On the application of the Theory of Critical Distances for prediction of fracture in fibre composites

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ABSTRACT. This paper is concerned with the fracture of composite materials containing stress concentration features such as notches and holes. In particular, it addresses the question of the use of the Theory of Critical Distances (TCD) – a method which is widely used for predicting notch effects in fatigue and fracture. The TCD makes use of a length constant, L, known as the critical distance, which is normally assumed to be a material property. However, many workers in the field of composite materials have suggested that the critical distance is not a constant, but rather is a function of notch size. I examined the evidence for this assertion, and concluded that it arises for four different reasons, two of which (process zone size and constraint) are real material effects whilst the other two (choice of test specimen and estimation of the stress field) arise due to errors in making the assessments. From a practical point of view, the assumption of a constant value for L leads to only small errors, so it is recommended for engineering design purposes.

KEYWORDS. Fibre composites; fracture; notch; hole; critical distance

INTRODUCTION

When engineering components fail, they almost always do so from stress concentration features: geometrical discontinuities such as holes, notches and corners. Fibre composite materials are no exception, and much work has been done over the years to understand and predict the effects of these features on the load-bearing capacity of these materials. This paper is concerned with one particular method of prediction, which goes by various names but which I will call the Theory of Critical Distances (TCD). Here I will consider the application of this theory to the broad range of long-fibre laminate-type composite materials, and from the outset I should point out that I do not consider myself an expert on this class of materials. In that respect the paper is being written from the outside looking in, and I apologise in advance for any errors or misunderstandings that may arise as a result.

My investigations into the TCD began in the field of metal fatigue, where the approach has been used for over half a century [1, 2]. Examination of the published literature revealed that the same methodology was also being applied to predict monotonic fracture in composites, since first being proposed by Whiney and Nuismer in 1974 [3]. Further reading showed that work in the two areas (metal fatigue and composite fracture) has proceeded on parallel lines for the last thirty years, both in fundamental research and in industrial applications, with workers in one field being apparently unaware of the activities of those in the other. As a result, the approach has developed some particular characteristics: for example in the field of metal fatigue it is generally assumed that the critical distance, L, which is the fundamental parameter in the theory, is a material constant, unaffected by the geometry of the notch. In composites research, however, it has become common to assume that L is not a material constant, but rather that it varies with the size of the notch.

This question is of fundamental importance because the theory is much easier to use if we can assume a constant value for L. If a constant value of L cannot be accepted then more fundamental studies are needed to develop a general approach which would allow L to be calculated for any problem. Some workers, including ourselves, have indeed proposed such approaches [4, 5] and they have been found to be necessary in certain other materials, such as concrete, where the critical distance can be so large as to be similar to the size of the test specimen.

In this paper, I consider the evidence for and against the use of a constant L value in continuous-fibre composite laminate materials, both from a fundamental scientific perspective and from the viewpoint of the practical engineering application of the TCD.

THE TCD: A BRIEF INTRODUCTION

For those not familiar with critical distance methods, here follows a brief introduction. A recent paper provides further information [6] and those interested in a more comprehensive review are directed to a recent book on the subject [7]. In the great majority of cases, the TCD is implemented using a linear elastic stress analysis. The point of maximum stress is located (e.g. at the root of a notch) and a line is drawn from this point which is known as the focus path. Stress is plotted as a function of distance, r, along this line. There are two different variants of the approach, which I refer to as the Point Method and the Line Method. In the Point Method, the stress is considered at a single point, located at a distance of r=L/2. In the Line Method, the stress to be considered is the average stress along the line from r=0 to 2L. Failure is predicted to occur if this stress is greater than some critical value, σ_o. Fig. 1 illustrates these approaches schematically.

There are other variants of the TCD; for example some workers use L in a modified form of linear elastic fracture mechanics LEFM) in which L is considered to be the length of an imaginary crack at the notch root, or alternatively the crack is considered to advance in finite growth steps of magnitude 2L. These methods do not concern us here, except in so far as they can be combined with the stress-based methods to give approaches in which L is no longer a constant.

The point and line methods have the great advantage of simplicity: they can be very easily used in conjunction with finite element analysis and applied to any type of stress concentration feature, including those on engineering components. Extensive research has shown that they can give very accurate predictions in a wide variety of materials, for those mechanisms of failure which involve cracking, such as brittle fracture and fatigue [7]. An important relationship exists between the two constants in the TCD and the material's fracture toughness, K_c. This relationship can be derived by assuming that the TCD is applicable to cracks as well as notches:

$$L = \frac{1}{\pi}\left(\frac{K_c}{\sigma_o}\right)^2 \tag{1}$$

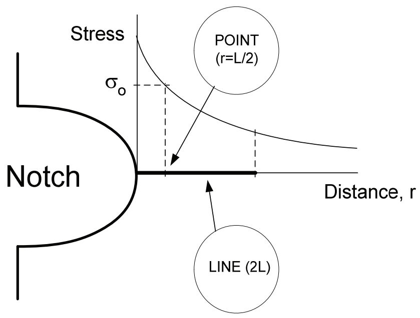

Figure 1: Schematic illustration of the Point Method and Line Method

APPLICATION OF THE TCD TO COMPOSITE MATERIALS

The use of this approach in the field of composites stems from the seminal paper by Whitney and Nuismer in 1974 [3] which was followed a short time later by a slightly more detailed treatment in a book by Whitney *et al* [8]. The original paper has been extremely influential in this field: at the time of writing there have been over 300 citations to this paper, in publications ranging from fundamental studies to engineering applications. The paper is very comprehensive, describing both the PM and the LM, which Whitney and Nuismer referred to as the Point Stress Criterion and the Average Stress Criterion. The validity of the method was tested against experimental data: Figs 2 and 3 are examples reproduced from the original paper. The theoretical link to K_c (as in Eq. 2 above) was also derived. Importantly, this paper established that the critical stress σ_o was equal to the ultimate tensile strength of the material as measured in tests on plain (i.e. unnotched) specimens. In other classes of materials this is not always the case but it appears to be true universally for composite laminates, and to my knowledge no one has proposed otherwise. A significant limitation of the paper was that only two types of notches were considered: circular holes located centrally in flat plates, and sharp edge notches in which the notch root radius was very small.

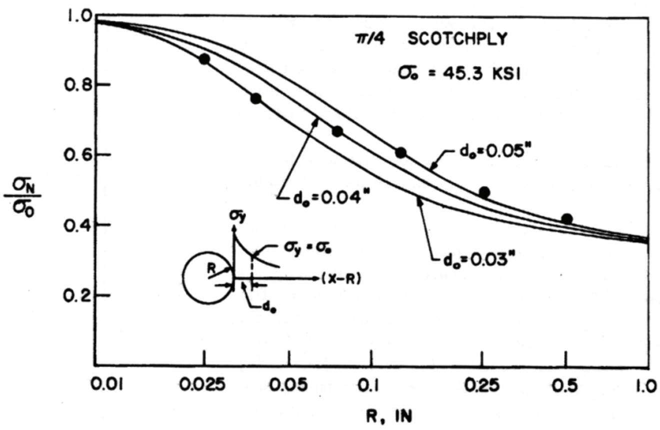

Figure 2: Data and predictions (using the point method) from Whitney & Nuismer [3], showing the effect of hole radius on fracture strength (nominal stress, normalised by the plain-specimen strength) in a quasi-isotropic glass/epoxy laminate. The three prediction lines were drawn using slightly different values of the critical distance, d_o which is equivalent to L/2.

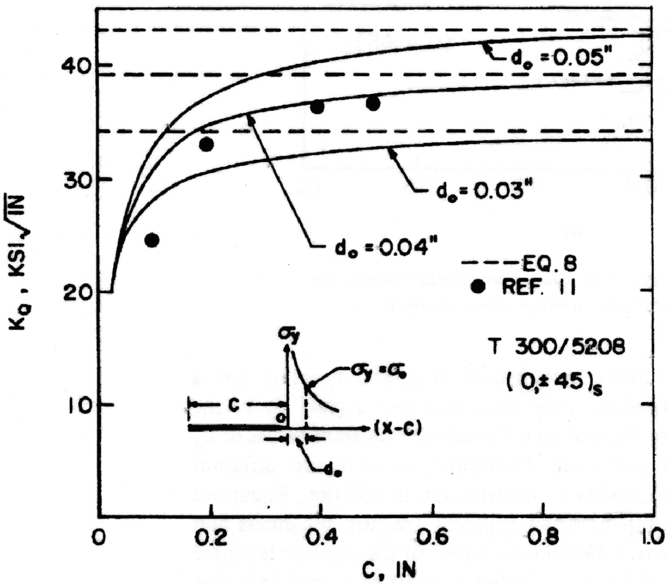

Figure 3: Data from Whitney and Nuismer [3] showing the effect of notch length (c) on fracture toughness, for sharp notches in a graph-epoxy laminate material. Prediction lines using the point method, drawn at different values of the critical distance d_o (equivalent to L/2).

Within a decade of the publication of this original paper, a lot of experimental data had been generated to demonstrate the truth of Whitney and Nuismer's proposal. Awerbuch and Madhukar [9] presented an enormous study covering over 2800 test results, and Wetherhold and Mahmoud [10] also considered a large set of data, both reports showing that the TCD could be applied successfully to a wide range of materials, mostly polymer-matrix long-fibre materials but also including some metal-matrix composites and some materials with discontinuous fibres. Interestingly, despite the wide range of strengths and toughnesses in these materials, it was found that L fell within a narrow range of values, being almost always between 1 and 5mm, sometimes as high as 15mm.
More recent literature has extended the subject in various ways, which I have reviewed elsewhere [7]. However, the overwhelming impression is that most workers have followed closely the lead taken by Whitney and Nuismer. As a result,

most papers are confined to studies of tensile test specimens containing either circular holes or very sharp edge notches. The limited range of test specimens studied is, in my view, the source of some misconceptions about the use of the TCD. This contrasts with work in the parallel field of metal fatigue, where different types of notches have been investigated, especially with regard to the effects of notch root radius, along with different types of loading.

DOES L VARY WITH NOTCH SIZE?

The large amount of data collected by Awerbuch and Madhukar allowed various correlations to be studied. One trend which emerged was that in some cases the value of L which best predicted the results tended to vary, increasing with the size of the hole or notch. These effects seemed quite significant, for example these workers show a case in which changing the length of a sharp notch from about 1mm to 20mm caused the best-fit value of L to approximately double in size. Further work was done by other researcher, to investigate this phenomenon, especially for the case of circular holes. As a result, two equations were developed which are now in common use. The first is that of Karlak [11], which relates L to the hole diameter (a) using a constant C_1, as follows:

$$L = C_1 a^{1/2} \tag{2}$$

The second equation is that proposed by Pipes *et al* [12] who developed a more general relationship including another constant m:

$$L = C_2 a^m \tag{3}$$

This second equation covers a wide range of possible conditions: two interesting cases are m=0, for which L becomes a material constant and m=1 which leads to a situation in which the size of the hole has no effect on the fracture strength, since L scales in direct proportion to a. In the example mentioned above, from Awerbuch and Madhukar, the value of m was 0.235. In fact, even the original data in Whitney and Nuismer shows something of this effect: for example in Fig.2 prediction lines were drawn using different values of the critical distance and one can see that the data tend to move from the smallest to the largest value with increasing hole size.
Investigating the data and the methods of analysis in some detail, I have come to the conclusion that there are four separate reasons for this effect, as follows.

Stress Analysis Errors

In calculating the stresses in the vicinity of the notch, for use in the Point Method or Line Method, Whitney and Nuismer used the following approximate method. They started from the equation for a notch in an infinite body: for example for a circular hole they used the well-known Airy equation for the stress σ(r) as a function of distance r :

$$\sigma(r) = \sigma\left(1 + \frac{1}{2}\left(\frac{a}{a+r}\right)^2 + \frac{3}{2}\left(\frac{a}{a+r}\right)^4\right) \tag{4}$$

They then modified this equation to take account of the finite width of the plate, W, multiplying it by the following factor Y:

$$Y = \frac{2 + (1 - a/W)^3}{3(1 - a/W)} \tag{5}$$

The same approach has been followed by many subsequent researchers in this field. Unfortunately, this approach is not precise, and leads to significant errors. Fig. 4 compares the stress/distance curve predicted by these equations to an accurate result obtained using finite element analysis, for the case of a hole with a/W = 0.375. The two curves begin to deviate significantly around r/a = 1. Unfortunately, many test specimens use a/W values equal to or greater than this, and in many cases the relevant values of r are quite large, given that L typically takes a value of several millimetres in these materials.
One can appreciate that this error will lead to a situation in which L appears to increase with notch size, because if notch size increases (at constant W) then the estimated stress at the point L/2 will deviate more and more from the actual stress,

leading to an erroneously low prediction of the fracture strength of the specimen, an error which can apparently be corrected by letting L increase.

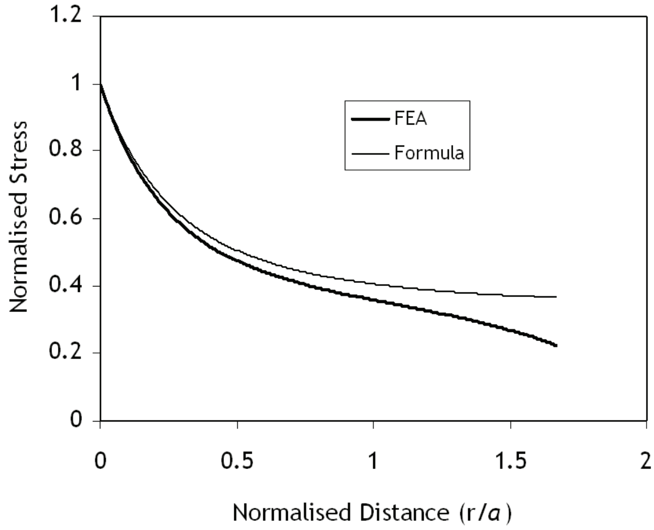

Figure 4: Stress as a function of distance from notch root, calculated using the approximate formula (Eq. 5), compared to an accurate result obtained from FEA.

Choice of Test Specimen

Fig. 5a shows stress/distance curves calculated for the specimens tested by Whitney and Nuismer (as reproduced here in Fig. 2), for loading conditions corresponding to fracture of the specimen in each case. If the Point Method is exactly correct then all of these curves should pass through a single point, at which r = L/2 and the stress is equal to the UTS (represented here by a horizontal dashed line). Based on this data one would conclude that there is a tendency for L to increase with increasing hole radius, by about a factor of 2. However, the situation changes drastically if we add data from specimens containing sharp notches (see Fig. 5b). The sharp-notch data shows no such trend, and the fracture strength of all the specimens can be predicted using a constant value of L, with a prediction error of no more than 10%.

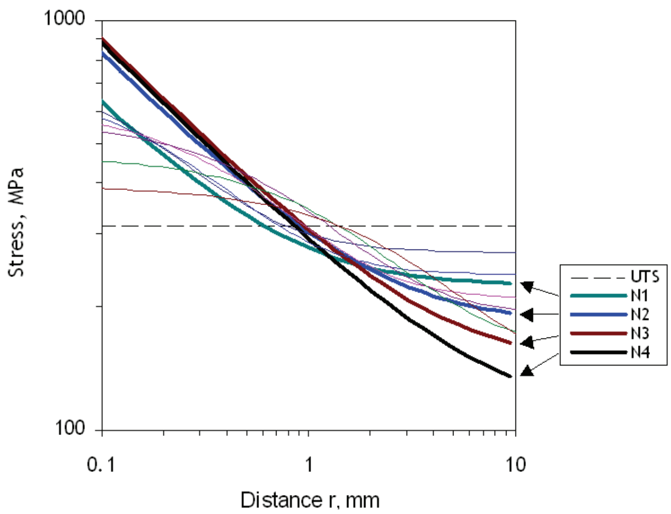

Figure 5a: Stress-distance curves at failure for specimens containing holes as shown in Fig.2. The symbols R1-R6 indicate increasing hole radius.

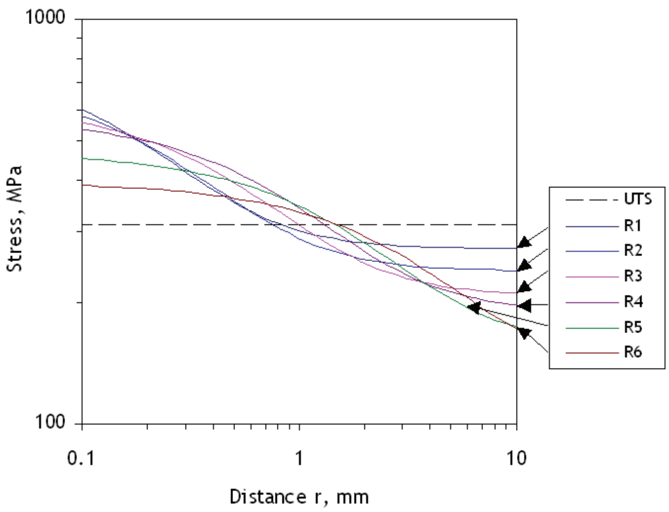

Figure 5b: The same data as in Fig.5a, plus lines representing the stresses in sharply-notched specimens of the same material. The symbols N1-N4 represent increasing notch depth.

Many workers have based their conclusions solely on data from circular holes. This is a mistake because the stress gradients in these specimens are quite shallow, so it is difficult to obtain an accurate value of L in any case, since the estimate relies on finding the point at which the stress/distance curve crosses the horizontal line representing the UTS.

Sharper notches are better in this respect because they give steeper gradients, and the best strategy is to use at least two different notch types, as shown here.

Process Zone Size

The above two effects arise essentially due to errors or inaccuracies, however there are also some reasons why the value of L would tend to increase in reality. The first of these relates to the size of the process zone. To illustrate this I have chosen some data from Kennedy et al [13], who tested centre-notched plates of an orthotropic graphite/epoxy composite, using very sharp, crack-like notches. I have chosen this data because it shows the largest change in L which I have been able to find. According to these workers, L changed by a factor of 3, from 8.4mm to 24.4mm, when the length of the notch was increased from 6.35mm to 305mm. The value of *a*/W was kept constant at 0.25, which is convenient because it means we can rule this out as a complicating factor.

The value of L gives an approximate estimate of the size of the process zone, or damage zone that occurs ahead of the notch prior to failure. From this we can conclude that the larger specimens were failing under LEFM conditions because the size of the damage zone at failure was much smaller than the remaining ligament (W-*a*). However this is not the case for the smaller specimens, for which L was a significant proportion of (W-*a*), and for the very smallest specimen it is likely that the process zone had spread completely across the specimen width before failure. We have encountered similar situations before, most obviously in the case of building materials such as concrete, which have equally large L values of the order of 5-10mm. If the specimen size is particularly small then this can lead to the absurd situation in which the critical point (or part of the critical line) lies outside the specimen. In such cases if the TCD can be used at all it must be with a smaller value of L. Approaches developed by myself and colleagues [5] and also by Leguillon [4], allow L to vary in such cases by using two failure criteria – one stress based and one stress-intensity based, which are assumed to apply simultaneously. The details of the approach are beyond the scope of the present paper: suffice it to say that the result is an L value which is constant when the remaining ligament (W-a) is much larger than L, but changes in size in such a way that it remains always smaller than (W-a).

These modified approaches can be applied to problems of the type shown above, and should be able to give improved predictions. However, it may not be worth the trouble. Regarding the data from Kennedy *et al*, which as I said showed the largest variation in L of any which I could find for composite materials, if we use a constant L value it is possible to predict all the data with errors no greater than 13% on stress. This seems strange at first but the anomaly is resolved by noting that the stress distance curves are quite shallow, even for relatively sharp notches, so a large change in distance r gives only a relatively small change in stress. Consequently it is permissible to make a relatively large error in the value of L because this will lead to only a small error in the predicted strength.

Constraint Effects

When reading articles on composite materials I was struck by the fact that little attention seems to be given to possible changes in constraint that arise when changing specimen thickness. In metallic materials the measured fracture toughness can change considerably if thickness is reduced in such a way as to reduce the out-of-plane constraint, changing from plane strain to plane stress conditions. Some workers have reported this effect in composite materials, but in most papers it is not mentioned, and Awerbuch and Madhukar actually reported a case of the opposite effect, whereby the measured toughness increased with increasing specimen thickness [9]. Given that most composite-laminate specimens tested are quite thin, one would expect that they are experiencing either plane stress or conditions which are intermediate between plane stress and plain strain. The change in K_c is due largely to changes in the degree of triaxiality in the plastic zone, and though the polymer and metal matrices of these composites will yield, it is possible that these effects are modified by the existence of microdamage in these zones.

Considering the relationship between fracture toughness and L (Eq. 1 above) one would expect L to increase on moving from plane strain to plane stress, and we showed previously that this is indeed the case for brittle fracture in metals [14]. A feature of small cracks in all materials is that they have lower fracture toughness values than large cracks. This effect occurs if the crack length is similar to, or less than, L. Such cracks will require less stress intensity to cause failure, so for a given specimen thickness they will experience more constraint, and hence can be expected to show a smaller value of L.

CONCLUSIONS

1) Some apparent changes in the critical distance L with notch size reported in the literature arise due to inaccuracies caused by the choice of test specimen and the use of imprecise methods of stress analysis.

2) We can, however, expect that L will change with notch size and other aspects of specimen geometry, if this causes changes in constraint or if the process zone size is a significant proportion of the remaining ligament.
3) Despite these changes, the assumption of a constant value of L leads to only small errors in the prediction of fracture stress in long-fibre composite laminate materials, so this approach is recommended for engineering applications.

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