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Review paper

VALUE AT RISK MEASURING AND EXTREME VALUE THEORY: EVIDENCE FROM MONTENEGRO

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Abstract. *The concept of value at risk (VaR) is a measure that is increasingly used for estimation of the maximum loss of financial position at a given time for a given probability. The aim of this manuscript is to show the most recent approaches for quantifying market risk. In particular, the manuscript investigates whether extreme value theory outperforms econometric calculation of VaR in emerging stock markets such as Montenegrin market. The paper is motivated by the desire that necessary attention is given to risks in Montenegro. Daily return of highly volatile stock EPCG (Elektroprivreda Crne Gore) from Montenegrin stock exchange is analysed for the period from January, 2004 – June, 2013. The sample of this structure and time dimension has not been discussed in empirical literature. Therefore, it is necessary to use the experience of the developed world's financial institutions which have studious approach to risk management, as well as the latest theoretical knowledge.*

Key Words: *Extreme value theory, Value at Risk, fat tails, GARCH model, peak over threshold, generalized Pareto distribution*

INTRODUCTION

The risk from extreme events is present in all fields of risk management. Methodology used for the assessment of financial markets participants' rate of exposition to risk, gives the estimation of value at risk. Value at risk (Value-at-risk, or abbreviated VaR) is the maximum loss of financial position over a given time period at a given confidence interval. It includes all types of financial risk and the application in the analysis of market risk is to be presented in this manuscript.

It is intended to show the latest approaches to quantification of market risk in this paper, in a theoretical and practical context. The aim of this manuscript is to present the

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estimation of VaR based on the analysis of specificities of financial time series, and to give empirical results of measuring Value at risk in Montenegrin financial market that is still developing. These include econometric evaluation, Riskmetrics methodology, quantile estimation and estimation based on extreme value theory. Econometric evaluation is derived from GARCH model, while Riskmetrics methodology uses IGARCH model.

There is a general opinion in literature data that there is no universal model giving the best estimation and forecast of VaR. Numerous papers observing the application of different approaches in developed financial markets confirm this – Caporin (2003), Christoffersen, et al. (2001), Angelidis, et al. (2004), Wong, et al. (2002), Alexander and Leigh (1997), Harimantzis, et al. (2006), Peters (2001), Embrechts, et al. (1999), McNeil, et al. (2000), Guermat and Harris (2002), So and Yu (2006).

On the other hand, there are very few papers observing the comparison of VaR models in developing financial markets. Gençay and Selçuk (2004) analized parameter models and quantile estimation of VaR of stock exchange indices in developing Central and Eastern European countries. These results show that generalized Pareto distribution and extreme value theory are basic tools in risk management in developing countries. Živković (2007) observed different approaches to VaR measuring on the example of new members and candidate countries for EU membership (Bulgaria, Romania, Croatia and Turkey). The conclusion of this research is that application of VaR models is not successful enough in financial markets of these countries because the returns show the existence of heavy tails, asymmetry and heteroscedasticity. Further researches followed in 2009, where Živković and Aktan analized VaR models of the returns of Turkish and Croatian stock-exchange indices with the onset of global financial crisis. It was concluded in this paper that extreme value theory and hybrid historical simulation are the best, while other models underestimate the level of risk. Anđelić, Djoković and Radišić (2010) observed Slovenian, Croatian, Serbian and Hungarian markets and concluded that under stable market conditions, the analized models give good forecasts of VaR estimations with 5% level of significance, while under the conditions of market volatility analized models give good estimations of VaR parameters with 1% level of significance. Nikolić-Đorić and Đorić (2011) observed the movement of stock-exchange index in Serbian financial market and concluded that GARCH models combined with extreme value theory – peaks over threshold method, decrease the mean value of VaR, as well as that given models are better than RiskMetrics method and IGARCH model. Also, Mladenović, Miletić and Miletić (2012), based on analysis of stock-exchange indices in Central and Eastern European countries (Bulgaria, Czech Republic, Hungary, Croatia, Romania and Serbia), came to conclusion that the methodology of extreme value theory is slightly better than GARCH model regarding the calculation of VaR, but general suggestion is to use both approaches for better measuring of market risk.

The purpose of this paper is to compare performance of econometric models, quantile estimation and extreme value theory in evaluting Value-at-Risk in Montenegrin stock exchange over long period that includes years of financial crisis. Results will be interesting given the recession period is included, and are relevant on micro and macroeconomic level. In particular, the manuscript investigates whether extreme value theory can outperform econometric calculation of VaR in emerging stock markets, and, in particular, Montenegrin stock market has not been discussed in empirical literature.

Now we are going to observe a portfolio of some risky assets and determine portfolio value as V_t at a moment in time t. Let us assume that we want to determine the level of risk over the period [t,$t+h$]. We mark the random variable of portfolio loss as $L_{t+h} = -(V_{t+h} - V_t) = \Delta V(h)$. Cummulative function of loss distribution is marked as F_L where $F_L(x) = P(L \le x)$. In this case, VaR at significance level α ($\alpha \in (0,1)$ - most often $\alpha = 0.01$ or $\alpha = 0.05$, i.e. 1% and 5%) is actually an α-quantile of distribution function F_L and represents the smallest real number satisfying the inequation $F_L(x) \ge \alpha$, i.e.:

$$VaR_\alpha = \inf(x | F_L(x) \ge \alpha). \tag{1}$$

Expected shortfall (ES) is a measure closely related to VaR and practically often indicated as a measure overcoming conceptual disadventages of VaR. For loss L, with its expected absolute value being definite, expected shortfall at significance level α is defined as

$$ES_\alpha = \frac{1}{1-\alpha} \int_\alpha^1 q_u(F_L) du, \tag{2}$$

where $q_u(F_L)$ is the quantile function of distribution function F_L. It is obvious that measure ES depends only on loss distribution, and $ES_\alpha \ge VaR_\alpha$. Therefore, this measure represents expected return value in case a marginal value (usually VaR) is exceeded.

1. Methodologies of VaR

The type of value at risk estimation can be: 1. Quantile estimation (historical simulation), 2. Econometric evaluation (GARCH models) and 3. Estimation based on extreme value theory.

1.1. GARCH model

Generalized autoregressive conditional heteroscedasticity (**GARCH**) model, introduced by Bollersev (1986) and Taylor (1986), represents the generalization of autoregressive conditional heteroscedasticity model – ARCH, developed by Engle in 1982. Log returns, usually expressed in percents, are marked as r_t. Innovation at moment t is $a_t = r_t - \mu_t$. Then, the model can be presented as follows (Tsay, 2010):

$$\begin{aligned} r_t &= \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + a_t - \sum_{j=1}^{q} \theta_j a_{t-j} \\ a_t &= \sigma_t \varepsilon_t \end{aligned} \tag{3}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{u} \alpha_i a_{t-i}^2 + \sum_{j=1}^{\upsilon} \beta_j \sigma_{t-j}^2. \tag{4}$$

Parameters of equation (3) representing autoregressive moving-average model (ARMA) of orders p and q, ARMA (p,q), are marked as $\phi_0, \phi_1, ..., \phi_p, \theta_1, ..., \theta_q$. The random member of the model, a_t, is the function of ε_t - series of independent and identically distributed random variables having a normal or t-distribution with zero mean and variance equal to 1. By the second equation in the model - (4), conditional variance of returns r_t is modeled, $\sigma_t^2 = E((r_t - E(r_t))^2 \left| \Omega_{t-1} \right.)$, where Ω_{t-1} is available data set with moment t-1 inclusive. In other words, conditional variance (volatility) is expected squared deviation of observations from the mean given the available data set.

Parameters $\alpha_0, \alpha_1, ..., \alpha_u, \beta_1, ..., \beta_\upsilon$ of conditional variance equation satisfy the conditions $\alpha_0 > 0,\ \alpha_1, ..., \alpha_u \geq 0,\ \beta_1, ..., \beta_\upsilon \geq 0,\ \sum_{i=1}^{\max(u,\upsilon)} (\alpha_i + \beta_i) < 1$.

If the series ε_t is a random variable with standardized normal distribution, i.e. $\varepsilon_t : N(0,1)$ then conditional distribution of random variable r_{h+1} for available data with the moment h inclusive, also has a normal distribution with mean $\hat{r}_h(1)$ and variance $\hat{\sigma}_h^2(1)$. Then, 5%-quantile of conditional distribution, representing the estimation of VaR at 95% confidence level and for forecast horizon 1 step ahead, is computed as:

$$\hat{r}_h(1) + 1,65\hat{\sigma}_h(1). \tag{5}$$

If random variable ε_t has Student's t distribution, with υ degrees of freedom, then the 5%-quantile of conditional distribution is computed as follows:

$$\hat{r}_h(1) + \frac{t_\upsilon(1-p)}{\sqrt{\frac{\upsilon}{\upsilon-2}}}\hat{\sigma}_h(1), \tag{6}$$

where $t_\upsilon(1-p)$ is the corresponding critical value of $(1 - p)$ quantile from t distribution with υ degrees of freedom.

GARCH(1,1) model has the following form:

$$\begin{aligned} r_t &= \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + a_t - \sum_{j=1}^{q} \theta_j a_{t-j} \\ a_t &= \sigma_t \varepsilon_t, \qquad\qquad \varepsilon_t : N(0,1) \end{aligned} \tag{7}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{8}$$

If the model GARCH(1,1) satisfies the parameters sum $\alpha_1 + \beta_1 = 1$, then the model describes the process of unlimited growth of conditional variability. Such a model is known as integrated GARCH model – IGARCH(1,1). It is in the basis of VaR estimation, representing the standard approach to risk measuring – **RiskMetrics**.

This methodology was developed by company J. P. Morgan (Longerstaey, 1995), and it implies that conditional distribution of the series of log daily returns is $r_t \left| \Omega_{t-1} \right. : N(\mu_t, \sigma_t^2)$, where μ_t is conditional mean, and σ_t^2 is conditional variance of series r_t. The following relations are valid for them:

$$\mu_t = 0, \qquad \sigma_t^2 = \alpha\sigma_{t-1}^2 + (1-\alpha) r_{t-1}^2, \qquad 0 < \alpha < 1. \tag{9}$$

Volatility forecast for one period ahead in time shows that $\sigma_{t+1}^2 = \alpha\sigma_t^2 + (1-\alpha) r_t^2$. The previous relation indicates that $Var(r_{t+i} \left| \Omega_t \right.) = \sigma_{t+1}^2$ for $i \geq 1$, and therefore, $\sigma_t^2[k] = k\sigma_{t+1}^2$. If the significance level is 5%, portfolio risk according to RiskMetrics methodology is computed using formula $1,65\sigma_{t+1}$, i.e. daily VaR value of the portfolio is

$$VaR = Value\ of\quad financial\quad position \times 1,65\sigma_{t+1}. \tag{10}$$

1.2. Quantile estimation (Historical simulation)

Historical simulation begins from the assumption that return distribution in the forecast period is the same as in the sampling period. Thus, the given return values of the sample are arranged according to size into a growing series in the form $r_{(1)} \le r_{(2)} \le ... \le r_{(n)}$ with the first minimal and last maximal value.

Let us assume that returns are independent and identically distributed random variables with constant distribution whose probability density function is $f(x)$, and corresponding function of cumulative distribution $F(x)$. Let x_p be p-quantile of the function $F(x)$. If $f(x_p) \neq 0$, then the statistic $r(l)$, where $l = np$, $0 < p < 1$, has approximately normal distribution with mean value x_p and variance $p(1-p)/n[f(x_p)]^2$, i.e.

$$r_{(l)} : N(x_p, \frac{p(1-p)}{nf^2(x_p)}), \qquad l = np. \tag{11}$$

1.3. Extreme value theory – Peak over threshold method (POT)

The extreme value theory is a very good methodological frame for the research of the trend of distribution tail. If we consider the problem of sample maximum, we come to the main mathematical problem which is in the basis of the extreme value theory.

Let X_1, X_2,... be the series of independent, non-degenerate random variables having an even distribution, with the common distribution function F. Let us observe the maximum values of variables ($M_1 = X_1$)

$$M_n = \max(X_1, ..., X_n), \tag{12}$$

where $n \ge 2$.

For the joint limiting distribution function of maxima M_n, based on the character of their independence, it is:

$$P(M_n \le x) = P(X_1 \le x, ..., X_n \le x) = \prod_{i=1}^{n} P(X_i \le x) = \prod_{i=1}^{n} F(x) = F^n(x). \tag{13}$$

We will mark *the right end* of distribution F with

$$x_F = \sup(x \in R : F(x) < 1). \tag{14}$$

Then, for every $x < x_F$,

$$P(M_n \le x) = F^n(x) \to 0, \quad n \to \infty, \tag{15}$$

and, if $x_F < \infty$, for $x \ge x_F$

$$P(M_n \le x) = F^n(x) = 1. \tag{16}$$

Therefore, distribution function, as $n \to \infty$, becomes degenerate. In order to obtain non-degenerate marginal distribution, it is necessary to carry out normalization (De Haan & Ferreira, 2006).

The problem comes to the determination of real constants $a_n > 0$ and b_n, so the variable $(M_n - b_n) / a_n$ has non-degenerate marginal distribution, as $n \to \infty$, i.e. $\lim_{n\to\infty} F^n(a_n x + b_n) = G(x)$. G represents the non-degenerate distribution function and such distributions are called extreme value distributions.

Let the real constants be a_n and b_n ($a_n > 0$), so for every n applies

$$\lim_{n\to\infty} P\{(M_n - b_n)/a_n \le x\} = \lim_{n\to\infty} F^n(a_n x + b_n) = G(x), \tag{17}$$

for non-degenerate distribution function $G(x)$. If this condition applies, it is said that *F is in the domain of attraction of maxima from G*, i.e. $F \in D(G)$.

Extreme value distribution includes three parameters - γ - *shape parameter*, β_n - *location parameter*, and $\alpha_n > 0$ is *scale parameter*. They can be assessed in two ways: using parametric or non-parametric methods. Traditional approach – block maxima method largely dissipates data because only maximum values from great blocks are used. This is reported as the biggest disadvantage of this model, so in practice it is increasingly being replaced with the method based on **peaks over threshold**, where all data representing extremes are used, in the context of exceeding some high level. The given method is to be exposed as follows.

If we mark a certain threshold as u, and we observe the series of daily log returns r_t, then if i^{th} excess happens on the i^{th} day, this model is focused on the data (t_i, $r_{t_i} - u$). The basic theory of this new approach observes conditional distribution from $r = x + u$ which is for $r \le x + u$ given that threshold is exceeded, $r > u$:

$$P(r \le x+u \mid r > u) = \frac{P(u \le r \le x+u)}{P(r > u)} = \frac{P(r \le x+u) - P(r \le u)}{1 - P(r \le u)}. \tag{18}$$

The main distribution used for the modeling of excess over the threshold is *generalized Pareto distribution*, defined in the following way:

$$G_{\gamma,\psi(u)}(x) = \begin{cases} 1 - (1 + \dfrac{\gamma x}{\psi(u)})^{-1/\gamma}, & \gamma \ne 0, \\ 1 - \exp(-\dfrac{x}{\psi(u)}), & \gamma = 0, \end{cases} \tag{19}$$

where $\psi(u) > 0$ and $x \ge 0$ for $\gamma \ge 0$, and $0 \le x \le -\psi(u)/\gamma$ when $\gamma < 0$. Therefore, we conclude that conditional distribution from r, if $r > u$, approximates well with generalized Pareto distribution with parameters γ and $\psi(u) = \alpha + \gamma(u - \beta)$. Parameter $\psi(u)$ is called *scale parameter*, and γ is *shape parameter*.

Generalized Pareto distribution has a very significant feature. If the excess distribution from r with the given threshold u_0 is generalized Pareto distribution with shape parameter γ and scale parameter $\psi(u_0)$, then for arbitrary threshold $u > u_0$, the given excess distribution for threshold u is also generalized Pareto distribution with shape parameter γ and scale parameter $\psi(u) = \psi(u_0) + \gamma(u - u_0)$.

When the parameter $\gamma = 0$, then generalized Pareto distribution is exponential distribution. Therefore, it is suggested to carry out a graphic examination of the tail behaviour using QQ plot. If $\gamma = 0$, then the graph of the excess is linear.

Peaks over thresholds model has a problem regarding the choice of an adequate threshold. This is how the given problem is usually solved in practice.

For the given high threshold u_0, let the excess $r - u_0$ follow generalized Pareto distribution with parameters γ and $\psi(u_0)$, where $0 < \gamma < 1$. Then, the *mean excess* over the threshold u_0:

$$E(r - u_0 \,|\, r > u_0) = \frac{\psi(u_0)}{1-\gamma}. \tag{20}$$

The mean excess function in the mark $e(u)$ is defined, for every $u > u_0$, as:

$$e(u) = E(r - u \,|\, r > u) = \frac{\psi(u_0) + \gamma(u - u_0)}{1-\gamma}. \tag{21}$$

Therefore, for the given value γ, the mean excess function is the linear function of excess $u - u_0$. Hence, for the determination of the given threshold u_0, a simple graphic model is used, forming the empirical mean excess function as

$$e_T(u) = \frac{1}{N_u} \sum_{i=1}^{N_u} (r_{t_i} - u), \tag{22}$$

where N_u is the number of returns exceeding the threshold u, and r_{t_i} are the values of given returns. Threshold u is chosen so the empirical mean excess function is approximately linear for $r > u$.

For the given probability p in the upper tail, (1-p)-quantile of log return r_t is

$$VaR = \begin{cases} \beta - \dfrac{\alpha}{\gamma}\{1 - [-D\ln(1-p)]^{-\gamma}\} & \gamma \neq 0, \\ \beta - \alpha \ln[-D\ln(1-p)] & \gamma = 0. \end{cases} \tag{23}$$

VaR evaluation is much more stable using the peaks over thresholds method because with the traditional approach, VaR is very sensitive to changes in the size of blocks n.

The measure of Expected shortfall, as an expected loss if VaR is exceeded, then can be defined as

$$ES_q = E(r \,|\, r > VaR_q) = VaR_q + E(r - VaR_q \,|\, r > VaR_q), \tag{24}$$

i.e.

$$ES_q = \frac{VaR_q}{1-\gamma} + \frac{\psi(u) - \gamma u}{1-\gamma}. \tag{25}$$

2. Empirical Results

The purpose of empirical analysis is the evaluation of risk measures for daily returns, for one stock in Montenegrin stock market. The best way is to choose a stock having showed a pronounced volatility in the previous period, able to illustrate advantages and disadvantages of each model. For these reasons, a stock of Elektroprivreda Crne Gore (EPCG) was chosen, illustrating models of VaR calculation. Time series of logarithmic returns of EPCG's stock, were observed on daily basis in the period from 9th January of

2004 until 18th June of 2013 (2338 data in total). Log daily returns (or continuously compounded returns), represent the difference between logarithmic levels of prices in two successive days. It can also be expressed in percents, when these differences are multiplied by 100. The data are taken from the website of Montenegro berza AD Podgorica, retrieved from http://www.montenegroberza.com. Empirical results are obtained using program package R.

Daily return of the EPCG stock is stationary (Fig. 1). Its empirical distribution differs from normal distribution, which is indicated by the skewness and curtosis, as well as the joint indicator of normality – Jarque-Bera test-statistic (JB). These descriptive statistics are shown in Table 1. The value in parenthesis next to the value of test-statistic is the corresponding *p*-value.

Table 1 Basic descriptive statistics of daily return for EPCG

Variance	Skewness	Curtosis	JB	Box-Ljung (m=8)	Box-Ljung (a_t^2)
23.96	0.6	19.88	38709.92 (<2.2e-16)	101.76 (<2.2e-16)	1032.92 (<2.2e-16)

The given return series is not too asymmetric, which can be seen from the skewness indicator, but the high curtosis indicates that the it is above normal, i.e. there are "heavy tails" – tails are heavier than those in normal distribution. Jarque-Bera (JB) normality test shows that the hypothesis of normality of returns can be abandoned even when the level of significance is 1%. JB test-statistic has an asymptotic χ^2 distribution with two degrees of freedom.

The next in Table 1. is Box-Ljung test-statistic (Box-Ljung). It is used for the determination of autocorrelation of order *m* between squared data and has asymptotic χ^2 distribution with *m* degrees of freedom. Null hypothesis in this test implies that the first *m* autocorrelation coefficients of squared data are zero and it is abondoned here. Value *m* is chosen in several ways and in practice the best form is $m \approx \ln(T)$, where T is the number of data of the observed variable (Tsay, 2010). In our case, for *m* this value is 8.

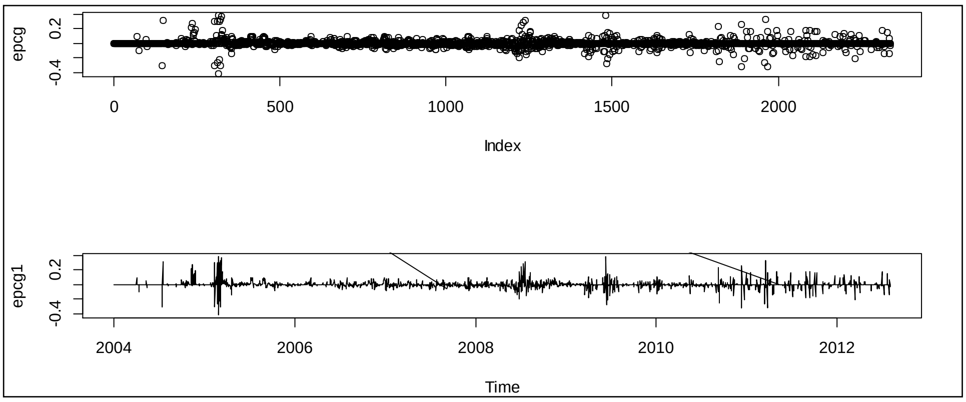

Fig. 1 Daily return of EPCG stock

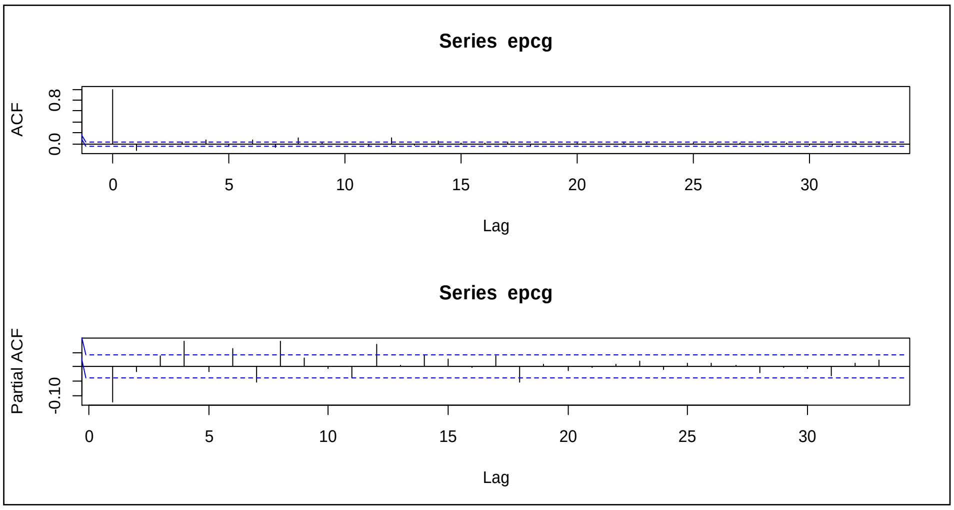

Fig. 2 Autocorrelation functions (ACF) and partial autocorrelations for EPCG series

To determine the existence of time-changing variability, the same Box-Ljung test-statistic is used, but for squared residual series (McLeod and Li, 1983, Tsay 2010). Return residual is defined as the difference between return level and mean of the return, i.e. $a_t = r_t - \mu_t$. For the return of EPCG, first the serial correlation was determined according to Box-Ljung test-statistic for the return data, and the same statistic for squared residuals also shows high volatility.

As daily return rate of EPCG stocks has an unstable variance, its dynamic is evaluated using GARCH model. Based on the specification analysis – sample functions of autocorrelation and partial correlation (Fig. 2) – it is estimated that the best model is ARMA(1,3). Volatility movement is well described by model GARCH(1,1).

Jointly estimated ARMA(1,3)-GARCH(1,1) model is:

$$r_t = -0.000016929 + 0.99696 r_{t-1} + a_t + 0{,}87183 a_{t-1} + 0.17137 a_{t-2} - 0.051543 a_{t-3},$$
$$\sigma_t^2 = 0.00018531 + 0.17587 a_{t-1}^2 + 0.74366 \sigma_{t-1}^2.$$

All model coefficients are already significant at significance level 1%, except free members in both equations, which are significant at level 10%.

The tests of residual normality, autocorrelation and conditional heteroscedasticity are given in Table 2. Therefore, it can be observed that the chosen model, which was the best of all econometric models (not all parameters were significant within other models), describes volatility really well. However, GARCH model did not remove autocorrelation successfully, which can be seen from Box-Ljung test. Autocorrelation was reduced enough, which can be concluded based on the autocorrelation of standardized residuals function, Figure 3.

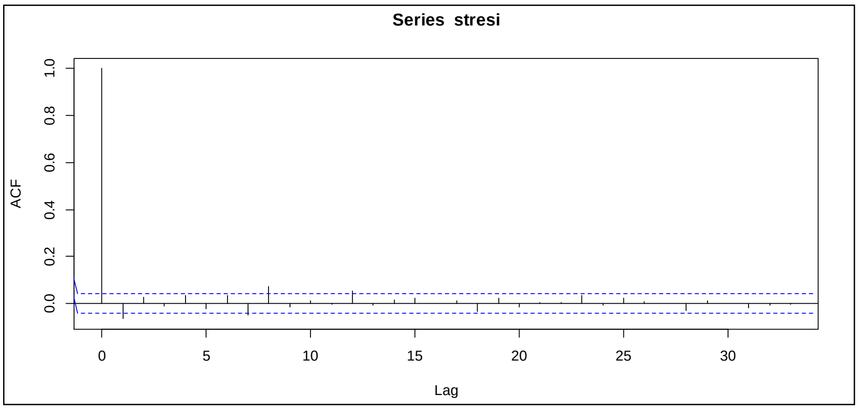

Fig. 3 Autocorrelation of standardized residuals function of model ARMA(1,3)-GARCH(1,1)

Table 2 Tests of ARMA(1,3)-GARCH(1,1) models: test-statistic and *p*-value

JB	Box-Ljung Q(10)	Box-Ljung (a_t^2)	LM ARCH Test
75584.7 (0)	37.75 (4,2e-05)	4.2146 (0.937)	5.46 (0.94)

Table 3. forecasts levels of return and volatility (conditional standard deviations) for one day time horizon, which are used for the assessment of VaR. The assessments are computed for level of confidence 95% and 99%.

Interpretation of the obtained result for VaR is as follows: if one possesses some value of EPCG stocks (for example, 1000€), then the possible loss for the owner of stocks for a one-day period does not exceed 7.483% of the value (74.83 €) with probability 95%. With the 99% probability, the estimation of the maximum loss is 10.65% of the value (106.5 €).

Table 3 Econometric evaluation of VaR for a one-day period (EPCG return)

Return forecast	Forecast of conditional st. dev.	VaR (95%)	VaR (99%)
-0.00092457	0.04619341	7.483%	10.65%

Riskmetrics method for the calculation of VaR assumes that conditional mean value is zero and that return volatility follows IGARCH(1,1) model. The adjusted model is

$$r_t = a_t, \quad a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = 0.041657\,\sigma_{t-1}^2 + (1-0.041657)a_{t-1}^2,$$

where ε_t is standard Gaussian series of white noise. Q statistic for squared standardized residuals is rather low (0.0005967), but not statistically significant.

According to the adjusted model, volatility forecast for one period in advance is $\hat{\sigma}(1) = 0.04951$, so 95% quantile of conditional distribution is $1.65 \times 0.04951 = 0.0816915$. VaR for 95% probability, one period in advance, for the position of, for example, 1000 €, will be

$$VaR = 1000€ \times 0.0816915 = 81.6915€.$$

According to the same principle, 99% quantile is $2.326 \times 0.04951 = 0.11516026$, so VaR, for the given probability is approximately 115.16€.

Quantile assessment of VaR is obtained as empirical 99% quantile, with the value of daily return for EPCG is 14.64244%, which means if we possess 1000 € worth EPCG stocks, the loss in one-day period does not exceed 146.4244 €, with 99% probability. The measure of expected shortfall for the same probability is 23.19685%, which means that if VaR is exceeded, for the same possession of 1000€ worth EPCG stocks, the loss expected in one-day period is 231.97 €. With confidence level 95%, VaR amounts to 5.395%, and the measure of expected shortfall is 11.69%.

The following is the evaluation of VaR based on the new approach of extreme value theory – **peaks over threshold method**. Negative logarithmic returns of EPCG stocks are observed, and according to literature for the series of stable returns, we usually choose 2.5% for threshold *u*. Fig. 4 shows, based on Q-Q plot, that the given returns derogate from normal distribution, so it is concluded that coefficient $\gamma \neq 0$. Also, the graph of mean excess function is linear up to threshold level 3%.

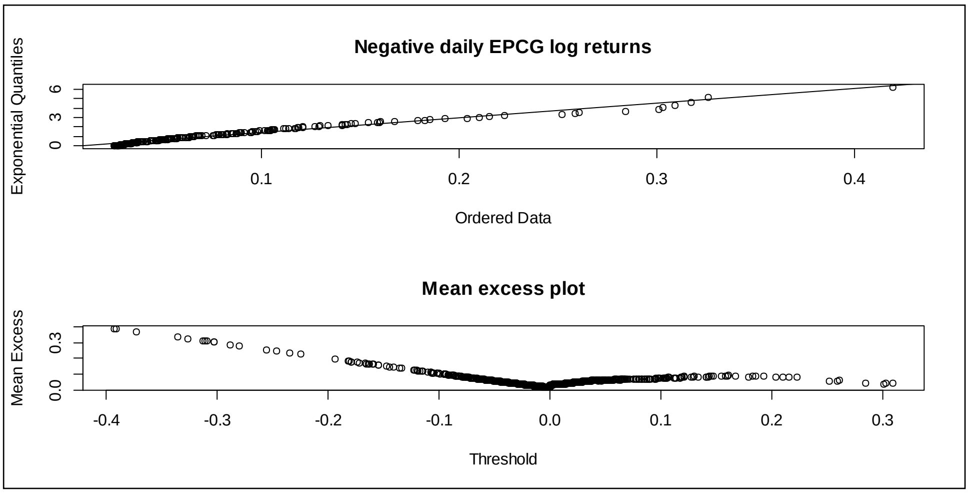

Fig. 4 Q-Q plot of excess over 2.5% threshold and mean excess function plot

Table 4 Result assessments of two-dimensional Poisson model of EPCG daily negative log returns[1]

Threshold	Exceeding	γ_n	α_n	β_n
3%	213	0.271 (0.084)	0.02 (0.0056)	-0.0383 (0.012)
2.5%	251	0.31 (0.078)	0.017 (0.04)	-0.03058 (0.008)
2%	315	0.437 (0.072)	0.0109 (0.002)	-0.015 (0.0045)

[1]Standard assessment errors are given in parentheses.

The set of extreme events exceeding the 2.5% threshold has 251 data. Based on this data set, the distribution of maximal negative logarithmic returns for EPCG stocks is modeled. Table 4 contains the estimation of parameters γ, α and β for the given data set, with the variation of threshold from 2% to 3%. Given parameters are used for the calculation of VaR and the adequacy of the given model can be based on plots which can be seen in Fig. 5.

The four plots show good accomodation of generalized Pareto distribution to the data. Q-Q plot (lower right graph) shows slight derogations from the straight line, which is also confirmed by tail probability assessment on the logarithmic scale (lower left graph), leading to conclusion that the modelling is appropriate.

Peaks over threshold method gives results for VaR and expected shortfall, summed up in Table 5. It is concluded that parameter results are more stable compared to econometric modeling (GARCH model and RiskMetrics), which shows parameter estimation variations depending on the choice of type of GARCH model (GARCH(1,1) or IGARCH(1,1)). It is evident here that results of VaR and expected shortfall differ less depending on different values of threshold excess, and with the same probability assessment. General conclusion is that this approach is superior to the econometric evaluation of VaR.

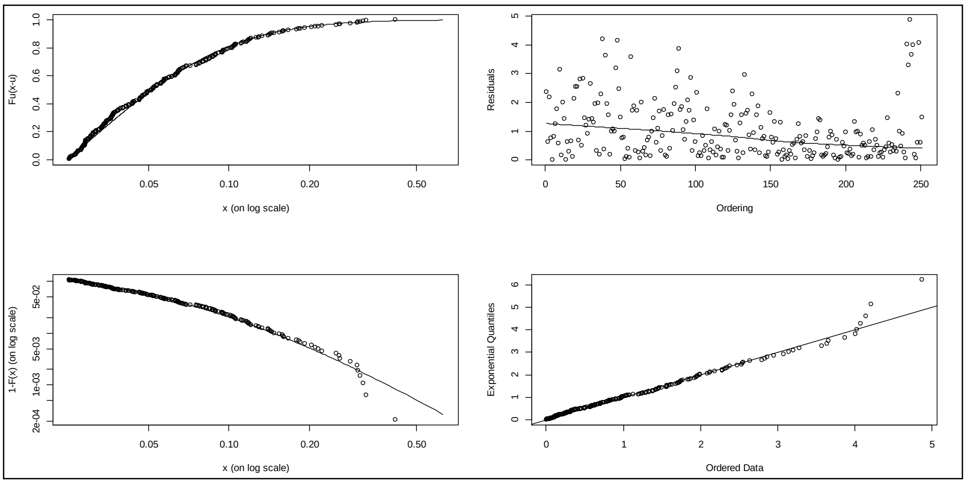

Fig. 5 Plots of generalized Pareto distribution adjustment to EPCG daily negative log returns

Table 5 Assessments of results of VaR and expected shortfall based on peaks over thresholds method

Threshold	*p*-value	VaR	Expected Shortfall
	0.05	0.05490012=5.49%	0.1185879=11.86%
2.5%	0.01	0.14661115=14.66%	0.2514971=25.15%
	0.001	0.389812=38.98%	0.603948=60.39%
	0.05	0.0526414=5.26%	0.1246470=12.46%
2%	0.01	0.1473389=14.73%	0.2927310=29.27%
	0.001	0.4724329=47.24%	0.8697588=86.98%
	0.05	0.05526823=5.53%	0.1179026=11.79%
3%	0.01	0.14737246=14.74%	0.2443104=24.43%
	0.001	0.37328683=37.33%	0.5543650=55.44%

Further, in order to compare the results, Value at Risk obtained using different calculation methods can be summed. If we possess 1000€ worth EPCG stocks, with 5% level of significance, meaning there is 95% probability the loss would be lower or the same as VaR for the following trading day, the parameter value is: 1) 74.83€ applying ARMA(1,3)-GARCH(1,1) model; 2) 81.6915€ applying RiskMetrics method; 3) 53.95€ by quantile estimation, and 4) 54.9€ applying peaks over threshold method (threshold is 2.5%).

The corresponding parameter values with 1% probability are: 1) 106.5€ applying ARMA(1,3)-GARCH(1,1) model; 2) 115.16€ applying RiskMetrics method; 3) 146.4244€ by quantile estimation, and 4) 146.6€ applying peaks over threshold method (threshold is 2.5%).

Due to different treatment in the estimation of statistic distribution tail behavior, there are different results obtained as well. The result of econometric assessment (ARMA-GARCH models and RiskMetrics), in case all assumptions for its applications are accomplished, depends on the chosen model. Therefore, it is necessary, as we have shown on the example, that a detailed analysis of the specification of potential models is the first phase in the performance of Value at Risk evaluation. It can be concluded that econometric estimation proved to be unstable, as they are on the upper bound at 5% significance level, and at 1% significance level they are on the lower bound of possible VaR movement interval.

Further, the choice of tail distribution probability also has an important role in the calculation of VaR. The value of the observed sample of 2338 data may be considered big enough for empirical quantiles with 99% and 95% probability for giving good parameter estimation. For both levels of significance, quantile Value at Risk evaluation is very close to the assessment of the new approach of extreme value theory. We note that these two assessments at 1% significance level are on the upper bound of the possible VaR parameter range.

Also, within the latter approach (Table 5), we can see that using a very low 0.1% probability, less reliable VaR evaluation are obtained. Therefore, that significance level was not used in other approaches.

CONCLUSION

Results of empirical analysis have multiple benefits. They show that *the assessments of Value at Risk based on extreme value theory are better than econometric evaluations.* It is obvious that econometric evaluations proved to be very unstable at the assessment of Value at Risk. Results showed that at 5% significance level, given evaluations are on the upper bound, and at 1% significance level, they are on the lower bound of possible Value at Risk movements. Therefore, it is not possible to say they either underestimate or overestimate the given parameter, but the estimation significantly changes depending on the level of confidence.

Taking these results into account, a suggestion can be given to financial institutions to quantify risk using several methods: peaks over thresholds method (the latest approach of extreme value theory), historical evaluation (quantile) – for large samples, and RiskMetrics method (containing econometric method). For the purpose of simplicity, risk estimation can be focused on these three methods as they have been proven to be the best regarding the range within which real value of VaR parameter can move. As it was said earlier, the real value of this parameter cannot be observed, so it is difficult to single out one estimation method as the best one.

Furthermore, these results refer to Montenegrin stock market, that is small emerging economy and the results obtained in the analysis should be limited on emerging economies and financial markets that are still developing. These markets are characterized by a greater influence of internal trade and high volatility compared to developed countries, so evaluation of VaR with standard methods that assume a normal distribution is much more difficult. Also, the observation period for measuring Value at Risk includes period of financial crisis, so that fact should be taken into account because of possible derogation of parameter results.

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MERENJE PARAMETRA VREDNOSTI PRI RIZIKU I TEORIJA EKSTREMNIH VREDNOSTI: DOKAZ NA PRIMERU CRNE GORE

Koncept vrednosti pri riziku je mera koja se sve više koristi za ocenu maksimalnog gubitka finansijske pozicije u određenom vremenskom periodu za datu verovatnoću. Cilj ovog rada je da pokaže najnovije pristupe merenja tržišnog rizika. Konkretno, rad ispituje da li je teorija ekstremnih vrednosti bolja od ekonometrijskog računanja VaR-a na berzanskim tržištima koja su u razvoju kao što je crnogorsko. Rad je motivisan željom da se riziku u Crnoj Gori posveti potrebna pažnja. Analiziran je dnevni prinos izraženo volatilne akcije EPCG (Elektroprivreda Crne Gore) sa Montenegroberze u periodu od januara 2004 do juna 2013. Ovakav uzorak i njegova vremenska dimenzija do sada nije razmatrana empirijski u literaturi. Stoga, neophodno je iskoristiti iskustva razvijenih svetskih finansijskih institucija koje studiozno pristupaju upravljanju rizikom, kao i najnovijih teorijskih znanja.

Ključne reči: *Teorija ekstremnih vrednosti, Vrednost pri riziku, teški repovi, GARCH model, prekoračenje iznad datog praga, generalizovana Pareto raspodela.*