Slope Reliability Analysis in Locating and Observing the Direction of Failure Propagation

Houcine Djeffal
Tissemsilt University, Algeria | Ecole Nationale Polytechnique, Algeria
houcine.djeffal@yahoo.com
(corresponding author)

Smain Belkacemi
Ecole Nationale Polytechnique, Algeria
smain.belkacemi@g.enp.edu.dz

Received: 22 November 2022 | Revised: 5 December 2022 | Accepted: 12 December 2022

ABSTRACT

Slope stability analysis is traditionally carried out with the ultimate equilibrium methods. In these approaches, the global assessment of slope stability is obtained through the evaluation of the factor of safety. The strength parameters that allow the evaluation of resistant forces along the hypothetical slip surface are considered unique and constant. The shape of the stress-strain curve of the soil may substantially affect the stability of the soil mass over, particularly in the case of the potential of a progressive failure. As a part of this work, the influence of uncertainties on the peak and post-peak strength parameters and the impact of a progressive failure on the failure probability of a hypothetical slip surface are discussed. The conventional method of Morgenstern-Price of slope stability analysis consisted of locating the critical slip surface by evaluating the minimum factor of safety. The probability of failure was estimated using three different techniques, i.e. FORM, FOSM, and Monte Carlo simulation. This study improved the assessment of the impact of progressive failure on the risk of slope failure. The reliability analysis of slices allowed for locating the failure area and observing the direction of failure propagation.

Keywords-limiting equilibrium; slope stability; slip surface; progressive failure; uncertainties; reliability index

I. INTRODUCTION

Slope failure has become one of the most important problems in geotechnical engineering. The limit equilibrium method is commonly used in slope stability analysis due to its simplicity and high efficiency. The assumptions and simplifications of conventional limit equilibrium slope stability methods may not be sufficient to represent the behavior of complex slope problems. Since this method is a statically indeterminate problem, assumptions on the inter-slice shear forces are employed to render the problem statically determinate. The classical limit equilibrium method only considers the ultimate limit state of the slope and provides no information on the development of progressive failure [1]. Some researchers [1-3] extended the limit equilibrium method to analyze the stability of strain-softening slopes, and assumed that the soil strength decreases directly to the residual value from the peak value. With the recent advancements in computational approaches, it is possible to model slope stability problems more realistically by adopting numerical simulation methods [4-6].

Due to depositional and post-depositional processes, soil characteristics such as density, mineral composition, moisture content, stress history, and shear strength change geographically. In addition, there are other factors, including the insufficient number of samples and measurement errors, which make more difficult to determine the soil properties precisely. In practice, measurements are taken only at selected locations, and thus, the soil properties are known only at these locations and can therefore be considered as random quantities. Then, utilizing their correlation structures, soil characteristics may be efficiently described in terms of their spatial variations [7, 8]. True security is unknown because the simplifications of these methods take into account neither the variability of soil strength nor the role of the deformation behavior of the mass of slip soil. The progressive failure mechanism is also dominant for the slope stability, because the interfaces between components in slopes exhibit strain-softening behavior [7]. The shape of the stress-strain curve of the soil may substantially affect the stability of the soil mass over, particularly in the case of a potential progressive failure. The many sources of uncertainty that emerge in these types of problems are systematically evaluated using a reliability approach to a slope stability analysis that takes into account the spatially correlated soil variables. Measurement errors and bias that often occur during soil investigations are now incorporated into the probabilistic model of soil properties in addition to the spatial variability and the effects of spatial averaging. Probabilistic methods are increasingly recognized as a useful tool to explicitly consider the effect of uncertainties on slope stability assessment. Many methods regarding slope reliability analysis have been developed [8-13]. Such approaches allow the engineers to better understand and quantify the risks.

In order to analyze slope stability, the current research suggests integrating several mechanical methodologies with reliability analysis methods. This paper examines the issue of slope stability in a probabilistic framework and aims to quantify the influence of uncertainties of strength parameters, and the incidence of progressive failure on stability. The analysis employed the Morgenstern-Price slope stability deterministic approach. Probabilistic methods FORM (First Order Reliability Method), FOSM (First Order Second Moment), and the Monte Carlo Simulation (MCS) technique were used to perform slope reliability analysis at different uncertainty levels of the basic parameters.

II. RELIABILITY INDEX AND PROBABILITY OF FAILURE

Soils are heterogeneous materials, created by complex geological processes. Their properties change from one point to another. Due to the uncertainty inherent spatial variability, and the limited information available, soil properties can be considered as random variables. The probability theory provides a mathematical framework for quantifying uncertainty. The probabilistic approach is based on a deterministic model, in which the various uncertain parameters are modeled as random variables $X = [X1, X2, X3, ..., Xn]$. Considering the simple case of a resistance R subjected to an S, consisting of independent random variables or dependent and distributed according to given laws of probability, the failure probability free operation, reliability, is equal to the probability of occurrence of the event $R \geq S$. R and S are characterized by their first moments, i.e. μ_R and μ_S, σ_R and σ_S, respectively [14].

A widely accepted and simple definition of the reliability index is the ratio of the expected safety margin to the standard deviation of the performance function. Several numerical methods have been developed for estimating the statistical moments of a performance function dependent on multiple random variables. For practical purposes, the two important statistical moments to be estimated are the expected value and the variance (square of standard deviation) [14-21]. The fundamental problem of reliability theory lies on the calculation of the integral multiple of probability to estimate the probability of failure P_f:

$$P_f = P[g(X) \leq 0] = \int_{g(X)\leq 0} f(x)dx \quad (1)$$

where $g(X)$ is the performance function associated with an operating rule, the relation $g(X) = 0$ constitutes the state limit.

Various solution methods have been proposed to estimate P_f and the Reliability Index. Among the most widely used methods are FOSM, FORM, and MCS [14, 15].

III. SLOPE STABILITY ANALYSIS

Once the geometry and conditions of the basement of a slope were determined, slope stability can be assessed. The main objectives of slope stability analysis include the assessment of rupture risk, through the calculation of the overall safety factor for a slope and the locations along the potential slip surface areas with high potential rupture [16]. There are several methods of slope stability analysis, based on the calculation of the limit equilibrium. Most of these methods use a technique called slices. In these methods, the safety factor is calculated using one or more equations of static equilibrium applied to the soil mass. The limit equilibrium methods need first to define the surface for which the safety factor will be evaluated. The safety coefficient calculations are repeated for a sufficient number of slip surfaces arbitrarily selected to locate the surface having the minimum safety factor. The slip surface shape depends on the geometry, stratigraphy of the problem, physical characteristics, and capabilities of the analytical method used. Forms of the slip surface may be circular or non-circular. The methods of static equilibrium decompose the soil mass above the sliding surface, in a finite number of slices. The analysis problem of slope stability is statically indeterminate. The various limit equilibrium methods use different assumptions or constraints to remove the indeterminacy.

IV. PROGRESSIVE FAILURE

Since the sixties, several authors have observed that the failure of slopes is a consequence of the development of large shear stresses well before the start of the failure, eventually leading to what is called the progressive failure [16-18]. More recently, many authors confirm that the rupture of the slopes is often a progressive nature [22]. The analysis of slope stability is often associated with the stress-strain curve of the soil for very large deformation which mobilizes a residual strength. If the material, along with a sliding surface, is best represented by a stress-strain curve having a peak, it will be impossible to engage the same shear strength on the entire surface. After the mobilization of the shear strength available to the peak, the mobilized shear strength decreases and tends towards the peak with shear strength increasing ground deformation. For simplicity, we assume that the strength reduction, the peak to residual, is instantaneous (Figure 1).

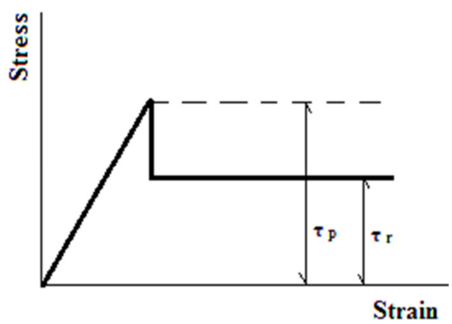

Fig. 1. Stress-strain curve used in the model.

Assessing the reliability of a slope in a progressive failure is a difficult task. When the shear strength mobilized locally corresponds to the maximum shear strength of the soil, there is a failure due to local stress concentrations likely to spread. The differential shear force is systematically transferred to the neighboring elements.

V. PROGRESSIVE FAILURE PROBABILITY

The slope stability problem can be considered as a system with multiple slip surfaces. The failure probability of the critical slip surface is considered as an estimate of the failure probability of the system. This approach assumes that the failure probabilities along the different slide surfaces are strongly correlated [23, 24]. The common approach in the probabilistic analysis is to locate the critical surface and then calculate the failure probability corresponding to this surface. However, the surface having a minimum safety factor may not be the surface of maximum failure probability [25]. The critical surface probability is associated with the highest failure probability or the lowest reliability [24, 26-29]. A model based on the bivariate failure probability of successive slices was presented in [17, 18]. This model has been developed to include the gradual overall probability by considering the slope of the spread out along the entire sliding surface. Fracture propagation is only a consequence of the interdependence of different age breakdowns of a sliding surface. The bivariant calculated probabilities relate to progressive successive events. This probability depends on the order in which the various tranches occur in sequence during the rupture propagation.

A. The Proposed Model

The proposed model treats the probabilistic progressive failure of a slope based on conditional probabilistic calculations of failure. The formulation of the statistical parameters of the safety factor, and in particular the correlation coefficient between the performance functions G_i and G_{i-1} of slices i and $i-1$, is laborious. Extending the problem to 4 strength parameters (C_P, ϕ_P), (C_r, ϕ_r), representing respectively the peak and the residual (post-peak) state, and their spatial variability is particularly cumbersome. Obviously, it is desirable to have a simpler model, in which complex calculations involving such correlations and joint probabilities of events are not required. The performance function G_i of a slice i is defined as the safety margin calculated assuming that the shear strength has reached the maximum peak:

$$G_i = (R_p)_i - S_i \tag{2}$$

The performance function G_j* of progressive failure is defined as the conditional safety margin related to failure propagation until the j th slice of the slip surface:

$$G_j^* = \left(R_p\right)_j + \left(R_r\right)_{1,j-1} - S_{1,j} \tag{3}$$

where $(R_P)_j$ is the shear strength of the slice j in peak, $(R_r)_{1,j\text{-}1}$ is the sum of the residual shear resistance of units 1 to (j-1), S_j is the shear stress of slice j, $S_{1,j}$ is the sum of shear stresses along the slices 1 to j.

B. Progressive Failure Probability

Let as assume F_j as the conditional event for failure, the mobilization of peak strength, developed at the slice j, given that the (j-1) previous installments mobilize the peak strength after the residual strength. The conditional event F_j is expressed by $F_j = \left(G_j^* < 0\right)$. E_j is the event of progressive failure of slices 1 to j in the order listed and about 1 to j represents the order of reaching the breaking of each slice. The probability that the failure develops gradually from slice 1 to slice j is a conditional probability. If we consider the situation in which the failure is propagated to the j th slice, this occurs only if the slices 1 to (j-1) are beyond the post-peak, this particular situation allows to write:

$$P\left[F_{j-1} / F_j\right] = 1 \tag{4}$$

knowing that:

$$P\left[F_{j-1} / F_j\right] = \frac{P\left[F_j F_{j-1}\right]}{P\left[F_j\right]} \tag{5}$$

$$P\left[F_j / F_{j-1}\right] = \frac{P\left[F_j F_{j-1}\right]}{P\left[F_{j-1}\right]} \tag{6}$$

Equations (4)-(6) are used to acquire (7):

$$P\left[F_j / F_{j-1}\right] = \frac{P\left[F_j\right]}{P\left[F_{j-1}\right]} \tag{7}$$

Hence:

$$P\left[E_j\right] = \frac{P\left[F_j\right]}{P\left[F_{j-1}\right]} P\left[E_{j-1}\right] \tag{8}$$

Failure propagation, based on conditional events, is a process in which a number of slices are already at the state of failure, having a failure probability that should be considered equal to unity. The likelihood of progressive failure of the system composed of n slices knowing that the first j slices are already beyond the state of failure is:

$$P\left[E_{(j+1)_n}\right] = P\left[E_n | E_1 = E_2 = \cdots = E_j = 1)\right] \tag{9}$$

Hence:

$$P\left[E_{(j+1)_n}\right] = P[E_n/E_{n-1}]P[E_{n-1}/E_{n-2}]P[E_{n-2}/E_{n-3}] \\ \ldots . P\left[E_{j+1}/E_j\right] \tag{10}$$

Calculations are performed in 3 distinct stages. In the first step, successive calculations of the probability or reliability index of the performance function G_i of each slice are made. After evaluating the failure probability of each slice, the direction of the rupture propagation is identified. A new index j is used to order the slices in descending order of failure probability of the slice. In the second stage, the calculation of conditional probabilities $P[F_j / F_{j\text{-}1}]$ and the probabilities $P[E_j]$ is performed. The final step in the simulation of the probability of progressive failure of the system composed of n slices, considering that for an increasing proportion, defined by a number of slices, the previously located surface potential shift, occurs beyond the peak strength and thereby engages only a residual strength.

VI. ILLUSTRATIVE EXAMPLE

The evaluation of the reliability index of slopes, taking into account the progressive failure, is made in two stages. In the first step, the limit equilibrium method of Morgenstern-Price is used to locate the critical slip surface. The geometrical parameters are assumed deterministic and their average values are shown in Table I. In the second step, a probabilistic analysis of the critical slip surfaces is carried out by means of physical models of the objective function for each of the developed considered deterministic methods. The analysis is made by taking into account the variation of the strength parameters and includes the impact of a progressive failure through an evolution of strength parameters of their initial values (C_p, ϕ_p) at their peak to residual values (C_r, ϕ_r). The probabilistic methods FORM, FOSM, and MCS were used. The parameters considered as random variables in the reliability analysis of slopes are shown in Table I. The change in the reliability index or the failure probability as a function of the variation coefficient of random variables is obtained while keeping the average random variables constant, and setting the coefficients of change in the other random variables at an arbitrary fixed value of 30%, and by changing the value of the variation coefficient of the variable into account. Random variable distributions are considered either normal or lognormal. Strength parameters to peak and post-peak are either independent or negatively correlated with a correlation coefficient ρ = -0.5.

TABLE I. PARAMETERS AND MEAN VALUES CONSIDERED IN THE SLOPE STABILITY ANALYSIS

Parameter	Symbol	Variable	Mean value
Cohesion term in peak	C_p	Random	20kN/m^2
Friction angle in peak	$\varnothing_p$	Random	25°
Cohesion term in post-peak	C_r	Random	10kN/m^2
Friction angle in post-peak	$\varnothing_r$	Random	10°
Weight	γ	Deterministic	18kN/m^3
Slope angle	β	Deterministic	26.56°
Slope height	H	Deterministic	20m

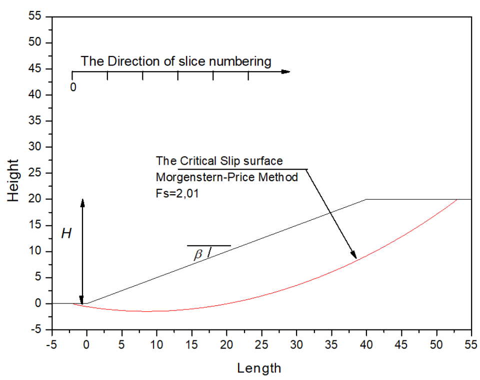

Fig. 2. The critical slip surface Morgenstern-Price method.

A. Deterministic Analysis of Slope Stability

The results of the deterministic study of the slope stability obtained by Morgenstern-Price method are represented below. The equilibrium of forces and moments is examined for each slice and the whole system, by dividing the soil mass into 250 slices. The critical slip surface is shown in Figure 2. The results of the deterministic analysis of slope stability have yielded a value of safety factor *Fs* = 2.01.

B. Probabilistic Analysis of Slope Stability

The results of the reliability analysis of each slice are shown in Figures 3-8. These figures show, for each value of the variation coefficient of strength parameters, variations in the reliability index of each slice of the system depending on the physical number of slices and the number of slices ordered in ascending order of reliability index. The main results are:

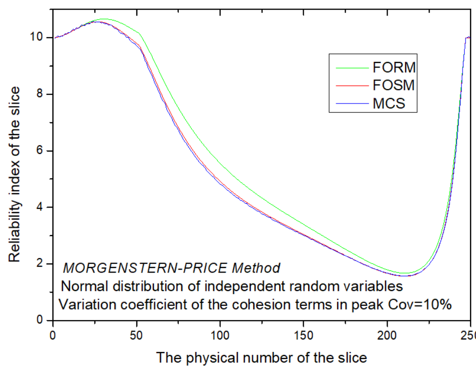

Fig. 3. Variation of the reliability index of the slice as a function of the physical number of the slice (normal distribution of independent random variables). Variation coefficient of the cohesion terms in peak *Cov* = 10%.

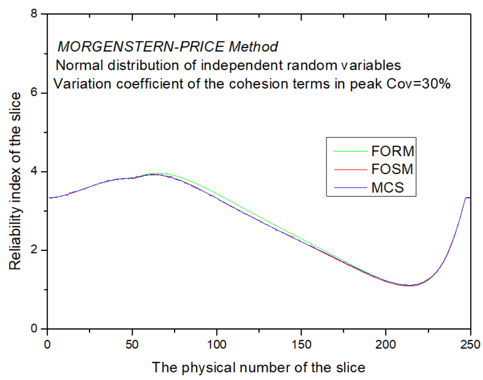

Fig. 4. Variation of the reliability index of the slice as a function of the physical number of the slice (normal distribution of independent random variables). Variation coefficient of the cohesion terms in peak *Cov* = 30%.

- The change in the shape of the reliability index as a function of the physical number of slices can locate the first slice passed in the failure state. It corresponds to the slice of the least reliability index.
- The change of the reliability index takes place simultaneously in both directions, relative to the first phase triggering failure.
- A strong decrease in the reliability index of slices is observed for high variation coefficients of strength parameters in post-peak. This decrease reflects the high sensitivity of the reliability index of a slice to the strength parameters in post-peak relatively to peak strength parameters.

- Taking into account the correlation between the strength parameters results in an increase in the reliability index of slices. This increase is more sensitive to low variation coefficients.
- For all cases considered, the FORM, FOSM and MCS methods give very similar results, regardless of the distribution type of random variables, normal or lognormal, and regardless of the correlation matrix considered.

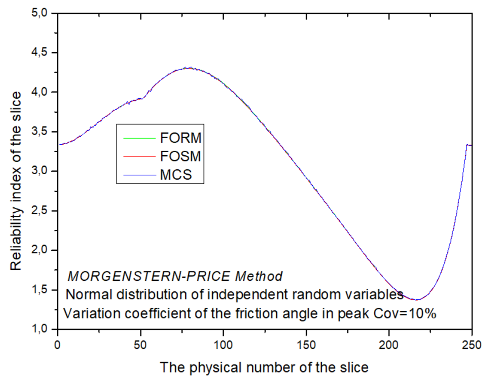

Fig. 5. Variation of the reliability index of the slice as a function of the physical number of the slice (normal distribution of independent random variables). Variation coefficient of the cohesion terms in peak *Cov* = 10%.

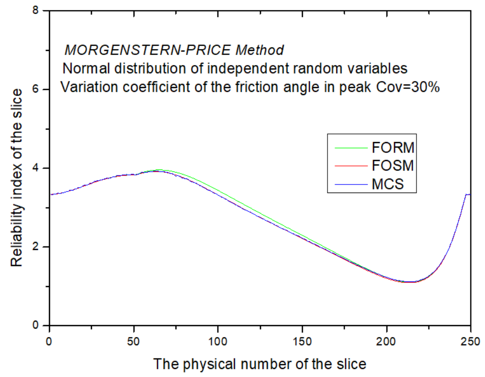

Fig. 6. Variation of the reliability index of the slice as a function of the physical number of the slice (normal distribution of independent random variables). Variation coefficient of the cohesion terms in peak *Cov* = 30%.

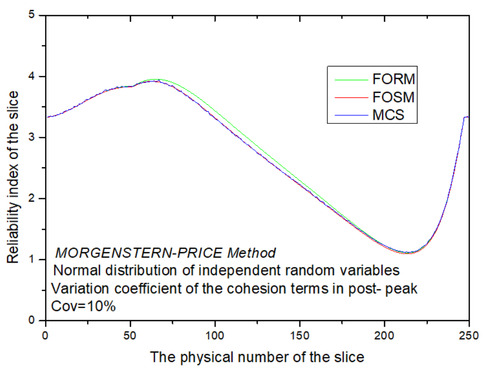

Fig. 7. Variation of the reliability index of the slice as a function of the physical number of the slice (normal distribution of independent random variables). Variation coefficient of the cohesion terms in post-peak *Cov* = 10%.

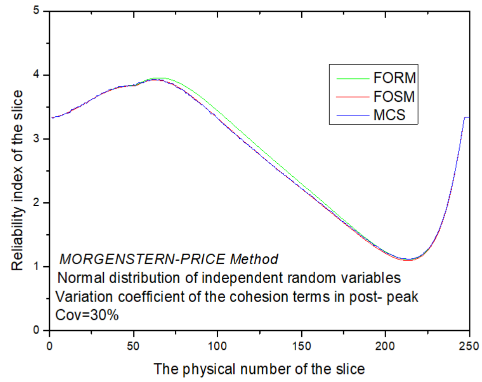

Fig. 8. Variation of the reliability index of the slice as a function of the physical number of the slice (normal distribution of independent random variables). Variation coefficient of the cohesion terms in post-peak *Cov* = 30%.

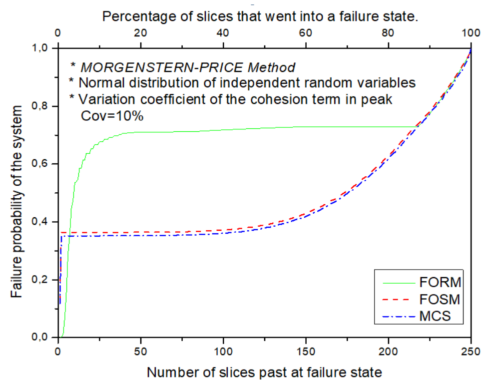

Fig. 9. Variation of the progressive failure probability of the system as a function of the number of slices past failure state (normal distribution of independent random variables). Variation coefficient of the cohesion term in peak *Cov* = 10%.

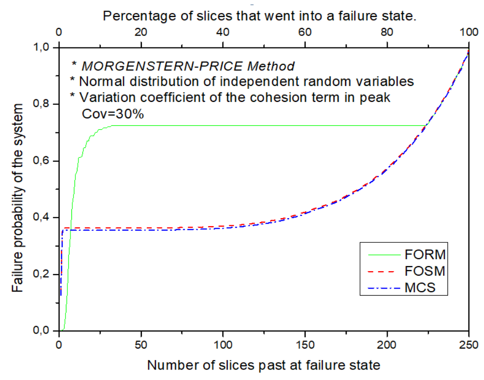

Fig. 10. Variation of the progressive failure probability of the system as a function of the number of slices past failure state (normal distribution of independent random variables). Variation coefficient of the cohesion term in peak *Cov* = 30%.

The probabilistic analysis results of the progressive failure of the system are shown in Figures 9-14. These figures show, for each value of the variation coefficient of strength parameters peak or post-peak changes in the failure probability of the system based on the number of slices past at failure state. The main derived results are:

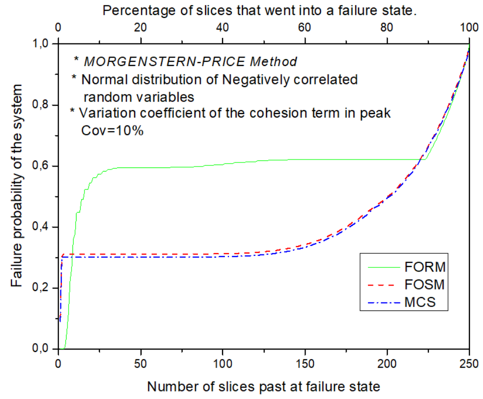

Fig. 11. Variation of the progressive failure probability of the system as a function of the number of slices past failure state (normal distribution of independent random variables). Variation coefficient of the cohesion term in peak *Cov* = 10%.

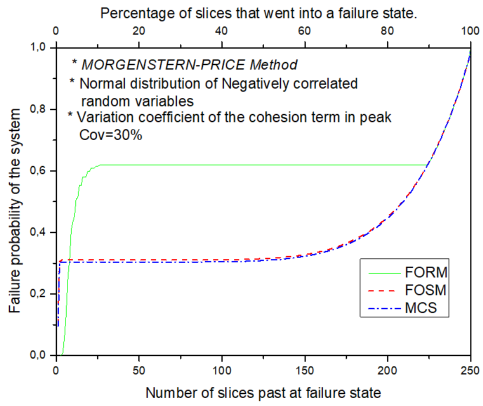

Fig. 12. Variation of the progressive failure probability of the system as a function of the number of slices past failure state (normal distribution of independent random variables). Variation coefficient of the cohesion term in peak *Cov* = 30%.

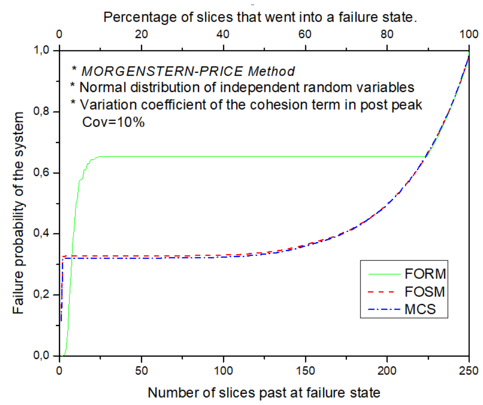

Fig. 13. Variation of the progressive failure probability of the system as a function of the number of slices past failure state (normal distribution of independent random variables). Variation coefficient of the cohesion term in post-peak *Cov* = 10%.

- For all the considered cases, FOSM and MCS give very similar results, regardless of the type of distribution of the random variables.
- The failure probabilities of the system obtained by the reliability methods FOSM and MCS differ from those obtained by FORM, and they become similar when over 90% of the slices of the system are under the condition of failure.
- The inclusion of a negative correlation between the strength parameters in probabilistic analysis, results in a decrease in the failure probability of the system.

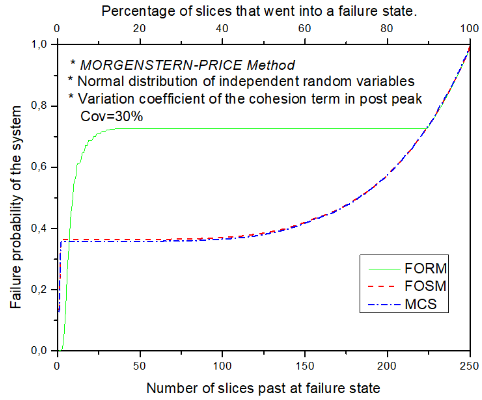

Fig. 14. Variation of the progressive failure probability of the system as a function of the number of slices past failure state (normal distribution of independent random variables). Variation coefficient of the cohesion term in post-peak *Cov* = 30%.

The results of the system reliability analysis are shown in Figures 15-20. These figures show, for each strength parameter of the limit state function, changes in the reliability index of the system depending on variation coefficient. The main derived results are:

- The reliability index is strongly dependent on the strength parameters in post-peak relative to the peak strength parameters, regardless of the type of distribution of the random variables and regardless of the considered correlation matrix.

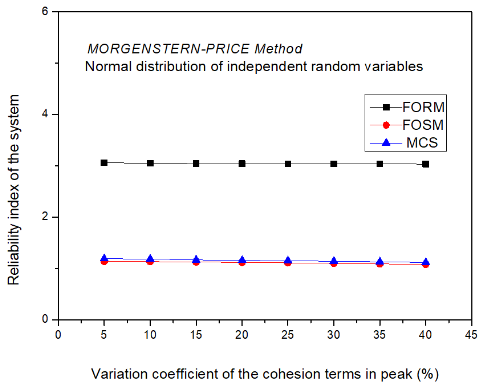

Fig. 15. Variation of the reliability index of the system as a function of the variation coefficient of the cohesion terms in peak. Independent random variables.

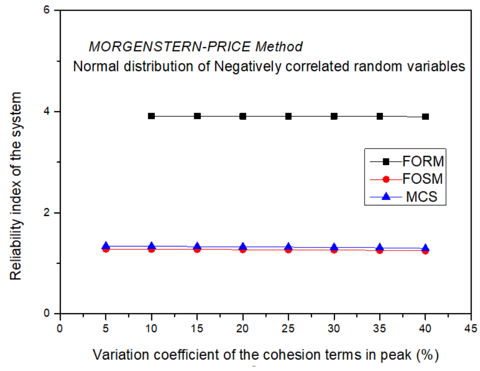

Fig. 16. Variation of the reliability index of the system as a function of the variation coefficient of the cohesion terms in peak. Negatively correlated random variables.

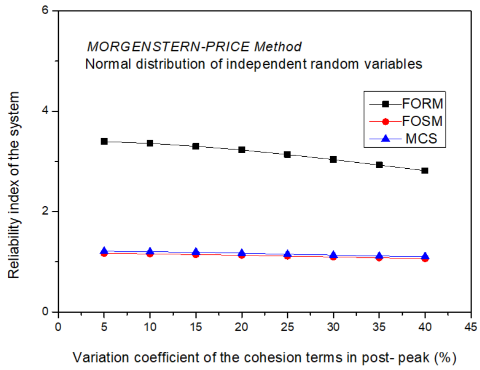

Fig. 17. Variation of the reliability index of the system as a function of the variation coefficient of the cohesion terms in post-peak. Independent random variables.

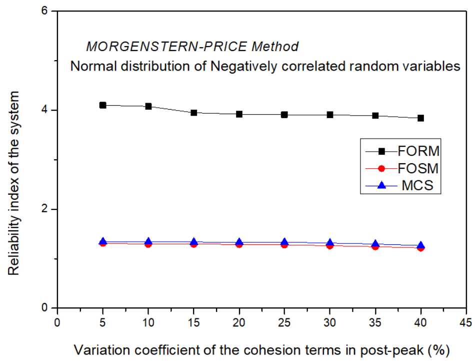

Fig. 18. Variation of the reliability index of the system as a function of the variation coefficient of the cohesion terms in post-peak. Negatively correlated random variables.

- In post peak, the cohesion term has a strong influence on the reliability index of the system with respect to the friction angle.
- The distribution of strength parameters has a strong influence on the reliability index.
- For all the considered cases, FOSM and MCS give very similar results, regardless of the distribution of the random variables.
- The reliability index of the system obtained by FOSM and MCS differs from that obtained by FORM.

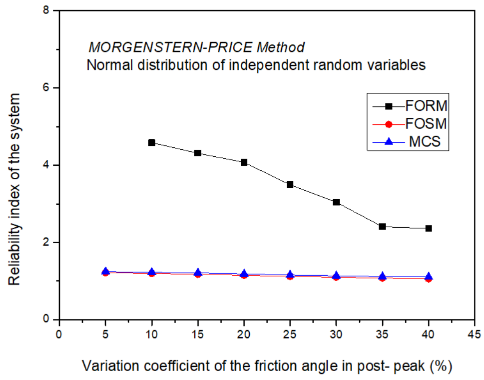

Fig. 19. Variation of the reliability index of the system as a function of the variation coefficient of the cohesion terms in post-peak. Independent random variables.

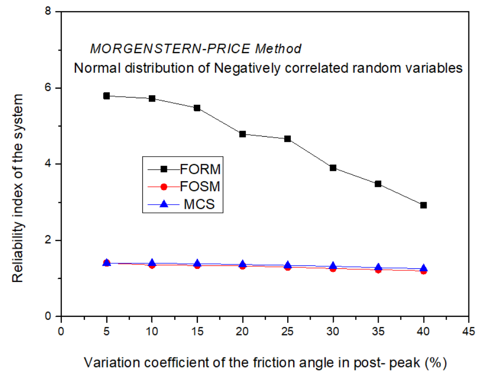

Fig. 20. Variation of the reliability index of the system as a function of the variation coefficient of the cohesion terms in post-peak. Negatively correlated random variables.

CONCLUSION

The study of reliability analysis of slope stability has, in the case of a progressive failure, to assess the effects of the uncertainties of strength parameters and the reliability index. The main conclusions from this paper are:

- The traditional design of slopes that uses safety factors can be misleading. The slopes designed with large safety factors are not exempt from failure risk.
- Reliability analysis provides a logical framework to introduce uncertainties in the design process and improve the perception of risk in traditional design methods.
- The limit state function considered in the probabilistic analysis of slope stability depends more on post-peak strength parameters that in peak strength parameters.
- The integration of the residual strength parameters of (Cr, ϕr) in the proposed model has better simulation results than the progressive failure process of slopes.

- The reliability analysis of slices allowed locating the area triggering a failure and observing the direction of failure propagation.
- The existence of a subsystem which limits the evaluation of the system behavior to that of the subsystem.
- The residual strength parameters of the soil are the most dominant variables and the order of influence of the other variables depends on the choice of distribution.
- The Monte Carlo Simulation method is simple and robust, but requires a large number of simulations.
- The difference between the results obtained by the reliability methods FORM, FOSM, and MCS decreases with increasing number of slices in failure.
- The probabilistic analysis is a tool for decision support under uncertainty. It helps the problem's structure and guides the engineer in his judgment.

REFERENCES

[1] Y. M. Cheng and C. K. Lau, *New Methods and Insight*. London, UK: CRC Press, 2008, https://doi.org/10.4324/9780203927953.

[2] K. T. Law and P. Lumb, "A limit equilibrium analysis of progressive failure in the stability of slopes," *Canadian Geotechnical Journal*, vol. 15, no. 1, pp. 113–122, Feb. 1978, https://doi.org/10.1139/t78-009.

[3] L. Hu, A. Takahashi, and K. Kasama, "Effect of spatial variability on stability and failure mechanisms of 3D slope using random limit equilibrium method," *Soils and Foundations*, vol. 62, no. 6, Dec. 2022, Art. no. 101225, https://doi.org/10.1016/j.sandf.2022.101225.

[4] M. Havaej, D. Stead, E. Eberhardt, and B. R. Fisher, "Characterization of bi-planar and ploughing failure mechanisms in footwall slopes using numerical modelling," *Engineering Geology*, vol. 178, pp. 109–120, Aug. 2014, https://doi.org/10.1016/j.enggeo.2014.06.003.

[5] G. Firpo, R. Salvini, M. Francioni, and P. G. Ranjith, "Use of Digital Terrestrial Photogrammetry in rocky slope stability analysis by Distinct Elements Numerical Methods," *International Journal of Rock Mechanics and Mining Sciences*, vol. 48, no. 7, pp. 1045–1054, Oct. 2011, https://doi.org/10.1016/j.ijrmms.2011.07.007.

[6] D. A. Mangnejo, S. J. Oad, S. A. Kalhoro, S. Ahmed, F. H. Laghari, and Z. A. Siyal, "Numerical Analysis of Soil Slope Stabilization by Soil Nailing Technique," *Engineering, Technology & Applied Science Research*, vol. 9, no. 4, pp. 4469–4473, Aug. 2019, https://doi.org/10.48084/etasr.2859.

[7] R. b. Gilbert and R. j. Byrne, "Strain-Softening Behavior of Waste Containment System Interfaces," *Geosynthetics International*, vol. 3, no. 2, pp. 181–203, Jan. 1996, https://doi.org/10.1680/gein.3.0059.

[8] J. T. Christian, C. C. Ladd, and G. B. Baecher, "Reliability Applied to Slope Stability Analysis, " *Journal of Geotechnical Engineering*, Vol. 120, pp. 2180-2207, 1994, https://doi.org/10.1016/j.compgeo.2017.07.020.

[9] D. A. Crum *et al.*, "Search Algorithm for Minimum Reliability Index of Earth Slopes," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 127, no. 2, pp. 194–200, Feb. 2001, https://doi.org/10.1061/(ASCE)1090-0241(2001)127:2(194.2).

[10] D. V. Griffiths and G. A. Fenton, "Probabilistic Slope Stability Analysis by Finite Elements," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 130, no. 5, pp. 507–518, May 2004, https://doi.org/10.1061/(ASCE)1090-0241(2004)130:5(507).

[11] H. El-Ramly, N. R. Morgenstern, and D. M. Cruden, "Probabilistic slope stability analysis for practice," *Canadian Geotechnical Journal*, vol. 39, no. 3, pp. 665–683, Jun. 2002, https://doi.org/10.1139/t02-034.

[12] K. Winkelmann, L. Zabuski, J. Przewłócki, and J. Górski, "Reliability-Based Stability Analysis of a Baltic Cliff by the Combined Response Surface Method," *Geotechnical and Geological Engineering*, vol. 38, no. 5, pp. 5549–5563, Oct. 2020, https://doi.org/10.1007/s10706-020-01384-5.

[13] S.-H. Jiang, D.-Q. Li, Z.-J. Cao, C.-B. Zhou, and K.-K. Phoon, "Efficient System Reliability Analysis of Slope Stability in Spatially Variable Soils Using Monte Carlo Simulation," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 141, no. 2, Feb. 2015, Art. no. 04014096, https://doi.org/10.1061/(ASCE)GT.1943-5606.0001227.

[14] G. Baecher and J. Christian, *Reliability and Statistics in Geotechnical Engineering*, 1st ed. Chichester, West Sussex, UK; Hoboken, NJ, USA: Wiley, 2003.

[15] H. AM and N. Lind, "An Exact and Invariant First Order Reliability Format," *Journal of Engineering Mechanics*, vol. 100, Jan. 1974.

[16] A. W. Skempton, "Long-Term Stability of Clay Slopes," *Géotechnique*, vol. 14, no. 2, pp. 77–102, Jun. 1964, https://doi.org/10.1680/geot.1964.14.2.77.

[17] R. N. Chowdhury and D. W. Xu, "Reliability index for slope stability assessment—two methods compared," *Reliability Engineering & System Safety*, vol. 37, no. 2, pp. 99–108, Jan. 1992, https://doi.org/10.1016/0951-8320(92)90002-3.

[18] R. N. Chowdhury, W. H. Tang, and I. Sidi, "Reliability model of progressive slope failure, " *Géotechnique*, Vol. 37, pp. 467-481, 1987, https://doi.org/10.1680/geot.1987.37.4.467.

[19] S. S. Kar and L. B. Roy, "Reliability Analysis of a Finite Slope Considering the Effects of Soil Uncertainty," *International Journal of Performability Engineering*, vol. 17, no. 5, May 2021, Art. no. 473, https://doi.org/10.23940/IJPE.21.05.P7.473483.

[20] Z. Cao, Y. Wang, and D. Li, "Probabilistic Approaches for Geotechnical Site Characterization and Slope Stability Analysis," 1st ed. New York, NY, USA: Springer, 2016.

[21] N. M. Okasha, "Reliability-Based Design Optimization of Trusses with Linked-Discrete Design Variables using the Improved Firefly Algorithm," *Engineering, Technology & Applied Science Research*, vol. 6, no. 2, pp. 964–971, Apr. 2016, https://doi.org/10.48084/etasr.675.

[22] A.-R. Yarahmadi-Bafghi, "La méthode des groupes-clef probabiliste," Ph.D. dissertation, Institut National Polytechnique de Lorraine, Nancy, France, 2003.

[23] G. M. Filz, J. J. B. Esterhuizen, and J. M. Duncan, "Progressive Failure of Lined Waste Impoundments," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 127, no. 10, pp. 841–848, Oct. 2001, https://doi.org/10.1061/(ASCE)1090-0241(2001)127:10(841).

[24] R. N. Chowdhury and D. W. Xu, "Geotechnical system reliability of slopes," *Reliability Engineering & System Safety*, vol. 47, no. 3, pp. 141–151, Jan. 1995, https://doi.org/10.1016/0951-8320(94)00063-T.

[25] A. I. Husein Malkawi, W. F. Hassan, and F. A. Abdulla, "Uncertainty and reliability analysis applied to slope stability," *Structural Safety*, vol. 22, no. 2, pp. 161–187, Jun. 2000, https://doi.org/10.1016/S0167-4730(00)00006-0.

[26] R. Y. Liang, O. K. Nusier, and A. H. Malkawi, "A reliability based approach for evaluating the slope stability of embankment dams," *Engineering Geology*, vol. 54, no. 3, pp. 271–285, Oct. 1999, https://doi.org/10.1016/S0013-7952(99)00017-4.

[27] R. Rackwitz, "Reliability analysis—a review and some perspectives," *Structural Safety*, vol. 23, no. 4, pp. 365–395, Oct. 2001, https://doi.org/10.1016/S0167-4730(02)00009-7.

[28] R. Y. Rubinstein and D. P. Kroese, *Simulation and the Monte Carlo Method*, 2nd ed. Hoboken, N.J, USA: Wiley-Interscience, 2007.

[29] S. S. Kar and L. B. Roy, "Probabilistic Based Reliability Slope Stability Analysis Using FOSM, FORM, and MCS," *Engineering, Technology & Applied Science Research*, vol. 12, no. 2, pp. 8236–8240, Apr. 2022, https://doi.org/10.48084/etasr.4689.