Probabilistic Based Reliability Slope Stability Analysis Using FOSM, FORM, and MCS

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Abstract-Soil uncertainties play an important part in the analysis and design of geotechnical structures. The effect of uncertainties on the geotechnical structures and their influence on the probability of failure or reliability of the structure is of great interest for geotechnical researchers. Probabilistic-based slope stability analysis incorporates the uncertainties present in the soil, as expressed in terms of mean, variance, and autocorrelation. In this paper, reliability analysis of a finite cohesive soil slope based on the probabilistic approach is presented using the First Order Second Moment (FOSM) method, First Order Reliability Method (FORM), and Monte Carlo Simulation (MCS) method. Stability analysis has been performed using the ordinary method of slices to calculate the Factor Of Safety (FOS) of the slope under undrained conditions. The reliability analysis has been implemented in the MS-excel spreadsheet environment and was mainly focused on the two models, namely the deterministic model for calculating the FOS of the slope and the uncertainty model for generating the random variables of uncertain soil parameters. The reliability index (β) of the soil slope and its corresponding probability of failure (P_f) was calculated using the above methods. The obtained result shows that the MCS method has significantly shown better performance than FOSM and FORM because of its robustness and simple approach to calculate P_f and β of the slope.

Keywords-reliability index; FOSM; FORM; MCS; probability of failure

I. INTRODUCTION

Soil characteristics are influenced by various factors (e.g. characteristics of their origin rock, erosion and weathering actions, and sedimentation condition) and therefore, its properties vary spatially at different depths, something that is also referred to as inherent spatial variation of the soil. These inherent variations of the soil cannot be reduced as they are independent in nature, therefore, classified as aleatory uncertainty [1]. Apart from the inherent variability of the soil, several other uncertainties such as measurement uncertainty, statistical uncertainty, and transformation model uncertainty affect the design and analysis work of geotechnical structures. The measurement uncertainties [2] arise due to system errors, sample mishandling, and testing errors which can be reduced by improving the knowledge on testing techniques and equipment, therefore, are classified as epistemic uncertainties [1]. The statistical uncertainty [2] is a part of the measurement uncertainty, which may arise due to the unavailability of the adequate number of sample data. The insufficiency of the model to represent the system's actual conditions, results into transformational model uncertainties [3]. These uncertainties can be reduced if proper correlations between the relevant parameters can be established. It has been shown that the epistemic uncertainties do not affect the geotechnical structures. The response of geotechnical structures is significantly affected by the inherent spatial variability of the soil [4-6]. These uncertainties affect the soil properties and ground stratification and subsequently influence the design of geotechnical structures. Slope stability determination is based on FOS. In the deterministic method of analysis, FOS is expressed as the ratio of the resisting moment to the overturning moment. When FOS > 1, the slope is considered as safe [7]. In the probabilistic approach of slope analysis, the uncertainty present in the soil is expressed in terms of its mean and standard deviation and is modeled using the autocorrelation function [8]. Probabilistic slope stability analysis is used to address the various uncertainties present in the soil [9, 10] and calculate the P_f and β of the slope. Many probabilistic based reliability methods have been developed to estimate the values of β and P_f for geotechnical structures specially for the slope problem, such as FOSM [1, 11-13], FORM [1, 13-15], and MCS [13, 16-18].

In this study, probabilistic based reliability analysis of a finite cohesive soil slope is done using FOSM, FORM, and MCS. The uncertainties present in the undrained shear strength of the soil at vertical depth have been considered. The spatial variations in undrained shear strength of the soil (S_u) are modeled by the one-dimensional random field theory and the autocorrelation function. The saturated unit weight (γ_{sat}), S_u, coordinates of the centre of slip surface (x_c, y_c) and the radii of the slip surface (r_c) are used to prepare the sample data. The slope stability model has been prepared and analyzed in a MS-Excel spreadsheet which is divided into two parts, the deterministic and the uncertainty model. The obtained β and P_f were compared.

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II. METHODOLOGY

A. Deterministic Model

Deterministic modeling is the process of estimating the FOS for a defined set of parameters using limit equilibrium methods. No concept of probability is used in the deterministic analysis. The FOS is calculated using the ordinary method of slices [19]:

$$FOS = \frac{\sum c' \Delta L + \sum (W \cos \alpha) \tan \emptyset'}{\sum W \sin \alpha} \quad (1)$$

where c' is the effective cohesion, ΔL is the length of the arc, $\emptyset'$ is the effective frictional angle, W is the weight of the slice, α is the inclination of the slice base. For cohesive soil under undrained condition, (1) can be further modified as:

$$FOS = \frac{\sum S_u \Delta L}{\sum W \sin \alpha} \quad (2)$$

where S_u is the undrained shear strength of the soil. Visual Basic Application (VBA) codes have been written for determining the FOS values with respect to different center coordinates (x, y) and radii of the slip surface (r) and the lowest value as FOS which corresponds to the critical slip surface having coordinates (x_c, y_c) and radius (r_c) were identified.

B. Uercertainty Model

The uncertainty model is used to generate the uncertain parameters which are considered as random variables in reliability analysis. Based on the distribution type, correlation details, and statistics of the random variables, the random samples of the uncertain parameters are obtained in the spreadsheet. In this study, the S_u of the soil is taken as an uncertain parameter with respect to the depth z_i. Let $\bar{S} = [S_u(z_1), S_u(z_2), \ldots, S_u(z_n)]^T$ represent the vector of S_u at different depths $z_i = z_1, z_2, \ldots, z_n$. When S_u is log-normally distributed, it can be represented as [9,20-21]:

$$\bar{S} = exp(\mu \bar{1} + \sigma \bar{L}\, \bar{N}) \quad (3)$$

where μ and σ are the mean and standard deviation (SD) of $ln[S_u(z)]$, $\bar{1}$ is the n-column unit vector, $\bar{N}$ is the n-dimensional standard normal vector, $\bar{L}$ is a $n \times n$ dimensional lower triangular matrix generated by the Cholesky decomposition of $\bar{R} = \bar{L}\bar{L}^T$. The correlation between $ln[S_u(z_i)]$ and $ln[S_u(z_j)]$ at respective depth z_i and z_j is given as:

$$\bar{R} = R_{ij} = e^{\left(-\frac{2|z_i - z_j|}{\lambda}\right)} \quad (4)$$

where $\bar{R}$ is the correlation matrix and λ is the correlation length. λ is defined as the length up to which the soil parameters are fully correlated.

III. RELIABILITY ANALYSIS

A. FOSM

The FOSM is a very simple method for calculating reliability based on the Taylor's first-order series expansion. The β is calculated using FOSM [1,8,13,22-23] as:

$$\beta = \frac{\mu_{FOS-1}}{\sigma_{FOS}} \quad (5)$$

where μ_{FOS} and σ_{FOS} are the mean and SD of FOS.

B. FORM

In FORM, β is calculated in terms of length as the shortest distance measured from the origin of the failure surface and the design point. β is expressed in matrix formulation [22] as:

$$\beta = \underbrace{min}_{X \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T [\bar{R}]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]}\ , \quad i = 1, 2, \ldots, n \quad (6)$$

where F represents the failure domain, x_i represents a set of random variables, μ_i represents the mean of the random variable, σ_i represents the SD of the random variable and $[\bar{R}]$ is the correlation matrix of uncertain parameters.

C. MCS

MCS is a mathematical procedure of continuously evaluating an empirical operator having a random variable of known probability distribution. For obtaining preferred accuracy level P_f, the number of samples to be generated by MCS should be at least equal to $10/P_f$ [18]. For example, for obtaining P_f of 0.001 accuracy, the total number of samples to be generated by MCS should be at least equal to 10,000. Figure 1 shows the flowchart of the slope stability analysis using MCS. The P_f of the slope is calculated as the ratio of number of samples having $FOS < 1$ to the total number of generated samples. The β corresponding to the P_f is calculated as:

$$\beta = \Phi^{-1}(1 - P_f) \quad (7)$$

where Φ is the standard normal cumulative distribution function.

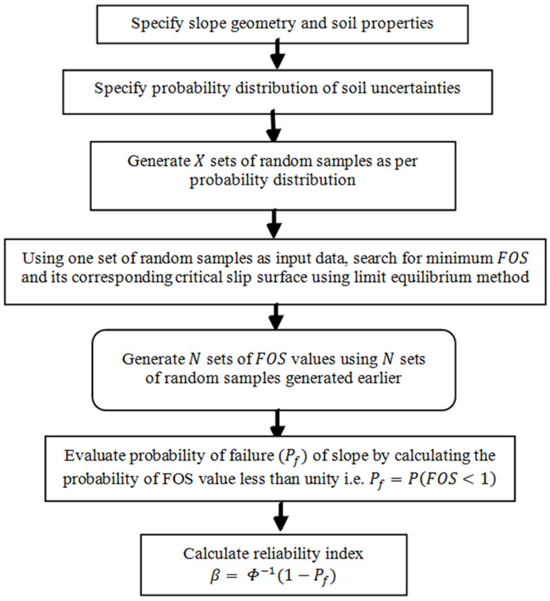


Fig. 1. Systematic representation of MCS for slope stability analysis.

IV. PROBLEM STATEMENT

A finite cohesive slope [21], has been taken in this study to assess its reliability having uncertainty in undrained shear strength in vertical depth. The cross-section and soil properties of the slope are shown in Figure 2. The slope stability analysis is carried out using the ordinary method of slices in undrained situation. Hard stratum is assumed to be present at 15.0m below the soil. The total 15.0m soil layer is divided into 30 layers, each having thickness equal to 0.5m. The undrained shear strength with depth is log-normally distributed having an exponential correlation. The correlation length is taken as 2.0m.

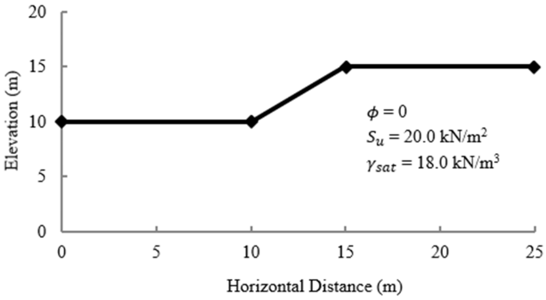


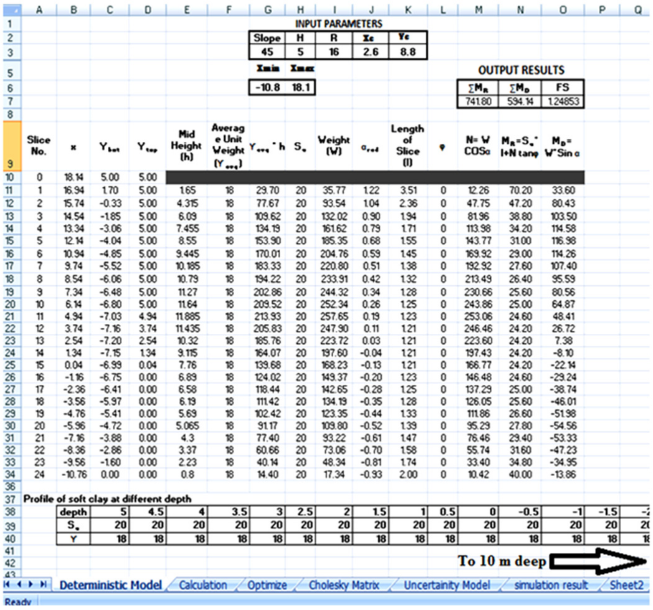
Fig. 2. Cross-section for the slope stability problem.

V. RESULTS AND DISCUSSION

The critical slip circle is obtained by choosing different combination of center coordinates (x, y) and radius of the slip surface (r) to obtain minimum FOS. Table I shows the range of (x, y) and r taken in this study.

TABLE I. RANGE OF CENTER COORDINATES AND RADIUS OF THE SLIP CIRCLE

Parameter	Minimum	Maximum	Range
Coordinate x (m)	1.0	4.0	3.0
Coordinate y (m)	7.0	10.0	3.0
Radius r (m)	11.0	16.0	5.0



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1							INPUT PARAMETERS								
2							Slope	H	R	Xc	Yc				
3							45	5	16	2.6	8.8				
5							Xmin	Xmax					OUTPUT RESULTS		
6							-10.8	18.1					ΣM_R	ΣM_D	FS
7													741.80	594.14	1.24853
8															
9	Slice No.	x	Y_{bot}	Y_{top}	Mid Height (h)	Average Unit Weight (Y_{avg})	Y_{avg} * h	S_u	Weight (W)	α_{rad}	Length of Slice (l)	φ	N= W COSα	M_R=S_u*l+N tanφ	M_D= W*Sin α
10	0	18.14	5.00	5.00											
11	1	16.94	1.70	5.00	1.65	18	29.70	20	35.77	1.22	3.51	0	12.26	70.20	33.60
12	2	15.74	-0.33	5.00	4.315	18	77.67	20	93.54	1.04	2.36	0	47.75	47.20	80.43
13	3	14.54	-1.85	5.00	6.09	18	109.62	20	132.02	0.90	1.94	0	81.96	38.80	103.50
14	4	13.34	-3.06	5.00	7.455	18	134.19	20	161.62	0.79	1.71	0	113.98	34.20	114.58
15	5	12.14	-4.04	5.00	8.55	18	153.90	20	185.35	0.68	1.55	0	143.77	31.00	116.98
16	6	10.94	-4.85	5.00	9.445	18	170.01	20	204.76	0.59	1.45	0	169.92	29.00	114.26
17	7	9.74	-5.52	5.00	10.185	18	183.33	20	220.80	0.51	1.38	0	192.92	27.60	107.40
18	8	8.54	-6.06	5.00	10.79	18	194.22	20	233.91	0.42	1.32	0	213.49	26.40	95.59
19	9	7.34	-6.48	5.00	11.27	18	202.86	20	244.32	0.34	1.28	0	230.66	25.60	80.56
20	10	6.14	-6.80	5.00	11.64	18	209.52	20	252.34	0.26	1.25	0	243.86	25.00	64.87
21	11	4.94	-7.03	4.94	11.885	18	213.93	20	257.65	0.19	1.23	0	253.06	24.60	48.41
22	12	3.74	-7.16	3.74	11.435	18	205.83	20	247.90	0.11	1.21	0	246.46	24.20	26.72
23	13	2.54	-7.20	2.54	10.32	18	185.76	20	223.72	0.03	1.21	0	223.60	24.20	7.38
24	14	1.34	-7.15	1.34	9.115	18	164.07	20	197.60	-0.04	1.21	0	197.43	24.20	-8.10
25	15	0.04	-6.99	0.04	7.76	18	139.68	20	168.23	-0.13	1.21	0	166.77	24.20	-22.14
26	16	-1.16	-6.75	0.00	6.89	18	124.02	20	149.37	-0.20	1.23	0	146.48	24.60	-29.24
27	17	-2.36	-6.41	0.00	6.58	18	118.44	20	142.65	-0.28	1.25	0	137.29	25.00	-38.74
28	18	-3.56	-5.97	0.00	6.19	18	111.42	20	134.19	-0.35	1.28	0	126.05	25.60	-46.01
29	19	-4.76	-5.41	0.00	5.69	18	102.42	20	123.35	-0.44	1.33	0	111.86	26.60	-51.98
30	20	-5.96	-4.72	0.00	5.065	18	91.17	20	109.80	-0.52	1.39	0	95.29	27.80	-54.56
31	21	-7.16	-3.88	0.00	4.3	18	77.40	20	93.22	-0.61	1.47	0	76.46	29.40	-53.33
32	22	-8.36	-2.86	0.00	3.37	18	60.66	20	73.06	-0.70	1.58	0	55.74	31.60	-47.23
33	23	-9.56	-1.60	0.00	2.23	18	40.14	20	48.34	-0.81	1.74	0	33.40	34.80	-34.95
34	24	-10.76	0.00	0.00	0.8	18	14.40	20	17.34	-0.93	2.00	0	10.42	40.00	-13.86
36															

37 Profile of soft clay at different depth

depth	5	4.5	4	3.5	3	2.5	2	1.5	1	0.5	0	-0.5	-1	-1.5
S_u	20	20	20	20	20	20	20	20	20	20	20	20	20	20
Y	18	18	18	18	18	18	18	18	18	18	18	18	18	18

To 10 m deep

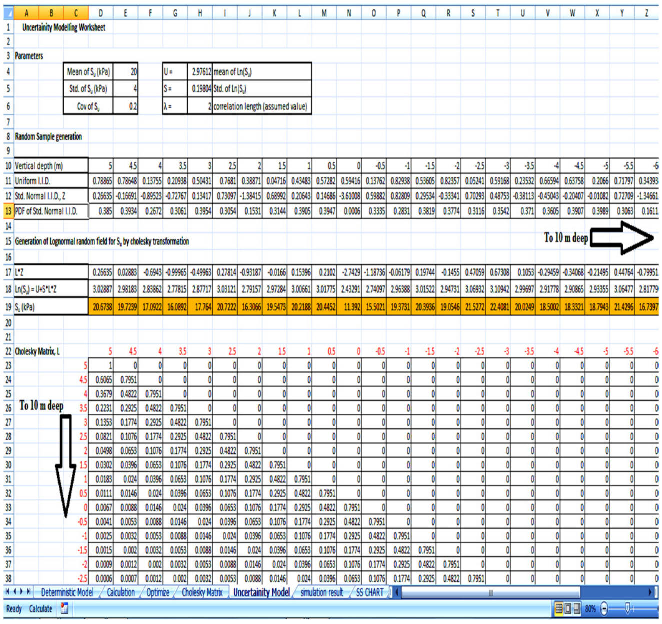
Fig. 3. Deterministic model developed in MS-Excel.

Based on this data, an area having the lowest FOS has been identified, and a grid of small intervals of 0.2m has been taken to further identify the critical slip surface. Figure 3 shows the deterministic model worksheet of the example problem. Table II shows the FOS, the center coordinates (x_c, y_c), and the radius (r) of the slip surface obtained with the deterministic model.

TABLE II. CRITICAL SLIP CIRCLE AND *FOS*

Method	FOS	r (m)	x_c (m)	y_c (m)
Ordinary method (MS-Excel)	1.248	16.0	2.6	8.8

After obtaining the FOS and critical slip surface, the uncertainty model is developed. Figure 4 shows the uncertainty model worksheet.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Uncertainity Modelling Worksheet																									
2																										
3	Parameters																									
4			Mean of S_u (kPa)		20		U =	2.97612	mean of Ln(S_u)																	
5			Std. of S_u (kPa)		4		S =	0.19804	Std. of Ln(S_u)																	
6			Cov of S_u		0.2		λ =	2	correlation length (assumed value)																	
7																										
8	Random Sample generation																									
9																										
10	Vertical depth (m)			5	4.5	4	3.5	3	2.5	2	1.5	1	0.5	0	-0.5	-1	-1.5	-2	-2.5	-3	-3.5	-4	-4.5	-5	-5.5	-6
11	Uniform I.I.D.			0.78865	0.78648	0.13755	0.20938	0.50431	0.7681	0.38871	0.04716	0.43483	0.57282	0.59416	0.13762	0.82938	0.53605	0.82357	0.05241	0.59168	0.23532	0.66594	0.63758	0.2066	0.71797	0.34393
12	Std. Normal I.I.D., Z			0.26635	-0.16691	-0.89523	-0.72767	0.13417	0.73097	-1.38415	0.68992	0.20643	0.14686	-3.61008	0.59882	0.82809	0.29534	-0.33341	0.70293	0.48753	-0.38113	-0.45043	-0.20407	-0.01082	0.72709	-1.34661
13	PDF of Std. Normal I.I.D.			0.385	0.3934	0.2672	0.3061	0.3954	0.3054	0.1531	0.3144	0.3905	0.3947	0.0006	0.3335	0.2831	0.3819	0.3774	0.3116	0.3542	0.371	0.3605	0.3907	0.3989	0.3063	0.1611
14																										
15	Generation of Lognormal random field for S_u by cholesky transformation																									
16																										
17	L*Z			0.26635	0.02883	-0.6943	-0.99965	-0.49963	0.27814	-0.93187	-0.0166	0.15396	0.2102	-2.7429	-1.18736	-0.06179	0.19744	-0.1455	0.47059	0.67308	0.1053	-0.29459	-0.34068	-0.21495	0.44764	-0.79951
18	Ln(S_u) = U+S*L*Z			3.02887	2.98183	2.83862	2.77815	2.87717	3.03121	2.79157	2.97284	3.00661	3.01775	2.43291	2.74097	2.96388	3.01522	2.94731	3.06932	3.10942	2.99697	2.91778	2.90865	2.93355	3.06477	2.81779
19	S_u (kPa)			20.6738	19.7239	17.0922	16.0892	17.764	20.7222	16.3066	19.5473	20.2188	20.4452	11.392	15.5021	19.3731	20.3936	19.0546	21.5272	22.4081	20.0249	18.5002	18.3321	18.7943	21.4296	16.7397
20																										
21																										
22	Cholesky Matrix, L			5	4.5	4	3.5	3	2.5	2	1.5	1	0.5	0	-0.5	-1	-1.5	-2	-2.5	-3	-3.5	-4	-4.5	-5	-5.5	-6
23			5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24			4.5	0.6065	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25			4	0.3679	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	To 10 m deep		3.5	0.2231	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27			3	0.1353	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28			2.5	0.0821	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29			2	0.0498	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30			1.5	0.0302	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31			1	0.0183	0.024	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32			0.5	0.0111	0.0146	0.024	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0	0
33			0	0.0067	0.0088	0.0146	0.024	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0	0
34			-0.5	0.0041	0.0053	0.0088	0.0146	0.024	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0	0
35			-1	0.0025	0.0032	0.0053	0.0088	0.0146	0.024	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0	0
36			-1.5	0.0015	0.002	0.0032	0.0053	0.0088	0.0146	0.024	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0	0
37			-2	0.0009	0.0012	0.002	0.0032	0.0053	0.0088	0.0146	0.024	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0	0
38			-2.5	0.0006	0.0007	0.0012	0.002	0.0032	0.0053	0.0088	0.0146	0.024	0.0396	0.0653	0.1076	0.1774	0.2925	0.4822	0.7951	0	0	0	0	0	0	0

To 10 m deep

Fig. 4. Uncertainty model developed in MS-Excel.

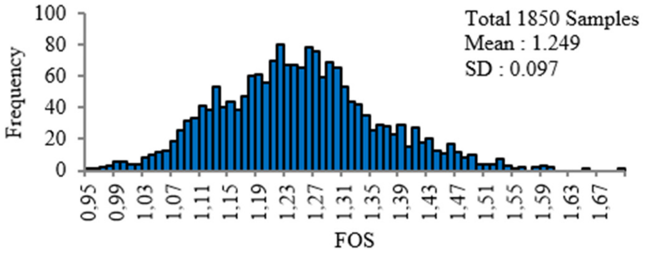
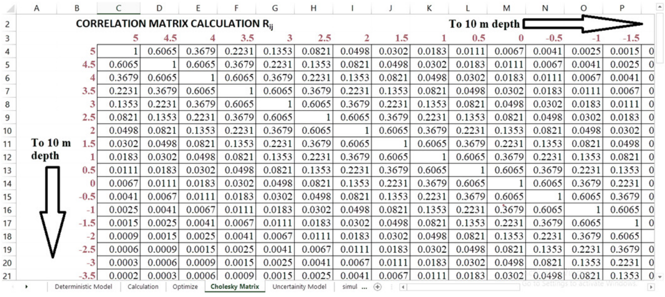


Fig. 5. *FOS* histogram showing the mean and the SD of the samples used in FOSM.

The S_u values obtained in the uncertainty model are copied to the S_u values in the deterministic model through linking of their input/output cell. By doing this task, the values of FOS generated in the deterministic model will become random and using the F9 key in MS-Excel will produce random values of FOS. By doing so, we could easily perform MCS, FORM, and FOSM by continuously pressing F9 key, but instead, a VBA macro code has been run in the MS-Excel to calculate the random values of FOS. For this study, a total of 1850 FOS samples were generated and reliability analysis has been performed using FOSM, FORM, and MCS. Figure 5 shows the mean and SD obtained from the samples of FOS in FOSM. The value of β was calculated as 2.56 and the value of P_f as $1 - \Phi(2.56) = 0.52\%$. The β is estimated using (6). Each term in (6) is computed using a code written in Matlab. The correlation matrix $[\overline{\mathrm{R}}]$, with dimensions of 31×31, was obtained using (4) and is shown in Figure 6. The reliability is calculated for all the 1850 FOS samples and the minimum value is reported as the reliability index. The β obtained using FORM is 2.62 which corresponds to a P_f value of 0.44%. Table III summarizes the result of MCS obtained for λ = 2m. All the 1850 samples were taken for MCS. For a specified target FOS (say 1.2), out of the 1850 samples, 374 samples got values less than 1.2. In other words, it can be stated that these 374 samples failed. Therefore, the P_f is calculated as 374/1850 = 2.76%. This corresponds to a value of β equal to 1.92.



CORRELATION MATRIX CALCULATION R_{ij}

To 10 m depth

To 10 m depth

	5	4.5	4	3.5	3	2.5	2	1.5	1	0.5	0	-0.5	-1	-1.5	
5	1	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067	0.0041	0.0025	0.0015	0
4.5	0.6065	1	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067	0.0041	0.0025	0
4	0.3679	0.6065	1	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067	0.0041	0
3.5	0.2231	0.3679	0.6065	1	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067	0
3	0.1353	0.2231	0.3679	0.6065	1	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0
2.5	0.0821	0.1353	0.2231	0.3679	0.6065	1	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0
2	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	1	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0
1.5	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	1	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0
1	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	1	0.6065	0.3679	0.2231	0.1353	0.0821	0
0.5	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	1	0.6065	0.3679	0.2231	0.1353	0
0	0.0067	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	1	0.6065	0.3679	0.2231	0
-0.5	0.0041	0.0067	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	1	0.6065	0.3679	0
-1	0.0025	0.0041	0.0067	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	1	0.6065	0
-1.5	0.0015	0.0025	0.0041	0.0067	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	1	0
-2	0.0009	0.0015	0.0025	0.0041	0.0067	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0.6065	
-2.5	0.0006	0.0009	0.0015	0.0025	0.0041	0.0067	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0.3679	0
-3	0.0003	0.0006	0.0009	0.0015	0.0025	0.0041	0.0067	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0.2231	0
-3.5	0.0002	0.0003	0.0006	0.0009	0.0015	0.0025	0.0041	0.0067	0.0111	0.0183	0.0302	0.0498	0.0821	0.1353	0

Fig. 6. Correlation matrix $[\overline{\mathrm{R}}]$.

TABLE III. SUMMARY OF MCS RESULTS

λ	Simulation technique	Samples	Target FOS	Number of Samples < target FOS	P_f (%)	β
2m	MCS	1850	1.1	51	2.76	1.92
		1850	1.2	374	20.22	0.83
		1850	1.3	977	52.81	-
		1850	1.4	1538	83.14	-
		1850	1.5	1764	95.35	-
		1850	1.6	1839	99.40	-
		1850	1.7	1848	99.89	-

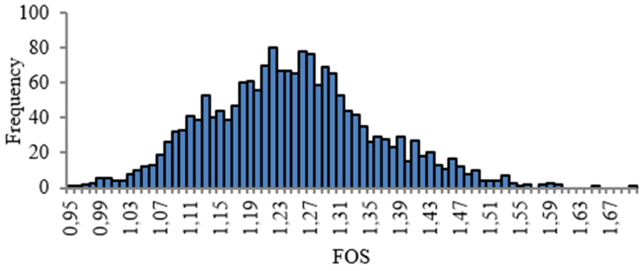


Fig. 7. FOS histogram from MCS.

The histogram of the FOS obtained from the 1850 MCS samples is illustrated in Figure 7. It can be seen that out of the 1850 samples, 13 have FOS values less than 1. The P_f is calculated as 13/1850 = 0.7%. This corresponds to a value of β equal to 2.46.

Table IV summarizes the results of the obtained β and P_f using the 3 different reliability methods. The value of P_f varies from 0.44% to 0.70% having maximum relative difference among different methods of about 37%. There is a decrease of 26% in P_f in FOSM as compared to MCS. Similarly, a difference of 37% in P_f in FORM as compared to MCS is observed. The β of the slope varies from 2.46 to 2.62.

TABLE IV. TESULTS OBTAINED FROM DIFFERENT RELIABILITY METHODS

Method	Samples	β	P_f(%)	Relative difference in P_f (%)
FOSM	1850	2.56	0.52	-26.0
FORM	1850	2.62	0.44	-37.0
MCS	1850	2.46	0.70	#

#Base value for calculating relative difference

VI. RESULT COMPARISON

Authors in [22] assessed the reliability of a cohesive soil slope having spatial inherent variation in the undrained shear strength. The height of the slope is considered 10m having a slope angle of 26.6°. The hard stratum is present 20m below the top of the soil. They analyzed the slope and calculated the FOS using the ordinary method of slices. Further, they calculated the reliability index of the slope using FOSM, FORM, and MCS. They concluded that various uncertainties can be taken into account rationally in probabilistic slope stability analysis through MCS. MCS method provides a robust and conceptually simple way to estimate the reliability index or slope failure probability.

Authors in [13] studied the reliability-based probabilistic method of analysis of a finite soil slope by considering the uncertainties in cohesion and angle of the internal friction of the soil. The height of the slope is considered to be 5m having a slope inclination of 1V:2H. The unit weight of the soil is taken as a constant and the hard stratum is present 15m below the top of the soil. They analyzed the slope using the ordinary method of slices and calculated the reliability index and its corresponding probability of failure of the slope using FOSM, FORM, and MCS. They found that the MCS method performed significantly better than FOSM and FORM.

VII. CONCLUSION AND FUTURE WORK

The current study mainly focuses on the reliability analysis of a finite cohesive soil slope in MS-Excel spreadsheet environment based on the probabilistic approach. The effect of uncertainties arising due to the spatial variability of the soil was examined. A comparative study on the results of slope analysis using the FOSM, FORM, and MCS reliability methods was presented. The obtained results show that MCS exhibited improved performance in comparison with the other methods due to its robustness and simple approach. When using the MCS method, it becomes very easy to generate any number of samples of the FOS and calculate their failure probability and its corresponding reliability index.

The study also presents the involvement of a large number of random variables modeled with the random field theory. The research also shows that the MCS method can assist in the understanding of the nature of complex problems and assessing the associated risk. This study will help in guiding the geotechnical practitioners dealing with slope stability analysis when various uncertainties are encountered in the soil and will help them in their decision-making process.

In the future, the study will be further extended for a cohesive-frictional soil slope and layered soil slope using other sophisticated limit equilibrium methods, i.e. the Bishop's simplified method and the Morgenstern-Price method.

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