Strength and Deflection Reliability Estimation of Girder Steel Portal Frames Using the Bayesian Updating Method

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Abstract-Uncertainty is ubiquitous in any engineering system at any stage of product development and throughout a product's life cycle. As information and sensor technology develops, more and more data about engineering systems are gathered. A strong technique for calibrating models using new information and observations is Bayesian updating. The applied loads, yield strength, plastic section modulus, span, cross-section dimensions, and modulus of elasticity from the international literature have been updated through the local literature and a data survey for the interior girder portal frames to investigate the reliability index and the probability of failure of the system. First Order Reliability Method (FORM) and Monte Carlo (MC) simulations have been used to estimate the reliability and probability of failure of strength and serviceability limit state function. The results reveal that for shear force, the reliability index increased significantly from 9.03 to 16.01. At the same time, the reliability index of the bending moment and deflection increased from 4.34 to 5.30 and from 6.90 to 7.66 respectively.

Keywords-interior supporting frame; Bayesian method; sample data; prior data; posterior data; FORM; MC; reliability

I. INTRODUCTION

Most structure engineering designs are based on deterministic variables and often do not consider the variations in the material properties and the geometry of the structure [1]. For the purpose of determining the structural system's reliability, the effects of uncertainties must be quantified and propagated. Uncertainty and reliability are vital to the analysis and design of an engineering system. The reliability of the system can be stated in reference to some performance criteria. The need for reliability analysis stems from the fact that there is a presence of uncertainty in the definition, understanding, modeling, and behavior prediction of the models that describe the system [2]. Increasing amounts of data on engineering systems are collected and stored. This information can and should be used to reduce the uncertainty in engineering models and optimize the management of these systems. A coherent and effective framework for merging new information with existing models is provided by Bayesian analysis, in which prior probabilistic models are updated with data and observations. The Bayesian framework enables the combination of uncertain and incomplete information with models from different sources and provides probabilistic information on the accuracy of the updated model [3]. The current study emphasizes on the reliability analysis of the strength and deflection of steel portal frames as new data are acquired for the independent variables of the system. The Bayesian method presents an efficient way to update the prior knowledge regarding the stiffness and strength of the frame. Finally, the FORM and MC simulations have been adopted to investigate the reliability of the deflection and strength.

II. THE BAYESIAN METHOD

In engineering, one often needs to use whatever information is available in formulating a sound basis for making decisions. This may include observed (field or experimental) data, information derived from theoretical models, and expert judgments based on experience. Moreover, the available information may need to be updated as new information or data are acquired [4]. The proper tool for combining and updating the available information is embodied in the Bayesian approach. Parameter estimation in the Bayesian approach is based on the updating formula:

$$f(\theta) = cL(\theta)p(\theta) \quad (1)$$

where $p(\theta)$ is the prior Probability Density Function (PDF) representing the initial state of knowledge about the unknown parameter θ, $L(\theta)$, is the likelihood function representing the knowledge gained from a set of observations, and the constant c is a normalizing factor. The posterior PDF, $f(\theta)$, represents the updated state of knowledge regarding the parameters θ.

A. *Prior Distribution*

The prior distribution represents the distribution of possible parameter values from which the parameter has been drawn. The selection of the prior distribution is an important matter in Bayesian modeling. Since the choice of the prior

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distribution has a major effect on the resulting inference, this choice must be conducted with the utmost care [5].

B. Posterior Distribution

The posterior probability distribution contains all the current information about the parameter. It merges the information in both prior distribution and likelihood. This typically results in the posterior representing stronger information than separate sources of information [6]. The prior and posterior densities are presented in Table I, the mean and variance of the posterior parameters are presented in Table II.

TABLE I. RANDOM VARIABLES AND THEIR PRIOR AND POSTERIOR DENSITIY DISTRIBUTIONS [4]

Basic Random Variables	**Parameter**	**Prior and Posterior Distribution**
Normal $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$	μ	Normal $\mu''_\mu = \frac{\mu'_\mu(\sigma^2/n) + \bar{x}{\sigma'}^2_\mu}{\sigma^2/n + (\sigma'_\mu)^2}$
Lognormal $f_X(x) = \frac{1}{\sqrt{2\pi}\xi x} exp\left[-\frac{1}{2}\left(\frac{\ln x-\lambda}{\xi}\right)^2\right]$	λ	Normal $f_\Lambda(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{1}{2}\left(\frac{\lambda-\mu}{\sigma}\right)^2\right]$

TABLE II. MEAN AND VARIANCE OF THE PARAMETERS AND THEIR POSTERIOR STATISTICS [4]

Mean and Variance of Parameters	**Posterior Statistics**
$E(\mu) = \mu_\mu$	$\mu''_\mu = \frac{\mu'_\mu(\sigma^2/n) + \bar{x}{\sigma'}^2_\mu}{\sigma^2/n + (\sigma'_\mu)^2}$
$Var(\mu) = \sigma^2_\mu$	$\sigma''_\mu = \sqrt{\frac{(\sigma'_\mu)^2(\sigma^2/n)}{(\sigma'_\mu)^2 + \sigma^2/n}}$
$E(\lambda) = \mu$	$\mu'' = \frac{u'(\xi^2/n) + \sigma^2 \ln \bar{x}}{\xi^2/n + \sigma^2}$
$Var(\lambda) = \sigma^2$	$\sigma'' = \sqrt{\frac{\sigma^2(\xi^2/n)}{\sigma^2 + \xi^2/n}}$

III. RELIABILITY ANALYSIS

The reliability analysis of an engineering structure requires the limit state function to evaluate its performance [7]. The concept of a limit state is used to help define failure in the context of structural reliability analysis. It is a boundary between desired and undesired performance of a structure. The limit state function can be defined as [8]:

$$g(R, Q) = R - Q \quad (2)$$

where R represents the resistance and Q represents the load effect. The state of the structure can be described using various parameters $X_1, X_2, \cdots, X_n$, load and resistance parameters. The limit $g(X) = 0$ separates the failure domain ($g(X) < 0$) and the safety domain ($g(X) > 0$) [9].

IV. RELIABILITY ANALYSIS METHODS

Due to the randomness of nature events, the complexity of the applied loads and initial imperfections, many times it is impossible to describe the response of structural systems. As a result, various analytical and numerical approaches have been developed. Taylor Series (TS)-based approaches, such as the FORM, and simulation-based methods, such as the MC simulation [4].

A. First-Order Reliability Method (FORM)

It is possible to expand the original model into infinite TS around the mean values:

$$g(X) = g(\mu_X) + (X - \mu_X)\frac{dg}{dX} + \frac{1}{2}(X - \mu_X)^2\frac{d^2g}{dX^2} + \cdots + \frac{1}{n!}(X - \mu_X)^n\frac{d^ng}{dX^n} \quad (3)$$

where the function and derivatives are evaluated at μ_X. It is common to include only linear terms, assuming that random input variables are independent. A function $g(X)$ of N independent random variables can be approximated by linear terms of the TS, which are [10]:

$$E(Y) \approx g(\mu_x) \quad (4)$$

$$Var(Y) \simeq Var(X - \mu_X)\left(\frac{dg}{dX}\right)^2 = Var(x)\left(\frac{dg}{dX}\right)^2 \quad (5)$$

The limit state function is $g(x) = R - Q$, where R, and Q both being random variables uncorrelated and assumed to be normally distributed. The reliability index β is evaluated as a function of mean and standard deviation of resistance R and load Q as given by [9]:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (6)$$

The probability of failure is given by:

$$P_f = \Phi(-\beta) \quad (7)$$

where Φ is the cumulative probability distribution function of the standard normal variate.

B. Monte Carlo (MC) Method

The MC method is considered one of the most powerful and accurate simulation tools to estimate the reliability and failure probability of uncertain structures numerically. It can be applied to many practical problems allowing the direct consideration of any type of probability distribution for the random variables. Using MC simulations, an estimate of the probability of structural failure is obtained by [11]:

$$P_f = \frac{N_f}{N} \quad (8)$$

where N_f is the number of samples in the failure domain and N is the total number of samples.

V. STATISTICAL CHARACTERISTICS OF INDEPENDENT VARIABLES

This study considers the applied loads, yield strength, plastic section modulus, girder span, cross-section dimensions, and modulus of elasticity as random variables. The sample data of the independent variables are summarized in Table III. The prior statistical characteristics of these variables collected from the literature are presented in Table IV. The posterior statistical data of the input variables have been calculated from the updating process of the prior data by the Bayesian method. The yield strength of steel F_y, has been considered as an example to explain that process. Updating mean, standard deviation, and coefficient of variance have been estimated based on the relations in (9)-(11). The posterior statistical characteristics of the independent variables are summarized in Table V.

$$\mu'' = \frac{\left[\frac{\bar{x}}{\left(\frac{\sigma}{\sqrt{n}}\right)^2}\right]+\left[\frac{\mu'}{(\sigma')^2}\right]}{\left[\frac{1}{\left(\frac{\sigma}{\sqrt{n}}\right)^2}\right]+\left[\frac{1}{(\sigma')^2}\right]} = \frac{\bar{x}(\sigma')^2+\mu'\left(\frac{\sigma^2}{n}\right)}{(\sigma')^2+\left(\frac{\sigma^2}{n}\right)} = \frac{(275)(275\times0.06)^2+(275)\left(\frac{(275\times0.07)^2}{1}\right)}{(275\times0.06)^2+\left(\frac{(275\times0.07)^2}{1}\right)} = 275\text{MPa} \quad (9)$$

$$\sigma'' = \sqrt{\frac{(\sigma')^2\left(\frac{\sigma^2}{n}\right)}{(\sigma')^2+\left(\frac{\sigma^2}{n}\right)}} = \sqrt{\frac{(275\times0.06)^2\left(\frac{(275\times0.07)^2}{1}\right)}{(275\times0.06)^2+\left(\frac{(275\times0.07)^2}{1}\right)}} = 12.5\text{MPa} \quad (10)$$

$$cov'' = \frac{\sigma''}{\mu''} = \frac{12.5}{275} = 0.04 \quad (11)$$

TABLE III. SAMPLE STATISTICAL DATA OF THE VARIABLES

Uncertainty in Variables	Mean/Nominal	COV	Distribution type	Reference
Dead load, P_D	1.03	0.08	Normal	[12]
Live load, P_L	1.00	0.1	Gumbel	[13]
Moment of inertia, I_x	0.96	0.03	Normal	Local Survey
Modulus of elasticity, E_s	1.025	0.05	Normal	[14]
Modulus of elasticity, E_c	0.98	0.07	Lognormal	[15]
Member span, L	1.00	0.07	Lognormal	[16]
Plastic section modulus, Z_x	1.04	0.05	Lognormal	[17]
Yield strength of steel, F_y	1.10	0.07	Lognormal	[17]

TABLE IV. PRIOR STATISTICAL DATA OF THE VARIABLES

Uncertainty in Variables	Mean/Nominal	COV	Distribution type	References
Dead load, P_D	1.03	0.08	Normal	[12]
Live load, P_L	1.00	0.1	Gumbel	[13]
Moment of inertia, I_x	0.96	0.05	Normal	[18]
Modulus of elasticity, E_s	1.00	0.02	Normal	[17]
Modulus of elasticity, E_c	1.00	0.03	Lognormal	[19]
Member span, L	1.00	0.004	Lognormal	[20]
Plastic section modulus, Z_x	1.00	0.04	Lognormal	[2]
Yield strength of steel, F_y	1.10	0.06	Lognormal	[18]

TABLE V. POSTERIOR STATISTICAL DATA OF THE VARIABLES

Uncertainty in Variables	Mean/Nominal	COV	Distribution type
Dead load, P_D	1.03	0.08	Normal
Live load, P_L	1.00	0.1	Gumbel
Moment of inertia, I_x	0.96	0.005	Normal
Steel modulus of elasticity, E_s	1.01	0.016	Normal
Concrete modulus of elasticity, E_c	1.00	0.02	Lognormal
Member span, L	1.00	0.003	Lognormal
Plastic section modulus, Z_x	1.00	0.03	Lognormal
Yield strength of steel, F_y	1.10	0.04	Lognormal

VI. PARAMETRIC STUDY

A steel single-story warehouse, as shown in Figure 1, with a total length of 45m and width of 15m, and consisting of 5 bays has been considered in this parametric study. The superimposed dead load has been determined based on the assumption of a flooring system of a concrete slab with a thickness of 100mm and a metal deck of 7.5mm, supported on floor beams of (IPE 300). The first inner frame shown in Figure 2 has been considered the most critical one as it includes the exterior face of the first interior support where the shear force is about 15% greater than the average value. The frame consists of IPE 600 steel girder and columns. The applied live loads on the girder have been determined based on the ASCE 7 specifications. The self-weight of the girder and columns have been determined depending on the cross-section dimensions and material density.

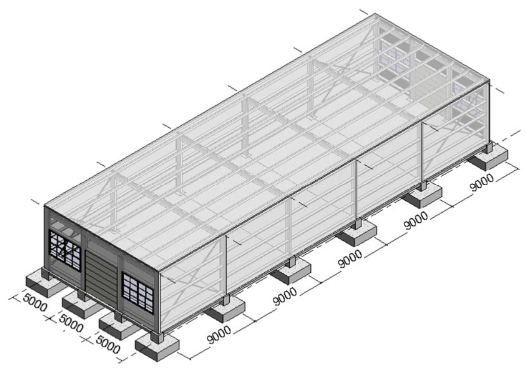


Fig. 1. Steel single-story warehouse building.

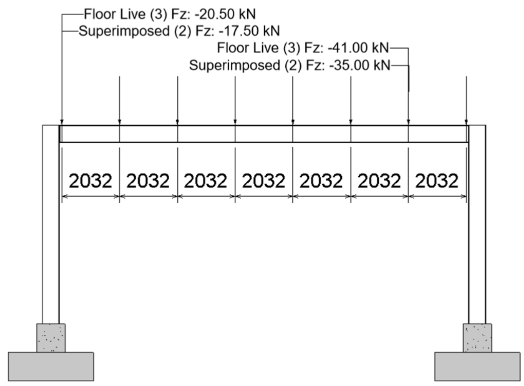


Fig. 2. Summary of the dead and live load on the interior supporting frame.

VII. DERIVED AND SIMULATED STATISTICAL PROPERTIES FOR THE STRENGTH AND DEFLECTION OF THE GIRDER

Two stages have been considered in the FORM and the MC simulation method. In the first stage, the randomness of the dependent variables has been deduced based on the prior knowledge of Table IV. In the second stage, they have been inferred based on the posterior knowledge of Table V.

A. First-Order Approximation Method

1) Flexure Analysis

The bending moment M is a function of the load P and the span L. With refereeing to (4) and (5), the mean and variance of the bending moment can be estimated by:

$$E(M) = f_m \times 0.904 \times \overline{PL} \quad (12)$$

$$Var(M) = Var(L)\left(\frac{\partial M}{\partial L}\right)^2 + Var(P)\left(\frac{\partial M}{\partial P}\right)^2 \quad (13)$$

where f_m is a deterministic dimensionless factor, equal to 0.56, that correlates the aforementioned bending moment to the bending moment of the corresponding simple span. The constant 0.904 is the mid-span bending moment due to the reactions from different floor beams. The nominal flexural strength M_n is a function of the yield strength F_y and the plastic section modulus Z_x. Refereeing to(4) and (5), the mean and variance of M_n can be estimated from:

$$E(M_n) = \bar{F}_y \bar{Z}_x \quad (14)$$

$$Var(M_n) = Var(F_y)\left(\frac{\partial M_n}{\partial F_y}\right)^2 + Var(Z_x)\left(\frac{\partial M_n}{\partial Z_x}\right)^2 \quad (15)$$

Based on the statistical characteristics of the independent variables illustrated in Tables IV and V, the prior and posterior statistical characteristics for bending moment M and nominal flexural strength M_n have been estimated and are presented in Tables VI and VII.

TABLE VI. PRIOR STATISTICAL CHARACTERISTICS OF THE GIRDER'S STRENGTH

Random Variables	Nominal (kN.m)	Mean (kN.m)	Standard deviation (kN.m)	COV
M	577.11	585.08	53.08	0.0907
M_n	878	965.8	69.64	0.072

TABLE VII. POSTERIOR STATISTICAL CHARACTERISTICS OF THE GIRDER'S STRENGTH

Random Variables	Nominal (kN.m)	Mean (kN.m)	Standard deviation (kN.m)	COV
M	577.11	585.08	53.06	0.0906
M_n	878	965.8	48.29	0.05

2) Shear Analysis

The shear force V, is a function of the load P. The mean and variance of the shear force are calculated by:

$$E(V) = f_v 3.5\bar{P} \quad (16)$$

$$Var(V) = Var(P)\left(\frac{\partial V}{\partial P}\right)^2 \quad (17)$$

where f_v is a deterministic dimensionless factor to correlate the aforementioned shear force to the shear force of the corresponding simple span and is equal to 1.0. The constant 3.5 is the shear at the support due to the reactions from different floor beams. The nominal shear force V_n is a function of the yield strength F_y and the area of the web A_w. The mean and variance of the nominal shear force are calculated by:

$$E(V_n) = 0.6\bar{F}_y \bar{A}_w \quad (18)$$

$$Var(V_n) = Var(F_y)\left(\frac{\partial V_n}{\partial F_y}\right)^2 + Var(A_w)\left(\frac{\partial V_n}{\partial A_w}\right)^2 \quad (19)$$

TABLE VIII. PRIOR STATISTICAL CHARACTERISTICS OF SHEAR FORCE

Random Variables	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
V	266	296.67	24.14	0.081
V_n	1011	1112	86.909	0.078

Based on the statistical characteristics of the independent variables illustrated in Tables IV and V, the prior and posterior statistical characteristics for V and V_n, have been estimated and are presented in Tables VIII and IX.

TABLE IX. POSTERIOR STATISTICAL CHARACTERISTICS OF SHEAR FORCE

Random Variables	Nominal (kN)	Mean (kN)	Standard deviation (kN)	COV
V	266	296.67	24.14	0.081
V_n	1011	1112	44.856	0.040

3) Deflection

The deflection Δ is a function of the load P, span L, modulus of elasticity E, and moment of inertia I. The mean and variance of the deflection are calculated by:

$$E(\Delta) = f_k \frac{613}{6000} \frac{\bar{P}\bar{L}^3}{\bar{E}\bar{I}} \quad (20)$$

$$Var(\Delta) = Var(I)\left(\frac{\partial \Delta}{\partial I}\right)^2 + Var(L)\left(\frac{\partial \Delta}{\partial L}\right)^2 + Var(E)\left(\frac{\partial \Delta}{\partial E}\right)^2 + Var(P)\left(\frac{\partial \Delta}{\partial P}\right)^2 \quad (21)$$

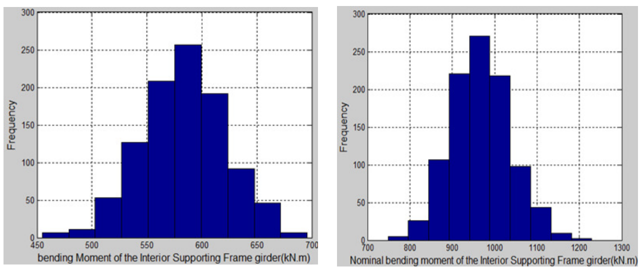
where f_k is a deterministic dimensionless factor that correlates the aforementioned deflection to the deflection of the corresponding simple span, equal to 0.32. The constant 613/6000 is the deflection at the mid-span due to the reactions from different floor beams. Based on Tables IV and V, the prior and posterior statistical characteristics for deflection are presented in Tables X and XI.

TABLE X. PRIOR STATISTICAL CHARACTERISTICS OF DEFLECTION

Random Variables	Nominal (mm)	Mean (mm)	Standard deviation (mm)	COV
Δ	45	48.08	5.05	0.105

TABLE XI. POSTERIOR STATISTICAL CHARACTERISTICS OF DEFLECTION

Random Variables	Nominal (mm)	Mean (mm)	Standard deviation (mm)	COV
Δ	45	48.57	4.49	0.092



(a) (b)

Fig. 3. Histogram for prior (a) M and (b) M_n sample of the typical interior girder.

B. Monte Carlo Simulation

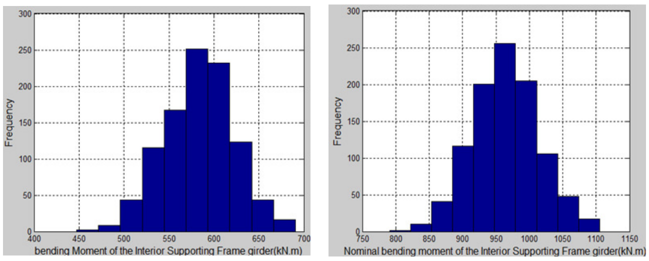
The MC method has been adopted to simulate the processes of the flexural strength, shear, and deflection for the interior supporting frame. A MATLAB code has been used to generate pseudo-random sampling with a size of 1000 for each input variable.

1) Flexure Analysis

The prior statistical characteristics of the bending moment M and nominal bending moment M_n are presented in Figure 3 and Table XII. The estimated posterior statistical characteristics are presented in Figure 3 and Table XIII.

TABLE XII. PRIOR STATISTICAL CHARACTERISTICS OF M AND M_n OF THE GIRDER

Random Variables	Mean (kN.m)	Standard deviation (kN.m)	COV	Distribution types
M	583.53	38.4	0.065	Normal
M_n	968.48	69.5	0.071	Lognormal



(a) (b)

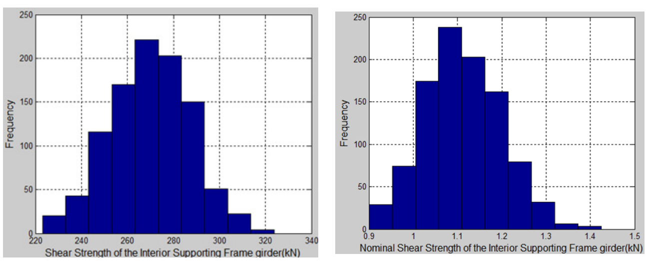
Fig. 4. Histogram for posterior prior (a) M and (b) M_n sample of the typical interior girder.

TABLE XIII. POSTERIOR STATISTICAL CHARACTERISTICS OF M AND M_N OF THE GIRDER

Random Variables	Mean (kN.m)	Standard deviation (kN.m)	COV	Distribution types
M	583.52	38.29	0.06	Normal
M_n	964.81	49.12	0.05	Lognormal

2) Shear Analysis

The prior statistical characteristics of the shear force V and nominal shear force V_n were estimated and are presented in Figure 5 and Table XIV. The posterior statistical characteristics of V and V_n were estimated and are presented in Figure 6 and Table XV.

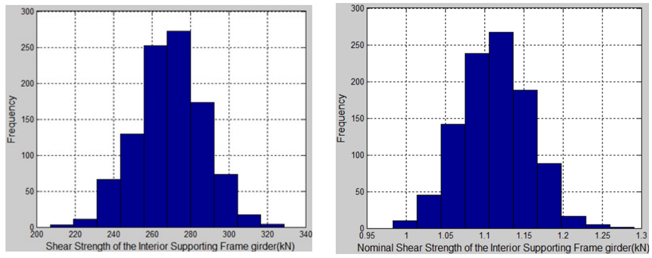


(a) (b)

Fig. 5. Histogram for prior (a) V and V_n of the typical interior girder.

TABLE XIV. PRIOR STATISTICAL CHARACTERISTICS OF V AND V_n

Random Variables	Mean (kN)	Standard deviation (kN)	COV	Distribution types
V	269.84	17.40	0.064	Normal
V_n	1111.5	86.300	0.077	Normal



(a) (b)

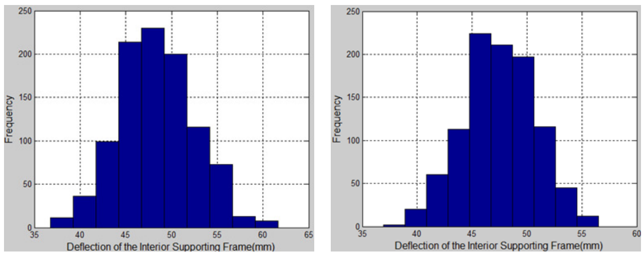
Fig. 6. Histogram for posterior (a) V and V_n of the typical interior girder.

TABLE XV. POSTERIOR STATISTICAL CHARACTERISTICS OF V AND V_n.

Random Variables	Mean (kN)	Standard deviation (kN)	COV	Distribution types
V	269.35	17.62	0.065	Normal
V_n	1112.8	43.70	0.039	Normal

3) Deflection

The estimated prior and posterior statistical characteristics of deflection are presented in Figure 7 and Table XVI.



(a) (b)

Fig. 7. Histogram for (a) prior and (b) posterior deflection sample of the girder.

TABLE XVI. PRIOR AND POSTERIOR STATISTICAL CHARACTERISTICS OF GIRDER DEFLECTION

Random Variables	Mean (mm)	Standard deviation (mm)	COV	Distribution types
Δ_{prior}	48.38	4.14	0.085	Normal
$\Delta_{posterior}$	47.51	3.33	0.070	Normal

VIII. RELIABILITY ANALYSIS FOR STRENGTH AND DEFLECTION OF THE GIRDER

The FORM has been adopted to estimate the reliability associated with girder's flexural strength, shear, and deflection. In the FORM, it has been assumed that all independent variables have a normal distribution. The mean and standard deviation of girder's flexural strength, shear, and have been used to evaluate the reliability index β and the probability of failure P_f due to the randomness in the load effects and resistance characteristics. Regarding flexure analysis, the mean and standard deviation of the bending moment (E_M, s_M) and the nominal flexural strength (E_n, s_n) have been estimated in Tables VI and VII. The prior and posterior β and P_f are summarized in Table XVII.

TABLE XVII. PRIOR AND POSTERIOR β AND P_f FOR THE FLEXURE STRENGTH OF THE GIRDER

Strength due to	p_f	β
$(P_D + P_L)_{prior}$	7.12414×10^{-6}	4.34
$(P_D + P_L)_{posterior}$	5.79013×10^{-8}	5.30

For shear analysis, the mean and standard deviation of the shear force (E_V, s_V) and the shear strength (E_n, s_n) were estimated and can be seen in Tables VIII and IX. The results for the prior and posterior β and P_f are summarized in Table XVIII.

TABLE XVIII. PRIOR β AND P_f FOR THE SHEAR OF THE GIRDER

Strength due to	p_f	β
$(P_D + P_L)_{prior}$	8.58359×10^{-20}	9.03
$(P_D + P_L)_{posterior}$	5.44049×10^{-58}	16.01

The estimated mean and standard deviation of deflection (E_Δ, s_Δ) and nominal deflection (E_{Δ_n}, s_{Δ_n}) shown in Tables X and XI comply to the limitations of IBC 2009. The results for the prior β and P_f are summarized in Table XIX.

TABLE XIX. PRIOR AND POSTERIOR β AND P_f FOR THE GIRDER DEFLECTION

Deflection due to	p_f	β
$(P_D + P_L)_{prior}$	2.60013×10^{-12}	6.90
$(P_D + P_L)_{posterior}$	9.29665×10^{-15}	7.66

IX. CONCLUSION

This paper has performed a prior and posterior statistical characteristics reliability analysis of the girder of the first inner frame. Through the application of the Bayesian method and the results of the prior and posterior analysis, it was shown that the greater the knowledge gained about the independent parameters, the less is the randomness in the dependent parameters, and thus enhances the reliability of the system. The results revealed that the reliability index for shear force significantly increased from 9.03 to 16.01, when the statistical parameters of the system were updated, while the reliability index of the bending moment and deflection increased from 4.34 to 5.30 and 6.90 to 7.66 respectively.

REFERENCES

[1] D. V. Ngo, K. V. Pham, D. D. Le, K. H. Le, and K. V. Huynh, "Assessing Power System Stability Following Load Changes and Considering Uncertainty," *Engineering, Technology & Applied Science Research*, vol. 8, no. 2, pp. 2758–2763, Apr. 2018, https://doi.org/10.48084/etasr.1892.

[2] P. E. Hess, D. Bruchman, I. A. Assakkaf, and B. M. Ayyub, "Uncertainties in Material and Geometric Strength and Load Variables," *Naval Engineers Journal*, vol. 114, no. 2, pp. 139–166, 2002, https://doi.org/10.1111/j.1559-3584.2002.tb00128.x.

[3] D. Straub and I. Papaioannou, "Bayesian Updating with Structural Reliability Methods," *Journal of Engineering Mechanics*, vol. 141, no. 3, Mar. 2015, Art. no. 04014134, https://doi.org/10.1061/(ASCE)EM.1943-7889.0000839.

[4] A. H.-S. Ang and W. H. Tang, *Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering, 2e Instructor Site: Emphasis on Applications to Civil and Environmental Engineering*, 2nd ed. New York, NY, USA: Wiley, 2006.

[5] A. Belay, E. OBrien, and D. Kroese, "Truck fleet model for design and assessment of flexible pavements," *Journal of Sound and Vibration*, vol. 311, no. 3, pp. 1161–1174, Apr. 2008, https://doi.org/10.1016/j.jsv.2007.10.019.

[6] A. Mansour, H. Jan, C. Zigelman, Y. N. Jen, and S. J. Harding, "Implementation of Reliability Methods to Marine Structures," *Transactions of the Society of Naval Architects and Marine Engineers*, vol. 92, pp. 353–382, 1984.

[7] A. U. Ebenuwa and K. F. Tee, "Reliability estimation of buried steel pipes subjected to seismic effect," *Transportation Geotechnics*, vol. 20, Sep. 2019, Art. no. 100242, https://doi.org/10.1016/j.trgeo.2019.100242.

[8] A. Nowak and K. Collins, *Reliability of Structures*. New York, NY, USA: McGraw-Hill, 2000.

[9] D. Mohammed and S. Al-Zaidee, "Deflection Reliability Analysis for Composite Steel Bridges," *Engineering, Technology & Applied Science Research*, vol. 12, no. 5, pp. 9155–9159, Oct. 2022, https://doi.org/https://doi.org/10.48084/etasr.5146.

[10] S. S. Kar and L. B. Roy, "Probabilistic Based Reliability Slope Stability Analysis Using FOSM, FORM, and MCS," *Engineering, Technology & Applied Science Research*, vol. 12, no. 2, pp. 8236–8240, Apr. 2022, https://doi.org/10.48084/etasr.4689.

[11] N. L. Tran and T. H. Nguyen, "Reliability Assessment of Steel Plane Frame's Buckling Strength Considering Semi-rigid Connections," *Engineering, Technology & Applied Science Research*, vol. 10, no. 1, pp. 5099–5103, Feb. 2020, https://doi.org/10.48084/etasr.3231.

[12] M. S. Darmawan, A. N. Refani, M. Irmawan, R. Bayuaji, and R. B. Anugraha, "Time Dependent Reliability Analysis of Steel I Bridge Girder Designed Based on SNI T-02-2005 and SNI T-3-2005 Subjected to Corrosion," *Procedia Engineering*, vol. 54, pp. 270–285, Jan. 2013, https://doi.org/10.1016/j.proeng.2013.03.025.

[13] S. G. Buonopane and B. W. Schafer, "Reliability of Steel Frames Designed with Advanced Analysis," *Journal of Structural Engineering*, vol. 132, no. 2, pp. 267–276, Feb. 2006, https://doi.org/10.1061/(ASCE)0733-9445(2006)132:2(267).

[14] A. Ghabussi, J. Asgari Marnani, and M. S. Rohanimanesh, "Improving seismic performance of portal frame structures with steel curved dampers," *Structures*, vol. 24, pp. 27–40, Apr. 2020, https://doi.org/10.1016/j.istruc.2019.12.025.

[15] P. Geyskens, A. D. Kiureghian, and P. Monteiro, "Bayesian Prediction of Elastic Modulus of Concrete," *Journal of Structural Engineering*, vol. 124, no. 1, pp. 89–95, Jan. 1998, https://doi.org/10.1061/(ASCE)0733-9445(1998)124:1(89).

[16] H. Q. Jebur and S. R. Al-Zaidee, "Non-deterministic Approach for Reliability Evaluation of Steel Portal Frame," *Civil Engineering Journal*, vol. 5, no. 8, pp. 1684–1697, Aug. 2019, https://doi.org/10.28991/cej-2019-03091363.

[17] K. Atua, I. Assakkaf, and B. M. Ayyub, "Statistical characteristics of strength and load random variables of ship structures," in *Probabilistic Mechanics and Structural Reliability, Proceeding of the Seventh Specialty Conference*, Worcester, MA, USA, 1996.

[18] H. Zhang, H. Liu, B. R. Ellingwood, and K. J. R. Rasmussen, "System Reliabilities of Planar Gravity Steel Frames Designed by the Inelastic Method in AISC 360-10," *Journal of Structural Engineering*, vol. 144, no. 3, Mar. 2018, Art. no. 04018011, https://doi.org/10.1061/(ASCE)ST.1943-541X.0001991.

[19] ACI Committee 318, *Building Code Requirements for Structural Concrete: (ACI 318-19); and Commentary (ACI 318R-19)*. Farmington Hills, MI, USA: American Concrete Institute, 2019.

[20] *Indian standard, Tolerances for fabrication of Steel Structures*. New Delhi, India: Bureau of Indian Standards, 1974.