \subseteq U$. Then $A \subseteq U$. Since A is gb* -closed, Then $int(cl(A)) \subseteq U$ .Now $int(cl(B)) \subseteq int(cl(A)) \subseteq U$ .Therefore B is gb*-closed set in X .

Theorem 3-9 : The intersection of a gb* -closed set and a closed set is a gb* -closed set.

Proof: Let A be a gb* -closed set and F be a closed set . Since A is gb* -closed set , $int(cl(A)) \subseteq U$ whenever $A \subseteq U$,where U is agb-open set . To show that $A \cap F$is gb*-closed set ,it is enough to show that $int(cl(A \cap F)) \subseteq U$ whenever $A \cap F \subseteq U$, where U is gb-open set . Let G =X – F then $A \subseteq U \cup G$ .Since G is open set , $U \cup G$ is gb-open set and A is gb* -closed set , $int(cl(A)) \subseteq U \cup G$. Now $int(cl(A \cap F)) \subseteq int(cl(A)) \cap int(cl(F)) \subseteq int(cl(A)) \cap F \subseteq (U \cup G) \cap F \subseteq (U \cap F) \cup (G \cap F) \subseteq (U \cap F) \cup \varnothing \subseteq U$. This implies that $(A \cap F)$ is gb* -closed set.

Theorem 3-10: If A and B are two gb* -closed sets defined for a non –empty set X, then their intersection $A \cap B$ is gb* -closed set in X.

Proof: Let A and B are two gb* -closed sets in X. Let $A \cap B \subseteq U$, U is gp-open set in X. Since A is gb* -closed , $int(cl(A)) \subseteq U$,whenever $A \subseteq U$, U is g-open set in X .Since B is gb* -closed , $int(cl(B)) \subseteq U$,whenever $B \subseteq U$, U is g-open set in X . hence $A \cap B$ is gb* -closed set .

Remark 3-11: The Union of two gb* -closed sets need not to be gb* -closed set.

Example3-12: Let X ={a,b,c} with T={X , ∅ , {a},{c},{a,c} } .If A= {a}, B= {c} are gb* -closed set in X . then $A \cup B$ is not a gb* -closed set .

Theorem 3-13: Every gb - closed set is gb* -closed set.

Proof : Assume that A be a g b - closed set in X. and let U be an open set such that $A \subseteq U$. Since every open set is gb-open set . Then $int(cl(A)) \subseteq bcl(A) \subseteq U$. hence A is gb*-closed set .

Remark 3-14: The converse of the Theorem [3-13] need not be true as seen by the following example.

Example3-15: let X={ a,b,c } with T={X , ∅ , {a} }.In this topological space , the subset A={ a,b} is gb* -closed set, but not gb- closed set.

Theorem 3-16:: Every gb*-closed set is b-closed set .

Proof: Assume that A is a gb* -closed set in X , and let U be an open set such that $A \subseteq U$ .since every open set is b-open set and A is gb*-closed set, then $int(cl(A)) \subseteq intcl((A))\cup cl(int(A)) \subseteq U$. Therefore A is b-closed set in X .

Remark 3-17: The converse of the Theorem [3-16] need not be true as the following example shows.

Example3-18: let X={ a,b,c } with T={X , ∅ , {a} ,{b} ,{a,b}}. In this topological space , the subset A={ a,c} is b -closed set but not gb*- closed set.

Theorem 3- 19: Every w-closed set is gb* -closed set .

proof: Assume that A is w-closed set in X , and U is semi-open set such that A ⊆ U, every semi- open set is gb-open set then cl(A) ⊆int(cl(A)) therefore A is gb* -closed set.

Remark 3- 20: The converse of the Theorem [3-19] need not be true as seen by the following example

Example 3-21: let X={ a,b,c } with T={X , ∅ , {a} ,{c} ,{a,c}}. In this topological spaces , the subset A={ a } is gb*- closed set but not w-closed set .

Theorem 3- 22: Every b* -closed set is g b* -closed set .

Proof: Assume that A is a b*-closed set in X , and U is b- open set such that A ⊆ U. every b-open set is g b-open set . Then $\mathrm{int}(\mathrm{cl}(A)) \subseteq U$, Therefore A is g b* -closed set.

Remark 3- 23: The converse of the Theorem [3-22] need not be true as seen by the following example.

Example3-24: let X={ a,b,c,d } with T={X , ∅,{b},{c,d},{ b,c,d} }.

In this topological spaces the subset A={ c} is gb*-closed set ,but not b*-closed set .

Theorem 3-25: Every gb*-closed set is g*b-closed set.

Proof: Assume that A is a g b*-closed set in X .Then int(cl(A) ⊆ U ,U is gb-open set such that A ⊆ U. Then bcl(A) $\subseteq intcl(A)$. Since every g-open set is gb-open set .Then bcl(A) $\subseteq U$, Uis g-open set .Therefore A is g*b-closed set.

Remark 3-26: The converse of the Theorem [3-25] need not be true as seen by the following example.

Example3-27: let X={ a,b,c } with T={X , ∅ }.In this topological spaces , the subset A={ a,b} is g*b -closed set but not gb*- closed set.

Theorem 3- 28 : Every gb*-closed set is sg -closed set .

proof: Assume that A is gb*-closed set in X , and U is open set such that A ⊆ U every open set is semi-open set, A is gb*-closed and U is gb-closed then int(cl(A)) ⊆ A ∪ scl(A) ⊆ U therefore A is sg -closed set.

Remark 3- 29: The converse of the Theorem [3-28] need not be true as seen by the following example.

Example3-30: let X= {a ,b ,c} ,T={X,∅,{a,b }, {c}} In this example A={a,b} is sg-closed set but not gb* -closed set.

Theorem 3- 31*:* Every gb* -closed set is gβ-closed set .

proof: Assume that A is g*b* -closed set in X , and U is open set such that A ⊆ U, every open set is gb-open set then $\text{int}(\text{cl}(A) \subseteq A \cup \beta - \text{closed} \subseteq U$ Therefore A is g*b* -closed set.

Remark 3- 32: The converse of the Theorem [3-31] need not be true as seen by the following example.

Example3-33*:* let X= {a ,b ,c} ,T={X,∅,{b}, {b,c} }.

In this example A={a,b} is gβ –closed set but not gb* -closed set.

theorem 3- 34*:* Every gb* -closed set is b** -closed set .

proof: Assume that A is g*b* -closed set in X , and U is open set such that A ⊆ U, every open set is gb-open set then $\text{int}(\text{cl}(A) \subseteq \text{cl}(\text{int}(\text{cl}(A))) \cup \text{int}(\text{cl}(\text{int}(A))) \subseteq U$.Therefore A is b**-closed set.

Remark 3- 35: The converse of the Theorem [3-34] need not be true as seen by the following example.

Example3-36*:* let X= {a ,b ,c} ,T={X,∅,{a , b}, { c} } .

In this topological spaces the subset A={b, c} is b**-closed set but not g b* -closed set.

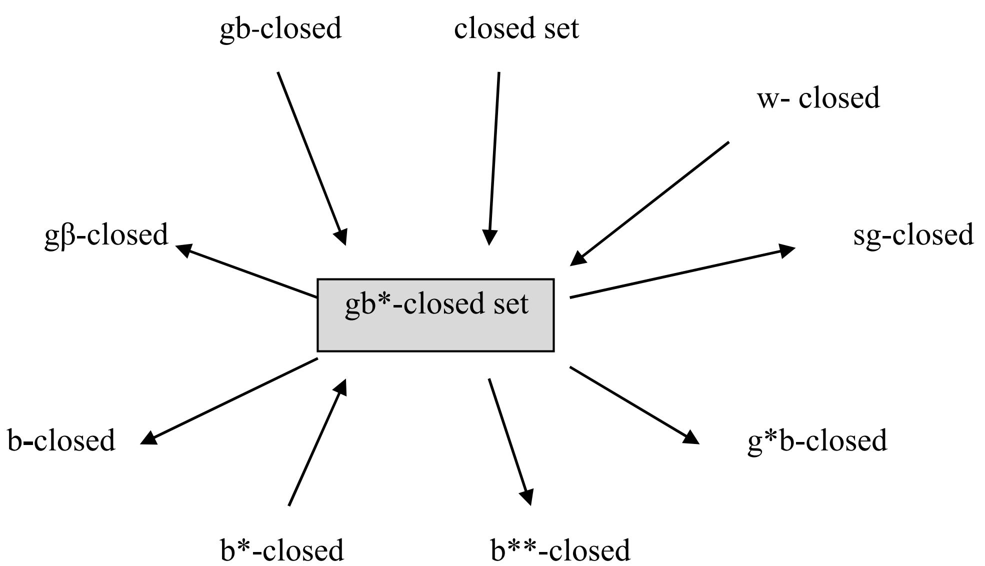

diagram (1)

4.gb*-closed set is independent of other closed sets

In this section ,we explain independency of gb*-closed set with some other closed sets.

Remark 4- 1: The following example shows that the concept of g-closed and gb*-closed sets are independent .

Example4-2: let X= {a ,b ,c} ,T={X,∅,{ b}, { b,c} } , In this topological space ,the subset A={a, b}is g-closed set but not gb*-closed set. And , in this topological space , the subset B={c}is gb*-closed set but not g-closed set.

Remark 4- 3: The following example shows that the concept of sb*-closed and gb*-closed sets are independent .

Example4-4 : let X= {a ,b ,c} ,T={X,∅,{ a},{c} ,{ a,c} } , In this topological space ,the subset A={a,c}is sb*-closed set but not gb*-closed set. And , in this topological space ,the subset B={c}is gb*-closed set but not sb*-closed set.

Remark 4- 5: The following example shows that the concept of g*-closed and gb*-closed sets are independent .

Example4-6: let X= {a ,b ,c} ,T={X,∅,{ b}, { b,c} } , In this topological space ,the subset A={a, b}is g*-closed set but not gb*-closed set. And , in this topological space ,the subset B={c}is gb*-closed set but not g*-closed set.

Remark 4- 7: The following example shows that the concept of gα-closed and gb*-closed sets are independent .

Example4-8: let X= {a ,b ,c} ,T={X,∅,{ a},{c} ,{ a,c} } , In this topological space ,the subset A={b,c}is gα -closed set but not gb*-closed set. And , in this topological space ,the subset B={a}is gb*-closed set but not gα -closed set

Remark 4- 9: The following example shows that the concept of gp-closed and gb*-closed sets are independent .

Example4-10: let X= {a ,b ,c} with the topology ,T1={X,∅,{ a,b},{,c} } , In this topological space ,the subset A={a,c}is gp -closed set but not gb*-closed set. For the topology T2={X,∅,{ a },{b}, {a,b} } topological ,the subset B={b}is gb*-closed set but not gp-closed set.

Remark 4- 11: The following example shows that the concept of wg-closed and gb*-closed sets are independent .

Example4-12: let X= {a ,b ,c} with the topology ,T1={X,∅,{ a } } , In this topological space ,the subset A={a,b}is wg-closed set but not gb*-closed set. For the topology T2={X,∅,{ a },{c}, {a,c } topological ,the subset B={a}is gb*-closed set but not wg-closed set.

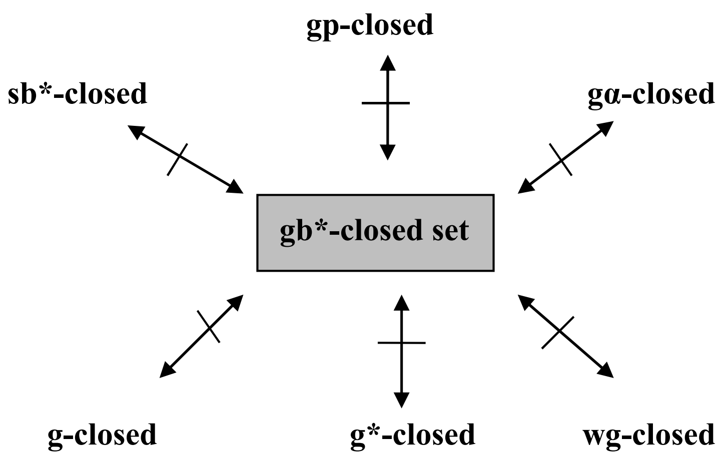

diagram (2)

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حول المجموعات المغلقة بالنمط –gb*

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الخلاصة

يعرض هذا البحث دراسة مفهوم جديد من المجموعات المغلقة يسمى المجموعات المعممة المغلقة gb* في الفضاءات التبولوجية كما نقوم بدراسة بعض الخصائص الأساسية، ودراسة العلاقات بينها وبين المجموعات المغلقة في الفضاء التبولوجي.

الكلمات ألمفتاحيه: المجموعات المعممة المغلقة -b* والمجموعات المعممة المغلقه -b والمجموعات المعممة المغلقة.