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MATHEMATICAL MODELING OF THERMAL PROCESSING OF MEAT PRODUCTS

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Abstract: *The article presents the mathematical modeling for sausage products heat treatment during thermal processing in steamer cooking chambers in meat processing plants.The sausage and meat cooking processing period has been divided into several stages, each stage duration being predefined by heat transfer analytical solving for any particularly sized sausage product. Sequential thermal process stages include drying stage, smoking stage and the cooking sausage product stage. The duration of each sausage thermal processing stage can be calculated by proposed empirical equations. Calculating the duration of the sausage product thermal cooking stages, the initial temperature condition of the product is considered to be characterized as the temperature averages. Specially elaborated thermal calculation algorithm for sausage product heat treatment allows estimating the thermal energy consumption for any given cooking processing, considering the changes in technological parameters.Thus, the mathematical model is proposed to be applied to thermal (structural) design and engineering calculations for the above named steamer cooking chambers.*

Keywords: *heat treatment, meat (sausage) products, mathematical modeling, thermal (structural) design calculation of the cooking chambers.*

1. Introduction

The process of heat treatment of meat (sausage) products is carried out in a steamer cooking chamber and contains three (3) steps: warming through(drying), (smoking)roasting and cooking. The process is completed when the core temperature reaches 72 ° C [1]. Selecting heat treatment modes(for different stages) not only the recommendations of equipment manufacturers have to be taken into account, but also theoretical knowledge for each stage of treating meat products is essential.

The aim of this work is to create a mathematical model of the meat (sausage) product thermal processing, and implement it to optimize of thermal and structural design procedure of steamercookingchambers.

2. Matherials and methods

Any sausage product can be considered to have oval-cylindrical shape,and be introduced as a figure (Fig.1), where Rh - radius.

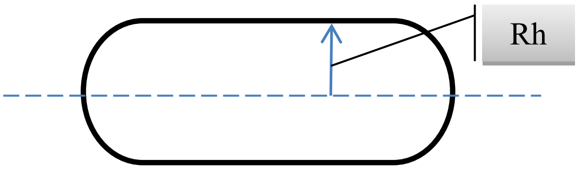

Fig. 1. Sausage product shape

Where D is sausage product diameter, i.e.*D=2·Rh*, and acceptinglength L being 1meter. In case the product has an oval, square or rectangular form in cross-section, we take the diameter of the smallest radius(Fig.2) for a diameter.

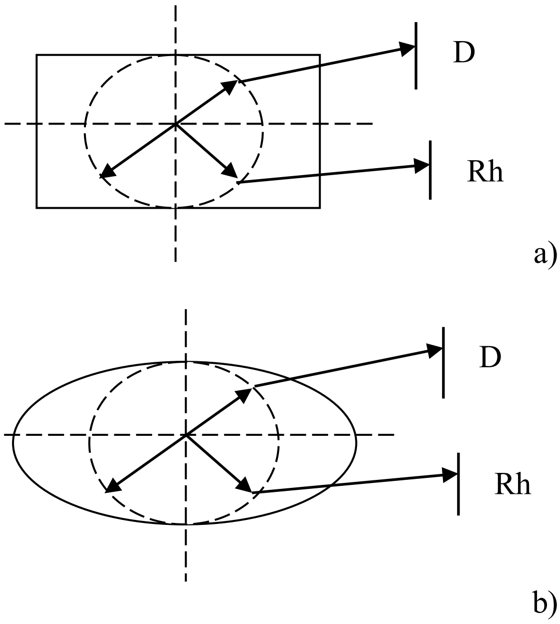

Fig. 2. Cross-section of the sausage product: a) rectangular shape; b) oval shape

Considering $Rh = R_0$, where R_0– is the diameter of the product at the initial stage of heat treatment.

The total duration of the heat treatment process of cured/boiled and/or smoked raw-cooked sausage products is specified using the following formula:

$$\tau_{total} = \omega_1 \cdot \tau_{drying} + \omega_2 \cdot \tau_{smoking} + +\omega_3 \times \tau_{cooking}, \qquad (1)$$

where τ_{drying}–the warming-though (drying) time; $\tau_{smoking}$–the smoking and roasting time; $\tau_{cooking}$–the time for sausages cooking, ω_i–availability ratio of I–stage of heat treatment (i=1 – warming-through (drying), i=2 – smoking and roasting; i=3 – cooking) in the product treatment processing. The numerical value of the coefficient is $\omega_i = [0.1]$ where $\omega_i = 0$ – in the non- heat treatment processing stage, and $\omega_i = 1$ – in the heat treatment processing stages.

At each stage of the sausage product heat treatment process its diameter (radius R_h)changes its value according to the i-stage of processing and is calculated by the empirical formula:

$$Rs_i = K_i \cdot R_0, \qquad (2)$$

where K_i– the coefficient characterizing the increase (growth) of the sausage product radius during its heat treatment. Therefore accepted values are $K_1 = 1$, (i=1), $K_2 = 1.023$ (i=2), $K_3 = 1.045$ (i=3), [1].

3. Results and discussion

Let us consider the duration of each stage of the heat treatment.

1) The drying sausage processing stage.

The duration of the stage of sausage product warming-through (drying) is the amount of time spent on the curingwarming-through and drying of the product (τ_{drying}) and the time required to evaporate condensation from sausage casing developinginchamber ($\tau_{condensation}$):

$$\tau_{drying} = = \tau_{warm-thr(drying)} + 60 \cdot \tau_{condensation}, \qquad (3)$$

Time calculating $\tau_{warm-thr(drying)}$is carried out using the following formula:

$$\tau_{(\text{warm-thr(drying)}} = \frac{F_{0,drying} \cdot Rs^2}{a} = = \frac{F_{0,drying} \cdot R_0^2}{a} = \frac{F_{0,drying} \cdot Rs_2^2}{a}, \quad (4)$$

where $F_{0,drying}$–the duration of drying in dimensionless form; a – temperature conductivity coefficient of the sausage product; for cured/boiled and/or smoked raw-cooked sausages $a=5\cdot10^{-4}$ m/min, [1].

Fourier criterion value $F_{0,\ drying}$, corresponding to the time required to reach the set-pointcore temperature of the product (in the center of sausage loaf) is calculated using the following formula:

$$F_{0,drying} = \left(\frac{Bi_{drying} + 4}{8 \cdot Bi}\right) \times \times \left(ln\left(\frac{2}{(Bi_{drying}+2)\cdot(1-T_{n,drying})}\right) + F_0^{'}\right), \quad (5)$$

where Bi_{drying}– Biot criteria for drying stage; $T_{n,drying}$– dimensionless value of the surface product temperature during drying; F_0'– the time of the "temperature front" passage, defined in nomogram [1,5].

The value of the Biot criteria is specified using the following formula:

$$Bi_{drying} = \frac{\alpha_{drying}}{\lambda} \cdot R_0 = 25.47 \cdot R_0, (6)$$

where α_{drying} – the heat-transfer coefficient during heat treatment; λ – thermal conductivity coefficient (λ=0.465 W/m*K is accepted for cured/boiled and/or smoked raw- cooked sausages), [1,6].

The heat- transfer coefficient can be specified by the following expression:

$$\alpha_{drying} = \alpha_{environment} \cdot (1 + 1.9 \cdot d) = = (6.16 + 4.49 \cdot W) \cdot (1 + 1.9 \cdot d) = = 11.84 \text{W/(m}^2\cdot\text{K)}, \quad (7)$$

where $\alpha_{environment}$ – the heat-transfer coefficient of the vapor environment in a heat chamber, defined by the empirical Yurhes formula [2,3]:

$$\alpha_{environment} = 6.16 + +4.49 \cdot W \, W/(m^2\cdot K),$$

where d – the moisture content of the product (sausage product) calculated in kg of moisture(water) per kg of the environment with chamber environment temperature of $t_{environment}$=100°C and relative air humidity φ=10 % the moisture content d=0,076 kg/kg.

F_0'–the time of the "temperature front" passage can be calculated by the above nomogram [1,6], but having defined the specification of the Bio criteriaBi_{drying} according to the formula (6), can also be specified using the following expression:

$$F_0' \approx 0.7 \cdot \left(\frac{1}{12} + \frac{1}{3\cdot Bi_{drying}} - \frac{2}{3\cdot(Bi_{drying})^2} \times \times ln(1 + 0.5 \cdot Bi_{drying})\right) = 0.7 \times \times \left(\frac{1}{12} + \frac{1}{3\cdot 25.47\cdot R_0} - \frac{2}{3\cdot(25.47\cdot R_0)^2} \times \times \ln(1 + 0.5 \cdot 25.47 \cdot R_0)\right) = 0.7 \cdot \left(\frac{1}{12} + \frac{1}{76.41\cdot R_0} - \frac{2}{1946.16\cdot R_0^2} \times ln(1 + 12.47 \cdot R_0)\right), \quad (8)$$

The dimensionless temperature value of the product surface while drying is specified using the following formula:

$$T_{n,drying} = \frac{t_{end,dtying} - t_{0,drying}}{t_{environment,dtying} - t_{0,dtying}}, \quad (9)$$

where $t_{end,\ drying}$– the temperature of the product surface (sausage loaf) at the end of the drying process, °C; $t_{0,\ drying}$–the initial temperature of the sausage product surface (it is assumed that $t_{0,\ drying}$=15°C),°C. The environment temperature $t_{environment,\ drying}$ is specified by the technological requirements foreachof appropriate stage of sausage product heat treatment.

Substituting the equations (6) and (9) in the formula (5), we obtain the Fourier criterion value corresponding to the time required to reach the set-point temperature in the center of sausage product:

$$F_{0,drying} = \left(\frac{25.47 \cdot R_0 + 4}{8 \cdot 25.47 \cdot R_0}\right) \times \left(ln\left(\frac{2}{(25.47\cdot R_0+2)\cdot\left(1-\frac{t_{end,dtying}-t_{0,drying}}{t_{environment}-t_{0,dtying}}\right)}\right) + F_0^{'}\right), \quad (10)$$

Otherwise, F_0'value can be determined using nomogram and taking into accountcriteria Bi_{drying} and the parameter $\rho = \frac{Bi_{drying} - T_{n,drying} \cdot (Bi_{drying} + 2)}{Bi_{drying} \cdot (1 - T_{n,drying})}$, that allow you to determine F_0'.

The time required for evaporation of condensate from the outer surface of the sausage product can be specified using the following formula:

$$\tau_{condensate} = (2 - 0.46 \cdot Bi_{drying}) \times \times (21 \cdot T_{n,drying} - 8) = = (2 - 0.46 \cdot 25.47 \cdot R_0) \times \times (21 \cdot T_{n,drying} - 8), \quad (11)$$

Taking into accountthe equations (4) and (11), the drying stageduration can be specified using the following formula:

$$\tau_{drying} = \frac{F_{0,drying} \cdot R_0^2}{a} + 60 \cdot \tau_{condensate} = \frac{\left(\frac{Bi_{drying}+4}{8 \cdot Bi_{drying}}\right) \cdot \left(ln\left(\frac{2}{(Bi_{drying}+2) \cdot (1 - T_{n,drying})}\right) + F_0'\right) \cdot R_0^2}{a} + 60 \cdot \tau_{condensate}, \quad (12)$$

2). *The smoking (roasting) stage of the sausage product*

The duration of the smoking(roasting) stage τ_{frying}can be specified using the following formula:

$$\tau_{flying} = \frac{F_{0,smoking} \cdot Rs_2^2}{\alpha_{smoking}}, \quad (13)$$

where$Rs_2 = R_0 \cdot K_2$–the average diameter (radius) of the product at the end of thesmoking(roasting) stage.

The value of the Biot criteria for the smoking(roasting) stage is specified using the following formula

$$Bi_{smoking} = \frac{\alpha_{smoking}}{\lambda} \cdot Rs_2 = \frac{\alpha_{smoking}}{\lambda} \cdot R_0 \cdot K_2, \quad (14)$$

Where $\alpha_{smoking}$ – is the heat-transfer coefficient of the vapor environment to the sausage product at the smoking (roasting) stage, defined by the above empirical Yurhesformula (7):

$$\alpha_{smoking} = \alpha_{drying} = \alpha_{environment} \times \times (1 + 1.9 \cdot d) = 11.84 \text{W/(m}^2\text{·K)};$$

$\lambda -$ is the thermal conductivity coefficientofcured/boiled and / or smokedsausages, which is also accepted for the warming-through and dryingstage λ = 0.465 W/m·K) [1].

The smoking/ roasting stage duration of sausage products can be finalized by the following expression:

$$F_{0,smoking} = \left(\frac{Bi_{smoking} + 4}{8 \cdot Bi_{smoking}}\right) \times \times \left(ln\left(\frac{2}{(Bi_{smoking}+2) \cdot (1 - T_{n,smoking})}\right) + F_0' - -F_{0,drying}\right), \quad (15)$$

where

$$T_{n,smoking} = \frac{t_{end,smoking} - t_{0,smoking}}{t_{smokingenvironment} - t_{0,smoking}}, \quad (16)$$

3). *The cooking stage of the sausage product.*

The dimensional cooking stage duration $\tau_{cooking}$ of the sausage products is specified using the formula (4), and the Biot criteria using the formula (6).

The diameter (radius) of the sausage product (loaft) at the cooking stage is specified using the empirical relation:

$$R_{cooking} = R_0 \cdot K_3 = 1.045 \cdot R_0$$

The heat-transfer coefficient value of the vapor environment to the outer sausage product surface can be specified using the following equation:

$$\alpha_{cooking} = \alpha_{environment} \cdot (1 + 1.9 \cdot d) = = 35.68 \text{W/(m}^2\text{·K)},$$

where d – the moisture content of the product (sausage product) with vapor environment temperature in the chamber; $t_{environment}$=85 °C and relative air humidity φ=90 %. According to the technological requirements for the heat treatment process of sausage products, the above parameters of the environment $t_{environment}$ and φ must be steadily maintained during the whole cooking stage [3]and have to meet following values: the moisture content of d=0.7653 kg/kg, $t_{environment}$=85 °C and φ=90%.

The value of the Biot criteria for the cooking stage is calculated usingthe following formula:

$$Bi_{cooking} = 79.42 \cdot R_0, \quad (17)$$

The Fourier criterion value for the cooking stage is given by formula (8), substituting though Biot value (the equation (17):

$$F_{0,cooking} = 0.7 \cdot \left(\frac{1}{12} + \frac{1}{238.26 \cdot R_0} - \right.$$
$$\left. - \frac{2 \cdot ln(1 + 39.71 \cdot R_0)}{18922.61 \cdot R_0^2}\right), \quad (18)$$

The sausage product cooking stage duration in the dimensionless value is specified using the following formula:

$$F_{0,cooking} = \left(\frac{Bi_{cooking} + 4}{8 \cdot Bi_{cooking}}\right) \times$$
$$\times \left[ln\left(\frac{t_{cookingenvironment} - t_{cooking0}}{t_{cookingenvironment} - t_{endcooking}}\right) + F_0^{'}\right]$$
$$= \left(\frac{79.42 \cdot R_0 + 4}{8 \cdot 79.42 \cdot R_0}\right) \times$$
$$\times \left[ln\left(\frac{t_{cookingenvironment} - t_{cooking0}}{t_{cookingenvironment} - t_{endcooking}}\right) + \right.$$
$$\left. + F_0^{'}\right], \quad (19)$$

where $t_{endcooking}$- the core product temperature(in the center of sausage loaf) at the end of the cooking stage.

Calculating the cooking stage duration of the sausage product, the initial condition of the product is considered to be characterized $t_{cooking,0}$ by the temperature averages:

$$t_{cookingenvironment,0} = \frac{t_{flying} + t_{n,flying}}{2}, (20)$$

The dimensional cooking stage duration $\tau_{cooking}$ (of the sausage loaf) is specified using the following formula:

$$\tau_{cooking} = 2142.45 \cdot F_{0,cooking} \cdot R_0^2, \quad (21)$$

4. Conclusion

Thus we obtain the mathematical heat treatment model for the entire thermal processing of cured/ boiled and/or smoked raw-cooked sausages as follows:

$$\tau = 2000 \cdot F_{0,drying} \cdot R_0^2 + (2 - 11.72 \times$$
$$\times R_0^2) \cdot (21 \cdot T_n - 8) + 2048.29 \times$$
$$\times F_{0,cooking} \cdot R_0^2 + 2142.45 \times$$
$$\times F_{0,cooking} \cdot R_0^2, \quad (22)$$

5. References

[1]. BRAZHNIKOV A., KARPYCHEV V., PELEEV A., Analytical Methods for the Study of Thermal Processing of Meat Products, *Learning guide*- M.: Food industry: 1–365, (1974)

[2]. IVASHKIN Yu., Modelling of Production Processes of Meat and Dairy Industry, *Learning guide*- M.: Agropromizdat: 1– 236, (1987)

[3]. PAVLOV K., ROMANKOV P., NOSKOV A., Examples and Problems on the Course of Processes and Devices of Chemical Technology, *Learning guide*, - L.: Chemistry: 1–576, (2007)

[4]. PAVELKO V., Heating Supply of the Meat and Dairy Processing Industries, *Learning guide*- Vinnytsia: NivaKnyha: 156–166, (2007)

[5]. PAVELKO V., SOKOLENKO O., ZASLSVSKY A., The Influence of SomeTechnologicalFactors on theDuration of theProcess of HeatTreatment of Sausages, - K.: Researchpapers NUFT: 31-36, (2012)

[6]. BRAZHNIKOV A., The Theory of Thermal Processing of Meat Products, - M.: Agropromisdat: 1-270, (1987)