Stability Analysis of FGCNT Beams with Imperfections

Pranav Kumar
Department of Civil Engineering, National Institute of Technology Patna, India
pranavk.phd18.ce@nitp.ac.in (corresponding author)

Ajay Kumar
Department of Civil Engineering, National Institute of Technology Delhi, India
sajaydce@gmail.com

Received: 12 April 2023 | Revised: 10 May 2023 and 16 May 2023 | Accepted: 25 May 2023

ABSTRACT

Functionally Graded Carbon Nanotube (FG CNT) reinforced curved beams are being used nowadays in different structural applications due to their properties although they have stability issues due to geometric imperfections that affect the load-bearing capacity of the beam and residual strength, hence a geometric imperfection is being introduced as a hollow trapezoidal layer is incorporated in the curved beam design to get better strength. Hence a new frictional endo-stratified thermal model is proposed to analyze the frictional effect between interface layers with the help of elastic and plastic deformation of the curved beam. Thus, analysis is done in ABAQUS using a finite element model to determine the stability due to frictional characteristics. The proposed design is giving better results than existing designs with higher specific gravity and tensile strength.

Keywords-functionally graded CNT; reinforced curved beams; friction behavior; delamination; thermal behavior

I. INTRODUCTION

Functionally Graded Materials (FGMs) are being developed as a tough contender to replace existing laminated composites and isotropic materials [1]. This is primarily due to inherent properties like thermal and flexural resistance and continuity, which eliminates abrupt transitions between distinct materials, which can cause problems like inter-laminar stresses, de-lamination, or cracking in multi-layer systems [2]. In terms of characteristics, this compositional diversity improves the system's thermal and mechanical performance. Elasticity, density, heat conduction, and other properties of FGM structures can be produced in a variety of ways to meet different needs. Beams, plates, and shells have all been used extensively in various types of engineering structures as fundamental components [3]. The beam element is one of the most useful and widely used elements in skeletal structures. Many researchers have become increasingly interested in analyzing the nonlinear behavior of structures in recent years. Geometric imperfections in engineering structures, such as beams and plates, are unavoidable during the manufacturing process due to a variety of production and environmental factors [4]. Hence the thermal stability of beams due to friction are concentrated and analyzed in this study to enhance its stability. The main contribution provided by this paper is that, the frictional endo-stratified thermal model is an effective method for determining the thermal characteristics of functionally graded reinforced curved beams. This model involves identifying the frictional behavior of the various layers in the beam, and can be used to accurately predict the thermal performance of the structure.

II. LITERATURE REVIEW

Authors in [5] presented a study of the effect of an applied methodology on out-of-plane interlaminar strength characterization. Additionally, the mechanical behavior of three carbon fiber-reinforced thermoplastic composites was compared using the curved beam strength test. Data evaluated using different standards gave statistically significantly different results. Further tests on the coupon with TPC matrix are planned for the future in order to expand the material database. Based on these tests, a material will be selected that could replace the thermosetting materials used in aircraft structures for interior panels. Authors in [6] discussed the existing literature on the area of application, stability, and free vibration analysis of FGM structures conducted by some recent research studies and to provide a comprehensive overview of the development, application, different numerical representation of materials, demonstrating procedures and arrangement technique and solution method of FGM rectangular plate. To expand the thermal analysis of various structures, including nonlinear effects, detailed research must be conducted. Authors in [7] investigated the nonlinear stability of the Functionally Graded Porous (FGP) cylinder reinforced by graphene nanofiller (GNF).

Authors in [8] proposed a unified and generic numerical modeling approach that accounts for both the intra and inter-laminar damage modes in stiffened CFRP panels. A 3D finite element based Progressive Damage Model (PDM) is proposed to simulate the collapse behavior of the single blade stiffened composite (SSC) CFRP panels with and without embedded de-bonding defects under uniaxial compression loading. A user-defined material subroutine based on 3D Hashin failure criteria was developed in Abaqus to study the evolution of intra-laminar damages in the SSC panel. The stability response and collapse load results obtained using the proposed PDM were compared with the experimental observations. Authors in [11] performed fiber-optic-based internal strain measurement and evaluated the residual deformation of L-shaped parts with inter-laminar resin layers. Through thickness cure shrinkage strain at the corner part was relaxed and the spring-in angle decreased by holding the parts at the cure temperature, indicating that viscoelasticity of the thermoplastic particles is important. Viscoelastic finite element analysis supported this finding and indicated that the effect of inter-laminar toughened layers on the residual deformation should be considered to optimize curing processes. In [5], a material was selected that could replace the thermosetting materials used in aircraft structures for interior panels. In [6], detailed research was conducted to expand the thermal analysis of various structures, including nonlinear effects. Authors in [7] reported that more detailed modeling of inter-laminar resin layers for quantitative comparison between experiments and simulation, and development of a method to reduce variations in residual deformation are required. The result of the numerical examples showed that changes in the stiffness of the elastic support have a significant effect on the displacement response of the beam. This is being studied in [14].

In this paper, we studied the Stochastic Iso-Geometric Analysis (SIGA) for a functionally graded plate under random loads with the assumption that random loads are a homogeneous Gaussian random field in the plate plane. The coefficient of variation of the deflection in the center of the plate predicted by SIGA was validated with the results of Monte Carlo simulation in [15]. This work is carried out to evaluate the effects of nanoparticle addition on the penetration and wear resistance of polyethylene at different temperatures (22.8 and 40 °C). A low volume of fractions of carbon nanotubes and nanoclays (0.5 wt.%) were embedded separately into the blend of polymeric materials to improve their mechanical properties, mainly wear and penetration resistance [16]. A homogenization strategy must be used to reduce the complicated heterogeneous microstructure of FGMs to examine them properly [17].

III. STABILITY ENHANCEMENT FOR GEOMETRICALLY IMPERFECT FG CNTR CURVED BEAMS AFFECTED BY FRICTIONAL BEHAVIOR

FGCNT have geometric imperfections that influence their buckling behavior and strength. With the help of this research, a unique design of induced imperfect hollow trapezoidal layer is incorporated to get better result for the residual strength of the FGCNT reinforced curved beam. Hence a novel frictional endo-stratified thermal model was used to analyze the thermal behavior due to the inter laminar frictional effect caused by the delamination between the induced imperfect hollow trapezoidal layer and the adjacent layer of the geometrically imperfect stacked foliate and is modeled using frictional effects due to elastic and plastic deformation of the curved beam at the peak and the trough, respectively, hence determining the thermal characteristics effectively. Figure 1 exhibits the block diagram of the proposed stability analysis and enhancement model in which thermal characteristics were analyzed with the determination of frictional effects on the surface of the curved beam. To further enhance the stability of the curved beam, the interaction among the molecules in FGCNT has been analyzed with the kinetic molecular theory for intermolecular forces with the consideration of the strength bearing capacity of the FGCNT.

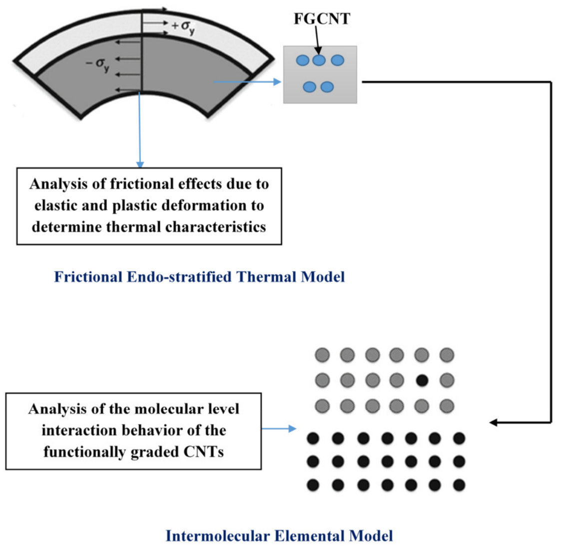

Fig. 1. Overall architecture of proposed model.

A. *The Frictional Endo-Stratified Thermal Model*

In the frictional endo-stratified thermal model, the frictional effect is considered during the analysis of thermal behavior in the curved beam, introducing an imperfect hollow trapezoidal layer in the design. A mixture of CNTs and polymeric matrix, blended based on the volume proportion of constituents makes up the FG CNT reinforcement curved beam. The volume fraction of carbon nanotubes in the FG CNT reinforcement curved beam mixture is dispersed in the thickness direction according to the (UD), (FG X), (FG V), (FG K), and (FG O) functions. Figure 2 illustrates the various intermolecular positions of the FGCNT in a curved beam. The volume fraction of CNT (v_C) under various functions is given in (1):

$$v_C(y) = \begin{cases} v_{UD}^* \\ 4v_{FGX}^*\left(\frac{|y|}{h}\right) \\ v_{FGV}^*\left(1+\frac{2y}{h}\right) \\ v_{FG\wedge}^*\left(1-\frac{2y}{h}\right) \\ 2v_{FGO}^*\left(1-\frac{2|y|}{h}\right) \end{cases} \quad (1)$$

where $v^*$is the total volume fraction of CNT, given by:

$$\frac{w_f}{w_f+\left(\frac{d}{d_m}\right)(1-w_f)}= v^* \qquad (2)$$

where d_m is the mass density of the matrix and d and w_f are the mass density and mass fraction of the CNT, respectively. The rule for FGCNT is based on the condition provided in (3):

$$1 = v_C(y) + v_m \qquad (3)$$

where $v_C(y)$ and v_m are the effective volume fractions of CNTs in position y along the thickness direction and the matrix mixture. The expanded rule of mixture is used to express the mixture of FG CNT reinforced curved beams and the definition of its mechanical properties as a function of y is expressed by:

$$E_1 = \mu_1 E_1^c v_C + E_m v_m \qquad (4)$$

where the Yong's moduli of CNTs are represented by the parameter E_1. E_m stands for the CNT matrix's elastic moduli. The efficiency parameter μ_1 is included to account for the scale-dependent material properties of CNTs. By comparing the elastic moduli that are discovered by molecular dynamic simulations and the law of mixture, it is feasible to estimate the efficiency parameters. It is possible to characterize the material properties of FG CNTs reinforced curved beam as a function of y for different distributions of CNTs in the thickness directions by incorporating (1) and (3) into (4). When the produced equivalent stresses in FG CNTs reinforced curved beam exceed its yield limit, the ductile metallic component primarily yields the FG structure plastically. Consequently, (5) is an expression for the multi-linear hardening elastoplastic material properties of FG in FG CNTs reinforced curved beam throughout thickness:

$$E(y) = \left[\left(\frac{Q+E_c}{Q+E_m}\right) E_m v_m + E_c v_c\right] / \left[\left(\frac{Q+E_c}{Q+E_m}\right) v_m + v_c\right] \qquad (5)$$

where E_c denotes the elastic modulus of the ceramic phase, and σ_y and h_m represents the yield limit and tangent modulus of the metallic phase, respectively. The ratio of stress to strain transfer is represented by Q.

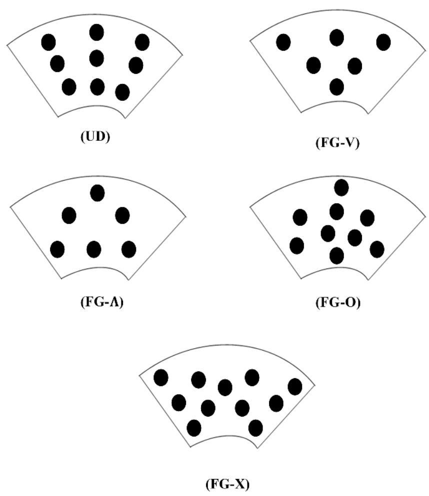

Fig. 2. CNT distribution in FG reinforcement curved beams.

B. The Finite Element Model

A finite element model is constructed in ABAQUS to analyze the thermal behavior due to frictional and inter molecular characteristics constituting the stability of the beam. To perform finite element analysis, initially the geometry of FGCNT reinforced curved beams is introduced by incorporating hallow trapezoidal layers and stability has been analyzed in this model with the consideration of friction and molecular interaction. The finite element algorithm uses the pre-processed data obtained from the geometry of FGCNT as input to generate and solve a scheme of linear or nonlinear equations. During the FEM process, the detailing friction effect and stresses at various points across the model are calculated with a typical presentation of colored outlines displaying the stress levels on the model. The elastic stresses and strains were examined using the functionally graded CNT model, which included deformations and von Mises stresses. The stress caused by the loading distribution was calculated using the elastic Von Mises stress. This parameter integrates the three main stresses to determine the plastic deformation in metals:

$$\sigma_p = \frac{1}{2}\left[(\sigma_{vs} - \sigma_m)^2 + (\sigma_m - \sigma_x)^2 + (\sigma_x - \sigma_{vs})^2\right]^{\frac{1}{2}} \qquad (6)$$

where σ_{vs}, σ_m, and σ_x are the principal stresses in hollow trapezoidal coordinates. To determine the effect of the friction on the curved beam, initially the volume fraction of FGCNT in various distributions was analyzed and then the material property was calculated. Coulomb's friction law is applied to determine the frictional behavior of the interface layers due to elastoplastic deformation. Also, using finite element exploration, the peak and trough thermal stability of a geometrically imperfect curved beam containing functionally graded CNTs is determined.

FG CNTs reinforced curved beams were subjected to the kinetic molecular theory of intermolecular forces to analyze the molecular interactions and the strength bearing capacity of the curved beam with various distributions of FGCNT reinforced composites. The steps used in the FEM are:

Step 1: Create the geometry of the curved beam with FGCNT reinforced composite.

Step 2: Define the material characteristics of the FGCNT.

Step 3: Assign appropriate material attributes to various distributions of FGCNT.

Step 4: Create an element section and assign it to each element involved when determining the curved beam parameters including the volume fraction of CNT.

Step 5: Make an element instance.

Step 6: Create analysis stages for the friction effect and molecular interaction.

Step 7: Apply the corresponding Neumann boundary conditions by providing dynamic load and energy constraints. The normal derivative at the boundary must be zero or a constant according to the Neumann boundary condition. In the context of a heat diffusion problem, zero normal derivative denotes an adiabatic boundary when the boundary is a plane normal to an axis, such as the x axis. At the boundary, the flux of conduction heat is zero, assuming that there is no bending moment on the free ends of the curved beam.

Step 8: Use the element's quadratic meshing to its full potential.

Step 9: Create a job and submit it for consideration.

Step 10: Result visualization.

These steps were performed to simulate the curved beam model with mesh characteristics and functionally graded CNT reinforced composite. Then, stability analysis of the curved beam was performed to evaluate the enhancement of the residual strength of the curved beam while considering the thermal characteristics.

IV. RESULTS AND DISCUSSION

This section details the implementation findings as well as the performance of the proposed curved beam design along with a comparative study to guarantee that this proposed design outperforms the existing ones.

A. Simulation Output and Performance Evaluation

This section of the research paper presents the simulation results and performance metrics of the proposed curved beam design. The performance of the proposed design was evaluated using various metrics, including elastic stress, strain, path points, and von Mises stress. The results are provided and analyzed.

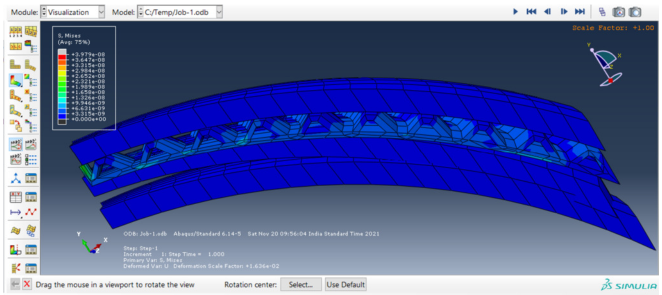

Fig. 3. Von-Mises stress in an FG CNT reinforced curved beam.

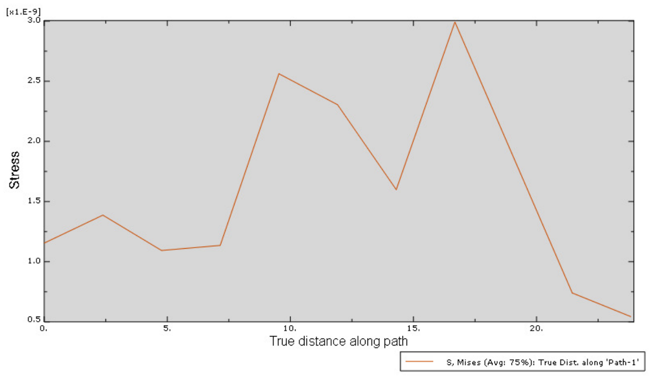

Fig. 4. Von mises stress of an FG CNT reinforced curved beam.

Figure 3 represents the Von-Mises stress of the FG CNTs reinforced curved beam with a hollow trapezoidal layer. The sudden increase and decrease of stresses in FG CNT reinforced curved beams have an impact on the strength bearing capacity. The maximum Von-Mises stress obtained was $+3.9791e^{-8}$, with a step time of 1.00 and a deformation scale factor of $+1.636e^{-02}$. The Von Mises stress of the FG CNT reinforced curved beam with hollow trapezoidal layer is shown in Figure 4. The maximum and the minimum obtained stress are 3 e^{-9} at 17.5 mm and -0.6 e^{-9} at 25 mm. As a result, the true distance along the path increases from 0.5 to 25 mm whereas the stress level varies non-linearly.

The stress obtained after analyzing the thermal characteristics based on the frictional effect in terms of time is shown in Figure 5. In thermal analysis, pressure, Von Misses stress, and strain increase as load pressure rises. Furthermore, when the true length along the path increases, the stress determined along the path points in terms of time decreases to an average of 75%. The yield stress and total deformation of the FG CNTs reinforced curved beam rise as the stress decreases.

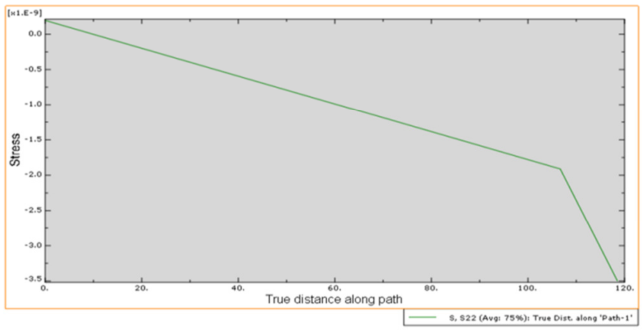

Fig. 5. Stress in terms of time function.

B. Comparison Analysis

To assess the effectiveness of the proposed FG CNT reinforced curved beam design, a comparison was made with existing designs, such as Poly Phenyl Sulfide (PPS), Poly-Ether-Ether-Ketone (PEEK), and Poly-Aryl-Ether-Ketone (PAEK) [5, 13]. The comparison was made in terms of various properties, including specific gravity (g/cm^3), melt temperature (°C), tensile strength (MPa), tensile modulus (GPa), elongation yield (%), and flexural strength (MPa). The purpose of this comparison was to determine how the proposed design performs compared to the existing designs and to identify any potential areas for improvement.

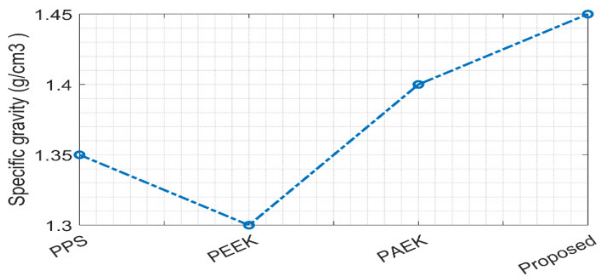

Fig. 6. Specific gravity comparison.

Figure 6 depicts the specific gravity comparison of the proposed FG CNTs reinforced curved beam design with the existing PPS, PEEK, and PAEK. It is noted that the proposed design has a high specific gravity of 1.45 g/cm^3, whereas PPS, PEEK, and PAEK have a low specific gravity of 1.35, 1.30, and 1.40 g/cm^3. Hence the proposed design has a high specific gravity whereas PEEK has a low specific gravity.

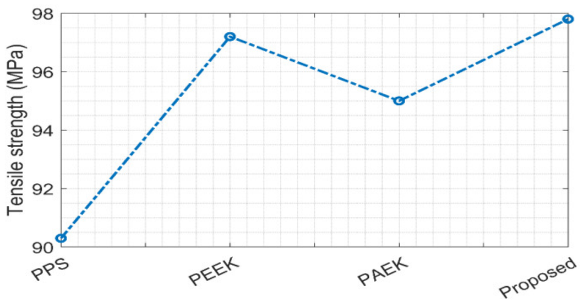

Fig. 7. Comparison of tensile strength.

Figure 7 depicts the tensile strength (MPa) comparison of proposed FG CNTs reinforced curved beam design with the existing designs such as PPS, PEEK, and PAEK. It is noted that the proposed design has a high tensile strength of 98 MPa due to the introduction of hollow trapezoidal layer and with the consideration of frictional effect in thermal analysis whereas the existing techniques such as PPS, PEEK, and PAEK have a low tensile strength of 90.5, 97, and 95 MPa. Hence, the proposed design has a high tensile strength whereas PPS has a low tensile strength.

Overall, the proposed FG CNTs reinforced curved beam design has the better Von Mises stress and maximum stress for molecular level interaction as 5.5 e^{-9}. Also, friction coefficient up to 1.5 and stress of 51,000 MPa were provided for the determination of thermal characteristics. The proposed design has a high specific gravity of 1.45 g/cm^3, a melt temperature of 360°C, and a tensile strength of 98 MPa, which overperform the PPS, PEEK, and PAEK.

V. CONCLUSION

This research paper presents a study on improving the stability of imperfect functionally graded carbon nanotube reinforced curved beams by incorporating a hollow trapezoidal layer design. The design was able to improve the tensile strength of the FG CNTs up to 98 MPa. The beams were subjected to loads of $L/h = 50$ in the radial direction, resulting in pure bending, and the stress was calculated using the Von Mises yield criteria. To determine the thermal characteristics of the beams, a Frictional Endo-stratified Thermal Model was developed using Coulomb's friction law. The model was able to simulate friction coefficients ranging from 0.2 to 1.5 and stress levels up to 51,000 MPa. The simulation results demonstrated that the proposed design outperforms the existing designs in terms of specific gravity, melt temperature, and elongation yield, with a high specific gravity of 1.45 g/cm^3, a melt temperature of 360 °C, and an elongation yield of 4%.

REFERENCES

[1] M. J. Mirzaali *et al.*, "Mechanics of bioinspired functionally graded soft-hard composites made by multi-material 3D printing," *Composite Structures*, vol. 237, Apr. 2020, Art. no. 111867, https://doi.org/10.1016/j.compstruct.2020.111867.

[2] Z. Cheng, Z. Sui, H. Yin, and H. Feng, "Numerical simulation of dynamic fracture in functionally graded materials using peridynamic modeling with composite weighted bonds," *Engineering Analysis with Boundary Elements*, vol. 105, pp. 31–46, Aug. 2019, https://doi.org/10.1016/j.enganabound.2019.04.005.

[3] M. Rezaiee-Pajand, E. Arabi, A. R. Masoodi, "Nonlinear analysis of FG-sandwich plates and shells," *Aerospace Science and Technology*, vol. 87, pp. 178–189, Apr. 2019, https://doi.org/10.1016/j.ast.2019.02.017.

[4] M. Malikan, V. A. Eremeyev, and K. K. Zur, "Effect of Axial Porosities on Flexomagnetic Response of In-Plane Compressed Piezomagnetic Nanobeams," *Symmetry*, vol. 12, no. 12, Dec. 2020, Art. no. 1935, https://doi.org/10.3390/sym12121935.

[5] R. Hron, M. Kadlec, and R. Ruzek, "Effect of the Test Procedure and Thermoplastic Composite Resin Type on the Curved Beam Strength," *Materials*, vol. 14, no. 2, Jan. 2021, Art. no. 352, https://doi.org/10.3390/ma14020352.

[6] E. K. Njim, M. Al-Waily, and S. H. Bakhy, "A Critical Review of Recent Research of Free Vibration and Stability of Functionally Graded Materials of Sandwich Plate," *IOP Conference Series: Materials Science and Engineering*, vol. 1094, no. 1, Oct. 2021, Art. no. 012081, https://doi.org/10.1088/1757-899X/1094/1/012081.

[7] Z. Li and J. Zheng, "Nonlinear stability of the encased functionally graded porous cylinders reinforced by graphene nanofillers subjected to pressure loading under thermal effect," *Composite Structures*, vol. 233, Feb. 2020, Art. no. 111584, https://doi.org/10.1016/j.compstruct.2019.111584.

[8] N. R. Kolanu, G. Raju, and M. Ramji, "A unified numerical approach for the simulation of intra and inter laminar damage evolution in stiffened CFRP panels under compression," *Composites Part B: Engineering*, vol. 190, Jun. 2020, Art. no. 107931, https://doi.org/10.1016/j.compositesb.2020.107931.

[9] S. Minakuchi, K. Sawaguchi, K. Takagaki, S. Niwa, and N. Takeda, "Effect of inter-laminar toughened layers on process-induced strain and deformation of L-shaped composites," *Advanced Composite Materials*, vol. 28, no. 5, pp. 445–461, Sep. 2019, https://doi.org/10.1080/09243046.2019.1573452.

[10] M. Arefi, S. Firouzeh, E. Mohammad-Rezaei Bidgoli, and O. Civalek, "Analysis of porous micro-plates reinforced with FG-GNPs based on Reddy plate theory," *Composite Structures*, vol. 247, Sep. 2020, Art. no. 112391, https://doi.org/10.1016/j.compstruct.2020.112391.

[11] F. Tornabene, M. Viscoti, R. Dimitri, and M. A. Aiello, "Higher order formulations for doubly-curved shell structures with a honeycomb core," *Thin-Walled Structures*, vol. 164, Jul. 2021, Art. no. 107789, https://doi.org/10.1016/j.tws.2021.107789.

[12] F. Tornabene, M. Viscoti, R. Dimitri, and J. N. Reddy, "Higher order theories for the vibration study of doubly-curved anisotropic shells with a variable thickness and isogeometric mapped geometry," *Composite Structures*, vol. 267, Jul. 2021, Art. no. 113829, https://doi.org/10.1016/j.compstruct.2021.113829.

[13] H. Luinge and L. L. Warnet, "On an application of multi-material composite laminates in the aerospace sector," *Advanced Composites and Hybrid Materials*, vol. 3, no. 3, pp. 294–302, Sep. 2020, https://doi.org/10.1007/s42114-020-00163-3.

[14] T. D. Hien, N. D. Hung, N. T. Hiep, G. V. Tan, and N. V. Thuan, "Finite Element Analysis of a Continuous Sandwich Beam resting on Elastic Support and Subjected to Two Degree of Freedom Sprung Vehicles," *Engineering, Technology & Applied Science Research*, vol. 13, no. 2, pp. 10310–10315, Apr. 2023, https://doi.org/10.48084/etasr.5464.

[15] N. T. Hiep, D. S. Dan, N. D. Diem, and D. N. Tien, "NURBS-based Isogeometric Analysis and Refined Plate Theory Application on a Functionally Graded Plate Subjected to Random Loads," *Engineering,*

Technology & Applied Science Research, vol. 13, no. 2, pp. 10243–10248, Apr. 2023, https://doi.org/10.48084/etasr.5478.

[16] A. S. Alghamdi, "Wear and Indentation Resistance of Polyethylene Nanocomposites at High Temperatures," *Engineering, Technology & Applied Science Research*, vol. 12, no. 4, pp. 9018–9022, Aug. 2022, https://doi.org/10.48084/etasr.4982.

[17] P. Kumar and A. Kumar, "Stability analysis of imperfect functionally graded CNTs reinforced curved beams," *Mechanics Based Design of Structures and Machines*, Sep. 2022, https://doi.org/10.1080/15397734.2022.2116340.