FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol. 15, N° 2, 2017, pp. 245 - 256
DOI: 10.22190/FUME170505012N

Original scientific paper

DECOUPLING CONTROL OF THE TITO SYSTEM SUPPORTED BY THE DOMINANT POLE PLACEMENT METHOD

UDC 62-5:681.5

Novak N. Nedić[1], Saša Lj. Prodanović[2], Ljubiša M. Dubonjić[1]

[1]University of Kragujevac, Faculty of Mechanical and Civil Engineering in Kraljevo, Serbia
[2]University of East Sarajevo, Faculty of Mechanical Engineering, Bosnia and Herzegovina

Abstract. *Appropriate approach to the nature of systems is a significant precondition for its successful control. An always actual issue of its mutual coupling is considered in this paper. A multivariable system with two-inputs and two-outputs (TITO) is in the focus here. The dominant pole placement method is used in trying to tune the PID controllers that should support the decoupling control. The aim is to determine parameters of the PID controllers which, in combination with decoupler, can obtain a good dynamical behavior of the system. Therefore, this kind of the centralized analytically obtained controller is used for object control. Another goal is to simplify the tuning procedure of PID controllers and enlarge the possibility for introducing the given approach into practice. But the research results indicate that the proposed procedure leads to the usage of P controllers because they enable the best performances for the considered object. Also, it is noticed that some differences from the usual rules in selection of the dominant poles gives better results. The research is supported by simulations and, therefore, the proposed method effectiveness, regarding the system behavior quality, is presented on several examples.*

Key Words: *Decoupling Control, PID Control, TITO Process, Dominant Pole Placement Method*

1. INTRODUCTION

Multivariable systems have been in focus of many research studies in recent decades. Their decoupling has been studied intensively in [1-5]. Neither type of decoupler is universal; hence which of them will provide for appropriate compensation of the mutual coupling depends on the object nature. In the present paper the static inverted decoupler is used for the investigated

Received May 05, 2017 / Accepted July 03, 2017
Corresponding author: Saša Lj. Prodanović
University of East Sarajevo, Faculty of Mechanical Engineering, Vuka Karadžića 30, 71123 East Sarajevo, Bosnia and Herzegovina
E-mail: sasa.prodanovic@ues.rs.ba

object, like in [6]. Cantilever beam as an object of control is taken into consideration. Its mathematical model is determined in [7]. Here the electrohydraulic servosystem designed for structural testing is considered as a system with two inputs and two outputs (TITO). The decoupling control enables taking this kind of system as a finite number (in this case two) of SISO (single-input single-output) systems. Having in mind this fact, a wider spectrum of methods can be used for the tuning of controller parameters. Therefore, the dominant pole placement method has also significant place as one of the tuning rules. Das et al. [8] tune PID controllers by using the guaranteed dominant pole placement method. Investigation of this method for the time delay systems was performed in [9-12]. Madady and Reza-Alikhani considered approaches for the first-order controller design using dominant pole placement, too [13]. Besides many other procedures for PID controller tuning, Åström and Hägglund in [14] presented the dominant pole placement method for several kind of objects. Filipović and Nedić in [15] showed procedures for PI and PID controller design based on this way. Q.-G. Wang et al. [16] dealt with the fourth-order object but without zeros. Nicolau [17] researched possibilities for PID controller design based on the pole placement technique in the combination with symmetrical optimum criterion. Consideration of tracking performance for a continuous-time PID controller (tuned using this method) with anti-windup compensator was described in [18]. The decoupling control, that contains controller designed according to the dominant pole placement, was described and tested in [19]. A further step in the method implementation was made by Rasouli et al. [20]. They made fractional order pole placement controller. Extension of the original dominant pole placement method for controller design to the multivariable systems is presented in [21-24].

In contrast to the aforementioned research studies, the present paper deals with controller design for the TITO object, whose decoupled loops are of the third-order with two left half plane zeros.

2. Decoupling of Object

General transfer function matrix of the considered object is given by Eq. (1):

$$\mathrm{G}(s)=\begin{bmatrix} \mathrm{g}_{11}(s) & \mathrm{g}_{12}(s) \\ \mathrm{g}_{21}(s) & \mathrm{g}_{22}(s) \end{bmatrix} \tag{1}$$

where $g_{ij}(s)$ are elements of the transfer function matrix.

The decoupling control strategy containing inverted decoupler in the combination with PID controllers is shown in Fig. 1, where Laplace operator s was omitted to make it simpler. Taking into account [5,6], decoupler $D(s)$ is calculated as a static decoupler using Eq. (2), in order to avoid introducing of additional dynamic into the system:

$$D(s)\Big|_{s=0}=\begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix}=\begin{bmatrix} 1 & \dfrac{-g_{12}(0)}{g_{11}(0)} \\ \dfrac{-g_{21}(0)}{g_{22}(0)} & 1 \end{bmatrix} \tag{2}$$

where $d_{ij}(s)$ are off-diagonal elements of the decoupler transfer matrix.

Apparent system of equations (3), that should be obtained after decoupling, enables considering of the TITO system as a finite number of SISO systems (in this case two SISO systems q_1(s) and q_2(s)).

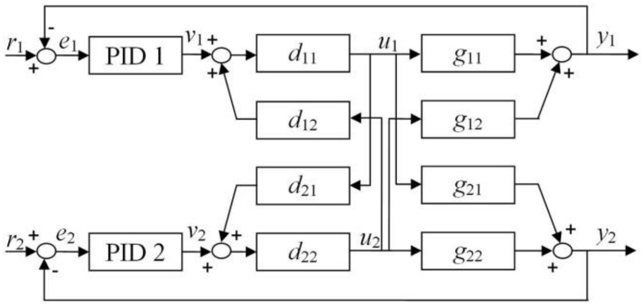

Fig. 1 Inverted decoupling control for the TITO object [1]

Actually, the inverted decoupling is applied because of its utilization of advantages of ideal (simple apparent system $Q(s)$) and simplified decoupling (simple decoupler). Hence,

$$Q(s) = G(s) \cdot D(s) = \begin{bmatrix} q_1(s) & 0 \\ 0 & q_2(s) \end{bmatrix} \tag{3}$$

Controllers will be designed based on diagonal elements of Eq. (3).

3. Controller Design

General expression for the decentralized PID controller for the TITO process is given by Eq. (4) and its elements (two single loop controllers $k_1(s)$ and $k_2(s)$) are presented with Eq. (5):

$$K(s) = \begin{bmatrix} k_1(s) & 0 \\ 0 & k_2(s) \end{bmatrix} \tag{4}$$

$$\begin{aligned} k_1(s) &= K_{p1} + \frac{K_{i1}}{s} + K_{d1} \cdot s \\ k_2(s) &= K_{p2} + \frac{K_{i2}}{s} + K_{d2} \cdot s \end{aligned} \tag{5}$$

where K_p, K_i and K_d are proportional, integral and derivative controller gains, respectively. In the inverted decoupling, controllers are designed for the diagonal elements of $Q(s)$ and hence: $q_1(s)=g_{11}(s)$ and $q_2(s)=g_{22}(s)$. Therefore, as previously stated, the PID controller design using the dominance pole placement method will be researched for the third-order transfer function with two left half plane zeros (Eq. (6)):

$$g_{ii}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + \mathrm{a}_2 s^2 + \mathrm{a}_1 s + \mathrm{a}_0} \tag{6}$$

where a_i and b_i are coefficients of the denominator and numerator, respectively. According to that, the characteristic equation of the single loop is expressed by Eq. (7-9):

$$1 + g_{ii}(s) \cdot k_i(s) = 0 \tag{7}$$

$$1 + \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0} \cdot \frac{K_{d1} \cdot s^2 + K_{p1} \cdot s + K_{i1}}{s} = 0 \tag{8}$$

$$s \cdot (s^3 + a_2 s^2 + a_1 s + a_0) + (b_2 s^2 + b_1 s + b_0) \cdot (K_{d1} \cdot s^2 + K_{p1} \cdot s + K_{i1}) = 0 \tag{9}$$

Equation (10) is a general form of the fourth-order characteristic equation. So, there are four poles: two conjugate complex Eq. (11) and two real. Since the PID controller has three parameters, three dominant poles should be determined.

$$(s + \alpha\omega_n) \cdot (s + \beta\omega_n) \cdot (s^2 + 2\xi\omega_n s + \omega_n^2) = 0 \tag{10}$$

$$s = -\omega_n \xi + j\omega_n \sqrt{1 - \xi^2} \tag{11}$$

Here ω_n is natural frequency and ξ is damping coefficient, while α and β are parameters which serve for pole placement.

Equalization of the Eq. (9) and Eq. (10) and large mathematical transformations lead to expressions for the PID controller gains Eq. (12):

$$\begin{gathered} K_p = \frac{(2\xi + \alpha + \beta)\omega_n - a_2}{b_2} \\ K_i = \frac{\alpha\beta\omega_n^4}{b_0} \\ K_d = \frac{(1 + 2\alpha\xi + 2\beta\xi + \alpha\beta) b_1 b_2 \omega_n^2 - a_1 b_1 b_2 - (\alpha + \beta + 2\alpha\beta\xi) b_2^2 \omega_n^3 + a_0 b_2^2}{b_0 b_1 b_2} \\ \frac{+(2\xi + \alpha + \beta) b_0 b_2 \omega_n - a_2 b_0 b_2 - (2\xi + \alpha + \beta) b_1^2 \omega_n + a_2 b_1^2}{b_0 b_1 b_2} \end{gathered} \tag{12}$$

4. EXAMPLES

The proposed procedure is illustrated through three examples that have been examined to check its sensitivity to the model uncertainties and at the same time to start investigation of its applicability to the different objects.

4.1. Example 1

Electrohydraulic servosystem for structural testing is shown in Fig. 2. Its mathematical model was obtained by means of an appropriate identification procedure and given by Eq. (13) [7]. The control system serves to enable defined load to the cantilever beam. Intensity and character of the forces on the piston rods are characteristics that should be controlled by flow rates through the servovalves. Forces F_{1r} and F_{2r} are reference values. Values F_1 and F_2 from their transducers are object outputs (controlled variables).

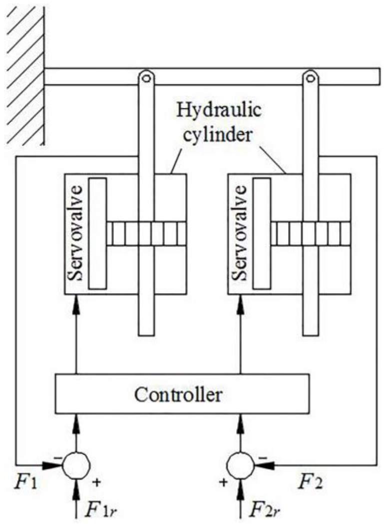

Fig. 2 Double actuator electrohydraulic servosystem for structural testing (scheme) [7]

$$G(s) = \frac{1}{\Delta(s)} \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$
$$g_{11}(s) = 2.926 \cdot 10^2 s^4 + 1.9152 \cdot 10^4 s^3 + 1.2667 \cdot 10^7 s^2 + 5.5825 \cdot 10^7 s + 4.7959 \cdot 10^9$$
$$g_{12}(s) = -3.8382 \cdot 10^4 s^3 - 1.7068 \cdot 10^7 s^2 - 8.3584 \cdot 10^7 s - 6.4967 \cdot 10^9 \tag{13}$$
$$g_{21}(s) = -4.4533 \cdot 10^3 s^3 - 3.2461 \cdot 10^6 s^2 - 1.4362 \cdot 10^7 s - 1.2403 \cdot 10^9$$
$$g_{22}(s) = 2.506 \cdot 10^2 s^4 + 1.6229 \cdot 10^4 s^3 + 6.6134 \cdot 10^6 s^2 + 3.0476 \cdot 10^6 s + 2.4813 \cdot 10^9$$
$$\Delta(s) = s^5 + 1.2308 \cdot 10^2 s^4 + 6.993 \cdot 10^4 s^3 + 1.5098 \cdot 10^6 s^2 + 3.5504 \cdot 10^8 s + 8.2333 \cdot 10^{-6}$$

Static inverted decoupler for the particular system Eq. (13) calculated according to Eq. (2) is:

$$D(s)\Big|_{s=0} = \begin{bmatrix} 1 & 1.35 \\ 0.5 & 1 \end{bmatrix} \tag{14}$$

Fifth-order elements of the transfer matrix $g_{11}(s)/\Delta(s)$ and $g_{22}(s)/\Delta(s)$ were reduced to the third-order using Matlab Toolbox. Effectiveness of the reducing procedure has been checked by comparison of step and sine responses of these elements. These graphics are shown in Fig. 3. Based on them, it is obvious that the reduced elements well represent identified transfer matrix. This is due to the appropriate matching of the step responses and excellent matching of the sine responses.

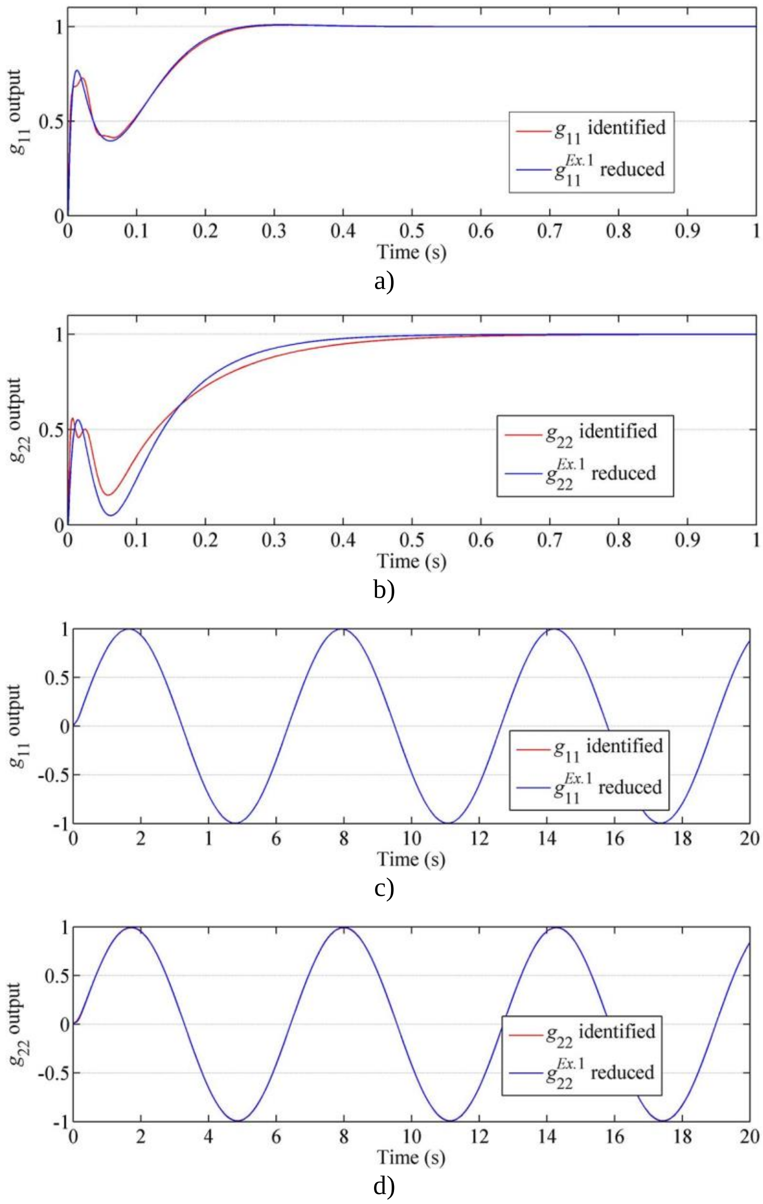

Fig. 3 Comparison of step (a, b) and sine (c, d) responses of identified [7] and reduced elements of the transfer matrix presented by Eq. (13)

Laplace operator *s* was omitted in Fig. 3 in order to improve its clarity.

Reduced elements are given by:

$$
\begin{aligned}
g_{11}(s)^{Ex.1} &= \frac{191\cdot s^2+593\cdot s+75911}{s^3+14.3\cdot s^2+5620.5\cdot s} \\
g_{22}(s)^{Ex.1} &= \frac{102\cdot s^2-1041\cdot s+39243}{s^3+16.8\cdot s^2+5614.8\cdot s}
\end{aligned}
\tag{15}
$$

Appropriate choice of parameters α, β and ξ defines the position of the poles in the complex plane. The other coefficients are known from Eq. (6). In the all three examples the following values of the parameters are taken α=12, β=1 and ξ=1. In this one, according to Eq. (15) natural frequency is ω_n=7.15 rad/s (for $g_{11}^{Ex.1}$) and ω_n=8.4 rad/s (for $g_{22}^{Ex.1}$). Controller parameters calculated from Eq. (12) are:

$$K_{p1}=0.4866\ ;\ K_{i1}=0.4131\ ;\ K_{p2}=1.0706\ ;\ K_{i2}=1.5224$$

Values for derivative gains are too high; knowing that they cause system instability, they are not taken into consideration for this system. This is a potential drawback of this procedure.

4.2. Example 2

In this example, polynomial coefficients in the Eq. (15) are increased for 20 % to obtain Eq. (16). Considering poles, two of three poles have been moved to the left in comparison with example 1. Their movement has been carried out because of their well-known influence to the system behavior. This and following example are used to examine the possibility for appropriate tuning of the controller when the mathematical model of the object is not completely accurate. This case is very often in practice due to changeable functioning conditions and also during process of identification. Thus these two examples actually represent variants of example 1.

$$
\begin{aligned}
g_{11}(s)^{Ex.2} &= \frac{229.2\cdot s^2+711.6\cdot s+91093.2}{s^3+17.16\cdot s^2+6744.6\cdot s} \\
g_{22}(s)^{Ex.2} &= \frac{122.4\cdot s^2-1249.2\cdot s+47091.6}{s^3+20.16\cdot s^2+6737.76\cdot s}
\end{aligned}
\tag{16}
$$

According to Eq. (16), natural frequency is ω_n= 8.58 rad/s (for $g_{11}^{Ex.2}$) and ω_n= 10.08 rad/s (for $g_{22}^{Ex.2}$). Afterwards, the controller gains from Eq. (12) are:

$$K_{p1}=0.4866\ ;\ K_{i1}= 0.7139\ ;\ K_{p2}=1.0706\ ;\ K_{i2}= 2.6308$$

Derivative gains are also too high for this system; that is the reason why they are omitted.

4.3. Example 3

Coefficients in the Eq. (15) are decreased for 20 % in this case. Here two of three poles have been moved to the right in comparison with example 1. Now diagonal elements of the Eq. (1), i.e. Eq. (3) are given by Eq. (17):

$$g_{11}(s)^{Ex.3} = \frac{171.9 \cdot s^2 + 533.7 \cdot s + 68319.9}{s^3 + 12.87 \cdot s^2 + 5058.45 \cdot s}$$
$$g_{22}(s)^{Ex.3} = \frac{91.8 \cdot s^2 - 936.9 \cdot s + 35318.7}{s^3 + 15.12 \cdot s^2 + 5053.32 \cdot s} \tag{17}$$

Here, natural frequency is ω_n= 6.435 rad/s (for $g_{11}^{Ex.3}$) and ω_n= 7.56 rad/s (for $g_{22}^{Ex.3}$). From Eq. (12) it follows:

$$K_{p1}=0.4866 \ ; \ K_{i1}= 0.3012 \ ; \ K_{p2}=1.0706 \ ; \ K_{i2}= 1.1098$$

Derivative gains have been avoided like in previous examples.

5. Simulation Results

Based on configuration in Fig.1, the proposed decoupling control is simulated using Matlab/Simulink. Simulations are carried out for the two cases regarding reference functions (signals) r_1 and r_2. In the first case (Fig. 4) r_1 is unit sine function and r_2 is unit step function, and *vice versa* in the second case.

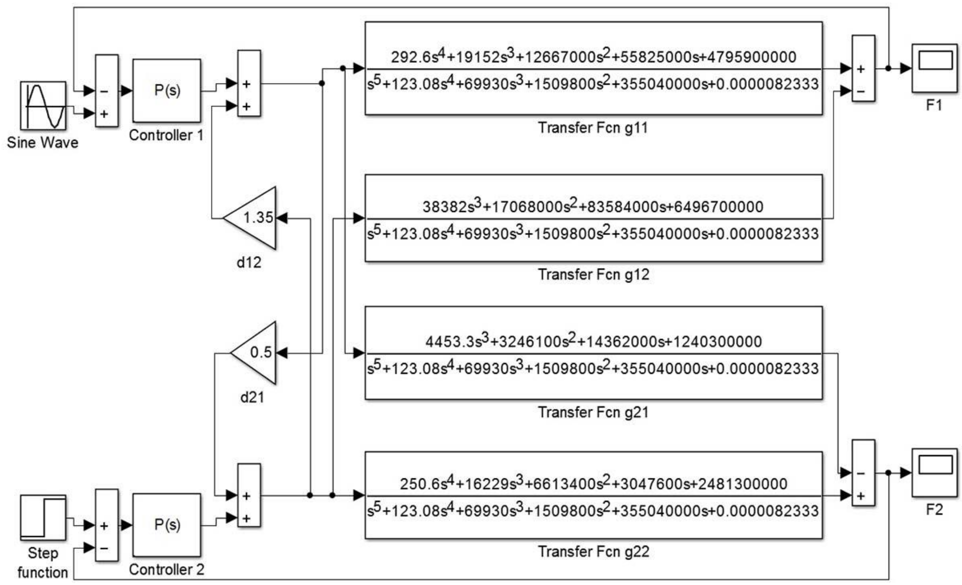

Fig. 4 Block diagram of the control algorithm for electrohydraulic servosystem

System responses and their enlarged views are shown in Figs. 5 and 6.

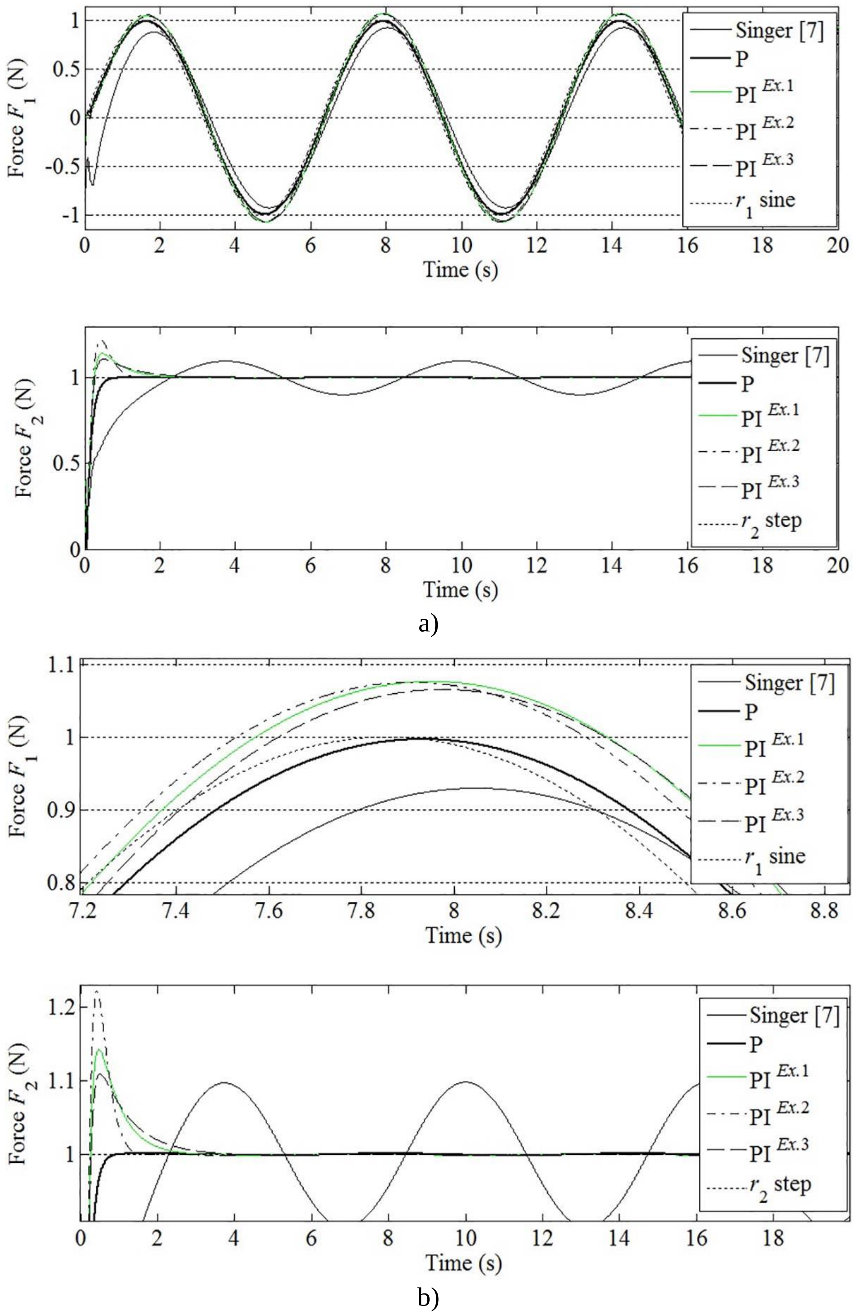

Fig. 5 a) Forces on the cylinders (r_1 unit sine function for F_1, r_2 unit step function for F_2) b) Enlarged view of characteristic response range

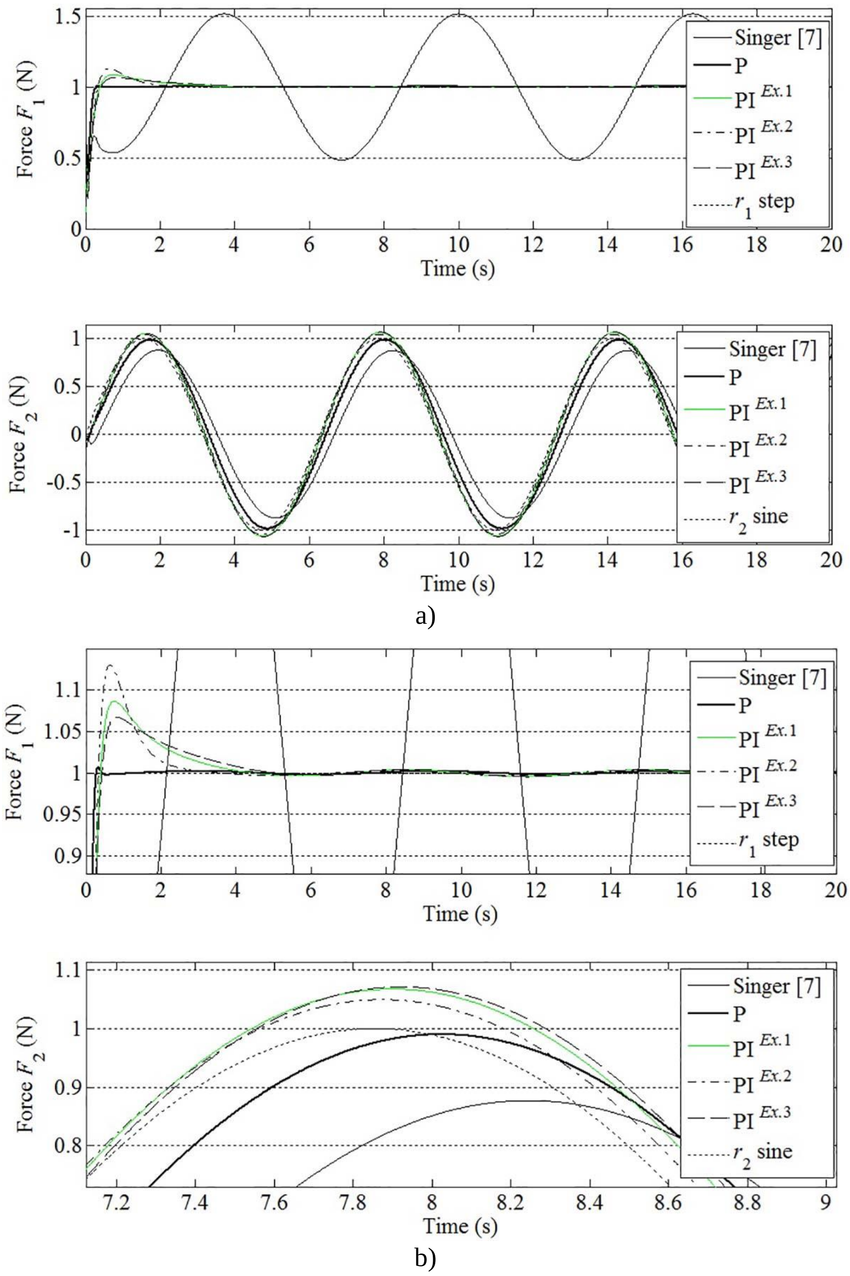

Fig. 6 a) Forces on the cylinders (r_1 unit step function for F_1, r_2 unit sine function for F_2)
b) Enlarged view of characteristic response range

These figures show responses for the four types of controllers in the combination with the static inverted decoupler and one response without decoupler that was controlled in [7]. It is noticeable that P controllers give the best reference tracking. This is confirmed by a very small deviation between reference signals r_1 and r_2 compared to the appropriate responses of the system with applied P controller. The described slight deviation is, in fact, an expected delay of the output in relation to the input signal. This fact cancels the aforementioned possible drawback of the proposed procedure because it is important that at least one type of controller can satisfy the defined requirements for the system dynamic behavior. Moreover, it leaves the possibility of its application to other objects.

The most appropriate value for proportional gain K_p is obtained when non-dominant pole has 12 times higher absolute value of the real part than the three dominant poles. PI controllers give a lower quality of responses. Observing the values of K_p in the examined three examples, it is also noticeable that the P controller is the least sensitive to the model perturbations, i.e. model uncertainties. In comparison with [7] (the case without decoupling), there is an obvious improvement in the compensation of interaction between loops.

6. Conclusions

The proposed procedure for the PID controller design is extension of the dominant pole placement method to the third-order objects with two left half plane zeros. After calculating controller gains, the most suitable controller type can be chosen. It is proved that, in some control algorithms, the ratio between the absolute values of the real part of non-dominant and dominant poles should be greater than four, which is the value usually suggested in the literature. Suitable reduction of the previously known (identified) transfer matrix, i.e. its diagonal elements, makes easier controller tuning. Effectiveness of the applied reduction is proved as good because the designed controllers were tested on the identified (initial) model of the system and they enabled appropriate system behavior. The controllers tuned on the basis of the presented approach are compatible with the previously decoupled objects. This is confirmed on the TITO electrohydraulic system for structural testing, where the P controller in the combination with static inverted decoupler enables good system performances, especially regarding reference tracking as well as cancellation of mutual coupling and reducing sensitivity to the mathematical model uncertainties. Omitting of the derivative terms due to their high values for the considered cantilever beam does not necessarily be a rule for other systems. This should be the subject of future research studies.

Acknowledgements: *The authors wish to express their gratitude to the Serbian MPNTR for partly financing of this paper through the project TR33026.*

References

1. Nordfeldt, P., Hägglund, T., 2006, *Decoupler and PID controller design of TITO systems*, J. Process Control, l (16), pp. 923-936.
2. Vázquez, F., Morilla, F., 2002, *Tuning decentralized PID controllers for MIMO systems with decouplers*, Proceedings of the 15th IFAC World Congress, Barcelona, Spain.

3. Morilla F., Vázquez, F., Garrido, J., 2008, *Centralized PID control by decoupling for TITO processes*, Proceedings of the 13th IEEE International Conference on Emerging Technologies and Factory Automation. Hamburg, Germany, pp. 1318-1325.
4. Garrido, J., Vázquez, F., Morilla, F., Hägglund, T., 2011, *Practical advantages of inverted decoupling*, Proceedings of the Institution of Mechanical Engineers, Part I: J. Systems and Control Engineering, 225, pp. 977-992.
5. Morilla, F., Garrido, J., Vázquez, F., 2013, *Control multivariable por desacoplo*, Revista Iberoamericana de Automática e Informática industrial, 10, pp. 3-17, (In Spanish).
6. Prodanović, S., Nedić, N., 2016, *Control improvement of a double actuator electrohydraulic servosystem for structural testing*, In Proceedings of the 15th Youth Symposium on Experimental Solid Mechanics YSESM 2016, Rimini, Italia.
7. Singer, G., Meashio, Y., 1995, *Analysis of a double actuator electrohydraulic system for structural testing*, IEE, Savoy Place, London.
8. Das, S., Halder, K., Pan, I., Ghosh, S., Gupta, A., 2012, *Inverse optimal control formulation for guaranteed dominant pole placement with PI/PID controllers*, International Conference on Computer Communication and Informatics (ICCCI -2012), Coimbatore, India.
9. Zítek, P., Fišer, J., Vyhlídal, T., 2013, *Dimensional analysis approach to dominant three-pole placement in delayed PID control loops*, J. Process Control, 23, pp. 1063-1074.
10. Branlea, I., Petrovic, I. and Peric, N., 2002, *Toolkit for PID dominant pole design*, 9th International Conference on Electronics, Circuits and Systems, 3, pp. 1247-1250.
11. Zítek, P., Fišer, J., Vyhlídal, T., 2012, *Ultimate-frequency based three-pole dominant placement in delayed pid control loop*, Proceedings of the 10-th IFAC Workshop on Time Delay Systems, The International Federation of Automatic Control Northeastern University, Boston, USA, pp.150-155.
12. Li, Y., Sheng, A., Qi, Q., 2011, *Further results on guaranteed dominant pole placement with PID controllers*, Proceedings of the 30th Chinese Control Conference, Yantai, China, pp. 3756-3760.
13. Madady, A., Reza-Alikhani, H.R., 2011, *First-order controllers design employing dominant pole placement*, 19th Mediterranean Conference on Control and Automation Aquis Corfu Holiday Palace, Corfu, Greece, pp.1498-1503.
14. Åström, K.J., Hägglund, T., 1995, *PID Controllers: Theory, Design and Tuning*, Research Triangle Park, NC: Instrumental Society of America.
15. Filipović, V.Ž., Nedić, N.N., 2008, *PID Controllers*, University of Kragujevac, Faculty of Mechanical Engineering, Kraljevo, (in Serbian).
16. Wang, Q-G., Zhang, Z., Åström, K.J., Chek, L.S., 2009, *Guaranteed dominant pole placement with PID controllers*, J. Process Control, 19, pp. 349-352.
17. Nicolau, V., 2013, *On PID Controller Design by Combining Pole Placement Technique with Symmetrical Optimum Criterion*, Math. Probl. Eng., (2013), pp. 1-8.
18. Sadalla, T., Horla, D., 2015, *Analysis of simple anti-windup compensation in pole-placement control of a second order oscillatory system*, Measurement Automation Monitoring, 61(02), pp. 54-57.
19. Nedić, N.N., Prodanović, S.Lj., Dubonjić, Lj.M., 2016, *Some Considerations on the Decoupling Control of TITO Systems*, Proc. XIII International SAUM Conference on Systems, Automatic Control and Measurements, Niš, Serbia.
20. Rasouli, H., Fatehi, A., Zamanian, H., 2015, *Design and implementation of fractional order pole placement controller to control the magnetic flux in Damavand tokamak*, Rev. Sci. Instrum., 86(033503), pp. 1-11.
21. Zhang, Y., Wang, Q-G., Åström, K.J., 2000, *Dominant pole placement for multi-loop control systems*, Proceedings of the American Control Conference, Chicago, Illinois, pp. 1965-1969.
22. Lee, J., Edgar T.F., 2006, *Multiloop PI/PID control system improvement via adjusting the dominant pole or the peak amplitude ratio*, Chem. Eng. Sci., 61, pp. 1658-1666.
23. Mokadam, H.R., Patre, B.M., Maghade, D.K., 2013, *Tuning of multivariable PI/PID controllers for TITO processes using dominant pole placement approach*, Int. J. Automation and Control, 7(1/2), pp. 21-41.
24. Maghade, D.K., Patre, B.M., 2014, *Pole placement by PID controllers to achieve time domain specifications for TITO systems*, Trans. Inst. Meas. Control, 36(4), pp. 506-522.