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SENSORLESS ROTOR POSITION DETECTION OF PMSM FOR AUTOMOTIVE APPLICATION

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For correct operation of the permanent magnet synchronous motors is essential the knowledge of the rotor position, which is usually given by Hall-sensors, encoders or resolvers. The aim of the present work is to set up such a general motor model, which takes into consideration the anisotropies in the magnetic structure of the motor.

Keywords: rotor position estimation, PMSM motor model, model validation, sensorless control methods

Introduction

The electrical drives can be found in every field of the modern life from the cooling fans in computers to the most modern electric tractions. Most of the cases where a variable-speed operation was required, DC motors were used extensively, since the flux and torque can be controlled easily by the field and armature current. However, the main problem of the conventional DC motors are the commutators and the brushes. The commutators have limited capability under high-speed operation.

A good alternative to the conventional DC motors is the permanent magnet synchronous motor (PMSM), which are widely used in industrial applications due to their high efficiency, high torque-to-inertia ratio, high reliability, low maintenance and longer lifetime [1]. Position sensors are placed in the motor for accurate control. For position sensing Hall-effect sensors, different encoder types or resolvers are used. The construction of permanent magnet synchronous motors yields the advantages mentioned above but they do not have only bright side. The position sensors have many drawbacks: they increase cost and size of the motor, the operating temperature range is limited and need some external electronic circuits to transmit the correct position. Thus in space restricted applications, in cost effective applications and in hostile environments the necessity of sensorless operation is brought up [1, 3, 6].

In this paper *slotless* motors will be analyzed, which are high power small size servo motors. These PMSMs cannot be ranked in any conventional group because the rotor is one *block magnet*. The other special characteristic is the absence of the iron core in stator windings, which results *ironless* stator windings. This is not a common motor type, so far sensorless control of slotless PMSMs is not discussed in literature.

Modelling the PMSMs

Electric drives are complex electromechanical systems. Their power supply and their control are usually electrical and their output is usually mechanical. Therefore the modeling usually is done only for the electrical and mechanical phenomena. But if the magnetic structure of the motor is important (anisotropies, saturation, hysteresis and the combination of them is not negligible) then it should be taken into consideration thoroughly. Our sensorless method is based on these magnetic phenomena, so their analysis is very important from both theoretical and experimental side.

Geometrical Model

Usually more than one coordinate systems are used to describe the motor geometry. One of these systems is fixed to the stator $(\alpha-\beta)$ and the other to the rotor $(d-q)$. Their origins are the same. The angular position of the $(d-q)$ system in the $(\alpha-\beta)$ system is defined by the angle θ.

The modeled motors have 3 phase windings. The phases and their parameters usually are indexed with a, b and c. By fixing an axis to each phase can be created the 3 phase or natural $(a-b-c)$ coordinate system of the motor. This system is fixed to the stator as the $(\alpha-\beta)$. It is comfortable to define the $(\alpha-\beta)$ in a way which grants the equality of the axes α and a.

Electrical Model

The voltage equations of the phases (the voltage reference point is the start point):

$$U_i = \frac{d\Phi_i}{dt} + R_i i_i \tag{1}$$

The flux in Phase *i* can be dissolved to flux components that are produced by the windings and the permanent magnet. For Phase *a*:

$$\Phi_a = \Phi_{aa} + \Phi_{ab} + \Phi_{ac} + \Phi_{Ma} \tag{2}$$

In the above equation Φ_{ij} represents the flux component in Phase *i* produced by Phase *j*, and Φ_{Mi} represents the flux component in Phase *i* produced by the permanent magnet. The voltage equation of Phase *a*:

$$U_a = \frac{d\Phi_{aa}+\Phi_{ab}+\Phi_{ac}}{dt} + \frac{d\Phi_{Ma}}{dt} + R_a i_a \tag{3}$$

The flux derivatives can be dissolved into products of inductances and current derivatives:

$$\frac{d\Phi_{aa}+\Phi_{ab}+\Phi_{ac}}{dt} = \frac{d\Phi_{aa}+\Phi_{ab}+\Phi_{ac}}{di_a}\frac{di_a}{dt} = \\ =(L_{aa} + L_{ab} + +L_{ac})\frac{di_a}{dt} = L_a\frac{di_a}{dt} \tag{4}$$

The change of the phase flux component induces the following voltage component, which can be considered proportional to the actual rotor speed:

$$E = \frac{d\Phi_{Ma}}{dt} \cong k_{E,a}\omega_m \tag{5}$$

Voltage equations for the phases:

$$U_a = L_a\frac{di_a}{dt} + k_{E,a}\omega_m + Ri_a \tag{6}$$

$$U_b = L_b\frac{di_b}{dt} + k_{E,b}\omega_m + Ri_b \tag{7}$$

$$U_c = L_c\frac{di_c}{dt} + k_{E,c}\omega_m + Ri_c \tag{8}$$

The flux equation, which describes the relation between the phase fluxes and phase currents:

$$\begin{bmatrix}\Phi_a\\ \Phi_b\\ \Phi_c\end{bmatrix} = \frac{1}{N}\begin{bmatrix}L_{aa} & L_{ab} & L_{ac}\\ L_{ba} & L_{bb} & L_{bc}\\ L_{ca} & L_{cb} & L_{cc}\end{bmatrix}\begin{bmatrix}i_a\\ i_b\\ i_c\end{bmatrix} + \begin{bmatrix}\Phi_{Ma}\\ \Phi_{Mb}\\ \Phi_{Mc}\end{bmatrix} \tag{9}$$

$$\mathbf{\Phi} = \frac{1}{N}[L]\mathbf{i} + \mathbf{\Phi}_M \tag{10}$$

In the development of mathematical background of the sensorless method it is an important task to determine the [*L*] inductance matrix and its dependencies on the other physical quantities, especially the rotor position.

In the magnetic modeling one of the main goals will be to write the inductance matrix into a form which uses the magnetic properties of the motor.

The inductance matrix is also important because it is the mathematical object which describes the relationship between the electric and magnetic parts of the model.

Mechanical Model

The torque equation of the motor:

$$\Theta_r\frac{d\omega_m}{dt} = M_m - M_T - k_S\omega_m \tag{11}$$

In the above equation Θ_r is the inertia of the rotor, M_m is the motor torque produced by the phase windings, and the M_T is the load torque. The $k_S\omega_m$ product is the torque produced by the friction forces.

The M_m motor torque can be computed from the equality of the incoming electrical energy and the outgoing mechanical energy ($P_{electr} = P_{mech}$):

$$P_{mech} = M_m\omega_m \tag{12}$$

$$P_{electr} = \sum_{i=a}^{c}[U_i i_i - R_i i_i^2] = \sum_{i=a}^{c}E_i i_i \tag{13}$$

Solving the above equation system yields:

$$M_m = \frac{\sum_{i=a}^{c}E_i i_i}{\omega_m} \tag{14}$$

Magnetic Model

The most important physical phenomena were the followings during the mathematical modeling of the motor's magnetic structure: *Saturation:* the rotor permanent magnets reluctance is flux dependent. *Hysteresis:* permanent magnets' B–H curve is a hysteresis loop. *Anisotropies:* the rotor permanent magnets reluctance and probably the saturation and the hysteresis are anisotropic quantities.

An other phenomenon, that should be considered is the eddy current on the surface of the permanent magnet. For geometrical reason the motor model should be divided into two parts:

- stator and air gap model
- rotor model.

Stator contains the air core windings, the air gap between the windings and the rotor, and the external iron cylinder. The air gap and the air cores can be discussed together as a simple reluctance.

The external iron cylinder, the air cores and the air gap can be modeled as constant reluctances. The stator windings are magnetomotive forces in the magnetic circuit.

The rotor is a permanent magnet, which is a cylindriform object.

The permanent magnet can be modeled as a magnetic circuit that contains reluctances and magnetomotive forces. First, is practical to model separately the reluctances and the magnetomotive forces to create a reluctance model and a magnetomotive model, and then connect them.

Supposing the rotor is perfectly cylindriform, the magnetic field of the rotor permanent magnet is symmetrical to the plane defined by the d axis and motor shaft. It is assumed that inside the magnet there is no leakage flux [2].

Reluctance Model

Hypothesis: The reluctance between two circumferential points opposite each other depends on the point pair and the flux flowing from one to the other. With other words, using the symbols of *Fig. 1*, the reluctance R_{AB} depends on the angle φ_1 and R_{CD} depends on φ_3 and both reluctances depend on the actual Φ flux flowing through the permanent-magnet.

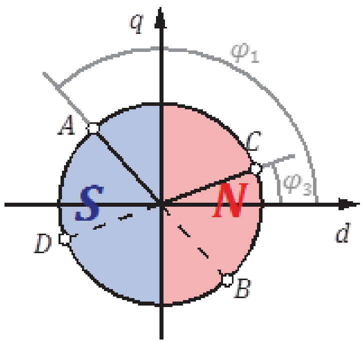

Figure 1: Reluctance between two surface points of a permanent magnet cylinder

So any φ angle and Φ flux defines a $R(\varphi, \Phi)$ “passing through” reluctance value, which can be dissolved into two reluctance components:

$$\mathcal{R}(\varphi_1, \Phi) = \mathcal{R}_M(\varphi_1, \Phi_1) + \mathcal{R}_M(\varphi_1 + +180°, -\Phi_1) \tag{15}$$

Where $R_M(\varphi, \Phi)$ is the rotor magnet’s “radial” reluctance function, which defines the “radial” reluctance between the center point and the surface point in the direction of the φ angle, when the flux flowing through is Φ.

The reluctance between non-opposite surface point can be dissolved to two “radial” reluctances. For example, the reluctance between the points A and C, when Φ flux flows through from point A to point C, is computed in the following way using the $R_M(\varphi, \Phi)$ function:

$$\mathcal{R}_{AC} = \mathcal{R}_M(\varphi_1, \Phi) + \mathcal{R}_M(\varphi_3, -\Phi) \tag{16}$$

The above method has some error, which is greater when the difference of the two angle (in the above example the difference of φ_1 and φ_3) is smaller. If the difference is 120° (between the center lines of the phase windings) the error of the method is acceptable.

The rotor can be modeled as reluctances connected in star using the $R_M(\varphi, \Phi)$ function, see *Fig. 2*.

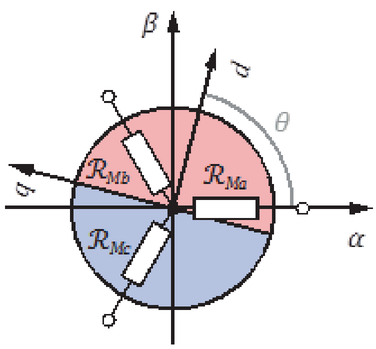

Figure 2: Reluctance circuit of the permanent magnet rotor. (In the $\alpha\beta$ coordinate system)

The phase reluctances and their pins are fixed in the standing coordinate system. The value of the phase reluctances can change because the rotor can rotate.

Describing the phase reluctances with the $R_M(\varphi, \Phi)$ function yields:

$$\mathcal{R}_{Mi}(\theta) = \mathcal{R}_M\left(-\theta + i\frac{2\pi}{3}, \Phi_i\right) \tag{17}$$

Complete Magnetic Circuit

The motor’s complete magnetic circuit with the inductances and phase windings is shown on *Fig. 3:*

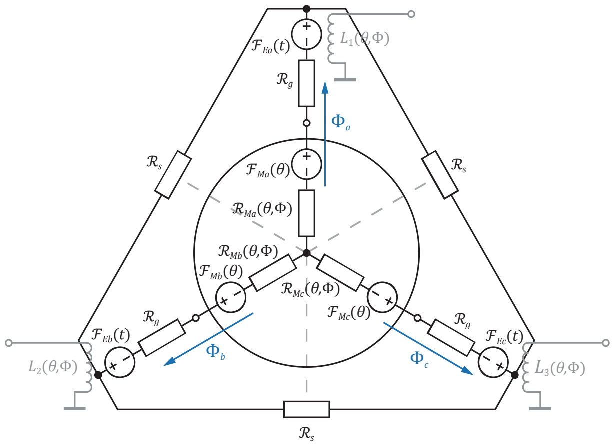

Figure 3: The magnetic circuit of the motor

The merged phase reluctances:

$$\mathcal{R}_i(\theta, \Phi_i) = \frac{\mathcal{R}_s}{3} + \mathcal{R}_g + \mathcal{R}_{Mi}(\theta, \Phi_i) \tag{18}$$

Using the above reluctance components the inductance matrix can be determined.

Flux Equations

The Φ_{aa} flux component, which is produced by the F_{Ea} magnetomotive force in Phase *a*:

$$\Phi_{aa} = \frac{\mathcal{F}_{Ea}}{\sum \mathcal{R}_a} = \frac{N i_a}{\sum \mathcal{R}_a} \tag{19}$$

The Φ_{ab} flux component, which is produced by the F_{Eb} magnetomotive force of Phase *b* in Phase *a*:

$$\Phi_{ab} = \frac{\mathcal{F}_{Eb}}{\sum \mathcal{R}_b}\frac{\mathcal{R}_c}{\mathcal{R}_a + \mathcal{R}_c} = \frac{N i_b}{\sum \mathcal{R}_b}\frac{\mathcal{R}_c}{\mathcal{R}_a + \mathcal{R}_c}\Phi_{aa} = \frac{\mathcal{F}_{Ea}}{\sum \mathcal{R}_a} = \frac{N i_a}{\sum \mathcal{R}_a} \tag{20}$$

Based on the above equations, flux equations can be written for the other two phases and can be written together in a vectorial form that contains the inductance matrix:

$$\begin{bmatrix}\Phi_a\\ \Phi_b\\ \Phi_c\end{bmatrix} = \begin{bmatrix} \frac{N}{\sum \mathcal{R}_a} & \frac{N}{\sum \mathcal{R}_b}\frac{\mathcal{R}_c}{\mathcal{R}_a + \mathcal{R}_c} & \frac{N}{\sum \mathcal{R}_c}\frac{\mathcal{R}_b}{\mathcal{R}_a + \mathcal{R}_b} \\ \frac{N}{\sum \mathcal{R}_a}\frac{\mathcal{R}_c}{\mathcal{R}_b + \mathcal{R}_c} & \frac{N}{\sum \mathcal{R}_b} & \frac{N}{\sum \mathcal{R}_c}\frac{\mathcal{R}_a}{\mathcal{R}_a + \mathcal{R}_b} \\ \frac{N}{\sum \mathcal{R}_a}\frac{\mathcal{R}_b}{\mathcal{R}_b + \mathcal{R}_c} & \frac{N}{\sum \mathcal{R}_b}\frac{\mathcal{R}_a}{\mathcal{R}_a + \mathcal{R}_c} & \frac{N}{\sum \mathcal{R}_c} \end{bmatrix} \begin{bmatrix} i_a\\ i_b\\ i_c\end{bmatrix} + \begin{bmatrix}\Phi_{Ma}\\ \Phi_{Mb}\\ \Phi_{Mc}\end{bmatrix} \tag{21}$$

Mathematical Description of Standstill

Position detection at standstill is critical in sensorless methods, thus it has high priority. During the measurement only one phase winding was excited through the star point of the motor and the difference of the induced voltages in the two other phases was measured. The two induced voltages were not equal, and the amplitude of the difference was depending on the rotor position. The results can be seen on *Fig. 4*.

The above voltage measurements prove the presence of anisotropies. Based on the amplitude of the difference in different rotor positions characteristic curves can be determined which are periodic with 180°, thus cannot give the exact rotor position.

Using the previously presented formalism, the following relation can be written for the exciting voltage, current and induced voltages:

$$U_a - Ri_a = -\frac{\mathcal{R}_b(\theta)+\mathcal{R}_c(\theta)}{\mathcal{R}_c(\theta)} U_b = -\frac{L_{aa}(\theta)}{L_{ba}(\theta)} U_b \quad (22)$$

Similarly to (22) the voltage equations can be written for the other phases. Altogether six equations can be obtained, from these with current measurement three reluctance value can be defined in one specified rotor position. From these the R_M reluctance function can be defined. To do this further high precision measurements are necessary.

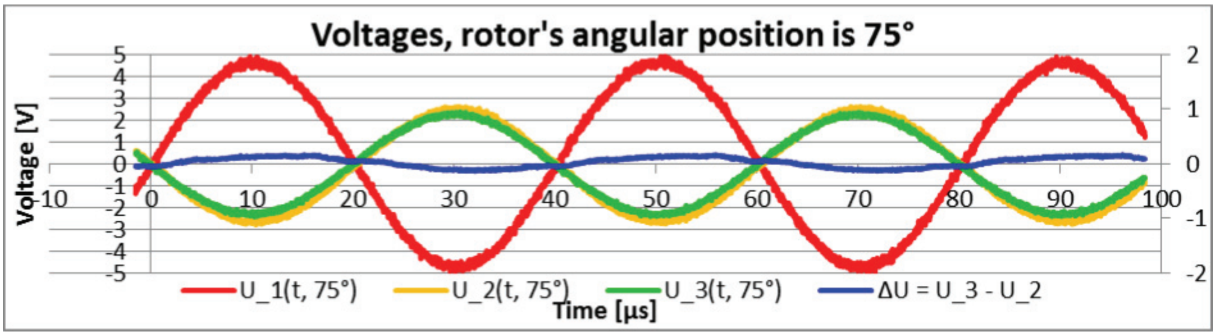

Figure 4: U1: exciting voltage, U2 and U3: voltages induced in Phase 2 and 3, ΔU = U3 - U2, the difference of the induced voltages. The U1 exciting voltage measured on the left-hand y axis.

Conclusions

According to the phase geometry the phase characteristic curves are shifted with 120° to each other, however they are not really uniform. All three curves have a period of 180°, which means that the angular position only by voltage measurement cannot be done. In that way only the line defined by the rotor magnet poles can be found, but the two poles cannot be distinguished from each other. The line mentioned above can be determined with a relatively small error (5°–10°).

The developed mathematical model and formalism makes possible a general description of permanent magnet synchronous motors. The formalism fits in the notation used by the literature.

The main goal, to prove the existence of the anisotropies and their measurability has been reached.

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