Revista Colombiana de Estadística
July 2016, Volume 39, Issue 2, pp. 167 to 184
DOI: http://dx.doi.org/10.15446/rce.v39n2.58914

Nonparametric Double EWMA Control Chart for Process Monitoring

Gráfico de control EWMA doble no paramétrico del proceso de supervisión

Muhammad Riaz[a], Saddam Akber Abbasi[b]

Department of Mathematics and Statistics, King Fahad University of Petroleum and Minerals, Dhahran, Saudi Arabia

Abstract

In monitoring process parameters, we assume normality of the quality characteristic of interest, which is an ideal assumption. In many practical situations, we may not know the distributional behavior of the data, and hence, the need arises use nonparametric techniques. In this study, a nonparametric double EWMA control chart, namely the NPDEWMA chart, is proposed to ensure efficient monitoring of the location parameter. The performance of the proposed chart is evaluated in terms of different run length properties, such as average, standard deviation and percentiles. The proposed scheme is compared with its recent existing counterparts, namely the nonparametric EWMA and the nonparametric CUSUM schemes. The performance measures used are the average run length (ARL), standard deviation of the run length (SDRL) and extra quadratic loss (EQL). We observed that the proposed chart outperforms the said existing schemes to detect shifts in the process mean level. We also provide an illustrative example for practical considerations.

***Key words*:** ARL, Control charts, DEWMA, EQL, Nonparametric, Process location, Run length sistribution, SDRL.

Resumen

En el seguimiento de los parámetros del proceso, asumimos normalidad de la característica de calidad de interés que es un supuesto ideal. En muchas situaciones prácticas, no podemos conocer el comportamiento de distribución de los datos y por lo tanto, surge la necesidad de técnicas no paramétricas. En este estudio, un gráfico de control EWMA doble paramétrico, a saber, la

[a]Professor. E-mail: riazm@kfupm.edu.sa
[b]Professor. E-mail: saddamaa@kfupm.edu.sa

carta NPDEWMA, se propone para una vigilancia eficaz en el parámetro de localización. El rendimiento del gráfico propuesto se evalúa en términos de propiedades diferentes de longitud de ejecución, como promedio, desviación estándar y percentiles. El esquema propuesto se compara con sus homólogos de los últimos existentes, a saber, la EWMA no paramétrico y los esquemas de CUSUM no paramétricas. Las medidas de desempeño utilizadas son la longitud promedio de carreras (ARL), la desviación estándar de la longitud de ejecución (SDRL) y pérdida cuadrática extra (EQL). Se observa que el gráfico propuesto supera a dichos regímenes existentes para detectar cambios en el proceso de nivel medio. También se proporciona un ejemplo ilustrativo para consideraciones prácticas.

***Palabras clave*:** ARL, gráficas de control, DEWMA, EQL, no paramétrica, ubicación proceso, ejecutar distribución de longitud, SDRL.

1. Introduction

Process monitoring plays an important role in improving quality of the final output. Control charts are important statistical process monitoring tools that help in differentiating un-natural variations from natural. The design structures of control charts may be memory-less (cf. Shewhart, Shewhart 1931) or memory (cf. CUSUM, Page 1954) and (cf. EWMA, Roberts 1959). The memory-less control charts are meant for larger shifts while the others are meant for the shifts of smaller magnitudes. The application of control charting techniques is not only limited the to manufacturing industry but it covers a wide range of disciplines, such as nuclear engineering (cf. Hwang, Lin, Liang, Yau, Yenn & Hsu 2008), health care (cf. Woodall 2006), education (cf. Wang & Liang 2008) and analytical laboratories (cf. Masson 2007, Abbasi 2010), among many others.

There are different types of classifications of control charts, such as parametric versus non-parametric, variable versus attributes, univariate versus multivariate and bayesian versus classical. The parametric charting structures rely on the assumption that the parent distribution of the quality characteristic of interest is known (more specifically normally distributed in many cases). This assumption is not very realistic as, in practice, one may not know the distributional form of the quality characteristic of interest.

This limitation restricts the use of parametric control charts for the monitoring of process parameters. In these scenarios, the parametric charts may lead to misleading results, and hence, we prefer to use the non-parametric control charts. The relevant literature is written by Chakraborti & Graham (2007), Das (2009), Qiu, Zou & Wang (2010), Human, Chakraborti & Smit (2010), Khilare & Shirke (2010), and Pawar & Shirke (2010), Abbasi & Miller (2013) and the references therein. The robust charting structures also provide a good alternative for non-normal situations (cf. Stoumbos & Reynolds 2000, Stoumbos & Sullivan 2002, and the references therein)

There are different types of non-parametric Shewhart, EWMA and CUSUM control charts. For an efficient detection of small shifts, nonparametric EWMA

(NPEWMA) and nonparametric CUSUM (NPCUSUM) control charts have also been proposed in the literature. In reference to several of these; Li, Tang & Ng (2010) proposed NPEWMA and NPCUSUM charts based on the Mann-Whitney statistic; Zou & Tsung (2011) proposed a multivariate EWMA control chart using the weighted version of the sign test; Graham, Chakraborti & Human 2011*a*, 2011*b* proposed nonparametric EWMA sign and signed-rank control charts to monitor the location parameter; Yang & Cheng (2011) proposed two NPEWMA control charts, namely the nonparametric EWMA sign (NPS_E) chart and the nonparametric Arcsine EWMA sign (NPAS_E) chart; Yang & Cheng (2011) proposed a nonparametric CUSUM (NPS_C) chart, using the sign statistics, for quick detection of shifts from the process target. Abbasi (2012) may also be seen in this direction.

The double EWMA (DEWMA) concept was investigated by Shamma & Shamma (1992), Zhang & Chen (2005), and Khoo, Teh & Wu (2010) for a normally distributed quality characteristic of interest. Zhang, Govindaraju, Lai & Bebbington (2003) also investigated the DEWMA chart for Poisson processes. This study is aimed at proposing a new nonparametric DEWMA chart, namely the NPDEWMA chart to efficiently monitor process location. The organization of the rest of the article is given as follows: Section 2 describes the design structure of the proposed NPDEWMA control chart. Section 3 evaluates the performance of the proposed chart using different run length characteristics. Section 4 provides a comparison of the NPDEWMA chart with recently proposed NPS_E, NPAS_E and NPS_C charts. Section 5 presents an example to illustrate the application of the proposed, and finally, the article ends with the concluding remarks in Section 6.

2. The Proposed Nonparametric Double EWMA (NPDEWMA) Chart

This section provides the design structure of the proposed NPDEWMA control chart to monitor the location parameter. Assume that $X_1, X_2, \ldots, X_n$ represent a random sample of size n from a process with the location parameter μ. Let us define a transformed random variable Y as $Y_i = X_i - \mu$. Let p be a probability measure defined as: $p = \mathrm{pr}(Y_i > 0)$. We assume that the process location is in-control if p takes the value p_0 and otherwise out-of-control for any other value. We intend to control the location parameter by monitoring the stability of p with reference to p_0.

In order to do this, we introduce here an indicator variable I,as $I = 1$ if $Y_i > 0$ and zero otherwise. Based on this indicator variable, Yang, Lin & Cheng (2011) defined $M = \sum_{i=1}^{n} I_i$, which follows a binomial distribution with parameters n and p_0. The arcsine transformation for M and its approximate distribution is given as

$$z = \sin^{-1}\sqrt{\frac{M}{n}} \sim N\left(\sin^{-1}\sqrt{p_0}, \frac{1}{4n}\right). \tag{1}$$

Based on the above mentioned variable, the exponentially weighted moving average (EWMA) statistic is defined as:

$$W_t = \lambda Z_t + (1-\lambda)W_{t-1} \tag{2}$$

where λ is a smoothing constant ($0 < \lambda < 1$), and $W_0 = sin^{-1}\sqrt{p_0}$.

We define here another EWMA statistic for W_t as:

$$D_t = \lambda W_t + (1-\lambda)D_{t-1} \tag{3}$$

where λ is defined above and $D_0 = sin^{-1}\sqrt{p_0}$.

We refer to this new statistic as the double EWMA (DEWMA) statistic. The DEWMA statistic defined above may be expressed in an alternative from, as (cf. Roberts 1959 and Montgomery 2009) have described

$$D_t = \lambda \sum_{i=0}^{t-1} (1-\lambda)^i W_{t-i} + (1-\lambda)^t D_0 \tag{4}$$

By substitutions and further simplifications we have

$$D_t = \lambda \sum_{i=0}^{t-1} (1-\lambda)^i (\lambda \sum_{j=0}^{t-i-1} (1-\lambda)^j Z_{t-i-j} + (1-\lambda)^{t-i} W_0) + (1-\lambda)^t D_0 \tag{5}$$

For the DEWMA statistic D_t, the expressions for the mean and the variance are respectively given as (cf. Zhang et al. 2003 and Zhang & Chen 2005)

$$\mu_D = \sin^{-1}\sqrt{p_0},$$

$$\begin{aligned}\sigma_D^2 = (\lambda^4/4n(1-(1-\lambda)^2)^3)(1 + (1-\lambda)^2 - (t+1)^2(1-\lambda)^{2t} \\ + (2t^2+2t-1)(1-\lambda)^{2t+2} - t^2(1-\lambda)^{2t+4})\end{aligned} \tag{6}$$

Based on these quantities, the L-sigma limits for the proposed NPDEWMA chart are defined as

$$LCL_D = \mu_D - L * \sigma_D \qquad \text{and} \qquad UCL_D = \mu_D + L * \sigma_D, \tag{7}$$

where L is the control limits coefficient that helps in fixing the average run length (ARL) for an in-control situation denoted by ARL_0. The values of L depend on the choices of n, λ and ARL_0 for the NPDEWMA control chart. The values of L are worked out for different combinations of the aforementioned quantities. Figure 1 presents plots of ARL_0 versus L for some representative values of n at $\lambda = 0.05$ for ARL_0 values ranging from 50 to 500. These plots help to choose the appropriate control chart multiplier to fix the ARL_0 for the NPDEWMA chart. The ARL_0 plots versus L may be obtained on the similar lines for other choices of n, λ and ARL_0.

By using the above mentioned structure of the control limits for the proposed NPDEWMA chart, an out-of-control signal (if any) is detected when D_t plots beyond the (LCL, UCL) limits. The NPDEWMA chart makes use of current as well as the past information to efficiently monitor the location shits (particularly of a smaller magnitude) that accumulate over time.

3. Performance Evaluations of the Proposed Chart

This section provides the performance evaluations of the proposed NPDEWMA chart to monitor the location parameter. The performance measures used in this study are the average run length (ARL) and extra quadratic loss (EQL). The ARL measure is defined as the average number of samples required before an out-of-control signal is issued in the process. It is classified by two forms, namely ARL_0 (when process is in-control) and ARL_1 (when process is out-of-control). Also, it is evaluated shift by shift (we will use δ to refer to the amount of shift in standard deviation units). A good chart should have larger values for ARL_0 and smaller values for ARL_1. The other measure, namely EQL is evaluated over the whole range of δ values (from the smallest (min) to the largest (max)). It is defined as (see cf. Ahmad, Lin, Abbasi & Riaz 2012, 2013*a*, 2013*b*) and the references therein for further details)

$$\mathbf{EQL} = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{max}}^{\delta_{min}} \delta^2 ARL(\delta) d\delta \tag{8}$$

where ARL(δ) is the average run length of a particular chart at δ. It is generally desirable to have smaller values of EQL for an efficient charting structure.

In order to evaluate the performance of the proposed NPDEWMA chart for different numbers of shifts, $p = 0.5$ is taken as the in-control value while $p \neq 0.5$ is the out-of-control value. Moreover, by computing the EQL, we define δ as a deviation of p from 0.5 (the in-control level) to some other value of p in standard deviation units. In this study, a Monte Carlo simulation with 10^4 iterations is used for the run length distribution of the proposed NPDEWMA chart with an acceptable error rate (cf. Lucas & Saccucci 1990, Maravelakis, Panaretos & Psarakis 2005, Abbasi 2010 and Abbasi & Miller 2013, Kim 2005 and Schaffer & Kim 2007). The run length properties investigated in this study include ARL, standard deviation of the run length (SDRL) and different quantile points ($Q_i = i^{th}$ percentile point) of the run length distribution. It should to be noted that $Q_{0.50}$ refers to median run length (MRL), which is also a useful run length measure.

The summary of the run length properties (in-control and out-of-control) for the proposed NPDEWMA chart is reported in the following form: ARL and SDRL (along with the controlling coefficient L) (cf. Table 1), different percentiles (cf. Table 2) and EQL (cf. Table 5) for some representative choices of n and at $ARL_0 =$ 370. For the other choices, the results can be obtained on similar lines. The relative standard errors of the results reported in the above mentioned tables are observed to be around 1.5%, and they are checked by repeating the simulations. This is quite acceptable in control chart studies (cf. Kim 2005 and Schaffer & Kim 2007).

TABLE 1: Run length characteristics of the proposed NPDEWMA control chart when $\mathbf{ARL}_0 = 370$.

				p														
λ	n		L	**0.05**	**0.15**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.85**	**0.95**
0.05	**5**	**ARL**	**0.484**	1.32	2.38	4.68	7.13	11.7	22.6	68.2	368	67.1	23.2	11.9	7.16	4.77	2.39	1.33
		SDRL		0.68	1.67	3.55	5.47	9.37	18.8	67	409	64.5	19	9.34	5.57	3.62	1.67	0.68
	7	**ARL**	**0.316**	1.32	2.07	3.65	5.46	8.9	17.6	51.9	371	52.7	17.9	9.07	5.43	3.65	2.04	1.32
		SDRL		0.5	1.07	2.41	3.96	6.8	14.1	47.1	411	47	14.2	7.03	3.93	2.43	1.05	0.51
	10	**ARL**	**0.210**	1.09	1.6	2.72	3.98	6.51	13.2	40.2	371	39.5	13.2	6.57	4.02	2.71	1.59	1.09
		SDRL		0.29	0.76	1.67	2.74	4.92	10.5	34.8	415	34.7	10.4	4.93	2.79	1.66	0.75	0.29
	15	**ARL**	**0.135**	1	1.19	1.87	2.64	4.44	8.94	28.4	369	28	8.95	4.38	2.67	1.86	1.2	1.01
		SDRL		0.07	0.42	1.1	1.8	3.37	7.22	24.2	417	24.1	7.22	3.34	1.82	1.09	0.43	0.08
0.25	**5**	**ARL**	**0.671**	1.51	2.96	6	9.26	16.8	38.6	129	371	130	38.3	16.9	9.33	6.01	2.96	1.51
		SDRL		1	2.09	4.4	7.28	14.6	37.5	132	401	134	36.6	14.5	7.19	4.45	2.11	1
	7	**ARL**	**0.433**	1.39	2.53	4.8	7.2	12.5	28	102	371	99.6	27.9	12.6	7.17	4.74	2.51	1.4
		SDRL		0.67	1.42	2.92	4.74	9.5	24.2	100	383	97.4	24.2	9.4	4.71	2.88	1.4	0.68
	10	**ARL**	**0.284**	1.1	1.78	3.4	5.03	8.49	18.8	72	369	72.2	19	8.67	5.05	3.4	1.8	1.1
		SDRL		0.33	1	2.05	3.21	5.93	15.7	69.9	371	69.4	15.5	6.02	3.18	2.05	1	0.34
	15	**ARL**	**0.183**	1.04	1.45	2.5	3.68	6.01	12.4	48.9	371	48.3	12.4	6.02	3.66	2.52	1.46	1.04
		SDRL		0.19	0.61	1.32	2.1	3.73	9.01	44.6	377	44	9.15	3.68	2.05	1.33	0.62	0.18

4. Comparative Analysis of the Results

In this section we provide a comparative discussion of the results obtained in Section 3 for the proposed NPDEWMA chart. Moreover, the efficiency of the proposed chart is also compared with the recently proposed NPS_E, NPAS_E and NPS_C charts in the literature. The summarized information of these charts, along with the proposed NPDEWMA chart, is provided in Table 3. For the details of these charting structures, see Yang et al. (2011), Yang & Cheng (2011) and Abbasi (2012). We evaluate performance of these competing charts for some selective values of n. The control charting parameters of different control charts (like K, H, k, L) are set in such a way that the ARL_0 value of all the charts is fixed at 370 for valid comparisons among different charts. The resulting ARL values for the competing charts are provided in Table 4. The EQL values are also evaluated for these competing charts on the same lines as the proposed NPDEWMA chart. The results for all charts are provided in Table 5.

TABLE 2: Percentile points of the run length distribution of the proposed NPDEWMA control chart when $\mathbf{ARL}_0 = 370$.

λ	n		p									
			0.5	**0.6**	**0.6**	**0.7**	**0.7**	**0.8**	**0.8**	**0.9**	**0.9**	**1**
0.05	**5**	$Q_{0.05}$	1	1	1	1	1	1	1	1	1	1
		$Q_{0.25}$	61	19	8	4	3	2	1	1	1	1
		$Q_{0.50}$	228	50	19	10	6	4	3	2	1	1
		$Q_{0.75}$	516	93	33	17	10	7	5	4	2	1
		$Q_{0.95}$	1171.95	195	59	30	18	12	8	6	4	3
	7	$Q_{0.05}$	3	2	2	1	1	1	1	1	1	1
		$Q_{0.25}$	79	17	7	4	2	2	2	1	1	1
		$Q_{0.50}$	250	40	15	7	4	3	2	2	2	1
		$Q_{0.75}$	538	73	26	13	7	5	4	2	2	2
		$Q_{0.95}$	1204.95	148	45	22	13	8	6	4	3	2
	10	$Q_{0.05}$	3	2	2	1	1	1	1	1	1	1
		$Q_{0.25}$	77.25	13	5	3	2	2	1	1	1	1
		$Q_{0.50}$	241	32	11	5	3	2	2	1	1	1
		$Q_{0.75}$	533	57	18	9	5	3	3	2	2	1
		$Q_{0.95}$	1227	106	33	16	9	6	4	3	2	2
	15	$Q_{0.05}$	2	2	1	1	1	1	1	1	1	1
		$Q_{0.25}$	65	9	3	2	1	1	1	1	1	1
		$Q_{0.50}$	233	22	7	3	2	2	1	1	1	1
		$Q_{0.75}$	532	40	13	6	4	2	2	1	1	1
		$Q_{0.95}$	1187.9	74	23	11	6	4	3	2	2	1
0.25	**5**	$Q_{0.05}$	1	1	1	1	1	1	1	1	1	1
		$Q_{0.25}$	94	33	13	7	4	3	1	1	1	1
		$Q_{0.50}$	253	88	27	13	8	5	4	3	1	1
		$Q_{0.75}$	533	182	53	22	13	8	6	4	3	1
		$Q_{0.95}$	1155.95	390	114	44	23	14	9	7	5	4
	7	$Q_{0.05}$	13	6	3	2	1	1	1	1	1	1
		$Q_{0.25}$	106	31	11	6	4	3	2	1	1	1
		$Q_{0.50}$	257	71	21	10	6	4	3	2	2	1
		$Q_{0.75}$	511	138	38	17	9	6	4	3	3	2
		$Q_{0.95}$	1109.9	299	77	31	16	10	7	5	4	3
	10	$Q_{0.05}$	11	5	2	1	1	1	1	1	1	1
		$Q_{0.25}$	105	23	8	4	3	1	1	1	1	1
		$Q_{0.50}$	258	52	15	7	5	3	2	1	1	1
		$Q_{0.75}$	516	99	25	11	7	5	3	3	2	1
		$Q_{0.95}$	1165	206	50	20	11	7	5	4	3	2
	15	$Q_{0.05}$	15	5	2	1	1	1	1	1	1	1
		$Q_{0.25}$	107	17	6	3	2	2	1	1	1	1
		$Q_{0.50}$	266.5	36	10	5	3	2	2	1	1	1
		$Q_{0.75}$	532	68	16	8	5	3	2	2	1	1
		$Q_{0.95}$	1143	139	31	13	7	5	4	3	2	1

TABLE 3: Design structure of different control charts.

Control Chart	Monitoring Statistic	Control limits
$NPDEWMA$	$D_t = \lambda W_t + (1-\lambda)D_{t-1}$	$LCL = \mu_D - L\sqrt{\sigma_D^2}$
		$UCL = \mu_D + L\sqrt{\sigma_D^2}$
NPS_E	$W_t' = \lambda M_t + (1-\lambda)W_{t-1}'$	$LCL = n/2 - K'\sqrt{\frac{\lambda}{2-\lambda}(n/4)}$
		$UCL = n/2 + K'\sqrt{\frac{\lambda}{2-\lambda}(n/4)}$
$NPAS_E$	$W_t = \lambda Z_t + (1-\lambda)W_{t-1}$	$LCL = sin^{-1}(\sqrt{0.5}) - K\sqrt{\frac{\lambda}{2-\lambda}(1/4n)}$
		$UCL = sin^{-1}(\sqrt{0.5}) + K\sqrt{\frac{\lambda}{2-\lambda}(1/4n)}$
NPS_C	$C_t^+ = \max(0, C_{t-1}^- + M_t - (np_0 + k))$	H
	$C_t^- = \min(0, C_{t-1}^+ + M_t - (np_0 - k))$	

Table 4: ARL of NPS_E, $NPAS_E$ and NPS_C charts when $ARL_0 = 370$.

		p														
λ	n	0.05	0.15	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.85	0.95
NPS_E Chart																
0.05	5	4.80	6.37	9.29	12.21	17.60	30.98	86.35	369.08	86.33	30.66	17.67	12.23	9.35	6.38	4.77
	7	4.17	5.35	7.69	10.02	14.24	24.71	68.29	370.99	66.94	24.33	14.16	9.98	7.70	5.36	4.17
	10	3.44	4.44	6.29	8.08	11.37	19.10	51.47	369.53	51.37	19.04	11.40	8.11	6.33	5.00	4.00
	15	3.01	3.59	5.07	6.48	8.99	14.64	38.12	371.55	38.19	14.68	8.99	6.46	5.07	4.00	3.00
0.25	5	3.21	4.54	7.94	12.19	21.70	50.27	152.51	368.72	154.54	50.24	21.92	12.21	8.07	4.54	3.21
	7	2.55	3.64	6.10	9.05	16.06	36.76	129.40	370.71	128.70	36.51	16.00	9.17	6.13	3.65	2.55
	10	2.12	2.89	4.62	6.55	11.09	24.89	95.31	371.33	93.24	25.01	11.00	6.55	4.65	2.89	2.12
	15	2.01	2.36	3.53	4.88	7.71	16.55	67.58	369.12	66.78	16.63	7.68	4.82	3.53	2.35	2.00
$NPAS_E$ Chart																
0.05	5	3.81	5.71	9.10	12.28	17.95	32.47	90.35	370.77	90.98	31.84	17.89	12.30	9.06	5.73	3.81
	7	3.30	4.74	7.40	9.86	14.36	25.04	70.38	371.60	70.13	25.09	14.33	9.78	7.40	4.73	3.31
	10	2.71	3.94	6.01	7.90	11.26	19.12	52.54	370.33	52.56	19.23	11.23	7.88	6.02	3.93	2.71
	15	2.31	3.27	4.92	6.32	8.88	14.62	38.84	369.02	38.37	14.65	8.85	6.31	4.89	3.27	2.31
0.25	5	2.58	4.46	8.80	14.12	26.21	58.78	174.46	369.03	170.97	60.30	25.94	13.90	8.86	4.44	2.58
	7	2.39	3.70	6.95	10.91	20.15	47.69	150.97	368.12	149.61	47.56	20.35	10.99	6.96	3.68	2.40
	10	1.47	2.58	4.65	6.99	12.37	29.13	109.55	368.93	110.84	29.65	12.29	7.02	4.68	2.62	1.47
	15	1.53	2.21	3.48	4.84	7.98	17.03	69.72	371.21	70.04	17.36	7.97	4.87	3.48	2.22	1.55
NPS_C Chart																
	5	5.42	7.04	10.31	13.47	19.30	33.71	95.86	371.56	94.90	33.84	19.32	13.46	10.32	7.05	5.42
	7	4.17	5.48	8.07	10.48	14.99	26.86	79.35	369.59	78.85	26.80	15.01	10.50	8.09	5.46	4.16
	10	4.19	5.17	7.02	8.87	12.35	21.39	64.55	370.27	64.30	21.33	12.36	8.87	7.05	5.16	4.19
	15	3.18	4.08	5.45	6.76	9.31	15.78	50.99	370.88	50.81	15.94	9.34	6.77	5.46	4.09	3.19

TABLE 5: EQL Values for NPDEWMA, NPS_E, NPAS_E and NPS_C charts.

n	**NPDEWMA**		$\boldsymbol{NPS_E}$		$\boldsymbol{NPAS_E}$		$\boldsymbol{NPS_C}$
	0.05	**0.25**	**0.05**	**0.25**	**0.05**	**0.25**	
5	2.7518	3.3328	8.3287	5.9710	7.0073	5.3460	9.3505
7	2.5297	2.9110	7.1468	4.7281	5.9519	4.6871	7.2190
10	2.0249	2.1859	6.3038	3.8190	4.8859	3.0224	7.0421
15	1.7108	1.9072	5.0796	3.3858	4.1101	2.8188	5.4128

For a comparative discussion of the different choices for the proposed NPDEWMA chart and the other competing charts, we have created some useful graphs for different values of n and λ at $\text{ARL}_0 = 370$. The similar graphs may be observed for the other choices of the said quantities. These graphs are provided in Figures 2-5 where p is plotted on horizontal axis and the ARL on the vertical axis. They are in logarithmic scale for better visual inspection.

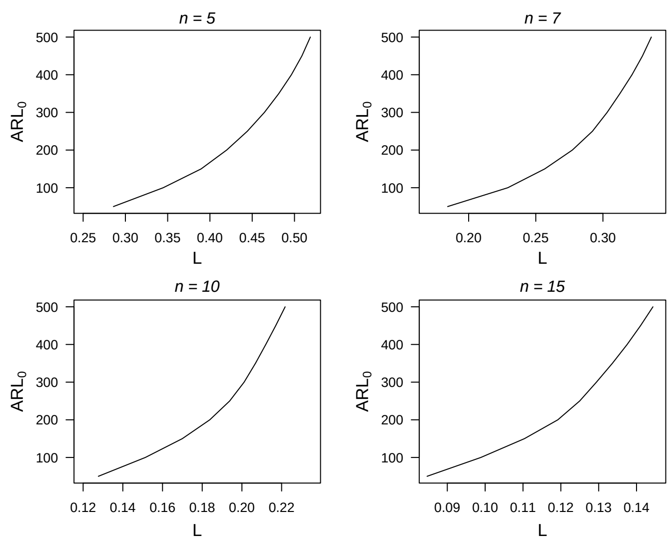

FIGURE 1: ARL_0 plots versus L for the proposed NPDEWMA chart.

In terms of different measures, the performance evaluations of the proposed and the other competing control charts, advocate the following:

i) the proposed NPDEWMA chart is easily implemented in practice for process monitoring as it simply involves the computation of the D_t statistic given in (4), and it is plotted it against the limits given in (7) (without relying on normality);

ii) the proposed NPDEWMA chart has the ability to efficiently detect smaller (for smaller choices of λ) as well as larger shifts (for larger choices of λ) (cf. Table 1 and Figures 2-3);

iii) the proposed structure is effective for both directions, i.e. increasing and decreasing shifts in the process parameter) (cf. Table 1 and Figures 2-3);

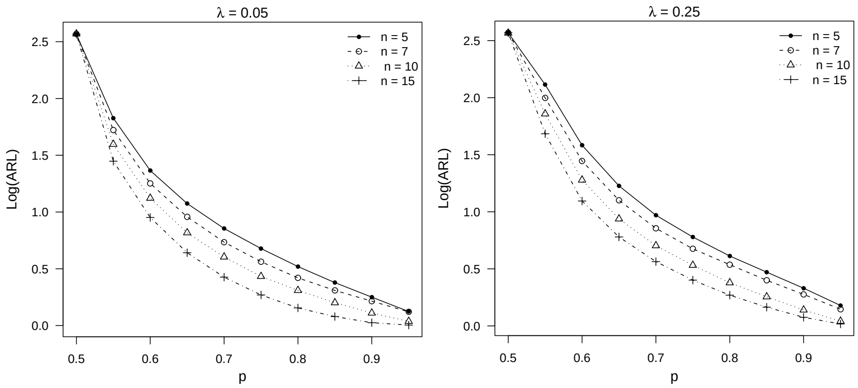

FIGURE 2: ARL comparison of the NPDEWMA chart for different values of n at $\mathrm{ARL}_0 = 370$.

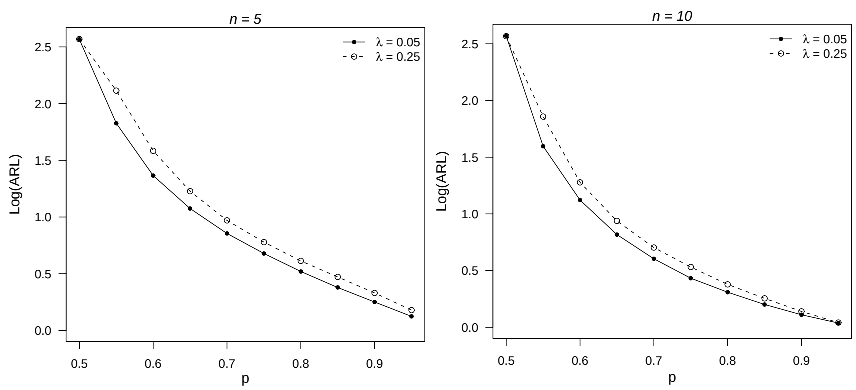

FIGURE 3: ARL comparison of the NPDEWMA chart for different values of λ at $\mathrm{ARL}_0 = 370$.

iv) the run length distribution of NPDEWMA is positively skewed;

v) the NPDEWMA chart shows decreasing behavior of ARL, SDRL and the percentile points with a decrease in the value of λ and an increase in the value of n and δ (cf. Tables 1-2 and Figures 2-3);

vi) the proposed NPDEWMA chart performs better than the competing counterparts NPS_E, NPAS_E and NPS_C charts in terms of run length efficiency (cf. Tables 1 and 4 and Figures 4-5);

vii) the proposed NPDEWMA chart outperforms the other competing charts in terms of its overall performance measure EQL (cf. Table 5).

viii) The order and superiority (in terms of ARL and EQL measures) of the different charts being researched in the study is: NPDEWMA, $\mathrm{NPAS}_E/\mathrm{NPS}_E$ and NPS_C.

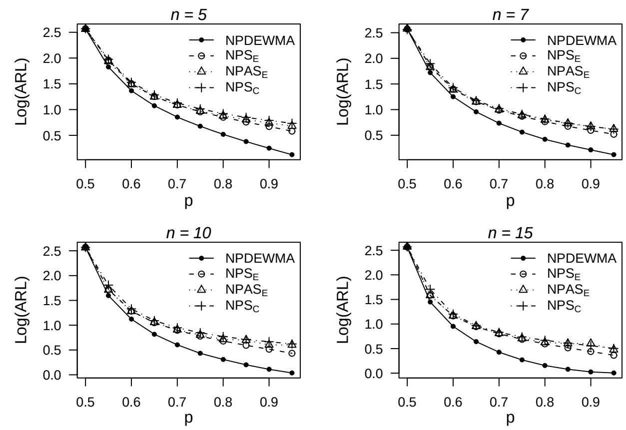

FIGURE 4: Comparison between NPDEWMA, NPS_E, NPAS_E and NPS_C charts when $\lambda = 0.05$ and $\text{ARL}_0 = 370$.

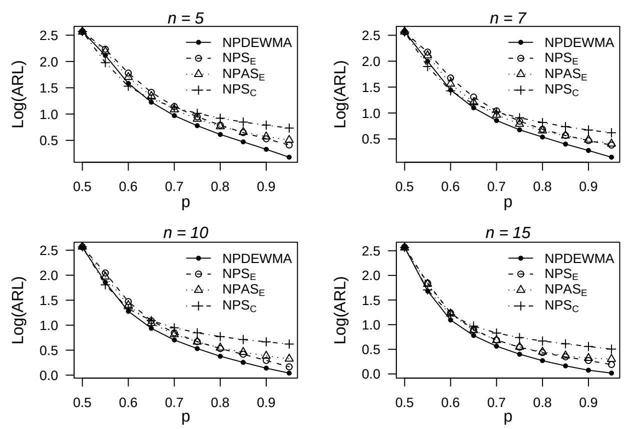

FIGURE 5: Comparison between NPDEWMA, NPS_E, NPAS_E and NPS_C charts when $\lambda = 0.25$ and $\text{ARL}_0 = 370$.

As the run-length distribution is skewed to the right, many researchers recommend examining a different number of percentiles including the 5^{th}, 25^{th}, median, 75^{th} and 95^{th} percentiles in order to better characterize the run-length distribution. Therefore, we have provided some additional measures for the proposed chart (cf. Table 2). In addition, we have provided some useful properties of the proposed scheme for smaller values of λ, including $\lambda = 0.01$, 0.025 and 0.10 (for $n = 5$) to highlight the effectiveness of the proposal for smaller shifts. These results can be seen in Table 6; they support the performance of the proposal for smaller and smaller shift values.

Moreover, motivated by Gan (1994), we have also set the design parameters by fixing the MDRL value at a specific level, MDRL_0 for example and have evaluated the performance of the proposed scheme. The resulting performance measures (with the desired nominal MDRL is equal to 350 i.e. $\text{MDRL}_0 = 350$, in the form

Table 6: ARL, MDRL and SDRL of the DNPEWMA chart at low levels of Lambda when $ARL_0 = 370$ and $n = 5$.

λ		k	p												
			0.05	0.25	0.3	0.35	0.4	0.45	**0.5**	0.55	0.6	0.65	0.7	0.75	0.95
0.01	ARL	0.335	1.23	3.26	4.90	7.97	15.64	45.74	370.48	45.70	15.94	8.01	4.83	3.31	1.24
	MDRL		1.00	2.00	3.00	5.00	9.00	25.00	588.84	24.00	10.00	5.00	3.00	2.00	1.00
	SDRL		0.45	2.62	4.41	7.88	16.90	54.65	105.00	54.72	17.05	7.92	4.39	2.66	0.46
0.025	ARL	0.414	1.30	4.10	6.13	10.07	19.99	58.55	371.96	58.35	20.10	10.12	6.19	4.14	1.30
	MDRL		1.00	3.00	5.00	8.00	15.00	43.00	464.78	43.00	15.00	8.00	5.00	3.00	1.00
	SDRL		0.60	3.12	5.09	8.80	18.28	58.38	215.00	57.85	18.31	8.93	5.12	3.17	0.61
0.1	ARL	0.563	1.33	5.33	7.96	13.23	26.49	83.31	368.32	83.75	26.59	13.18	8.01	5.30	1.32
	MDRL		1.00	5.00	7.00	11.00	22.00	61.00	409.68	59.00	22.00	11.00	7.00	5.00	1.00
	SDRL		0.71	3.94	5.86	10.13	22.35	82.05	247.00	84.36	22.35	10.15	5.93	3.95	0.69

of ARL, MDRL and SDRL) are provided in Table 7. This means that there is at least a 50% chance that the first out-of-control signal will be observed by the 350^{th} sample despite the process actually being in-control. The results of Table 7 offer attractive properties for the proposed chart by fixing the in-control run-lengths to be greater than or equal to 350. These results are given for λ =0.01, 0.025 and 0.10 for $n = 5$. Similar results may be obtained for other choices of n,λ and MDRL_0 values.

5. Illustrative Example

In this section, we provide an application of the proposed NPDEWMA chart for a real data set to illustrate how it is implemented. The data set is based on an important quality characteristic in a production process, namely the fill volume of soft-drink beverage bottles (cf. Montgomery 2009, Yang et al. 2011 and Yang & Cheng 2011). A coded scale is used to measure the volume by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle. The correct fill height is indicated by a reading of zero on this scale. The data consists of fifteen samples of size $n = 10$ that are measured on the fill volume. Figure 6 represents the data in graph form.

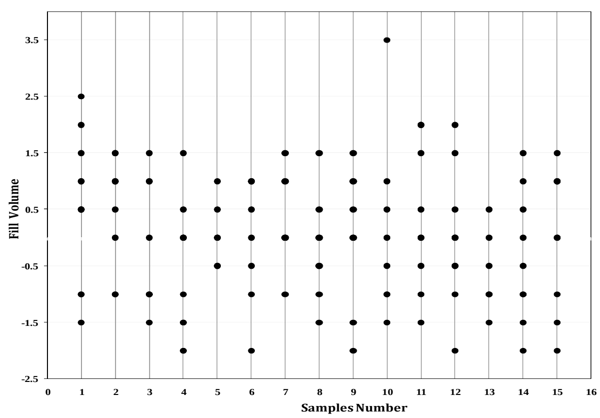

FIGURE 6: Graphical display for the example's Fill Volume data.

For the aforementioned data set, the charting statistic (D_t) values of the proposed NPDEWMA chart are computed using λ= 0.25. The control limits are set such that $\text{ARL}_0 = 370$ for which the control limits multiplier for the proposed NPDEWMA chart $L = 0.284$ is used. The charting statistics are plotted against these control limits. The same is done for the dataset under discussion, in the three other competing control charts NPS_E, NPAS_E and NPS_C of this study. The resulting control chart displays are shown in Figure 7 for all the four charts. We observed that the proposed NPDEWMA chart gives six out-of-control signals for the location parameter. The other charts show the out-of-control signals as: NPS_E three, NPAS_E one and NPS_C none. This detection ability in the example data set is in agreement with the dominance order that the previous section establishes.

Table 7: ARL, MDRL and SDRL of the DNPEWMA chart at low levels of when $\mathbf{MDRL}_0 = 350$ and $n = 5$.

			p												
λ		k	0.05	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.95
0.01	ARL	0.397	1.29	3.90	6.03	9.94	20.45	63.55	727.91	61.58	20.70	10.09	5.88	3.94	1.27
	MDRL		1.00	3.00	4.00	7.00	15.00	43.00	350.00	41.00	15.00	7.00	4.00	3.00	1.00
	SDRL		0.58	3.10	5.23	9.12	20.09	67.22	974.37	64.96	20.13	9.29	5.05	3.13	0.56
0.025	ARL	0.459	1.29	4.58	6.85	11.65	23.30	71.17	571.90	68.92	23.67	11.66	7.02	4.50	1.30
	MDRL		1.00	3.00	5.00	9.00	18.00	54.00	349.00	53.00	19.00	9.00	6.00	3.00	1.00
	SDRL		0.61	3.60	5.59	10.00	20.42	68.07	677.32	65.13	20.53	9.82	5.84	3.60	0.61
0.1	ARL	0.596	1.49	5.70	8.55	14.28	29.40	101.48	526.73	101.87	29.60	14.43	8.56	5.71	1.48
	MDRL		1.00	5.00	8.00	13.00	24.00	71.00	351.00	72.00	24.00	13.00	8.00	5.00	1.00
	SDRL		0.93	4.19	6.26	10.83	24.25	100.42	562.51	100.44	24.61	10.66	6.27	4.14	0.92

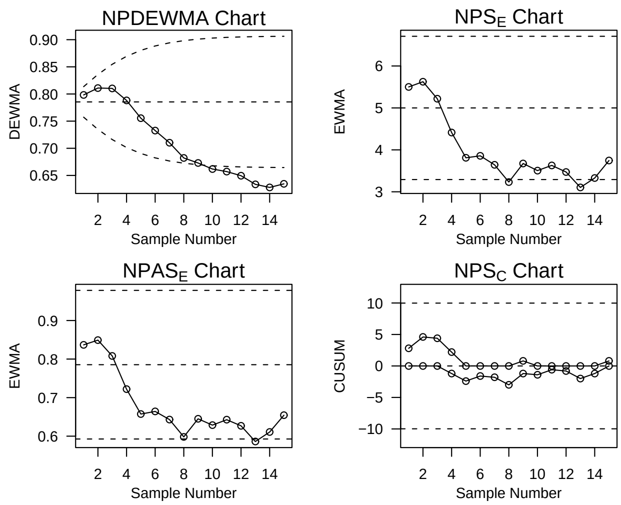

FIGURE 7: Control chart plots of the NPDEWMA, NPS_E, NPAS_E and NPS_C charts for the example dataset.

6. Concluding Remarks

This study has proposed a nonparametric double EWMA control chart for efficiently monitor to the location parameter that has an unknown distributional behavior. The design structure of the proposed NPDEWMA chart depends on the sample size n and the smoothing constant λ. The controlling coefficients L are worked out in irder to fix ARL_0 at the prespecified level. Also, the control limits are developed for the proposed chart to improve the monitoring of process location when the parametric distribution of the quality characteristic of interest is not known. The performance of the proposed chart is evaluated in terms of different run length properties, including ARL, SDRL and some useful percentile points. The EQL measure is also computed as an overall performance measure of the proposed chart.

Comparisons of the proposal were also carried out with some existing counterparts, namely the NPS_E, NPAS_E and NPS_C charts that were proposed in the literature. The comparisons revealed the superiority of the proposed NPDEWMA chart over the competing counterparts in terms of improved run length properties. For example, when p shifts to 0.55 at $n = 5$, $\lambda = 0.05$ and $\text{ARL}_0 = 370$, the ARL_1 performance of the proposed NPDEWMA chart is 67.10. This is followed by NPS_E with $\text{ARL}_1 = 86.33$, then NPAS_E with $\text{ARL}_1 = 90.98$, and finally NPS_C with $\text{ARL}_1 = 94.90$. For this particular change in p a gain of almost 22%, 26% and 29% for the proposed NPDEWMA chart in comparison to NPS_E, NPAS_E and NPS_C charts, respectively. The similar gains may be seen at other values of p for different combinations of n and λ.

The quality control practitioners may benefit from the proposal for better detection of out-of-control signals during in the monitoring of process location parameter. The scope of the study may be extended to double CUSUM charts. Moreover, multivariate extensions of these charts to monitor the vector of location parameters for more than one quality characteristics of interest is another potential topic for future research.

Acknowledgements

The authors are thankful to the anonymous reviewers for their constructive comments that helped to improve the latest version of the paper. The authors are also indebted to the King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia for providing excellent research facilities and a dynamic research environment and for authorizing project number IN131019.

[Received: November 2014 — Accepted: July 2015]

References

Abbasi, S. A. (2010), 'On the performance of EWMA chart in the presence of two component measurement error', *Quality Engineering* **22**(3), 199–213.

Abbasi, S. A. (2012), 'A new nonparametric EWMA sign control chart', *Expert Systems with Applications* **39**(9), 1–1.

Abbasi, S. A. & Miller, A. (2013), 'Mdewma chart: an efficient and robust alternative to monitor process dispersion', *Journal of Statistical Computation and Simulation* **83**(2), 247–268.

Ahmad, S., Lin, Z., Abbasi, S. A. & Riaz, M. (2012), 'On efficient monitoring of process dispersion using interquartile range', *Open Journal of Applied Sciences* **2**, 39–43.

Ahmad, S., Riaz, M., Abbasi, S. A. & Lin, Z. (2013*a*), 'On efficient median control charting', *Journal of the Chinese Institute of Engineers* **37**(3), 358–375.

Ahmad, S., Riaz, M., Abbasi, S. A. & Lin, Z. (2013*b*), 'On monitoring process variability under double sampling scheme', *International Journal of Production Economics* **142**(2), 388–400.

Chakraborti, S. & Graham, M. A. (2007), *Nonparametric control charts. in Encyclopedia of Statistics in Quality and Reliability*, Wiley, New York.

Das, N. (2009), 'A comparison study of three non-parametric control charts to detect shift in location parameters', *International Journal of Advanced Manufacturing Technology* **41**(7-8), 799–807.

Gan, F. F. (1994), 'An optimal design of cumulative sum control chart based on median run length', *Communications in Statistics: Simulation and Computation* **23**(2), 485–503.

Graham, M. A., Chakraborti, S. & Human, S. W. (2011*a*), 'A nonparametric EWMA sign chart for location based on individual measurements', *Quality Engineering* **23**(2), 227–241.

Graham, M. A., Chakraborti, S. & Human, S. W. (2011*b*), 'A nonparametric exponentially weighted moving average signed-rank chart for monitoring location', *Computational Statistics and Data Analysis* **55**(8), 2490–2503.

Human, S. W., Chakraborti, S. & Smit, C. F. (2010), 'Nonparametric shewhart-type sign control charts based on runs', *Communications in Statistics - Theory and Methods* **39**(11), 2046–2062.

Hwang, S. L., Lin, J. T., Liang, G. F., Yau, Y. J., Yenn, T. C. & Hsu, C. C. (2008), 'Application control chart concepts of designing a pre-alarm system in the nuclear power plant control room', *Nuclear Engineering and Design* **238**(12), 3522–3527.

Khilare, S. K. & Shirke, D. T. (2010), 'A nonparametric synthetic control chart using sign statistic', *Communications in Statistics - Theory and Methods* **39**(18), 3282–3293.

Khoo, M. B. C., Teh, S. Y. & Wu, Z. (2010), 'Monitoring process mean and variability with one double EWMA chart', *Communications in Statistics - Theory and Methods* **39**(20), 3678–3694.

Kim, M.-J. (2010), Number of replications required in control chart Monte Carlo simulation studies, Ph.d. dissertation, University of Northern Colorado.

Li, S. Y., Tang, L. C. & Ng, S. H. (2010), 'Nonparametric CUSUM and EWMA control charts for detecting mean shifts', *Journal of Quality Technology* **42**(2), 209–226.

Lucas, J. M. & Saccucci, M. S. (1990), 'Exponentially weighted moving average control schemes: Properties and enhancements', *Technometrics* **32**(1), 1–12.

Maravelakis, P., Panaretos, J. & Psarakis, S. (2005), 'An examination of the robustness to non normality of the EWMA control charts for the dispersion', *Communications in Statistics-Simulation and Computation* **34**(4), 1069–1079.

Masson, P. (2007), 'Quality control techniques for routine analysis with liquid chromatography in laboratories', *Journal of Chromatography* **1158**(1-2), 168–173.

Montgomery, D. C. (2009), *Introduction to statistical quality control*, 6 edn, Wiley, New York.

Page, E. S. (1954), 'Continuous inspection schemes', *Biometrika* **41**, 100–115.

Pawar, V. Y. & Shirke, D. T. (2010), 'A nonparametric shewhart-type synthetic control chart', *Communications in Statistics: Simulation and Computation* **39**(8), 1493–1505.

Qiu, P., Zou, C. & Wang, Z. (2010), 'Nonparametric profile monitoring by mixed effects modeling', *Technometrics* **52**(3), 265–277.

Roberts, S. W. (1959), 'Control chart tests based on geometric moving averages', *Technometrics* **1**(3), 239–250.

Schaffer, J. R. & Kim, M. J. (2007), 'Number of replications required in control chart monte carlo simulation studies', *Communications in Statistics-Simulation and Computation* **36**(5), 1075–1087.

Shamma, S. E. & Shamma, A. K. (1992), 'Development and evaluation of control charts using double exponentially weighted moving averages', *International Journal of Quality and Reliability Management* **9**(6), 18–25.

Shewhart, W. (1931), *Economic Control of Quality Manufactured Product*, American Society for Quality Control in 1980, New York.

Stoumbos, Z. & Reynolds, M. R. J. (2000), 'Robustness to non-normality and autocorrelation of individuals control charts', *Journal of Statistical Computation and Simulation* **66**(2), 145–187.

Stoumbos, Z. & Sullivan, J. H. (2002), 'Robustness to non-normality of the multivariate EWMA control chart', *Journal of Quality Technology* **34**(3), 260–276.

Wang, Z. & Liang, R. (2008), Discussion on applying SPC to quality management in university education, *in* 'Proceedings of the 9th International Conference for Young Computer Scientists', ICYCS 2008, pp. 2372–2375.

Woodall, W. H. (2006), 'The use of control charts in health-care and public-health surveillance', *Journal of Quality Technology* **38**(2), 89–104.

Yang, S. F. & Cheng, S. W. (2011), 'A new nonparametric CUSUM sign control chart', *Quality and Reliability Engineering International* **27**(7), 867–875.

Yang, S. F., Lin, J. S. & Cheng, S. W. (2011), 'A new nonparametric EWMA sign control chart', *Expert Systems with Applications* **38**(5), 6239–6243.

Zhang, L. & Chen, G. (2005), 'An extended EWMA mean chart', *Quality Technology & Quantitative Management* **2**(1), 39–52.

Zhang, L., Govindaraju, K., Lai, C. D. & Bebbington, M. S. (2003), 'Poisson DEWMA control chart', *Communications in Statistics-Simulation and Computation* **32**(4), 1265–1283.

Zou, C. & Tsung, F. (2011), 'A multivariate sign EWMA control chart', *Technometrics* **53**(1), 84–97.