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LIQUID-SOLID HEAT TRANSFER WITH THE PHASE-CHANGE OF SOLID

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This paper presents a study of the heat transfer for a singular spherical particle melting in a stagnant liquid phase. The proposed mathematical model allows the calculation of process duration, the radius of melting front and the degree of melting. In order to verify the mathematical model, the degree of melting, the melting rate and the heat transfer coefficient were experimentally determined, using spherical ice particles at 267 K. The melting medium was distilled water at temperatures of 288, 298 and 318 K. The experimental results have been compared with those corresponding to relation: $Nu = 2 + 0.6Ra^{1/4}\,\mathrm{Pr}^{1/3}$. The agreement is good.

Keywords: melting, heat transfer, heat transfer coefficient, melting rate

Introduction

Technical literature provides a large number of papers regarding the heat transfer in liquid – solid systems. Part of these papers [1-9] approach theoretically and experimentally the heat transfers at the melting process by direct contact with a liquid phase of a singular particle or assembly of particles.

The available studies regard equally to heat transfer by natural and forced convection.

Thereby, Woods [2] presents a review about the dissolution and melting of solids in contact with a melted substance.

Jochem and Koerber [3] studied theoretically the heat and mass transfer of ice, melting in sodium chloride solution and glycerin. Using an iterative method based on the Newton algorithm; they solved the differential equations of the melting process.

Fukusako et all [4] approach experimentally the heat transfer by natural convection at the melting of an ice cylinder in 3.5% saline solution. They determined the local heat transfer coefficient for the range of 274.8-292,8 K temperatures.

Okada et all [5] studied experimentally the melting of a fix bed of spherical ice particles using water as melting medium.

Gobin and Bernard [6] treated the metal melting by natural convection and analyzed the influence of Prandtl and Rayleigh numbers.

Other investigations [7,8] approached theoretically and experimentally the heat transfer at the melting of a spherical ice, moving upwards through a column with water. The authors established a mathematical model, which was applied to the determination of temperature distribution, in the inner of the particle, as function of radius and time. Also, they determined experimentally the heat transfer coefficient.

This paper presents a mathematical model allowing the determination of melting front radius, degree of phase change and process duration at the melting of a singular spherical particle in a stagnant liquid phase. The heat transfer coefficient was experimentally determined. The influence of the liquid phase temperature on the rate melting and heat transfer coefficient was also studied. The proposed model was verified.

Mathematical formulation

The melting process of a solid, being in direct contact with a liquid phase, when the particle temperature is different of that of the melting temperature, has two stages:

In the first stage, the heating of the particle takes place until the temperature at the solid-liquid interface becomes equally to the melting temperature. At that moment, the second stage begins, that is, the proper melting. If the initial temperature of the particle is close to the melting temperature value, the duration of the primary stage is short and can be neglected.

In the case of the melting of a spherical particle containing a single A component in a solution

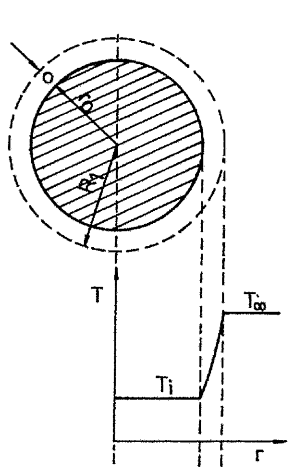

Fig.1 Physical model

containing also A component, the following three elementary processes are involved:

- the heat transfer from the solution to the particle surface
- the proper melting at the solid-liquid interface
- the mass transfer of the A component from the particle surface to the liquid phase.

The physical model is presented in the *Fig.1*. According to the physical model, at the initial moment, the radius of particle is R and decreases as the melted region grows. For a given moment, the radius of the particle is r_0. The temperature is T_i at the solid-liquid interface and T_∞ in the bulk solution.

Since the aim of the mathematical modelling is to establish the equation of process duration or the radius of melting front (degree of transformation) in time, one considers the energy equation for the radial direction:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\lambda}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) \tag{1}$$

The boundary conditions are:

$$t \le 0, \quad R \le r \le R_1, \quad T = T_\infty \tag{2}$$

$$t > 0, \quad r = r_0, \quad T = T_i$$

$$\lambda \frac{\partial T}{\partial r} = -\rho_s \Delta H_m \frac{dr_0}{dt} \tag{3}$$

$$t > 0, \quad r = R_1, \quad T = T_\infty \tag{4}$$

To simplify the solution of the differential equations, when the temperature is constant on an infinitesimal time, one considers the quasi-steady state. Thus, the process can mathematically be described by the equation:

$$\frac{\lambda}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0 \tag{5}$$

or:

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0 \tag{6}$$

By integration of the *Eq.(5)* one obtains:

$$\frac{dT}{dr} = \frac{1}{r^2} C_1 \tag{7}$$

$$T = -\frac{C_1}{r} + C_2 \tag{8}$$

The integration constants, C_1 and C_2, result from the boundary conditions:

$$T_i = -\frac{C_1}{r_0} + C_2 \tag{9}$$

$$T_\infty = -\frac{C_1}{R_1} + C_2 \tag{10}$$

From relations (9) and (10) one obtains:

$$T_i - T_\infty = \Delta T = -C_1\left(\frac{1}{r_0} - \frac{1}{R_1}\right) \tag{11}$$

or:

$$C_1 = -\frac{\Delta T}{\frac{1}{r_0} - \frac{1}{R_1}} \tag{12}$$

Substitution of *Eq.(12)* into *Eq.(10)* gives:

$$C_2 = T_\infty + \frac{\Delta T}{R_1}\left(\frac{1}{r_0} - \frac{1}{R_1}\right)^{-1} \tag{13}$$

Also, substitution of the C_1 constant into (7) gives:

$$\frac{dT}{dr} = -\frac{1}{r^2}\frac{\Delta T}{\frac{1}{r_0} - \frac{1}{R_1}} \tag{14}$$

The boundary condition (3) associated with *Eq.(14)* is:

$$\lambda \frac{1}{r_0^2}\frac{\Delta T}{\frac{1}{r_0} - \frac{1}{R_1}} = -\rho_s \Delta H_m \frac{dr_0}{dt} \tag{15}$$

Since,

$$\frac{1}{r_0} - \frac{1}{R_1} = \frac{\delta}{r_0(r_o + \delta)}$$

and

$$\frac{\lambda}{\delta} = \alpha$$

Eq.(15) becomes:

$$\frac{\alpha \Delta T (r_0 + \delta)}{r_0} = -\rho_s \Delta H_m \frac{dr_0}{dt} \tag{16}$$

The integration of *Eq.(16)*, between 0 - t and R – r_0, leads to:

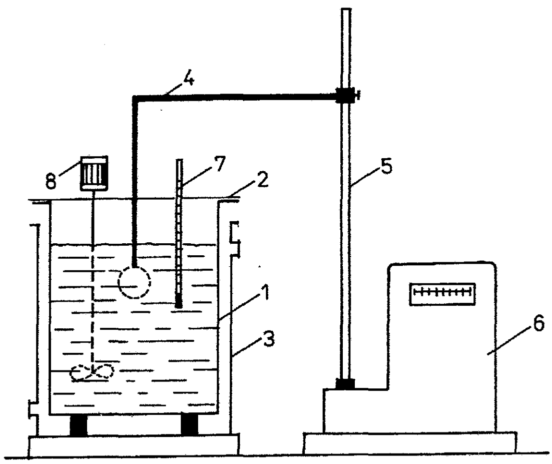

Fig.2 Experimental installation. 1 – cylindrical glass vase, 2 – cover, 3 – glass recipient, 4 – rod, 5 – support, 6 – semiautomatic scales, 7 – control thermometer, 8 – agitator

$$\frac{\alpha \Delta T}{\rho_s \Delta H_m} = (R - r_0) + \delta \ln \frac{R+\delta}{r_0+\delta} \qquad (17)$$

For small values of δ, results:

$$r_0 = R - \frac{\alpha \Delta T}{\rho_s \Delta H_m} t \qquad (18)$$

The radius (the location) of the melting front can be written as function of the degree of melting, η:

$$r_0 = R(1-\eta)^{1/3} \qquad (19)$$

Replacing (19) into (18) gives:

$$t = \frac{R\rho_s \Delta H_m}{\alpha \Delta T}\left[1-(1-\eta)^{1/3}\right] \qquad (20)$$

The relations (18) and (20) allow the calculation of the melting front radius, respectively, the process duration.

Experimental Apparatus and Procedure

According to *Fig.2*, the experimental arrangement consists of a cylindrical glass vase (1) with a cover (2), a glass recipient (3), rod (4), support (5), semiautomatic scales (6), control thermometer (7). The vase (1) represents the melting chamber, where the melting of particle ice takes place. The vase is provided with a dismountable agitator (8) for the temperature homogenization of the melting medium (distilled water). During of the particles melting, the agitator is removed from the melting chamber. A thermostat connected to recipient (3) supplies the necessary thermal energy for the temperature homogenization.

For investigations, the spherical ice particles have been used. The ice particles have been obtained by freezing of distilled water at 267 K, using a special device. The melting medium was distilled water of temperatures of 288, 298, 308 and 318 K.

In the experiments, the mass variations function in time has been determined separately, for every particle.

Table 1 The variation of the particle mass with time

T=288 K		T=298 K		T=308 K		T=318 K	
t (s)	$10^3\Delta m$ (kg)	t (s)	$10^3\Delta m$ (kg)	t (s)	$10^3\Delta m$ (kg)	t (s)	$10^3\Delta m$ (kg)
30	4.5	30	6.0	20	6.9	15	7.6
60	7.5	60	10.8	40	12.4	30	13.6
90	9.4	90	17.2	60	17.5	45	19.3
120	12.0	120	20.3	80	21.7	60	24.0
150	15.1	150	25.7	100	27.5	75	28.8

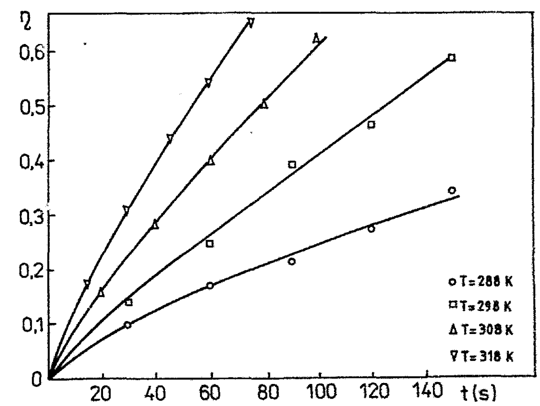

Fig.3 Variation of the degree of melting with time

Results and Discussion

Experimental results for the mass variation of the ice particle in time are presented in *Table 1.* These results have been used for the calculation of the melting level, η, with the following relation:

$$\eta = \frac{\Delta m}{m_0} \qquad (21)$$

Graphically, the η values are shown in *Fig.3.* As may be seen the degree of melting is increasing with the temperature increase. The medium melting temperature has a positive influence on the melting process:

In the following, using the above data and relations:

$$v_m = \frac{m_0 \eta}{S \Delta t} \qquad (22)$$

$$q = \alpha(T_\infty - T_i) \qquad (23)$$

the melting rate (v_m) and the heat transfer coefficient (α) have been determined. The melting rate has been calculated taking into account the degree of melting

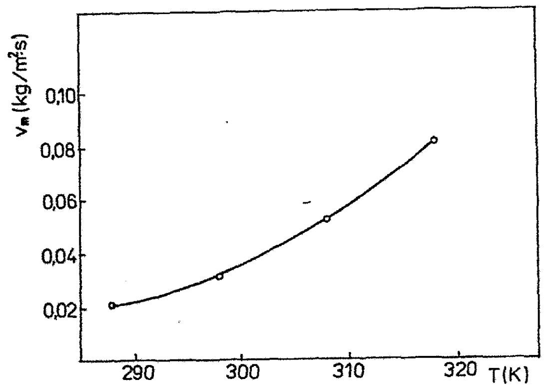

Fig.4 Temperature influence on melting rate

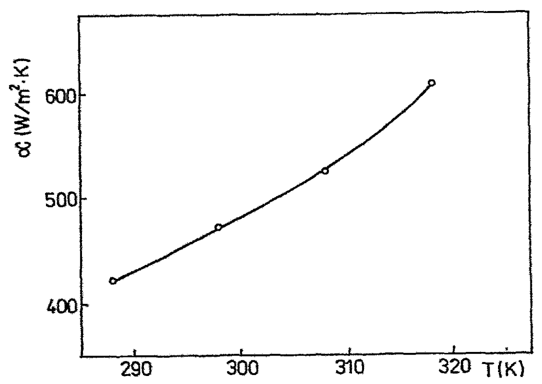

Fig.5 Temperature influence on heat transfer coefficient

corresponding to a 60-sec. duration. The diagram in *Fig.4* shows an increase of the melting rate with the temperature. Also, the water temperature has a positive effect on the heat transfer coefficient (*Fig.5*). The temperature increase amplifies the natural circulation of water around the particle ice. Consequently, the transferred heat flux from water to ice particle will increase and will amplify the heat transfer coefficient and the melting rate.

The proposed mathematical model represented by *Eq.(20)* is verifying. Based on the values of the heat transfer coefficient, using the relation (20), the process duration values have been calculated. For the degree of melting, the values corresponding to the 60-sec. duration have been considered. The obtained results are presented in *Table 2*. According to this table, one may see that the calculated values of the process duration are close to the experimental values (60 sec.). Therefore, one may assert that experimental data verify the proposed mathematical model.

Further on, the experimental data obtained in this study agree with the other authors data. To this purpose, one considers the dimensionless equation [1]:

$$Nu = 2 + 0.6Ra^{1/4}\,\mathrm{Pr}^{1/3} \qquad (24)$$

The experimental values of Nusselt number have been determined for more values of Rayleigh and

Table 2 Verifying of the mathematical model

Temperature (K)	288	298	308	318
Degree of melting	0.172	0.247	0.400	0.549
Experimental time (s)	60	60	60	60
Calculated time (s)	59.61	59.08	58.72	58.93

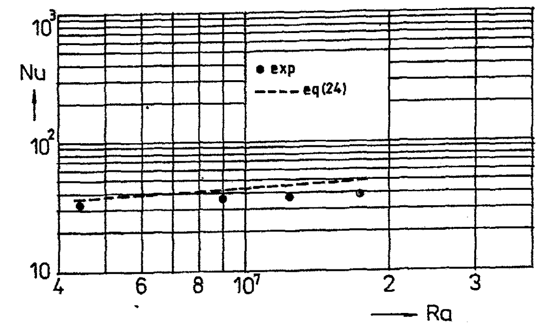

Fig.6 The dependence of the experimental Nusselt numbers with the Rayleigh numbers

Prandtl numbers. These values are graphically represented in *Fig.6*. This figure contains also the calculated values, using *Eq.(24)*, for Nusselt number. Data from *Fig. 6* indicate small differences between experimental data obtained by our study and those obtained using *Eq.(24)*.

Conclusions

In this paper a mathematical model of the heat transfer at melting of a spherical particle by direct contact with a stagnant liquid phase has been established. The model allows determination of the process duration, the melting front radius and the degree of melting.

The proposed mathematical model has been experimentally verified. For this purpose, the experimental values of the melting rate and the heat transfer coefficient have been determined. The experiments were carried out in a laboratory installation, using spherical ice particles and distilled water at different temperatures, as melting medium.

The comparison of the calculated values, based on the proposed model, with those obtained experimentally were in good agreement.

The experimental data obtained in this work were also verified with those of other authors. There can be noted a good agreement.

Based on experimental data, the influence of the melting medium temperature on the melting rate and on the heat transfer coefficient was studied.

SYMBOLS

c_p — specific heat, $\mathrm{Jkg^{-1}K^{-1}}$
d — particle diameter, m
g — gravitation acceleration, $\mathrm{ms^{-2}}$

m_0	particle mass at time t=0, kg
$q=v_m \Delta H_m$	specific heat flux, Wm^{-2}
r	radial coordinate, m
S	external surface area of particle, m^2
t	time, s
T_i	temperature at solid – liquid interface, K
T_∞	temperature in the bulk solution, K
v_m	melting rate, kg m^{-2} s^{-1}
ΔH_m	latent heat of melting, J kg^{-1}
Δm	variation of particle mass, kg
Ra = Gr Pr	Rayleigh number
$\mathrm{Re} = \dfrac{\rho v d}{\mu}$	Reynolds number
α	heat transfer coefficient, W m^{-2} K^{-1}
β	thermal expansion coefficient, K^{-1}
λ	heat conductivity, $Wm^{-1}K^{-1}$
η	degree of melting
μ	liquid viscosity, Nsm^{-2}
ρ	density of liquid, kg m^{-3}
ρ_s	density of particle, kg m^{-3}

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