The Improved CURLI Method for Multi-Criteria Decision Making

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ABSTRACT

Multi-Criteria Decision Making (MCDM) investigates the best available choice in the presence of multiple conflicting criteria, whereas the Collaborative Unbiased Rank List Integration (CURLI) method has been proposed recently and has been applied in various fields of daily life. However, most previous works concentrated on analyzing cases in which the factor of a criterion is a specific quantity. The present paper proposes an approach developed from the original CURLI method, named Improved CURLI. This improvement helps solve a problem when the factors of the criteria can be linguistic variables or a data set. The proposed method is applied to rank the alternatives for two case studies: choosing the best grinding wheel and the best service suppliers. The ranking results are compared to those obtained using other methods. Furthermore, sensitivity analysis is also conducted to examine the stability and reliability of the ranking results in various scenarios. The results demonstrate the validity of the Improved CURLI method and prove that it is applicable for making decisions in various fields.

Keywords-MCDM; CURLI method; improved CURLI method; data set

I. INTRODUCTION

The concept of MCDM is increasingly being used in various fields [1, 2]. MCDM problems are mentioned as MCDA or MADM. The essence of MCDM is evaluating and ranking the available options in order to select the best and avoid the worst. Researchers have suggested different MCDM approaches, and many of them have been applied in a variety of contexts [1, 3]. However, the original versions of the MCDM approaches are deemed inadequate for circumstances in which the criteria are expressed as a set of factors or linguistic variables [4, 5]. The main reason for this phenomenon is that the uncertainty about the object depends on a lot of factors, like experts' opinions, time, location, the way of data collection, etc. [6]. To deal with this issue, approaches that combine the fuzzy method with MCDM have been proposed. That combination is referred to as fuzzy - MCDM. Many studies are progressively employing the fuzzy - MCDM methods. For instance, TOPSIS integrated with fuzzy is used in many applications such as supplier selection [7], project selection [8], healthcare software [9], supply chain management [10], manager selection [11], etc. In [12], a fuzzy VIKOR – MDCM is proposed to analyze and evaluate the quality of information security policies as well as the content of press agencies in Gulf countries. Realizing this method's exploitation potential, numerous investigations have been launched in different tasks like warehouse location selection [13], mobile services [14], and sustainable development in Islamic countries [15]. In other approaches, the fuzzy MARCOS method is utilized for road traffic risk analysis [16], assessment of drone-based city logistics [17], e-service quality in airline industry [18], and so on.

When applying fuzzy MCDM, it is necessary to determine the weights of the criteria which play an important role to make the final decision [19-21]. In this manner, calculating the weights for the criteria is relatively complex, time-consuming, and difficult in situations requiring prompt decision-making. Furthermore, the ambiguity of the decision makers (or surveyed experts) regarding the research object influences the weights determined by these methods. Hence, the accuracy of the weights after the calculation is not guaranteed. As a result, no longer will the ranking of the options be certain [22-24]. In addition, most fuzzy MCDM methods still inherit some stages from the original MCDM methods, one of which is data normalization. But the data normalization processes of the MCDM methods are also not the same, leading to the ranking results of the alternatives also being highly dependent on the data normalization and sensitive to change [25-27].

Generally, despite the widespread application of fuzzy MCDM approaches, they still have limitations to some extent. If a suggested MCDM method can solve two major problems, it will make significant contributions to this field. Due to the mentioned limitations associated with the right determination for the criteria and the normalization of data, it is necessary to propose an MCDM approach that does not require the determination of weights for the criteria and the metric normalization. To overcome these issues, the CURLI method was first introduced in 2016. It is used to rank applicants for medical programs [28]. Despite the fact that it has existed for six years, only a handful of research projects about this technique have been carried out. The applying CURLI method is proposed to inspect the quality of the X12 steel grinding process [29]. A turning test progress based on the CURLI approach was designed in [30]. The result reveals that the proposed method is as precise as the PEG method and better than the PSI method. In addition, recent works have shown that the CURLI method is just as accurate as the R and CODAS approaches to ranking robots, just as accurate as the R, SAW, WASPAS, TOPSIS, VIKOR, MOORA, COPRAS, and PIV approach to rating the turning process, and just as accurate as the R and MABAC approaches to ranking bridge construction [31]. However, all the above mentioned studies are only considered when the criteria are clearly defined (not fuzzy set). In this paper, an approach based on the original CURLI method, which allows overcoming the mentioned drawbacks is proposed. The key contributions of the current paper are:

- This paper proposes an improved CURLI method to allow solving the MCDM problems, when the factors of the criteria can be linguistic variables or a data set.
- The proposed method does not need the input data to be normalized or the weights of the criteria to be evaluated.
- The proposed approach is the first step towards enhancing the CURLI method to optimize MDCM problems as well as applying it to various fields of daily life.

II. THE IMPROVED CURLI METHOD

The original *CURLI* method involves four steps as follows [28-31]:

Step 1: Establish a decision-making matrix with m options and n criteria as in Table I. C_{ij} is the factor of criterion j^{th} in option i^{th}, where $i = 1 \div m, j = 1 \div n$.

TABLE I. DECISION-SCORE MATRIX

Alternatives	Criteria			
	C_1	C_2	C_j	C_n
A_1	C_{11}	C_{12}	C_{1j}	C_{1n}
A_2	C_{21}	C_{22}	C_{2j}	C_{2n}
A_i	C_{i1}	C_{i2}	C_{ij}	C_{in}
A_m	C_{m1}	C_{m2}	C_{mj}	C_{mn}

Step 2: Create score-point matrices of level m. In criterion j, the entry corresponding to row t and column v $(1 \le t, v \le m)$ is determined based on the following regulations:

- If the factor of criterion j in alternative A_t is worse than that of alternative A_v, the entry in the corresponding row and column will be scored as -1.
- If the factor of criterion j in alternative A_t is better than that of alternative A_v, the entry in the corresponding row and column will be scored as 1.
- If the factor of criterion j in alternative A_t is equal to that of alternative A_v, the entry in the corresponding row and column will be scored as 0.

All the entries on the main diagonal will be blank. After this step, n square score-point matrices are established in total.

Step 3: Create the process score-point matrix. The entries of this matrix are calculated by summing all the corresponding entries of the score-point matrix in step 2.

Step 4: Rearrange the process score-point matrix. The arrangement will be performed by moving the rows and columns of the process score-point matrices by the following rules: the order of columns from the left to the right corresponding to the order of the rows from the top to the bottom. The number of the negative and zero entries above the main diagonal is maximal. After sorting, the solution in the first row will be the best choice. Priority of selection reduces from the first to the last row.

The CURLI approach has been used extensively to solve optimization problems in several disciplines, in which the factors of each criterion at an alternative are a specific quantity. However, in reality, a criterion may be represented by linguistic variables or a data set. This paper proposes an approach based on the original CURLI method for handling these issues as follows:

Step 1: Establish a decision-making matrix as in Table II.

Step 2: Create score-point matrices. This step is implemented similarly to the original CURLI. However, the score-point matrices will be determined with every factor of the criteria. The score point of each criterion is denoted as p^f $(1 \le f \le k)$, where k is the number of factors of a criterion.

TABLE II. DECISION-MAKING MATRIX WITH MULTI-CHOICE OF CRITERIA IN EACH SOLUTION

Alternatives	Criteria			
	C_1	C_2	C_j	C_n
A_1	$(C^1{}_{11}, C^2{}_{11},\ldots, C^k{}_{11})$	$(C^1{}_{12}, C^2{}_{12},\ldots, C^k{}_{12})$	$(C^1{}_{1j}, C^2{}_{1j},\ldots, C^k{}_{1j})$	$(C^1{}_{1n}, C^2{}_{1n},\ldots, C^k{}_{1n})$
A_2	$(C^1{}_{21}, C^2{}_{21},\ldots, C^k{}_{21})$	$(C^1{}_{22}, C^2{}_{22},\ldots, C^k{}_{22})$	$(C^1{}_{2j}, C^2{}_{2j},\ldots, C^k{}_{2j})$	$(C^1{}_{2n}, C^2{}_{2n},\ldots, C^k{}_{2n})$
A_i	$(C^1{}_{i1}, C^2{}_{i1},\ldots, C^k{}_{i1})$	$(C^1{}_{i2}, C^2{}_{i2},\ldots, C^k{}_{i2})$	$(C^1{}_{ij}, C^2{}_{ij},\ldots, C^k{}_{ij})$	$(C^1{}_{in}, C^2{}_{in},\ldots, C^k{}_{in})$
A_m	$(C^1{}_{m1}, C^2{}_{m1},\ldots, C^k{}_{m1})$	$(C^1{}_{m2}, C^2{}_{m2},\ldots, C^k{}_{m2})$	$(C^1{}_{mj}, C^2{}_{mj},\ldots, C^k{}_{mj})$	$(C^1{}_{mn}, C^2{}_{mn},\ldots, C^k{}_{mn})$

Step 3: Build the process score-point matrix. The entries of this matrix are calculated based on the sum of all points in corresponding marking point matrices. The detailed formulation is expressed as:

$$p_j = \frac{\sum_{1}^{k} p_j^f}{k} \quad (1)$$

Step 4: Rearrange the process score-point matrix. This step is the same as in the original CURLI method.

III. VERIFICATION STUDY AND DISCUSSION

A. Case Study 1

In this sub-section, the proposed method is applied to determine which grinding wheel to choose in an MCDM problem [32, 33]. There are 8 different types of grinding wheels (A1 ÷ A8), and each wheel is described by 7 criteria (C1 ÷ C7). Each criterion has 3 alternatives. The decision is made to satisfy the conditions: (a) for C7 the smallest is the best and (b) for the other criteria, the biggest is the best. The objective of MCDM is to identify the alternative that simultaneously assures that C7 is the smallest and the remaining criteria (from C1 to C6) are maximal. This work has also been accomplished by employing the Fuzzy TOPSIS method [33] and 6 variations of the VIKOR method [32]. The results of rating the alternatives using these two ways will be compared to the results of ranking the alternatives using the proposed method in this study.

In the first step, the decision-making matrix is composed as in Table III. In the second step, scoring for the criteria will be performed. In this case, the value of each criterion at each alternative has three levels of values. The number of alternatives that need to be ranked is 8. Thus, the score for each alternative (for each criterion) is a set of numbers as shown in (2):

$$P_j^f = \left\{P_j^1, P_j^2, P_j^3\right\} \text{ with } \left\{j \in \mathbf{N} \middle| 1 \le j \le 8\right\} \quad (2)$$

After the scoring process, the decision-score matrixes are illustrated from Table IV to Table X. In the next step, the process score-point matrix is determined based on (1) and is presented in Table XI. The process score-point matrix is rearranged based on the mentioned rules in Section 2. Following the arranging procedure, the alternative in the first row could be seen as the best choice. The final arrangement is shown in Table XII.

TABLE III. DECISION-SCORE MATRIX FOR CHOOSING A GRINDING WHEEL [32, 33]

Alternatives	Criteria						
	C1	C2	C3	C4	C5	C6	C7
A1	(2700, 3200, 3700)	(391, 451, 511)	(2925, 3475, 4025)	(581, 756, 931)	(12, 17, 22)	(2.65, 4.15, 5.65)	(12, 18, 24)
A2	(2000, 2400, 2800)	(590, 690, 790)	(4275, 4975, 5675)	(1099, 1324, 1549)	(68, 98, 128)	(2.2, 3, 3.8)	(45, 60, 75)
A3	(4400, 5000, 5600)	(725, 850, 975)	(6000, 6900, 7800)	(1282, 1532, 1782)	(9, 13, 17)	(3.55, 4.5, 5.45)	(714, 864, 1014)
A4	(2600, 3000, 3400)	(350, 400, 450)	(3200, 3800, 4400)	(729, 879, 1029)	(21, 30, 39)	(3.15, 4, 4.85)	(107, 152, 197)
A5	(7300, 8000, 8700)	(818, 953, 1088)	(5900, 6700, 7500)	(4188, 4688, 5188)	(950, 1200, 1450)	(6.45, 8.6, 10.8)	(1050, 1300, 1550)
A6	(2150, 2550, 2950)	(370, 440, 510)	(3950, 4600, 5250)	(400, 480, 560)	(150, 200, 250)	(2.6, 3.1, 3.6)	(6.5, 10, 13.5)
A7	(2400, 2800, 3200)	(385, 460, 535)	(1421, 1721, 2021)	(425, 600, 775)	(55, 90, 125)	(1.95, 2.5, 3.05)	(36, 50, 64)
A8	(900, 1200, 1500)	(115, 160, 205)	(1350, 1750, 2150)	(495, 620, 745)	(1.4, 2.2, 3)	(5.75, 8.2, 10.7)	(33, 45, 57)

TABLE IV. SCORE-POINT MATRIX FOR CRITERION C1

Alternatives	Points							
	P1	P2	P3	P4	P5	P6	P7	P8
A1		-1, -1, -1	1, 1, 1	-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A2	1, 1, 1		1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1
A3	-1, -1, -1	-1, -1, -1		-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A4	1, 1, 1	-1, -1, -1	1, 1, 1		1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A5	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1		-1, -1, -1	-1, -1, -1	-1, -1, -1
A6	1, 1, 1	-1, -1, -1	1, 1, 1	1, 1, 1	1, 1, 1		1, 1, 1	-1, -1, -1
A7	1, 1, 1	-1, -1, -1	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1		-1, -1, -1
A8	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	

TABLE V. SCORE-POINT MATRIX FOR CRITERION C2

Alternatives	Points							
	P1	P2	P3	P4	P5	P6	P7	P8
A1		1, 1, 1	1, 1, 1	-1, -1, -1	1, 1, 1	-1, -1, -1	-1, 1, 1	-1, -1, -1
A2	-1, -1, -1		1, 1, 1	-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A3	-1, -1, -1	-1, -1, -1		-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A4	1, 1, 1	1, 1, 1	1, 1, 1		1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1
A5	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1		-1, -1, -1	-1, -1, -1	-1, -1, -1
A6	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1	1, 1, 1		1, 1, 1	-1, -1, -1
A7	1, -1,-1	1, 1, 1	1, 1, 1	-1, -1, -1	1, 1, 1	-1, -1, -1		-1, -1, -1
A8	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	

TABLE VI. SCORE-POINT MATRIX FOR CRITERION C3

Alternatives	Points							
	P1	P2	P3	P4	P5	P6	P7	P8
A1		1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1	-1, -1, -1
A2	-1, -1, -1		1, 1, 1	-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A3	-1, -1, -1	-1, -1, -1		-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A4	-1, -1, -1	1, 1, 1	1, 1, 1		1, 1, 1	1, 1, 1	-1, -1, -1	-1, -1, -1
A5	-1, -1, -1	-1, -1, -1	1, 1, 1	-1, -1, -1		-1, -1, -1	-1, -1, -1	-1, -1, -1
A6	-1, -1, -1	1, 1, 1	1, 1, 1	-1, -1, -1	1, 1, 1		-1, -1, -1	-1, -1, -1
A7	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1		-1, 1, 1
A8	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, -1, -1	

TABLE VII. SCORE-POINT MATRIX FOR CRITERION C4

Alternatives	**Points**							
	P1	P2	P3	P4	P5	P6	P7	P8
A1		1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A2	-1, -1, -1		1, 1, 1	-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A3	-1, -1, -1	-1, -1, -1		-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A4	-1, -1, -1	1, 1, 1	1, 1, 1		1, 1, 1	-1, -1, -1	-1, -1, -1	-1, -1, -1
A5	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1		-1, -1, -1	-1, -1, -1	-1, -1, -1
A6	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1		1, 1, 1	1, 1, 1
A7	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1		1, 1, -1
A8	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1	-1, -1, 1	

TABLE VIII. SCORE-POINT MATRIX FOR CRITERION C5

Alternatives	**Points**							
	P1	P2	P3	P4	P5	P6	P7	P8
A1		1, 1, 1	-1, -1, -1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1
A2	-1, -1, -1		-1, -1, -1	-1, -1, -1	1, 1, 1	1, 1, 1	-1, -1, -1	-1, -1, -1
A3	1, 1, 1	1, 1, 1		1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1
A4	-1, -1, -1	1, 1, 1	-1, -1, -1		1, 1, 1	1, 1, 1	1, 1, 1	-1, -1, -1
A5	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1		-1, -1, -1	-1, -1, -1	-1, -1, -1
A6	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1	1, 1, 1		-1, -1, -1	-1, -1, -1
A7	-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	1, 1, 1	1, 1, 1		-1, -1, -1
A8	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	

TABLE IX. SCORE-POINT MATRIX FOR CRITERION C6

Alternatives	**Points**							
	P1	P2	P3	P4	P5	P6	P7	P8
A1		-1, -1, -1	1, 1, -1	1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	1, 1, 1
A2	1, 1,		1, 1, 1	1, 1, 1	1, 1, 1	1, 1, -1	-1, -1, -1	1, 1, 1
A3	-1, -1 , 1	-1, -1, -1		-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	1, 1, 1
A4	-1, 1, 1	-1, -1, -1	1, 1, 1		1, 1, 1	-1, -1, -1	-1, -1, -1	1, 1, 1
A5	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1		-1, -1, -1	-1, -1, -1	-1, -1, -1
A6	1, 1, 1	-1, -1, 1	1, 1, 1	1, 1, 1	1, 1, 1		-1, -1, -1	1, 1, 1
A7	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1		1, 1, 1
A8	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1	

TABLE X. SCORE-POINT MATRIX FOR CRITERION C7

Alternatives	**Points**							
	P1	P2	P3	P4	P5	P6	P7	P8
A1		-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1	1, 1, 1	-1, -1, -1	-1, -1, -1
A2	1, 1, 1		-1, -1, -1	-1, -1, -1	-1, -1, -1	1, 1, 1	1, 1, 1	1, 1, 1
A3	1, 1, 1	1, 1, 1		1, 1, 1	-1, -1, -1	1, 1, 1	1, 1, 1	1, 1, 1
A4	1, 1, 1	1, 1, 1	-1, -1, -1		-1, -1, -1	1, 1, 1	1, 1, 1	1, 1, 1
A5	1, 1, 1	1, 1, 1	1, 1, 1	1, 1, 1		1, 1, 1	1, 1, 1	1, 1, 1
A6	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1	-1, -1, -1		-1, -1, -1	-1, -1, -1
A7	1, 1, 1	-1, -1, -1	-1	-1, -1, -1	-1, -1, -1	1, 1, 1		1, 1, 1
A8	1, 1, 1	-1, -1, -1	-1	-1, -1, -1	-1, -1, -1	1, 1, 1	-1, -1, -1	

TABLE XI. SCORE-POINT MATRIX FOR THE EVALUATION PROCESS

Alternatives	**Points**							
	P1	P2	P3	P4	P5	P6	P7	P8
A1		1	2.3333	-0.333	5	-1	-3.6667	-5
A2	-1		3	-3	5	0.3333	-3	-3
A3	-2.3333	-3		-3	3	-3	-3	-3
A4	0.3333	3	3		5	1	-1	-3
A5	-5	-5	-3	-5		-5	-5	-5
A6	1	-0.3333	3	-1	5		-1	-3
A7	3.6667	3	3	1	5	1		-0.3333
A8	5	3	3	3	5	3	0.3333	

Table XII indicates that all the entries above the principal diagonal are negative, therefore A5 is the best choice and A8 is the worst one. The rank of alternatives is as follows: A5 > A3 > A6 > A2 > A1 > A4 > A7 > A8. In Table XIII, the ranking results of the present study are compared to Fuzzy TOPSIS method [33] and 6 variants of the VIKOR method.

TABLE XII. THE PROCESS SCORE-POINT MATRIX AFTER REARRANGEMENT

Alternatives	**Points**							
	P5	P3	P6	P2	P1	P4	P7	P8
A5		-3	-5	-5	-5	-5	-5	-5
A3	3		-3	-3	-2.3333	-3	-3	-3
A6	5	3		-0.3333	-1	-1	-1	-3
A2	5	3	0.3333		-1	-3	-3	-3
A1	5	2.3333	1	1		-0.3333	-3.6667	-5
A4	5	3	1	3	0.3333		-1	-3
A7	5	3	1	3	3.6667	1		-0.3333
A8	5	3	3	3	5	3	0.3333	

TABLE XIII. THE COMPARISON AMONG DIFFERENT METHODS FOR CHOOSING THE GRINDING WHEEL

Alternatives	**Methods**							
	Original VIKOR	Comprehensive VIKOR	Fuzzy VIKOR	Regret VIKOR	Modified VIKOR	Interval VIKOR	Fuzzy TOPSIS	Improved CURLI
A1	6	6	6	6	6	5	6	5
A2	3	3	3	5	3	4	3	4
A3	2	2	2	2	2	2	2	2
A4	4	5	4	3	5	3	4	6
A5	1	1	1	1	1	1	1	1
A6	5	4	5	4	4	6	5	3
A7	7	7	7	7	7	8	7	7
A8	8	8	8	8	8	7	8	8

The comparison results reveal a high degree of correlation between the methodologies. All approaches offer the same optimal and secondary options. In addition, the proposed method produces 7th and 8th alternatives similar to the other methods. There is a variation in the arrangement of alternatives between the 3rd and 6th. However, the difference is small, and this does not significantly impact the overall conclusion. It is evident that analyzing the sensitivity of the method plays an important role in solving the MCDM problem [34]. In this article, we inspect the sensitivity of the proposed approach by ranking the alternatives in the case of withdrawing at least one alternative out of the group randomly. The rank of the alternatives then is compared to the ideal order. The term ideal means that there is no reverse occurring if an alternative is eliminated. In the first scenario, the worst choice A8 is withdrawn from the calculation (Figure 1).

Fig. 1. The order of the solution after removing A8.

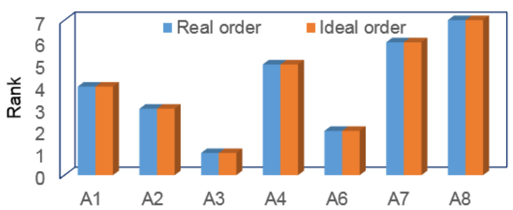

Fig. 2. The order of the solution after removing A5.

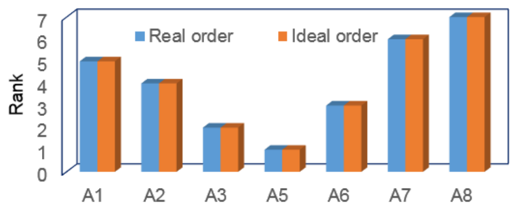

Fig. 3. The order of the solution after removing A4.

After that, the best alternative is eliminated from the list (Figure 2), and finally, the alternative in the middle of the rank is eliminated (Figure 3). It is observed that in all the considered cases, the order of the alternatives is exactly the same as the ideal order. This proves that the proposed method is reliable and applicable in solving MCDM problems.

B. Case Study 2

In the second case study, the proposed method is applied to find out the best service suppliers. Each criterion is demonstrated by linguistic variables and the decision is made based on 5 criteria: product quality (C1), price (C2), delivery process (C3), service quality (C4), and past efficiency (C5). Each criterion in an alternative row has 3 sub-options and the meaning of linguistic variables is illustrated in Table XIV [35]. The smallest criterion C2 is the best, and for the others, the largest is the best.

The Fuzzy TOPSIS method is applied to rank the alternatives [35]. The statistics of this alternate ranking approach will be compared to those of the proposed method. Applying the improved CURLI method to estimate the alternatives in Table XIV is the same as in the case study 1, and the ranking results are compared to the Fuzzy TOPSIS [35] in Table XV.

TABLE XIV. DECISION-SCORE MATRIX FOR CHOOSING SERVICE SUPPLIER [35]

Alternatives	**Criteria**				
	C1	C2	C3	C4	C5
A1	(M, G, G)	(VG, M, G)	(G, G, G)	(SB, G, G)	(G, SG, VG)
A2	(G, G, VG)	(SG, G, G)	(VG, VG, VG)	(M, G, G)	(G, G, VG)
A3	(G, G, M)	(SG, VG, SG)	(G, SG, SB)	(G, G, G)	(G, VG, SG)
M – medium; G – good; VG – very good; SG – small good; SB – small bad					

TABLE XV. COMPARISON BETWEEN THE IMPROVED CURLI AND THE FUZZY TOPSIS FOR CHOOSING THE SERVICE SUPPLIERS

Alternatives	**Methods**	
	Fuzzy TOPSIS	**Improved CURLI**
A1	2	2
A2	1	1
A3	3	3

It can be seen that the ranks of the alternatives of the two methods are precisely the same: A2 is the best and A3 is the worst alternative. This once again confirms the reliability of the proposed method. The sensitivity analysis of the alternative ranking is also performed and evaluated in detail. Figures 4-6 indicate the chart of the solution ranking after eliminating A1, A2, and A3.

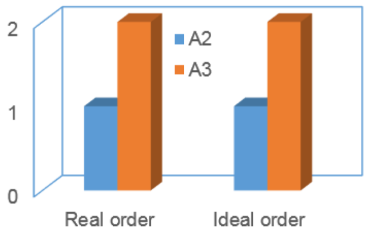

Fig. 4. The order of the solution after removing A1.

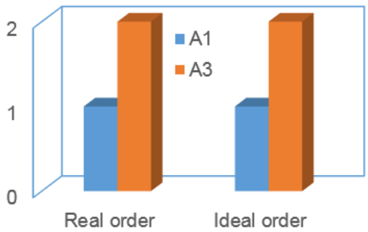

Fig. 5. The order of the solution after removing A2.

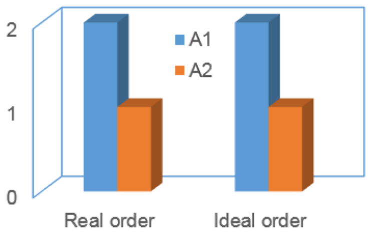

Fig. 6. The order of the solution after removing A3.

The comparison shows that there is no reverse phenomenon appearing in all considered scenarios. This once again confirms the success of the proposed improved CURLI method in the determination of the priority order.

IV. CONCLUSION

This paper proposes the improved CURLI approach based on the original CURLI method. Two case studies were implemented to evaluate the methodology efficiency with different types of variables. The ranking results are compared to those of other methods to verify the reliability of the proposed method. The following conclusions are drawn:

- It is notable that there was no major difference between the top and bottom positions. Additionally, the sensitivity of the improved CURLI was examined in the case of removing some random alternatives. The results indicate that there is no reverse of rank in any of the considered case studies. This is strong evidence that supports the idea that the improved CURLI method is better than the old MCDM when there are problems with data and uncertainty.

- It is also interesting that the proposed method can rank problems in which multiple factors influence each criterion at an alternative level without the requirement that the input data should be normalized or the criteria weights be evaluated.

- The improved CURLI method allows solving MCDM problems when the factors of the criteria can be linguistic variables or a data set. This study is the first step towards enhancing the understanding of the CURLI method to optimize MDCM problems.

LIST OF ACRONYMS

MCDM	Multi-Criteria Decision-Making
CURLI	Collaborative Unbiased Rank List Integration
MCDA	Multiple-Criteria Decision Analysis
MADM	Multi-Attribute Decision-Making
TOPSIS	Technique for Order Performance by Similarity to Ideal Solution
VIKOR	Vlsekriterijumska optimizacija i KOmpromisno Resenje (in Serbian)
MARCOS	Measurement Alternatives and Ranking according to Compromise Solution
PEG	Pareto-Edgeworth Grierson
CODAS	COmbinative Distance-based ASsessment
R	Ranking of the attributes and alternatives
SAW	Simple Additive Weighting
WASPAS	Weighted Aggregates Sum Product Assessment
MOORA	Multi-Objective Optimization on the basis of Ratio Analysis
COPRAS	COmplex PRroportional ASsessment
PIV	Proximity Indexed Value
MABAC	Multi-Attributive Border Approximation area Comparison

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