HUNGARIAN JOURNAL
OF INDUSTRIAL CHEMISTRY
VESZPRÉM
Vol. 39(2) pp. 173-176 (2011)

CONNECTION THEORY OF CONICAL WORM GEAR DRIVES

S. BODZÁS[1], I. DUDÁS[2]

[1,2]College of Nyíregyháza, Department of Technical Preparatory and Production Engineering
H-4400 Nyíregyháza, Sóstói u. 9-11., HUNGARY
[1,2]University of Miskolc, Department of Production Engineering, H-3515 Miskolc, Egyetemváros, HUNGARY
E-mail: bodzassandor@nyf.hu
E-mail: illes.dudas@uni-miskolc.hu

We worked out a mathematical modell for the production geometry and mathematical analysis of spiroid worm gear drives. This modell is adapted for analysis of spiroid worm gear drives with random profile. Using this modell it could be possible for defining of the equations of cog surfaces, the surface normal vector, contact curves and connection surface in one concrete case.

Keywords: spiroid, transformation matrix, normal vector

Introduction

In technical practice conical worm surfaces, which can be used in many ways, are most widely applied as a function surface of conical worms. The conical worm – crown wheel pairs spiroid drive, can be used for example as jointless drives of robots and tool machines [1].

The jointless drives are attained by simply shifting (setting) the worm in an axial direction. The cog surface of the conical worm of the spiroid drives *(Fig. 1)* can be attained the same way as that of the cylindrical worm, but besides the axial shift of the hob, a tangential shift must be done depending on the conicity of the worm. Different – evolvent, Archimedean and convolute – helical surfaces can be defined in case of spiroid worm surface similar to the line surface cylindrical worm.

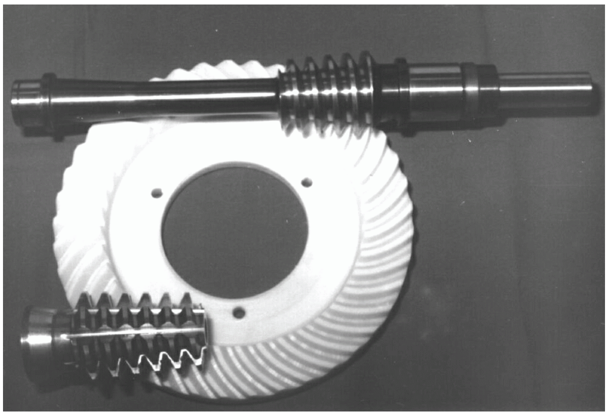

Figure 1: Spiroid worm gear drive

The dentation of crown wheel is produced with hob which tiler surface is similar to conical worm surface. This is called direct motion mapping.

With these modern drive pairs, which are characterized by favourable hidrodinamic conditions, great strength and high efficiency, the energy loss in the gear can be reduced significantly [1].

In power dissipation it is important to apply those cog geometrical characteristics which result in good connection terms.

Defining of the spatial coordination systems

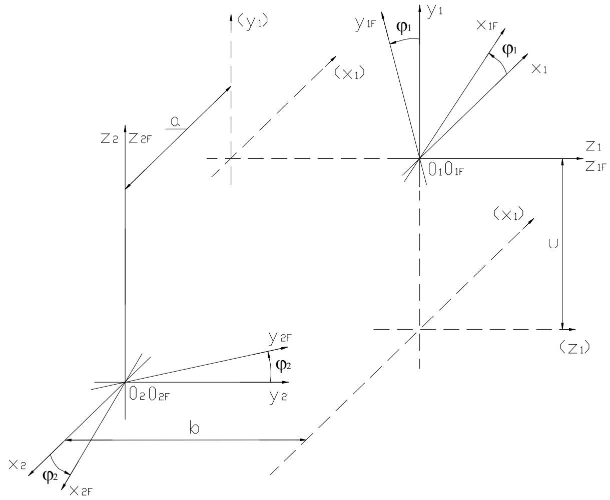

Figure 2: Evading rotation axis coordinate system for defining of cog surfaces

We worked out this model based on the General mathematical model of Dr. Illés Dudás [1, 2]. Defining of minimum four coordinate systems are needed for analysing of motion transmission between evading axis and defining of cog surface describing spatial coordinates: two fixed rotation coordinate systems for the first part K_{1F} (x_{1F}, y_{1F}, z_{1F}) and the second part K_{2F} (x_{2F}, y_{2F}, z_{2F}) and two standing coordinate systems for the first part K_1 (x_1, y_1, z_1) and the second part K_2 (x_2, y_2, z_2), where the positions of the rotating coordinate systems can be defined *(Fig. 2)*.

The rotation axis of the elements are z_1 and z_2, the turning direction is positive watching from the directions of the axis (opposite for the clock working), the turning angles, the motion parameters are φ_1 and φ_2.

The transformation matrixes between the rotation coordinate system K_{1F} (x_{1F}, y_{1F}, z_{1F}) for the first part and the standing coordinate system K_1 (x_1, y_1, z_1) for the first part are:

$$M_{1,1F} = \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$M_{1F,1} = \begin{bmatrix} \cos\varphi_1 & \sin\varphi_1 & 0 & 0 \\ -\sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The transformation matrixes between the standing coordinate system K_1 (x_1, y_1, z_1) for the first part and the standing coordinate system K_2 (x_2, y_2, z_2) for the second part are:

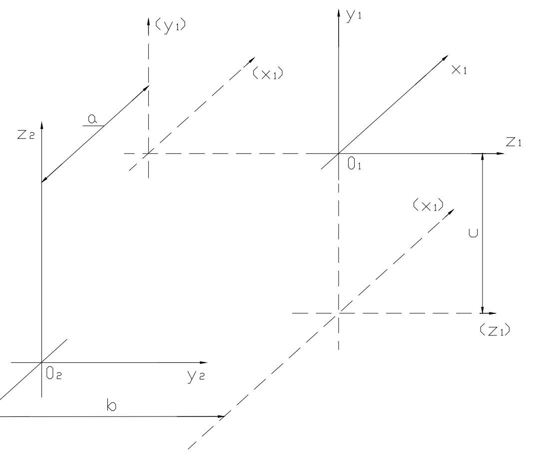

Figure 3: Connection between the K_1 (x_1, y_1, z_1) and the K_2 (x_2, y_2, z_2) standing coordinate systems

$$M_{2,1} = \begin{bmatrix} -1 & 0 & 0 & -a \\ 0 & 0 & 1 & b \\ 0 & 1 & 0 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$M_{1,2} = \begin{bmatrix} -1 & 0 & 0 & -a \\ 0 & 0 & 1 & -c \\ 0 & 1 & 0 & -b \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The transformation matrixes between the standing coordinate system K_2 (x_2, y_2, z_2) for the second part and the rotation coordinate system K_{2F} (x_{2F}, y_{2F}, z_{2F}) for the second part are:

$$M_{2F,2} = \begin{bmatrix} \cos\varphi_2 & \sin\varphi_2 & 0 & 0 \\ -\sin\varphi_2 & \cos\varphi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$M_{2,2F} = \begin{bmatrix} \cos\varphi_2 & -\sin\varphi_2 & 0 & 0 \\ \sin\varphi_2 & \cos\varphi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

The transformation matrixes between the rotation coordinate system K_{1F} (x_{1F}, y_{1F}, z_{1F}) for the first part and the rotation coordinate system K_{2F} (x_{2F}, y_{2F}, z_{2F}) for the second part are:

$$M_{2F,1F} = M_{2F,2} \cdot M_{2,1} \cdot M_{1,1F} =$$

$$= \begin{bmatrix} -\cos\varphi_2 \cdot \cos\varphi_1 & \cos\varphi_2 \cdot \sin\varphi_1 & \sin\varphi_2 & -a \cdot \cos\varphi_2 + b \cdot \sin\varphi_2 \\ \sin\varphi_2 \cdot \cos\varphi_1 & -\sin\varphi_1 \cdot \sin\varphi_2 & \cos\varphi_2 & a \cdot \sin\varphi_2 + b \cdot \cos\varphi_2 \\ \sin\varphi_1 & \cos\varphi_1 & 0 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$M_{1F,2F} = M_{1F,1} \cdot M_{1,2} \cdot M_{2,2F} =$$

$$= \begin{bmatrix} -\cos\varphi_2 \cdot \cos\varphi_1 & \cos\varphi_1 \cdot \sin\varphi_2 & \sin\varphi_1 & -a \cdot \cos\varphi_1 - c \cdot \sin\varphi_1 \\ \cos\varphi_2 \cdot \sin\varphi_1 & -\sin\varphi_2 \cdot \sin\varphi_1 & \cos\varphi_1 & a \cdot \sin\varphi_1 - c \cdot \cos\varphi_1 \\ \sin\varphi_2 & \cos\varphi_2 & 0 & -b \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

The equations of conical thread surface

The $\vec{r}_g$ leading curve is given in the K_0 (, ,) tool coordinate system and its equation of the η coordinate function. That is:

$$\vec{r}_g = \vec{r}_g(\eta) \quad (9)$$

Since we consider the coordinate an independent variable, the equation of the leading curve is:

$$\vec{r}_g = \xi(\eta) \cdot \vec{i} + \eta \cdot \vec{j} + \zeta(\eta) \cdot \vec{k} \quad (10)$$

Carrying out a p_a axial and p_r radial helical motion of the K_0 (, ,) coordinate system – which includes the $\vec{r}_g$ leading curve – along the z axis and the y axis alternatively includes, the leading curve touches a conical helical surface in the K_{1F} (x_{1F}, y_{1F}, z_{1F}) an independent position and equals K_0 coordinate system before the helical motion *(Fig. 4)*.

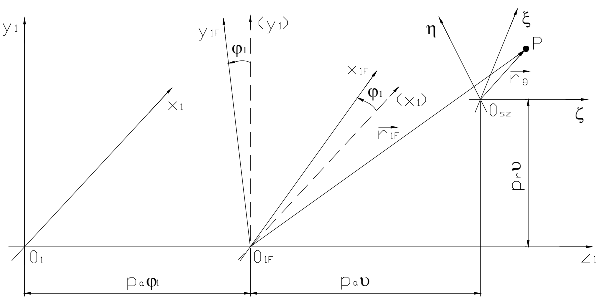

Figure 4: Touched thread surface by leading curve

The helical surface touched by $\vec{r}_g$ curve in the K_{1F} (x_{1F}, y_{1F}, z_{1F}) coordinate system is:

$$\vec{r}_{1F} = M_{1F,0} \cdot \vec{r}_g \tag{11}$$

The transformation matrix between the two coordinate systems is:

$$M_{1F,0} = \begin{bmatrix} \cos\vartheta & -\sin\vartheta & 0 & 0 \\ \sin\vartheta & \cos\vartheta & 0 & p_r \cdot \vartheta \\ 0 & 0 & 1 & p_a \cdot \vartheta \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

That is:

$$\left.\begin{aligned} x_{1F} &= \xi(\eta)\cdot\cos\vartheta - \eta\cdot\sin\vartheta \\ y_{1F} &= \xi(\eta)\cdot\sin\vartheta + \eta\cdot\cos\vartheta + p_r\cdot\vartheta \\ z_{1F} &= \zeta(\eta) + p_a\cdot\vartheta \end{aligned}\right\} \tag{13}$$

We gave the equations of the helical surface in the K_{1F} (x_{1F}, y_{1F}, z_{1F}) rotation coordinate system (13).

The given describing thread surface $\vec{r}_{1F} = \vec{r}_{1F}(\eta,\vartheta)$ two parameters vector – scalar function can be transformed from the K_{1F} coordinate system to the K_{2F} coordinate system:

$$\left.\begin{aligned} \vec{r}_{2F} &= M_{2F,1F}\cdot\vec{r}_{1F} \\ \varphi_2 &= i_{21}\cdot\varphi_1 \end{aligned}\right\} \tag{14}$$

Direct case

The $\vec{r}_{1F} = \vec{r}_{1F}(\eta,\vartheta)$ is given, the two parametric vector-scalar function in the coordinate system K_{1F} (x_{1F}, y_{1F}, z_{1F}) for the surface to be generated.

In the difference geometry for the independence of the η and θ parameters are essential condition:

$$\frac{\partial\vec{r}_{1F}}{\partial\eta}\times\frac{\partial\vec{r}_{1F}}{\partial\vartheta} \neq 0 \tag{15}$$

The η and θ parameters are the curve line coordinates of the surface.

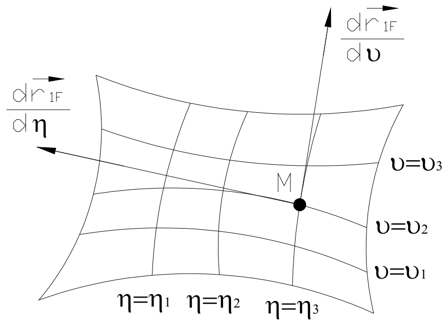

Figure 5: Definition of the surface normal vector

We suppose the surface is continuous on the working parts of cogs, it is continuous function of η and θ parameters; two coordinate lines have to be crossed throught on every M point of the surface: a) η = const, b) θ = const the tangents of this lines do not coincide in this point. The working parts of the cog surfaces could contain only general points.

The normal vector belong to the surface and the tangent plane will be decided only in the general point.

The plane defined by the tangents of the $\frac{\partial\vec{r}_{1F}}{\partial\eta}$ and $\frac{\partial\vec{r}_{1F}}{\partial\vartheta}$ parameter lines is the tangent plane of the surface in the given point. The surface normal vector $\vec{n}_{1F}$ is perpendicular for the tangent plane and it can be defined:

$$\vec{n}_{1F} = \frac{\partial\vec{r}_{1F}}{\partial\eta}\times\frac{\partial\vec{r}_{1F}}{\partial\vartheta} \tag{16}$$

The normal vector in the K_{1F} coordinate system is:

$$\vec{n}_{1F} = \frac{\partial\vec{r}_{1F}}{\partial\eta}\times\frac{\partial\vec{r}_{1F}}{\partial\vartheta} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial x_{1F}}{\partial\eta} & \frac{\partial y_{1F}}{\partial\eta} & \frac{\partial z_{1F}}{\partial\eta} \\ \frac{\partial x_{1F}}{\partial\vartheta} & \frac{\partial y_{1F}}{\partial\vartheta} & \frac{\partial z_{1F}}{\partial\vartheta} \end{vmatrix} \tag{17}$$

The relative velocity of the two surfaces can be determined in coordinate system K_{2F} using the transformation between K_{1F} (x_{1F}, y_{1F}, z_{1F}) coordinate system for worm and the K_{2F} (x_{2F}, y_{2F}, z_{2F}) coordinate system for worm gear:

$$\vec{v}_{2F}^{(12)} = \frac{d}{dt}\cdot\vec{r}_{2F} = \frac{d}{dt}\left(M_{2F,1F}\right)\cdot\vec{r}_{1F} \tag{18}$$

The vector $\vec{v}_{2F}^{(12)}$ should be transformed into coordinate system K_{1F} (x_{1F}, y_{1F}, z_{1F}) to determine the necessary connection surface, so:

$$\vec{v}_{1F}^{(12)} = M_{1F,2F}\cdot\vec{v}_{2F}^{(12)} = M_{1F,2F}\cdot\frac{d}{dt}\left(M_{2F,1F}\right)\cdot\vec{r}_{1F} = P_1\cdot\vec{r}_{1F} \tag{19}$$

where:

$$P_1 = M_{1F,2F}\cdot\frac{d}{dt}\left(M_{2F,1F}\right) \tag{20}$$

the matrix for kinematic generation.

$$P_1 = \begin{bmatrix} 0 & -1 & -i\cdot\cos\varphi_1 & -b\cdot i\cdot\cos\varphi_1 \\ 1 & 0 & i\cdot\sin\varphi_1 & b\cdot i\cdot\sin\varphi_1 \\ i\cdot\cos\varphi_1 & -i\cdot\sin\varphi_1 & 0 & a\cdot i \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

On the connecting cog surfaces of elements, as on tiler each other surfaces the contact curve can be defined by the concomitant solving of expression of Connection I. statement

$$\vec{n}_{1F}\cdot\vec{v}_{1F}^{(12)} = \vec{n}_{2F}\cdot\vec{v}_{2F}^{(12)} = \vec{n}\cdot\vec{v}^{(12)} = 0 \quad (22)$$

contact equation and describing cog surfaces vector – scalar function.

The connection equation expresses the correlation between the η and θ surface parameters and the φ_1 motion parameter, that is

$$F_{1F}(\eta,\vartheta,\varphi_1) = 0 \quad (23)$$

Defining of the contact curves of the $\sum_1$ and $\sum_2$ cog surfaces in the K_{1F} coordinate system is:

$$\left.\begin{aligned} F_{1F}(\eta,\vartheta,\varphi_1) &= 0 \\ \vec{r}_{1F} &= \vec{r}_{1F}(\eta,\vartheta) \end{aligned}\right\} \quad (24)$$

The tiler surfaces of contact curves forming the equations of cog surface of the second part are in the K_{2F} coordinate system:

$$\left.\begin{aligned} F_{1F}(\eta,\vartheta,\varphi_1) &= 0 \\ \vec{r}_{1F} &= \vec{r}_{1F}(\eta,\vartheta) \\ \vec{r}_{2F} &= M_{2F,1F}\cdot\vec{r}_{1F} \end{aligned}\right\} \quad (25)$$

The connection equation is in the K_{1F} coordination system:

$$\vec{n}_{1F}\cdot\vec{v}_{1F}^{(12)} = 0 \quad (26)$$

The relative velocity is:

$$\vec{v}_{1F}^{(12)} = P_1\cdot\vec{r}_{1F} = \\ = \begin{bmatrix} -y_{1F} - z_{1F}\cdot i\cdot\cos\varphi_1 - b\cdot i\cdot\cos\varphi_1 \\ x_{1F} + z_{1F}\cdot i\cdot\sin\varphi_1 + b\cdot i\cdot\sin\varphi_1 \\ x_{1F}\cdot i\cdot\cos\varphi_1 - y_{1F}\cdot i\cdot\sin\varphi_1 + a\cdot i \\ 0 \end{bmatrix} \quad (27)$$

with which the connection equation in the K_{1F} coordinate system is:

$$\begin{aligned} & n_{1Fx}\cdot(-y_{1F} - z_{1F}\cdot i\cdot\cos\varphi_1 - b\cdot i\cdot\cos\varphi_1) + \\ & n_{1Fy}\cdot(x_{1F} + z_{1F}\cdot i\cdot\sin\varphi_1 + b\cdot i\cdot\sin\varphi_1) + \\ & n_{1Fz}\cdot(x_{1F}\cdot i\cdot\cos\varphi_1 - y_{1F}\cdot i\cdot\sin\varphi_1 + a\cdot i) = 0 \end{aligned} \quad (28)$$

The application of the model

We designed a conical worm of which we carried out the virtual model and using the mathematical model *(Fig. 6)* we carried out the virtual model of the connecting worm gear *(Fig. 7)*.

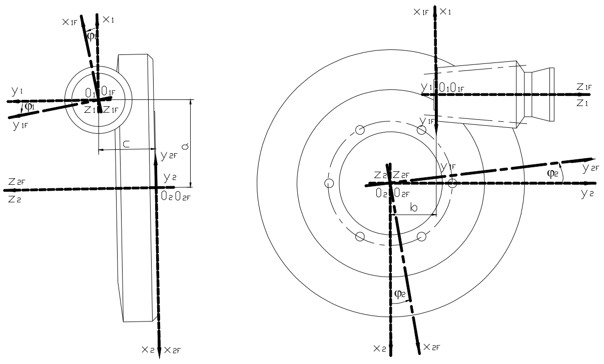

Figure 6: Defining of the coordinate systems

Figure 7: Our designed worm gear drive model

Summary

We worked out a mathematical model for production geometry and mathematical analysis of spiroid worm. This modell is appropriate for every spiroid worm gear drives with random profile.

We designed a spiroid worm gear drive and using this model we carried out the virtual model of this drive pair.

REFERENCES

1. I. DUDÁS: The Theory and Practice of Worm Gear Drives. Penton Press, London, 2000. (ISBN 1 8571 8027 5)
2. I. DUDÁS: Gépgyártástechnológia III., Miskolci Egyetemi Kiadó 2005. (ISBN 963 661 572 1)
3. H. JÓZSEF: Untersuchugen zur Anwendung von Spiroidgetrieben. Diss. A. TU. Desden, 1988