

Best Fit versus Default Distribution and the Impact on the Reliability over the Design Lifetime of Hydraulic Structures

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ABSTRACT

In the present study, Flood Frequency Analysis (FFA) is performed on the daily inflows of a reservoir dam taken as a case study. The Peaks-Over-Threshold (POT) approach was adopted. A comparison between the default generalized Pareto distribution and the best distribution fitted to the data has been carried out. After the risk analysis, the reliability of the structure decreases to 25.60% for the chosen threshold values if the best distribution is adopted instead of the default fit.

Keywords-flood frequency analysis; peaks-over-threshold; generalized Pareto distribution; best fit; risk analysis

I. INTRODUCTION

Flood Frequency Analysis (FFA) [1, 2] plays a major role in the design of hydraulic structures because it affects both safety and cost of the structure. The principal objective of FFA is to construct a relationship between the flood magnitude and the return period by the estimation of the probability of exceedance [3]. Hydrologists generally apply two types of approaches to perform FFA, Annual Maximum (AM) and Peaks-Over-Threshold (POT), also called partial duration series approach. The AM series approach is the most used method in FFA due to its simplicity in the sampling process [4, 5]. It uses only one maximum discharge value from each year. However, defining the samples in such a way eliminates a large portion of the data and results in loss of useful information, e.g. the second highest flow data in a year (which could be higher than many data points in the AM series) is not selected [4, 5]. The POT approach consists in retaining all peak values that are above a certain level usually called the threshold. The POT approach is more useful to the analysis of extreme values. The

main advantage of the POT approach is that it allows the selection of an enriched series of events to be considered as floods [7] and controls the number of flood occurrences to be used in the analysis, unlike the AM approach which includes only one event per year. Authors in [7] prepared a guide for the use of the POT approach. Authors in [6] indicate that the POT method can provide adequate and comparable estimates of N-year discharges for more stations with short temporal coverage. Authors in [8] reviewed recent advances in the POT approach from a statistical perspective. Authors in [4] reported that the POT method has better results than the AM method after comparing the results of both methods for data from the Litija 1 gauging station on the Sava River in Slovenia. Authors in [5] reviewed and summarized the current status of the POT model and identified the difficulties in applying it in FFA. Ensuring the independence of the data series and choosing an appropriate threshold value are two of the main difficulties associated with the POT approach [4]. The threshold should be high enough to maintain the assumption of flood independence, but should not be so high as to increase the variance by reducing the number

of events needed for flood analysis [9]. Therefore, the selection of an appropriate threshold is the subject of several studies and many methods, either graphical or analytical, have been reported. Graphical methods are widely used [10]. There are several probability distributions for modeling flood extreme events, the selection of the best fit distribution and the associated parameter estimation is an important step in FFA [11]. Generalized Pareto Distribution (GPD) is broadly used to model extreme floods over a threshold and the distribution of POT can be approximated by the GPD [12]. The annual number of events above the threshold can be approximated by either Poisson or Negative binomial distributions [13, 14].

In the current study, the FFA method, applied to the daily inflows of the Koudiat Medouar reservoir and the POT approach are adopted. Several additional distributions are tested to find the best one that fits the data. A risk analysis is then conducted.

II. STUDY AREA

Located 7km north-east of Timgad and 35km from Batna, the Koudiat Medouar dam was built in 1994 on the Oued Rebôa. It has a height of 48m. The reservoir is a part of the great strategic hydraulic complex Beni Haroun Transfer, with a capacity of 70 million m³. It supplies drinking and irrigation water to several towns. Figure 1 shows the Koudiat Medouar reservoir location.

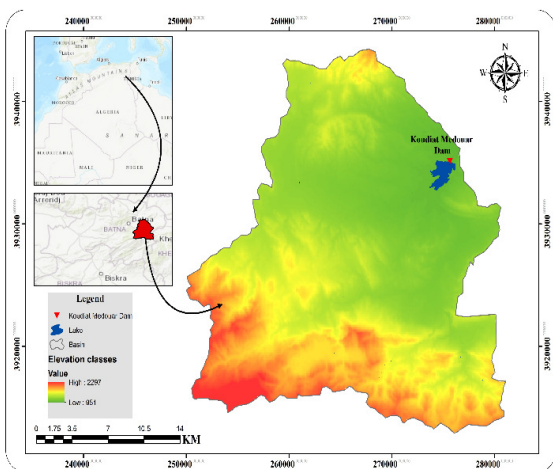


Fig. 1. Location of the Koudiat Medouar reservoir.

III. METHODOLOGY

FFA is a statistical method that consists of studying past events and characteristics of a given process (hydrological or other) to define the probability of future occurrence by a probability distribution using the AM or POT series. The POT approach consists in the constitution of a series of data by extracting the values superior to a well determined threshold. The selection of an appropriate threshold value is an important phase, Mean Residual Life Plot (MRLP) and the stability graph of the parameters are recommended in [15]. The threshold value is chosen upon a certain stability of the scale and shape graphs. Threshold is taken at the tail of the MRLP when the function of the mean value begins to be almost linear. The POT

flood series are adjusted by the distribution of GPD [16]. The adjustment of excesses above a threshold according to the GPD model hypothesis must be ensured by two discrete distributions, Poisson or negative binomial. The main step in the application of the POT method is the choice of the threshold. Indeed, selecting a very high threshold has the effect of decreasing the size of the series and thus increasing the sampling uncertainties. The Number of Maximums per Year (NMpY ≈ 1) subsequently gives a very short AM series [17]. On the other hand, a threshold that is too low (NMpY ≥ 6) gives series artificially enriched with information but one will be confronted on the problem of obtaining dependent series. Therefore, a trade-off must be found between acquiring information and obtaining independent series (Figure 2). However, there is no universal criterion for identifying such a threshold value [18]. In this paper, we focus on the following criteria for threshold selection:

- MRLPs.
- The stability of scale and modified shape graphs.
- Fixing the NMpY.

Once the preselection has been performed according to the above criteria, the basic hypothesis tests of FFA, i.e. homogeneity, independence, and stationarity, must be verified. To verify these hypotheses, three non-parametric statistical tests are used: [19] to verify the independence, [20, 21] to verify whether the data come from the same distribution or not, and [22, 23] to verify the stationarity of the data.

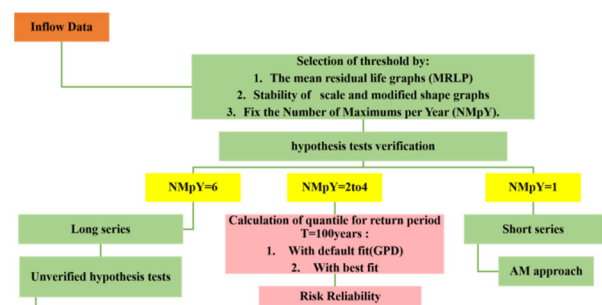


Fig. 2. Methodology chart.

A. Model Fitting

Several types of distribution are used to estimate flow extremes. The reasons for the operational use of a particular distribution type in many countries are often subjective or historical [24]. Table I illustrates the probability distributions mostly used to fit inflows. The choice of the appropriate estimator is one of the most important issues in FFA [25, 28]. The most commonly used are: the Method Of Moments (MOM), the Method of L-moments (ML), and the Maximum Likelihood Estimation (MLE). The ML is more used in statistical hydrology for estimating various hydro-meteorological variables [29, 30]. The objective of the FFA is to find reliable estimates of quantiles that will help us in the design of structures. After the selection of the thresholds, and defining the best fit, the quantiles are evaluated for a return period of 100 years.

TABLE I. USUAL PROBABILITY DISTRIBUTIONS FUNCTIONS

| Distribution | PDF |
|---------------------------------|--|
| Normal | $f(x; \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ |
| Log normal | $f(x; x_0, \mu, \sigma) = \frac{1}{(x-x_0)\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln(x-x_0)-\mu}{\sigma}\right]^2}$ |
| Gumbel | $f(x) = \frac{\exp\left[-e^{-\frac{x-x_0}{\alpha}}\right]}{\alpha} \exp\left(-\frac{x-x_0}{\alpha}\right)$ |
| Pearson 3 | $f(x; \lambda, \beta, \gamma) = \frac{1}{\lambda\Gamma\beta} \left(\frac{x-\gamma}{\lambda}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\lambda}\right)}$ |
| Generalized extreme value | $f(x; \alpha, \kappa, \mu) = \frac{1}{\alpha} \left[1 - \frac{\kappa}{\alpha}(x-\mu)\right]^{\frac{1}{\kappa}-1} \exp\left\{-\left[1 - \frac{\kappa}{\alpha}(x-\mu)\right]^{\frac{1}{\kappa}}\right\}$ |
| Generalized Pareto distribution | $f(x; \lambda, \beta, \gamma) = \begin{cases} \frac{1}{\lambda} \left[1 + \gamma\left(\frac{x-\beta}{\lambda}\right)\right]^{-1-\frac{1}{\gamma}}, \gamma \neq 0 \\ \frac{1}{\lambda} \exp\left[-\left(\frac{x-\beta}{\lambda}\right)\right], \gamma = 0 \end{cases}$ |
| Kappa | $f(x; \alpha, \kappa, \mu, h) = \alpha^{-1} \left[1 - \frac{\kappa(x-\mu)}{\alpha}\right]^{\frac{1}{\kappa}-1} [F(x)]^{1-h}$, where $F(x; \alpha, \kappa, \mu, h) = \left\{1 - h \left[1 - \frac{\kappa(x-\mu)}{\alpha}\right]^{\frac{1}{\kappa}}\right\}^{\frac{1}{h}}$ |

α : scale parameter, μ : location parameter, κ : shape parameter, γ : second scale parameter, h : second shape parameter, $\Gamma(\cdot)$: gamma function, $F(\cdot)$: cumulative distribution function

B. Risk Analysis

Water-control design requires the consideration of risk [31]. Risk and reliability analysis have great importance when it comes to natural phenomena such as floods. Risk analysis of future events requires a probabilistic approach [32]. This natural hydrological risk can be calculated using the following equations:

$$p(X \geq x_T) \text{ at least once in } N \text{ years} = 1 - \left(1 - \frac{1}{T}\right)^N \tag{1}$$

$$\bar{R} = 1 - \left[1 - p(X \geq x_T)\right]^N \tag{2}$$

where:

$$p(X \geq x_T) = \frac{1}{T} \tag{3}$$

$$\bar{R} = 1 - \left[1 - \frac{1}{T}\right]^N \tag{4}$$

where N is the expected life of the structure (in our case $N=50$ years), T is the return period (usually the life expectancy of a hydraulic structure is 100 years), and \bar{R} is the probability that an event $X \geq x_T$ will occur at least once in N years. The reliability Re is defined by:

$$Re = 100 - \bar{R} \tag{5}$$

IV. RESULTS AND DISCUSSION

A. Data Description

The used database in this study was collected from the National Agency for Dams and Transfer (ANBT). Daily inflows of Koudiat Medouar reservoir spanning the period from 2004 to 2019 were calculated according to the following water balance equation:

$$\text{Inflow} = (V_{ini} - V_{fin}) + (DEV + DWS + DRV + DSV + DFV + DLV) \tag{6}$$

where Inflow is the daily inflow, V_{ini} and V_{fin} represent the initial and the volume in the reservoir, DEV is the daily evaporated volume, DWS the daily volume allocated to water supply, DRV the daily volume allocated to irrigation, DSV the daily spilled volume, DFV the daily flashed volume, and DLV is the daily leakage volume.

Daily inflows are not normally distributed. The box plot in Figure 3 shows that two values $117.608\text{m}^3/\text{s}$ and $110.915\text{m}^3/\text{s}$ appear as outliers, but in fact they are real occurred events. Figure 4 shows the interannual variations of daily inflows at the Koudiat Medouar reservoir. Even if the stability of scale and shape parameters are confirmed (Figure 5), it is important to verify the hypothesis tests (homogeneity, independence, and stationarity). Between the thresholds of $5\text{m}^3/\text{s}$ and $11\text{m}^3/\text{s}$, even with that of $14\text{m}^3/\text{s}$, the tests were not verified.

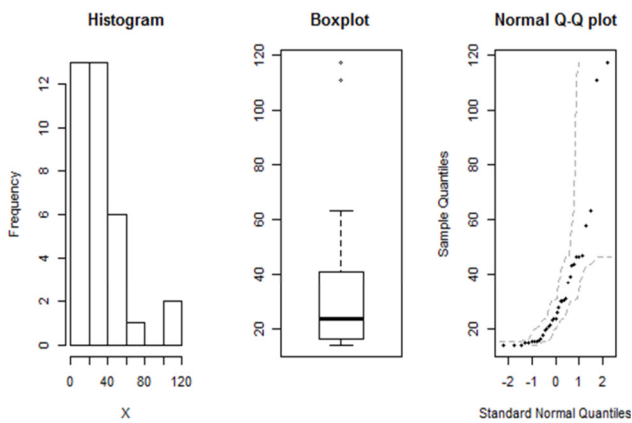


Fig. 3. Daily inflows description of Koudiat Medouar reservoir.

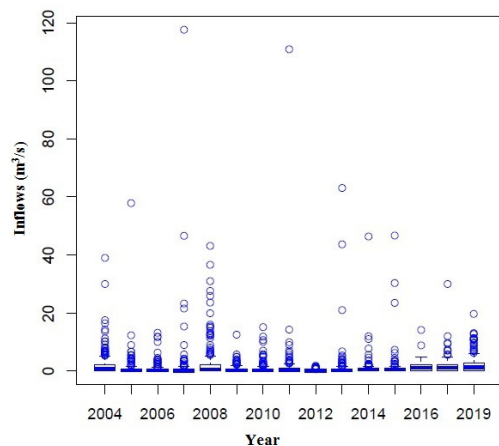


Fig. 4. Time series of the daily inflows at Koudiat Medouar reservoir (2004-2019).

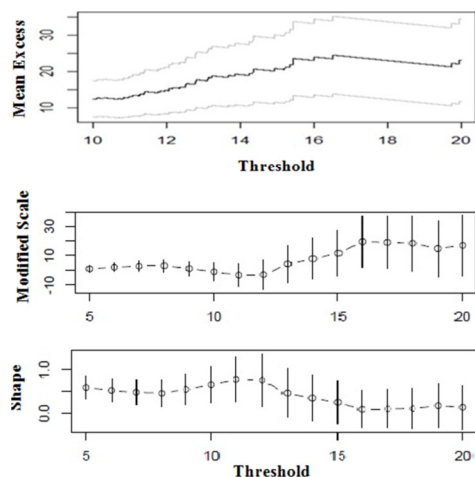


Fig. 5. Mean residual life plot for the daily inflows of the Koudiat Medouar reservoir.

This led us to keep only the values of 12, 13, and 15 m³/s respectively (Table II). Using the R code RStudio [33] and for the previously chosen thresholds, we tested whether the different distributions fit. The results show that GPD

distribution is not the best fit for 12 m³/s and 13 m³/s threshold (Figure 6). The threshold of 15 m³/s just ensures an exceedance number of 2 per year, for this purpose, this threshold is presented as a limit of choice beyond which we find the statistical situation of the AM. For thresholds below 12 m³/s, the opposite situation arises: these are samples artificially enriched in size with a greater chance of losing the satisfaction of the hypothesis tests, in particular the homogeneity and the randomness required for any reliable frequency analysis without statistical violation (Table III). The frequency distribution results are shown in Figure 6.

TABLE II. STATISTICAL PARAMETERS CORRESPONDING TO POT TIME SERIES

| Statistic | $u=12\text{m}^3/\text{s}$ | $u=13\text{m}^3/\text{s}$ | $u=15\text{m}^3/\text{s}$ |
|--------------------------|---------------------------|---------------------------|---------------------------|
| Sample size | 50 | 40 | 31 |
| Min | 12.08 | 13.12 | 15.22 |
| 1 st quantile | 13.34 | 15.31 | 19.85 |
| Median | 17.06 | 22.41 | 27.67 |
| Mean | 26.81 | 30.40 | 35.20 |
| 3 rd quantile | 30.27 | 37.26 | 43.42 |
| Max | 117.60 | 117.60 | 117.60 |
| Wald-Wolfowitz | p-value | 0.086 | 0.337 |
| Wilcoxon-Mann-Whitney | p-value | 0.851 | 0.814 |
| Mann-Kendal | p-value | 0.536 | 0.954 |

TABLE III. ESTIMATED QUANTILES FOR RETURN PERIOD T=100 YEARS

| Threshold | Type of fit | Distribution | Quantiles (m ³ /s) | RMSE (m ³ /s) |
|-----------------------------|------------------------|--------------|-------------------------------|--------------------------|
| $u = 12\text{m}^3/\text{s}$ | Best fit | Pearson 3 | 159.285 | 0.024 |
| | Default fit | GPD | 211.924 | 0.049 |
| $u = 13\text{m}^3/\text{s}$ | Best fit | Kappa | 169.318 | 0.026 |
| | Default fit | GPD | 190.416 | 0.037 |
| $u = 15\text{m}^3/\text{s}$ | Best fit = default fit | GPD | 174.899 | 0.028 |

The frequency distribution results are shown in Figure 6. The plots indicate that the Pearson 3 (RMSE=0.024) and Kappa (RMSE=0.026) distributions are the best fit for $u = 12$ and $13\text{m}^3/\text{s}$, respectively. Using (4) and (5), the risk reliability is calculated for a return period of 100 years and is presented for best fit and GPD distribution results. The quantile corresponding to 100 years return period for Pearson 3 distribution is equivalent to a 48 years return period of the GPD for the 12 m³/s threshold. Respectively, quantile corresponding to 100 years return period for Kappa distribution is equivalent to a return period of 70 years of the GPD fit for the 13 m³/s threshold (Table IV).

By selecting the best fit, the reliability of the structure decreases by 25.60% and 11.80% for thresholds of 12 m³/s and 13 m³/s, respectively, compared to the default fit. Quantile Pearson 3= 159.285 m³/s for $T=100$ years, corresponding to $T=48$ years quantile in GPD (reliability=-25.60%) and quantile Kappa= 169.318 m³/s for $T=100$ years in GPD corresponding to $T=70$ years (reliability=-11.80%). We note that when the reliability decreases, the risk increases, so we have under dimensioned hydraulic structures.

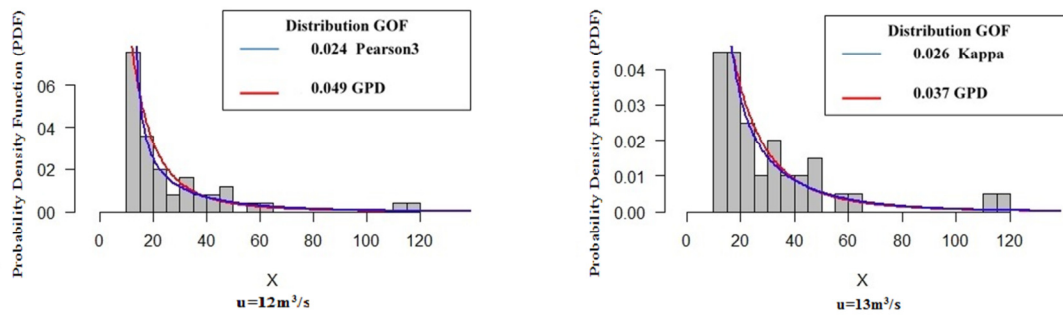


Fig. 6. Probability density function of best and default fit for the selected thresholds.

TABLE IV. RISK RELIABILITY

| | | | Pearson 3 | GPD |
|--------------------|-------------|-------------------------|-----------|--------|
| u (m^3/s) | 12 | Return period (T) | 100 | 48 |
| | | Risk (for $N=50$ years) | 39.50% | 65.10% |
| | | Reliability | 60.50% | 34.90% |
| | 13 | | Kappa | GPD |
| | | Return period (T) | 100 | 70 |
| | | Risk (for $N=50$ years) | 39.50% | 51.30% |
| | Reliability | 60.50% | 48.70% | |

V. CONCLUSION

This article presents high flood analysis in relation to the data from daily inflows of the Koudiat Medouar reservoir for the period from 2004 to 2019. FFA was carried out with the POT series. One of the main disadvantages concerning the POT approach is the selection of the threshold value. In FFA, choosing a very high threshold Number of Maximums per Year ($NMPY \approx 1$), has the effect of decreasing the size of the series and therefore increasing the sampling uncertainty which subsequently gives a very short series. On the other hand, a too small threshold gives series artificially enriched information ($NMPY \geq 6$). For this reason, we have retained only the series of 2 to 4 max per year. After the verification of the basic hypothesis tests, the thresholds retained were 12, 13, and $15m^3/s$. In this study, the GPD distribution is not systematically the best fit of the daily inflows. The study showed that the Pearson 3 and Kappa distributions are better suited than GPD for thresholds equal to 12 and $13m^3/s$, respectively, for the estimation of flood discharge.

The carried out risk analysis shows that the choice of GPD distribution decreases the reliability of the structure compared to the case of choosing the best fit distribution. The results presented in the current paper are helpful to the planning and optimization of the dimensions of hydraulic structures.

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