

# The Effect of Pilot Reuse Factor on Massive MIMO Spectral Efficiency

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## ABSTRACT

Due to their much increased spatial resolution and array gain, large-scale antenna arrays have the potential to significantly enhance the spectral efficiency of wireless systems. Recent research in the area of massive Multiple-Input Multiple-Output (MIMO) antennas shows that user channels are enhanced as the number of antennas at the Base Stations (BSs) rises, allowing achieving high signal improvement with minimum inter-user interference. The combined effects of the pilot reuse factor and the BS number of antennas on average overall spectral efficiency in huge MIMO antennas will be investigated. To examine the SE of these receivers, several channel estimators will be utilized, including LMMSE, ZF, and MR methods. Theoretically, BS could offer a substantially higher average total SE if it had more antennas.

**Keywords-**massive MIMO; L-MMSE; spectral efficiency; base station; pilot reuse factor; zero forcing; machine to machine; channel state information

## I. INTRODUCTION

The amount of mobile data traffic worldwide in 2017 was 12.6 times greater than what it was in 2012. The entire amount of mobile traffic has grown by a factor of 10 between 2019 and 2022 [1], and this rise continues today. Therefore, the cellular infrastructure of the future must be ready to serve new uses and, specifically, to satisfy the users' needs and meet the current wireless communication standards. Particularly, the use of smart devices and machine-to-machine (M2M) communications systems has dramatically increased mobile data traffic over the past ten years across most applications and industries. The International Telecommunication Union (ITU) has recently predicted that mobile traffic worldwide will grow by a factor of 660 between 2010 and 2030. Every decade, mobile technologies undergo a dramatic development and the demand for wireless data is only going to increase. The more data we use, the more we need networks with faster speeds and wider coverage to keep up. Each new technology generation has offered considerable performance improvements. These quick variations are reactions to the needed capacity from the significant data increase over the previous decade, mostly from video streaming. The resolution of video streaming capabilities is also rising, and phones supporting 4K video will require a higher data rate per user. Users' watching duration is also rising; watching long TV shows and movies via streaming video is now common. This tendency appears to have no end in sight.

Massive or big-scale Multiple Input Multiple Output (MIMO) technology has emerged as a viable technique for future wireless networks [2]. Despite the fact that massive MIMO demonstrates enormous capacity of supporting a substantial volume of data rate and wireless links, it is not that

easy to implement such a system due to the high cost of hardware and higher consumption of power. Researchers have been investigating energy efficient initiatives to maximize system efficiency in order to establish sustainable and clean wireless systems. Massive MIMO is a combination many innovations that have been combined in order to create a solution that is able to deliver incredible results. What all these technologies have in common is that they are able to significantly improve the performance of Base Stations (BSs) by providing them with the ability to increase their capacity at cell sites. They do this primarily by improving the link between users and BSs through transmitting and receiving more data from users at cell sites on both sides of the connection simultaneously.

Wireless networks have limited available electromagnetic spectrum while the need for higher data rates continues to increase. To reach future demands, wireless communications must be industrialized with the introduction of new and sophisticated technologies [3]. One of the recent proposed techniques is massive MIMO, which has a more promising potential compared to other techniques. Basically, the idea of this technology is about providing a BS with a big number of antennas to cover a much smaller number of users [4]. The Time Division Duplex (TDD) mode of operation is used to obtain accurate Channel Status Information (CSI) [5], which is vital for maximum effectiveness and getting the advantages of this technique. Since the number of training sequences is directly proportional to the number of users, any number of antennas in the BS can be achieved.

Large-scale MIMO is a system that consists of a couple of hundreds antennas serving a large number of end users at the same time while using the same frequency [6]. This kind of

technology has several advantages such as increasing Spectral Efficiency (SE) to achieve the future needs, mainly in dense areas. Also, it will provide a secure network to maintain information privacy [7]. The cost of the transmitter/receiver components will be reduced due to its lower complexity and the power efficiency should be improved while the signal processing will be much simpler.

The accuracy of channel estimation is a critical factor that has the ability to affect SE and for that reason it grabs the attention of researchers [8, 9]. Some estimation methods are utilized in channel estimation, one of them is the Minimum Mean Square Error (MMSE). This method is known for its good performing behavior that meets with any value of Signal to Noise Ratio (SNR). Other methods, such as Maximum Ratio (MR) and Zero-Forcing (ZF), will be investigated. To obtain maximum performance we need to have perfect channel matrix knowledge [10].

## II. LITERATURE REVIEW

Several studies have investigated the effect of the pilot reuse factor on the spectral efficiency of massive MIMO systems. In [11], it was shown that increasing the pilot reuse factor can improve the spectral efficiency up to a certain point, after which the performance saturates. The authors also proposed a new pilot design scheme based on the use of orthogonal pilot sequences, which can further improve the performance of massive MIMO systems. Authors in [12] analyzed the impact of pilot reuse factor on the downlink spectral efficiency of massive MIMO systems using different channel estimation techniques. The results showed that the optimal pilot reuse factor depends on system parameters such as the number of antennas, the channel coherence time, and the receiver noise level. The authors also proposed a new pilot allocation scheme based on the use of non-uniform pilot density, which can further improve the spectral efficiency of massive MIMO systems. Authors in [13] investigated the effect of pilot reuse factor on the uplink spectral efficiency of massive MIMO systems using a realistic channel model. The results showed that increasing the pilot reuse factor can improve SE up to a certain point, after which the performance saturates. The authors also proposed a new pilot allocation scheme based on the use of clustered pilot sequences, which can further improve the SE of massive MIMO systems.

Overall, the literature suggests that the choice of pilot reuse factor is an important design parameter for massive MIMO systems, and that different pilot allocation schemes can be used to optimize the system performance. However, further research is needed to investigate the optimal pilot reuse factor for different system configurations and its impact on other performance metrics. In this study, we were able to identify the ideal value for the pilot reuse factors for 3 distinct channel estimators based on the number of antennas in the BS and the number of active users. Our proposed system will always run at its greatest SE due to these ideal pilot reuse factor values.

Notation: In this study, column vectors, such as  $\mathbf{x}$ , are denoted by lower-case, bold notations. Matrix symbols  $\mathbf{X}$ , however, are upper-case, bold notations. The symbols for transpose, conjugate, and conjugate transpose  $\mathbf{X}$  are  $\mathbf{X}^T$ ,  $\mathbf{X}^*$ ,

and  $\mathbf{X}^H$ , respectively.  $\mathbf{I}$  is the representation of the identity matrix of appropriate dimensions.  $\mathbb{E}\{\mathbf{x}\}$  stands for the  $\mathbf{x}$  operation expected value, and  $\mathbb{E}\{\mathbf{x}|\mathbf{y}\}$  stands for  $\mathbf{x}$  conditional expected value that is related to  $\mathbf{y}$ . In the expression  $\mathbf{x} \sim \mathcal{CN}(\bar{\mathbf{x}}, \mathbf{Q})$ , the covariance matrix is  $\mathbf{Q}$  and the mean is  $\bar{\mathbf{x}}$ .

## III. CHANNEL AND SYSTEM MODEL

The primary goal of our study is to examine the related average sum spectral efficiency for the UL channel that is susceptible to interference due to the behavior of wireless channels. The system modeling comprises of a large antenna number in the BS terminal coupled to a small number of user terminals. The system used is a subset of massive MIMO, where the protocol utilized between the two sides of the link is TDD [14]. Because the accuracy of estimation in TDD does not depend on  $N$ , we can use a high number of BS antennas [15].

We maintain continuous CSI to identify UL data information by using reciprocity benefit of the channel. From one terminal to the other,  $\mathbf{h}$  represents the block-fading and it has a Rayleigh model as  $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ , while  $\mathbf{R} = \mathbb{E}\{\mathbf{h}\mathbf{h}^H\}$

The BS terminal is aware of the statistical distribution. Pilot for channel estimate serves as the foundation for the UL system model, and  $\mathbf{y}$  symbolizes the signal that is received by the BS.

$$\mathbf{y} = \mathbf{n} + \mathbf{h}d \quad (1)$$

where the received signal is denoted by  $d \in \mathbb{C}$ . It could be the data signal or a predetermined pilot. The average power of the signal is:

$$p = \mathbb{E}\{|d|^2\} \quad (2)$$

Channel noise is represented by  $\mathbf{n}$ , and is composed of the following parts:

$$\mathbf{n} = \mathbf{n}_{\text{noise}} + \mathbf{n}_{\text{if}} \quad (3)$$

where  $\mathbf{n}_{\text{if}}$  is the interfering noise caused by other messages and  $\mathbf{n}_{\text{noise}}$  is the independent receiver noise with zero mean and  $(\sigma_{BS}^2 \mathbf{I})$  covariance.

We assume that  $\mathbf{n}_{\text{if}}$  has a zero mean,  $\mathbb{E}\{\mathbf{n}_{\text{if}}\mathbf{n}_{\text{if}}^H\}$  covariance, and is represented by  $\mathbf{S}$ .  $\mathbf{Q}_{\mathcal{H}} = \mathbb{E}\{\mathbf{n}_{\text{if}}\mathbf{n}_{\text{if}}^H|\mathcal{H}\}$  is the definition of the conditional covariance matrix, while  $\mathcal{H}$  represents the realization of the channel. We are supposing that  $\mathbf{R}$  spectral norm is bounded uniformly independent of the BS's antenna number [16]. The matrix of channel covariance is produced by the exponential correlation model, which is described in greater detail in [17]. This correlation model presupposes that the value of  $\mathbf{R}$  has the following elements  $(i, j)$ :

$$[\mathbf{R}] = \begin{cases} \delta r^{j-i}, & i \leq j \\ \delta (r^{j-i})^*, & i > j \end{cases} \quad (4)$$

where  $r$  represents a correlation coefficient between adjacent channels and  $\delta$  is the scaling factor.

The factor of correlation value is restricted to the range from 0 to 1, and its phasor  $\angle r$  denotes the angle of departure or arrival. This obviously is not the most suitable model for scenarios in the actual world. However, it is still a straightforward model with a manageable amount of

parameters, making it possible to investigate how the correlation factor affects the estimation of the huge MIMO channel [18]. The correlation factor  $|r|$  in  $\mathbf{R}$  represents the eigenvalue spread, and  $\mathcal{L}r$  defines the associated eigenvectors. We take  $r$  to be a real number since the the mean square error is not affected by the angle of  $r$ .

By comparing the signal received by BS  $\mathbf{y}$  in (1) to the known pilot uplink signal  $d$ , where user equipment power corresponds to  $(p^{UE})$ , we may estimate the channel  $\mathbf{h}$ . In this paper, different types of estimators are used, the first one is MMSE. The following is a representation of the estimation of channel realization:

$$\hat{\mathbf{h}} = d^* \mathbf{R} \bar{\mathbf{Y}}^{-1} \mathbf{y} \quad (5)$$

The  $\mathbf{R}$  matrix, the average power, the received signal covariance, and the noise, make up the  $\mathbf{y}$  covariance matrix, which is shown as follows:

$$\bar{\mathbf{Y}} = \mathbb{E}\{\mathbf{y}\mathbf{y}^H\} = \mathbf{R} p^{UE} + \mathbf{S} + \sigma_{BS}^2 \mathbf{I} \quad (6)$$

The following is a representation of the covariance matrix of error that is used to calculate the overall MSE:

$$\mathbf{C} = \mathbb{E}\{(\hat{\mathbf{h}} - \mathbf{h})^H (\hat{\mathbf{h}} - \mathbf{h})\} = \mathbf{R} - p^{UE} \mathbf{R} \bar{\mathbf{Y}}^{-1} \mathbf{R} \quad (7)$$

We may get the entire value of MSE by finding the  $\mathbf{C}$  value, which is the error covariance matrix shown in (8):

$$\mathbf{MSE} = \text{tr}(\mathbf{C}) = \mathbb{E}\{\|\hat{\mathbf{h}} - \mathbf{h}\|_2^2\} \quad (8)$$

The channel may be described as a mixture of two components based on the prior observations:

$$\mathbf{h} = \hat{\mathbf{h}} + \boldsymbol{\epsilon} \quad (9)$$

where the process of unknown error estimation is represented by  $\boldsymbol{\epsilon}$ , and the value of the MMSE estimate is marked by  $\hat{\mathbf{h}}$ . The mean of the values,  $\hat{\mathbf{h}}$  and  $\boldsymbol{\epsilon}$ , is 0 for both and they are uncorrelated [19]. The  $\hat{\mathbf{h}}$  covariance matrix is generated as  $\mathbb{E}\{\hat{\mathbf{h}}\hat{\mathbf{h}}^H\} = \mathbf{R} - \mathbf{C}$ , and the  $\boldsymbol{\epsilon}$  covariance matrix as  $\mathbb{E}\{\boldsymbol{\epsilon}\boldsymbol{\epsilon}^H\} = \mathbf{C}$ .

We can disregard all the correlation matrices in the event that the conditions of the channel are favorable and the signal's interferences from nearby cells are faint. By applying that, ZF combining can be generated [20]. By knowing that the combining vector's normalization has no effect on the instantaneous up link signal to noise ratio, we can generate MR combining. The primary difference is that we no longer employ exact channels and instead use estimated channels (which are unknown in practice). In contrast to the preceding systems, MR does not need any matrix inversion. Since not all UEs in actuality have a very low SNR, it is anticipated that MR scheme will yield lower SE compared to other schemes.

It is assumed that there are 16 square cells in a 4x4 grid, a wrap-around topology is taken into consideration, in which every cell has numerous locations and any arbitrary BS and UE are always separated by the 9 combinations' shortest distance. The shortest path is used to derive the angular characteristics. Four distinct methods of reusing pilot sequences between cells

are depicted, which means that a higher pilot reuse factor denotes the need for more orthogonal pilot sequences.

The distribution and reuse of the  $\tau_p$  pilot sequences among the UEs and cells can be done in a variety of ways [21]. Unless otherwise indicated, we assume  $\tau_p = fK$  pilots, where  $f$  is an integer known as pilot reuse factor. That indicates that we have  $f$  times more pilots than user equipment in each cell and that a fraction of  $1/f$  of the cells reuses the same subset of pilots. In the running example, we consider  $f \in \{1, 2, 4, 8, 16\}$ , where the same pilot group is considered to consist of the cells that employ the same pilots. Every UE in a cell has a pilot allocated at random, thus the  $k$ th UE in two adjacent cells that is related to the same group of pilot utilizes the same pilot.

#### IV. NUMERICAL ILLUSTRATIONS

In this part, we provide illustrative simulations to demonstrate how the SE performs when based on various receive combining schemes and different pilot reuse factors. The use of the communication system toolbox in MATLAB allows us to analyze our model. To create the Figures where the analysis is done, several scripts were built. The system parameters are shown in Table I.

#### V. SYSTEM PARAMETERS

Parameter	Value
Cells number	16
Number of BS antennas	$M=40, 70, 100$
Number of users/cell	$K=10$
Pilot reuse factor	$f=1, 2, 4, 8, 16$
Uplink and downlink transmit power	20 dBm

For the downlink and uplink channels, the average signal to noise ratios are  $p^{BS} \text{tr}(\mathbf{R}) / N \sigma_{UE}^2$  and  $p^{EU} \text{tr}(\mathbf{R}) / N \sigma_{BS}^2$ , respectively. When using low and high SNR values, we take the value of SNR into account. We presume that the percentages of uplink and downlink data are identical and set at 0.45. Using this assumption, we obtain equal uplink and downlink.  $K=10$  user equipment per cell and various numbers of BS antennas were taken into account in the simulations. Each block contains  $fK$  pilots, with the remaining  $\tau_p - fK$  samples being utilized for uplink data transfer. We utilized the channel model with Gaussian local scattering.

Figure 1 shows the average uplink SE as a function of the pilot reuse factor with 100 BS antennas. The L-MMSE combination provides the highest SE compared to ZF and MR schemes. By increasing the pilot reuse factor from 1 to 2, both schemes L-MMSE and ZR are having higher SE while the MR is having a lower average sum SE. Once the value of  $f$  is increased from 2 to 4, the SE keeps increasing slightly for L-MMSE scheme while it is decreasing for the two other schemes, ZF and MR. As the pilot reuse factor is raised from 4 to 8 and 16, all the 3 schemes deteriorate quickly.

By using 70 antennas at the BS, Figure 2 shows lower average sum SE compared to the 100 antennas of Figure 1 due to the increase interference that cannot be canceled easily. In comparison to ZF and MR combining methods, the SE provided by the L-MMSE scheme is the greatest. The L-MMSE and ZR systems have both greater SE as a result of raising the pilot reuse factor from 1 to 2, whereas the MR has a

lower average sum SE. When  $f$  is increased from 2 to 4, the SE for the L-MMSE scheme continues to marginally grow while falling for the 2 other schemes. All the schemes quickly degrade as the pilot reuse factor is increased from 4 to 8 and 16 respectively. Due to the increased interference that is difficult to be eliminated when utilizing 40 antennas at the BS, Figure 3 displays the lowest average total SE. Raising  $f$  from 1 to 2 results in increased SE for the L-MMSE and ZF systems, while the MR has a lower average sum SE. All 3 schemes rapidly degrade as the  $f$  is increased from 2 to 4, 8, and 16 respectively. Even though the L-MMSE scheme provides higher SE, it has much more complexity than ZF and MR.

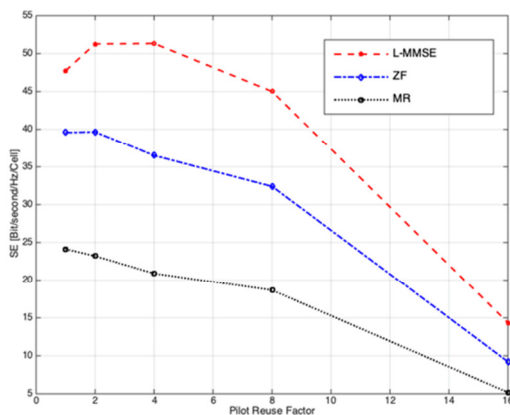


Fig. 1. Average uplink sum spectral efficiency as a function of pilot reuse factor for 100 BS antennas.

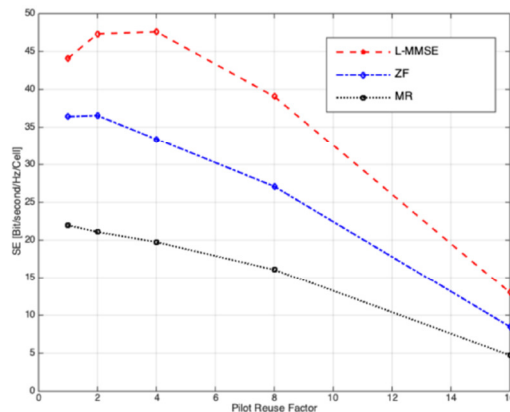


Fig. 2. Average uplink sum spectral efficiency as a function of pilot reuse factor for 70 BS antennas.

By comparing Figures 1-3 in terms of using the L-MMSE scheme, it is obvious that the higher the number of antennas in the BS, the higher the SE. When the number of antennas is equal to 100 (Figure 1), the SE exceeds 51 [Bit/second/Hz/Cell] while the pilot reuse factor is equal to 2 or 4. If the number of antennas in the BS is equal to 70 and the  $f$  is either 2 or 4 (Figure 2), the SE is less than 48 [Bit/second/Hz/Cell]. On the other hand, when the number of antennas in the BS is equal to 40 and  $f=2$  (Figure 3), the SE is less than 33 [Bit/second/Hz/Cell].

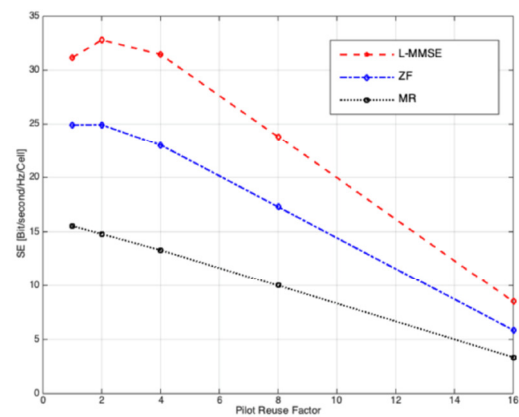


Fig. 3. Average uplink sum spectral efficiency as a function of pilot reuse factor for 40 BS antennas.

If we compare the Figures 1-3 considering the ZF scheme, it is also clear that the higher the number of antennas in the BS, the higher the SE. When the number of the antennas is equal to 100 in the BS (Figure 1) the SE is equal to 40 [Bit/second/Hz/Cell] if the pilot reuse factor is equal to 2 or 4. When the number of antennas in the BS is equal to 70 and the pilot reuse factor is either 2 or 4 (Figure 2), the SE is less than 37 [Bit/second/Hz/Cell], and when the number of antennas in the BS is equal to 40 and the pilot reuse factor is equal to 2 or 4 (Figure 3), the SE is equal to 33 [Bit/second/Hz/Cell].

When comparing Figures 1-3 while considering the MR scheme, it is clear again that if there are more antennas in the BS, the SE will be higher. When the number of antennas is equal to 100 in the BS (Figure 1) the SE is equal to 24 [Bit/second/Hz/Cell] if the pilot reuse factor is equal to 2. If the number of antennas in the BS is equal to 70 and the pilot reuse factor is 2 (Figure 2), the SE is less than 22 [Bit/second/Hz/Cell]. Finally, when the number of antennas in the BS is equal to 40 and the pilot reuse factor is equal to 2 or 4 (Figure 3), the SE is less than 33 [Bit/second/Hz/Cell].

## VI. CONCLUSION

In this paper, we examined the combined impact of the pilot reuse factor and the base station number of antennas on the average total spectral efficiency in massive MIMO. Different channel estimators, such as LMMSE, ZF, and MR schemes, have been used to investigate the spectral efficiency of these receivers. According to the simulation results, a larger number of antennas in BS would provide a significantly higher average total SE. On a larger number of base station antennas, raising the pilot reuse factor from 1 to 2 for both L-MMSE and ZR schemes would give greater spectral efficiency. In addition, it is noticeable that the increase in pilot reuse factor has not a positive impact on the MR scheme's spectral efficiency. Although the L-MMSE scheme has a greater spectral efficiency than the ZF and MR schemes, it is more complex.

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