Revista Colombiana de Estadística
January 2016, Volume 39, Issue 1, pp. 63 to 79
DOI: http://dx.doi.org/10.15446/rce.v39n1.55139

Enhancing the Mean Ratio Estimators for Estimating Population Mean Using Non-Conventional Location Parameters

Mejoras a los estimadores de razón de medias con el fin de estimar la media poblacional usando parámetros de localización no convencionales

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Abstract

Conventional measures of location are commonly used to develop ratio estimators. However, in this article, we attempt to use some non-conventional location measures. We have incorporated tri-mean, Hodges-Lehmann, and mid-range of the auxiliary variable for this purpose. To enhance the efficiency of the proposed mean ratio estimators, population correlation coefficient, coefficient of variation and the linear combinations of auxiliary variable have also been exploited. The properties associated with the proposed estimators are evaluated through bias and mean square errors. We also provide an empirical study for illustration and verification.

Key words: Bias, Correlation Coefficient, Coefficient of Variation, Hodges-Lehmann Estimator, Mean Square Error, Tri-Mean.

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Resumen

Las medidas convencionales de localización son a menudo usadas con el fin de desarrollar estimatores de raz ón. Sin embargo, en este artículo, se hace un intento por usar algunas medidas de localización no convencionales. Se incorpora la trimean, el estimador de Hodges-Lehmann y el rango medio de la variable auxiliar con este propósito. Para mejorar la eficiencia de los estimadores de razón de medias propuestos, el coeficiente de correlación poblacional, el coeficiente de variación y combinaciones lineales de variables auxiliares también han sido explotados. Las propiedades asociadas con los estimadores propuestos son evaluadas a través del sesgo y el error cuadrático medio. Un studio empírico es presentado con fines de ilustración y verificación.

***Palabras clave*:** coeficiente de correlación poblacional, coeficiente de variación, error cuadrático medio, estimador de Hodges-Lehmann, sesgo, trimean.

1. Introduction

A mechanism, in statistical analysis, in which a predetermined number of observations are taken from a statistical population is called sampling. Sampling plays a vital role in all kinds of disciplines. The purpose is to reduce the cost and/or the amount of work that it would take to survey the entire target population. The variable of interest or the variable about which we want to draw some inference is called a study variable.

In survey research, there are situations in which when the information is available on every unit in the population. If a variable, that is known for every unit of the population, is not a variable of direct interest but instead employed to improve the sampling plan or to enhance estimation of the variables of interest, then it is called an auxiliary variable. The auxiliary information is commonly associated with the use of ratio, product and regression estimation methods and to improve the efficiency of the estimators in survey sampling. The ratio estimator is most effective for estimating population mean when there is a linear relationship between study variable and auxiliary variable and they have a positive correlation.

Consider a finite population $U = \{U_1, U_2, \ldots, U_N\}$ of N distinct and identifiable units. Let Y be the study variable with value Y_i measured on U_i, $i = 1, 2, \ldots, N$ giving a vector $Y = \{Y_1, Y_2, \ldots, Y_N\}$. The objective is to estimate population mean $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, on the basis of a random sample. When the population parameters of the auxiliary variable X such as population mean, coefficient of variation, co-efficient of kurtosis, co-efficient of skewness, median, quartiles, population correlation coefficient, deciles etc., are known then a number of estimators available in the literature which performs better than the usual simple random mean under certain conditions. Below we provide here the complete list of notations that are used in this paper:

Nomenclature

Roman

N	Population size
n	Sample size
X	Auxiliary variable
$f = \frac{1}{N}$	Sampling fraction
$\overline{x}, \overline{y}$	Sample means
S_x, S_y	Population standard deviations
x, y	Sample totals
C_x, C_y	Coefficient of variation
$B(.)$	Bias of the Estimator
$MSE(.)$	Mean squared error of the estimator
$\overline{Y}_i$	Existing estimators
$\overline{Y}_j$	Proposed estimators
M_d	Median of X
$HL = \text{median}\left((X_j + X_k)/2, 1 \leq j \leq k \leq N\right)$	Hodges-Lehmann estimator
$MR = \frac{X_{(1)} + X_{(N)}}{2}$	Mid-range
$TM = \frac{Q_1 + 2Q_2 + Q_3}{4}$	Tri-mean
$QD = \frac{Q_3 - Q_1}{2}$	Quartile Deviation

Subscript

i	For existing estimators
j	For proposed estimators

Greek

ρ	Coefficient of correlation
$\beta_1 = \frac{N\sum_{i=1}^{N}(X_i - \overline{X})^3}{(N-1)(N-2)S^3}$	Coeff. of skewness of auxiliary variable
$\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i - \overline{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$	Coeff. of kurtosis of auxiliary variable
$\widehat{\overline{Y}}_i$	Existing modified ratio estimator of $\overline{Y}$
$\widehat{\overline{Y}}_{pj}$	Proposed modified ratio estimator of $\overline{Y}$

Based on the above mentioned notations, the mean ratio estimator for estimating the population mean $\overline{Y}$ of the study variable Y is defined as

$$\widehat{\overline{Y}}_r = \frac{\overline{y}}{\overline{x}}\overline{X} = \widehat{R}\overline{X}, \tag{1}$$

where $\widehat{R} = \frac{\overline{y}}{\overline{x}} = \frac{y}{x}$ is the estimate of $R = \frac{\overline{Y}}{\overline{X}} = \frac{Y}{X}$.

The bias, constant and the mean square error of the mean ratio estimator is given by:

$$B(\widehat{\overline{Y}}_r) = \frac{(1-f)}{n}\frac{1}{\overline{X}}\left(RS_x^2 - \rho S_x S_y\right)$$
$$R = \frac{\overline{Y}}{\overline{X}}.$$

$$MSE(\widehat{\overline{Y}}_r) = \frac{(1-f)}{n}\left(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y\right)$$

The mean ratio estimator given in (1) is used to improve the precision of the estimate of the population mean in comparison with the sample mean estimator whenever a positive correlation exists between the study variable and the auxiliary variable. Moreover, we mode improvements by introducing a large number of modified ratio estimators with the use of the known coefficient of variation, coefficient of kurtosis, co- efficient of skewness, deciles etc. Cochran (1940) suggested a classical ratio type estimator for the estimation of a finite population mean using one auxiliary variable under a simple random sampling scheme. Murthy (1967) proposed a product type estimator to estimate the population mean or total of study variable y by using auxiliary information when the coefficient of a correlation is negative. Rao (1991) introduced a difference type estimator that outperforms conventional ratio and linear regression estimators. Singh & Tailor (2003) proposed a family of estimators using known values of some parameters by using SRSWOR to estimate the population mean of the study variable. Singh, Tailor, Tailor & Kakran (2004) and Sisodia & Dwivedi (2012) used a coefficient of variation of the auxiliary variate. Upadhyaya & Singh (1999) derived ratio type estimators using a coefficient of variation and coefficient of kurtosis of the auxiliary variate.

The organization of the rest of the article is as follows: Section 2 provides a description of the existing estimators. The structure of the proposed mean ratio estimator and the efficiency comparison of the proposed estimator with the existing estimator are presented in Section 3. Section 4 consists of an empirical study of proposed estimators. Finally, Section 5 summarizes the findings of this study.

2. Existing Modified Ratio Estimators

Kadilar & Cingi (2004) suggested the following ratio estimators for the population mean $\overline{Y}$ in simple random sampling using some auxiliary information.

$$\begin{aligned}
\widehat{\overline{Y}}_1 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\overline{x}}\overline{X} \\
\widehat{\overline{Y}}_2 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x} + C_x\right)}\left(\overline{X} + C_x\right) \\
\widehat{\overline{Y}}_3 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x} + \beta_2\right)}\left(\overline{X} + \beta_2\right) \\
\widehat{\overline{Y}}_4 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x}\beta_2 + C_x\right)}\left(\overline{X}\beta_2 + C_x\right) \\
\widehat{\overline{Y}}_5 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x}C_x + \beta_2\right)}\left(\overline{X}C_x + \beta_2\right)
\end{aligned}$$

The biases, constants and the mean squared errors of estimators of Kadilar & Cingi (2004) are given by:

$$B(\overline{Y}_1) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_1^2 \qquad R_1 = \frac{\overline{Y}}{\overline{X}} \qquad MSE(\overline{Y}_1) = \frac{(1-f)}{n}\left(R_1^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$B(\overline{Y}_2) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_2^2 \qquad R_2 = \frac{\overline{Y}}{(\overline{X}+C_x)} \qquad MSE(\overline{Y}_2) = \frac{(1-f)}{n}\left(R_2^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$B(\overline{Y}_3) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_3^2 \qquad R_3 = \frac{\overline{Y}}{(\overline{X}+\beta_2)} \qquad MSE(\overline{Y}_3) = \frac{(1-f)}{n}\left(R_3^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$B(\overline{Y}_4) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_4^2 \qquad R_4 = \frac{\overline{Y}\beta_2}{(\overline{X}\beta_2+C_x)} \qquad MSE(\overline{Y}_4) = \frac{(1-f)}{n}\left(R_4^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$B(\overline{Y}_5) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_5^2 \qquad R_5 = \frac{\overline{Y}C_x}{\overline{X}C_x+\beta_2} \qquad MSE(\overline{Y}_5) = \frac{(1-f)}{n}\left(R_5^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

They showed that the above ratio estimators are more efficient than traditional ratio estimators to estimate the population mean.

Kadilar & Cingi (2006) has developed some modified ratio estimators using the correlation coefficient, which are shown below:

$$\begin{aligned}
\widehat{\overline{Y}}_6 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\overline{x}}\left(\overline{X} + \rho\right)\\
\widehat{\overline{Y}}_7 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{(\overline{x}C_x + \rho)}\left(\overline{X}C_x + \rho\right)\\
\widehat{\overline{Y}}_8 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{(\overline{x}\rho + C_x)}\left(\overline{X}\rho + C_x\right)\\
\widehat{\overline{Y}}_9 &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{(\overline{x}\beta_2 + \rho)}\left(\overline{X}\beta_2 + \rho\right)\\
\widehat{\overline{Y}}_{10} &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{(\overline{x}\rho + \beta_2)}\left(\overline{X}\rho + \beta_2\right)
\end{aligned}$$

The biases, constants and the mean squared errors in Kadilar & Cingi (2006) are given by:

$$B(\overline{Y}_6) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_6^2 \qquad R_6 = \frac{\overline{Y}}{\overline{X}+\rho} \qquad MSE(\overline{Y}_6) = \frac{(1-f)}{n}\left(R_6^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$B(\overline{Y}_7) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_7^2 \qquad R_7 = \frac{\overline{Y}C_x}{\overline{X}C_x+\rho} \qquad MSE(\overline{Y}_7) = \frac{(1-f)}{n}\left(R_7^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$B(\overline{Y}_8) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_8^2 \qquad R_8 = \frac{\overline{Y}\rho}{\overline{X}\rho+C_x} \qquad MSE(\overline{Y}_8) = \frac{(1-f)}{n}\left(R_8^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$B(\overline{Y}_9) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_9^2 \qquad R_9 = \frac{\overline{Y}\beta_2}{\overline{X}\beta_2+\rho} \qquad MSE(\overline{Y}_9) = \frac{(1-f)}{n}\left(R_9^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$B(\overline{Y}_{10}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_{10}^2 \qquad R_{10} = \frac{\overline{Y}\rho}{\overline{X}\rho+\beta_2} \qquad MSE(\overline{Y}_{10}) = \frac{(1-f)}{n}\left(R_{10}^2S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

These five estimators proved to be more efficient than all the existing ratio estimators, as is evident from their application to some natural populations.

Yan & Tian (2010) proposed the following two modified ratio estimators using coefficient of skewness and kurtosis;

$$\widehat{\overline{Y}}_{11} = \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x} + \beta_1\right)}\left(\overline{X} + \beta_1\right)$$
$$\widehat{\overline{Y}}_{12} = \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x}\beta_1 + \beta_2\right)}\left(\overline{X}\beta_1 + \beta_2\right)$$

The biases, constants and the mean squared errors for Yan & Tian (2010) are given by:

$$B(\overline{Y}_{11}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_{11}^2 \qquad R_{11} = \frac{\overline{Y}}{(\overline{X}+\beta_1)} \qquad MSE(\overline{Y}_{11}) = \frac{(1-f)}{n}\left(R_{11}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$
$$B(\overline{Y}_{12}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_{12}^2 \qquad R_{12} = \frac{\overline{Y}\beta_1}{(\overline{X}\beta_1+\beta_2)} \qquad MSE(\overline{Y}_{12}) = \frac{(1-f)}{n}\left(R_{12}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

The above-defined estimators that use the skewness and kurtosis coefficient respectively provide better estimates of the population mean in comparison with traditional ratio estimators.

Subramani & Kumarapandiyan (2012*a*, 2012*b*, 2012*c*) have introduced estimators with the use of population median, skewness, kurtosis and coefficient of variation for auxiliary information in simple random sampling to estimate population mean.

$$\widehat{\overline{Y}}_{13} = \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x} + M_d\right)}\left(\overline{X} + M_d\right)$$
$$\widehat{\overline{Y}}_{14} = \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(C_x\overline{x} + M_d\right)}\left(C_x\overline{X} + M_d\right)$$
$$\widehat{\overline{Y}}_{15} = \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(B_{1(x)}\overline{x} + M_d\right)}\left(B_{1(x)}\overline{X} + M_d\right)$$
$$\widehat{\overline{Y}}_{16} = \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(B_{2(x)}\overline{x} + M_d\right)}\left(B_{2(x)}\overline{X} + M_d\right)$$

The biases, constants and the mean squared errors for Subramani & Kumarapandiyan (2012*a*, 2012*b*, 2012*c*) are given by;

$$B(\overline{Y}_{13}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_{13}^2 \qquad R_{13} = \frac{\overline{Y}}{\overline{X}+M_d} \qquad MSE(\overline{Y}_{13}) = \frac{(1-f)}{n}\left(R_{13}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$
$$B(\overline{Y}_{14}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_{14}^2 \qquad R_{14} = \frac{\overline{Y}C_x}{\overline{X}C_x+M_d} \qquad MSE(\overline{Y}_{14}) = \frac{(1-f)}{n}\left(R_{14}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$
$$B(\overline{Y}_{15}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_{15}^2 \qquad R_{15} = \frac{\overline{Y}B_{1(x)}}{\overline{X}B_{1(x)}+M_d} \qquad MSE(\overline{Y}_{15}) = \frac{(1-f)}{n}\left(R_{15}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$
$$B(\overline{Y}_{16}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_{16}^2 \qquad R_{16} = \frac{\overline{Y}B_{2(x)}}{\overline{X}B_{2(x)}+M_d} \qquad MSE(\overline{Y}_{16}) = \frac{(1-f)}{n}\left(R_{16}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

Jeelani, Maqbool & Mir (2013) suggested an estimator with the use of the coefficient of skweness and quartile deviation of auxiliary information in simple random sampling to estimate population mean.

$$\widehat{\overline{Y}}_{17} = \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x}\beta_1 + QD\right)}\left(\overline{X}\beta_1 + QD\right)$$

The bias, constant and the mean square error for Jeelani et al. (2013) is as follows;

$$B(\overline{Y}_{17}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_{13}^2 \quad R_{17} = \frac{\overline{Y}\beta_1}{(\overline{X}\beta_1 + QD)} \quad MSE(\overline{Y}_{17}) = \frac{(1-f)}{n}\left(R_{17}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

3. The Proposed Modified Ratio Estimators

In this section, we propose different modified ratio type estimators using the population tri-mean, mid-range, Hodges-Lehmann, coefficient of variation and population correlation coefficient. The midrange is defined as $MR = \frac{X_{(1)}+X_{(N)}}{2}$, where $X_{(1)}$ and $X_{(N)}$ are the lowest and highest order statistics in a population of size N. It is highly sensitive to outliers as its design structure is based on only extreme values of data (cf. Ferrell (1953) for more details). We also include the measure based on the median of the pairwise Walsh averages which is defined as: $HL = \text{median}\left((X_j + X_k)/2, 1 \leq j \leq k \leq N\right)$. The main advantage of the HL is that it is robust against outliers. For more properties of HL see Hettmansperger & McKean (1988). The HL is also known as the Hodges-Lehmann estimator. The next measure included in this study is the trimean, which is the weighted average of the population median and two quartiles. It is defined as: $TM = \frac{Q_1+2Q_2+Q_3}{4}$, where Q_p $(p = 1, 2, 3)$ denote one of the three quartiles in a population. For detailed properties of trimean (TM) see Wang, Li & Cui (2007) and Nazir, Riaz, Ronald & Abbas (2013).

The proposed estimators are given below:

$$\begin{aligned}
\widehat{\overline{Y}}_{p1} &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x} + TM\right)}\left(\overline{X} + TM\right) \\
\widehat{\overline{Y}}_{p2} &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x}C_x + TM\right)}\left(\overline{X}C_x + TM\right) \\
\widehat{\overline{Y}}_{p3} &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x}\rho + TM\right)}\left(\overline{X}\rho + TM\right) \\
\widehat{\overline{Y}}_{p4} &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x} + MR\right)}\left(\overline{X} + MR\right) \\
\widehat{\overline{Y}}_{p5} &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x}C_x + MR\right)}\left(\overline{X}C_x + MR\right) \\
\widehat{\overline{Y}}_{p6} &= \frac{\overline{y} + b\left(\overline{X} - \overline{x}\right)}{\left(\overline{x}\rho + MR\right)}\left(\overline{X}\rho + MR\right)
\end{aligned}$$

$$\begin{aligned}\widehat{\overline{Y}}_{p7} &= \frac{\overline{y}+b\left(\overline{X}-\overline{x}\right)}{\left(\overline{x}+HL\right)}\left(\overline{X}+HL\right)\\ \widehat{\overline{Y}}_{p8} &= \frac{\overline{y}+b\left(\overline{X}-\overline{x}\right)}{\left(\overline{x}C_x+HL\right)}\left(\overline{X}C_x+HL\right)\\ \widehat{\overline{Y}}_{p9} &= \frac{\overline{y}+b\left(\overline{X}-\overline{x}\right)}{\left(\overline{x}\rho+HL\right)}\left(\overline{X}\rho+HL\right)\end{aligned}$$

The biases, constants and MSEs of proposed estimators are given below.

$$B(\widehat{\overline{Y}}_{p1}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_1^2 \quad R_1 = \frac{\overline{Y}}{(\overline{X}+TM)} \quad MSE(\widehat{\overline{Y}}_{p1}) = \frac{(1-f)}{n}\left(R_1^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

$$B(\widehat{\overline{Y}}_{p2}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_2^2 \quad R_2 = \frac{\overline{Y}}{(\overline{X}C_x+TM)} \quad MSE(\widehat{\overline{Y}}_{p2}) = \frac{(1-f)}{n}\left(R_2^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

$$B(\widehat{\overline{Y}}_{p3}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_3^2 \quad R_3 = \frac{\overline{Y}}{(\overline{X}\rho+TM)} \quad MSE(\widehat{\overline{Y}}_{p3}) = \frac{(1-f)}{n}\left(R_3^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

$$B(\widehat{\overline{Y}}_{p4}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_4^2 \quad R_4 = \frac{\overline{Y}}{(\overline{X}+MR)} \quad MSE(\widehat{\overline{Y}}_{p4}) = \frac{(1-f)}{n}\left(R_4^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

$$B(\widehat{\overline{Y}}_{p5}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_5^2 \quad R_5 = \frac{\overline{Y}}{(\overline{X}C_x+MR)} \quad MSE(\widehat{\overline{Y}}_{p5}) = \frac{(1-f)}{n}\left(R_5^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

$$B(\widehat{\overline{Y}}_{p6}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_6^2 \quad R_6 = \frac{\overline{Y}}{(\overline{X}\rho+MR)} \quad MSE(\widehat{\overline{Y}}_{p6}) = \frac{(1-f)}{n}\left(R_6^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

$$B(\widehat{\overline{Y}}_{p7}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_7^2 \quad R_7 = \frac{\overline{Y}}{(\overline{X}+HL)} \quad MSE(\widehat{\overline{Y}}_{p7}) = \frac{(1-f)}{n}\left(R_7^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

$$B(\widehat{\overline{Y}}_{p8}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_8^2 \quad R_8 = \frac{\overline{Y}}{(\overline{X}C_x+HL)} \quad MSE(\widehat{\overline{Y}}_{p8}) = \frac{(1-f)}{n}\left(R_8^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

$$B(\widehat{\overline{Y}}_{p9}) = \frac{(1-f)}{n}\frac{S_x^2}{\overline{Y}}R_9^2 \quad R_9 = \frac{\overline{Y}}{(\overline{X}\rho+HL)} \quad MSE(\widehat{\overline{Y}}_{p9}) = \frac{(1-f)}{n}\left(R_9^2S_x^2+S_y^2\left(1-\rho^2\right)\right)$$

4. Efficiency Comparisons

In this section, the condition under which the proposed modified ratio estimators will have minimum mean square error compared to usual ratio estimators and existing modified ratio estimators to estimate the finite population mean have been derived algebraically.

4.1. Comparison with Usual Mean Ratio Estimator

From the expression of proposed and usual ratio estimator MSEs, we have derived the conditions for which the proposed estimators are more efficient than the ratio estimator as:

$$MSE\left(\widehat{\overline{Y}}_{pj}\right) \leq MSE\left(\widehat{\overline{Y}}_{r}\right)$$

$$\frac{1-f}{n}\left(R_{pj}^2S_x^2+S_y^2\left(1-\rho^2\right)\right) \leq \frac{(1-f)}{n}\left(S_y^2+R^2S_x^2-2R\rho S_xS_y\right)$$

$$R_{pj}^2 S_x^2 - \rho^2 S_y^2 \leq R^2 S_x^2 - 2R\rho S_x S_y,$$

where $j = 1, 2, \ldots, 9$.

4.2. Comparison with Existing Modified Ratio Estimators

From the expression of proposed and existing MSEs, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as:

$$MSE\left(\widehat{\overline{Y}}_{pj}\right) \leq MSE\left(\widehat{\overline{Y}}_{i}\right)$$

$$\frac{1-f}{n}\left(R_{pj}^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right) \leq \frac{1-f}{n}\left(R_i^2 S_x^2 + S_y^2\left(1-\rho^2\right)\right)$$

$$R_{pj}^2 S_x^2 \leq R_i^2 S_x^2$$

$$R_{pj} \leq R_i$$

where $j = 1, 2, \ldots, 9$ and $i = 1, 2, \ldots, 17$.

5. Empirical Study

The performance of the proposed modified mean ratio estimators and the existing modified ratio estimators are evaluated by using 5 natural populations. The population 1 and 2 are taken from Singh & Chaudhary (1986) page 177, population 3 is taken from Cochran (1940) page 152 and population 4 and population 5 are taken from Murthy (1967) page 228. The characteristics of the five populations are given below in Table 1, whereas the constants and the biases are given in Tables 2-5. The MSEs of the existing and proposed modified ratio estimators are given in Tables 6 and 7. The percentage relative efficiencies (PREs) of the proposed estimators (p), with respect to the existing estimators (e), are computed as

$$PRE(e, p) = \frac{MSE(e)}{MSE(p)} * 100$$

and are given in Tables 8-12.

Tables 2-5 reveals that the constants and bias for the proposed ratio estimators are smaller compared to the existing ratio estimators. It is evident in Table 6 and 7 that the MSEs of the proposed ratio estimators with regard to the existing ones are much smaller, which indicates that proposed modified mean ratio estimators are more efficient. It can be seen that the proposed ratio estimators perform well in comparison with the existing estimators in terms of PREs (cf. Tables 8-12).

Table 1: Characteristics of the Populations.

Parameter	Population 1	Population 2	Population 3	Population4	Population5
N	34	34	49	80	80
n	20	20	20	20	20
$\overline{Y}$	856.4117	856.4117	127.7959	5182.637	5182.637
$\overline{X}$	208.8823	199.4412	103.1429	285.125	1126.463
ρ	0.4491	0.4453	0.981742	0.915	0.941
S_y	733.1407	733.1407	123.1212	1835.659	1835.659
C_y	0.8561	0.8561	0.963405	0.354	0.354
S_x	150.5059	150.2150	104.4051	279.429	845.610
C_x	0.7205	0.7531	1.012237	0.948	0.751
β_2	0.0978	1.0445	5.141245	1.301	-0.063
β_1	0.9782	1.1823	2.255276	0.698	1.050
M_d	150	142.50	64.00	148.000	757.500
QD	80.25	89.375	38.50	179.375	588.125
TM	162.25	165.562	72.75	206.937	931.562
MR	284.5	320	254.5	573	1795.5
HL	190	184	77.25	249	1040.5

Table 2: The constants of existing ratio estimators.

Estimator	Constant				
	Population 1	Population 2	Population 3	Population 4	Population 5
$\widehat{\overline{Y}}_r$	4.100	4.294	1.239	18.177	4.601
$\widehat{\overline{Y}}_1$	4.100	4.294	1.239	18.177	4.601
$\widehat{\overline{Y}}_2$	4.086	4.278	1.227	18.116	4.598
$\widehat{\overline{Y}}_3$	4.098	4.272	1.180	18.132	4.601
$\widehat{\overline{Y}}_4$	3.960	4.279	1.237	18.090	4.650
$\widehat{\overline{Y}}_5$	4.097	4.264	1.181	18.130	4.601
$\widehat{\overline{Y}}_6$	4.091	4.285	1.227	18.119	4.597
$\widehat{\overline{Y}}_7$	4.088	4.281	1.228	18.115	4.596
$\widehat{\overline{Y}}_8$	4.069	4.258	1.227	18.111	4.598
$\widehat{\overline{Y}}_9$	4.012	4.285	1.237	18.094	4.662
$\widehat{\overline{Y}}_{10}$	4.096	4.244	1.179	18.128	4.601
$\widehat{\overline{Y}}_{11}$	4.081	4.269	1.213	18.094	4.597
$\widehat{\overline{Y}}_{12}$	4.098	4.275	1.212	18.143	4.601
$\widehat{\overline{Y}}_{13}$	2.3863	2.5046	0.765	11.966	2.751
$\widehat{\overline{Y}}_{14}$	2.053	2.204	0.768	11.747	2.427
$\widehat{\overline{Y}}_{15}$	2.364	2.676	0.971	12.991	2.804
$\widehat{\overline{Y}}_{16}$	0.489	2.550	1.105	10.422	0.478
$\widehat{\overline{Y}}_{17}$	2.9438	3.1138	1.063	12.251	3.073

Table 3: The biases of existing ratio estimators.

Estimator	Bias				
	Population 1	Population 2	Population 3	Population 4	Population 5
$\widehat{\overline{Y}}_r$	4.270	4.940	0.254	115.09	60.877
$\widehat{\overline{Y}}_1$	9.154	10.002	3.875	174.83	109.52
$\widehat{\overline{Y}}_2$	9.091	9.927	3.800	173.67	109.37
$\widehat{\overline{Y}}_3$	9.145	9.898	3.516	173.98	109.53
$\widehat{\overline{Y}}_4$	8.539	9.930	3.860	173.18	111.86
$\widehat{\overline{Y}}_5$	9.142	9.865	3.520	173.93	109.53
$\widehat{\overline{Y}}_6$	9.115	9.957	3.802	173.71	109.34
$\widehat{\overline{Y}}_7$	9.099	9.943	3.803	173.65	109.27
$\widehat{\overline{Y}}_8$	9.015	9.834	3.799	173.57	109.36
$\widehat{\overline{Y}}_9$	8.763	9.960	3.861	173.23	112.46
$\widehat{\overline{Y}}_{10}$	9.135	9.771	3.509	173.90	109.53
$\widehat{\overline{Y}}_{11}$	9.069	9.885	3.711	173.24	109.31
$\widehat{\overline{Y}}_{12}$	9.145	9.914	3.709	174.17	109.53
$\widehat{\overline{Y}}_{13}$	3.101	3.403	1.476	75.76	39.15
$\widehat{\overline{Y}}_{14}$	2.296	2.634	1.489	73.02	30.47
$\widehat{\overline{Y}}_{15}$	3.043	3.886	2.383	89.31	40.69
$\widehat{\overline{Y}}_{16}$	0.130	3.526	3.085	57.48	1.186
$\widehat{\overline{Y}}_{17}$	4.719	5.260	2.852	79.42	48.85

Table 4: The constants of proposed ratio estimators.

Estimator	Constant				
	Population 1	Population 2	Population 3	Population 4	Population 5
$\widehat{\overline{Y}}_{p1}$	2.3076	2.3463	0.727	10.532	2.518
$\widehat{\overline{Y}}_{p2}$	1.9730	2.0427	0.730	10.297	2.189
$\widehat{\overline{Y}}_{p3}$	1.5021	1.4993	0.721	10.136	2.449
$\widehat{\overline{Y}}_{p4}$	1.7358	1.6487	0.357	6.039	1.774
$\widehat{\overline{Y}}_{p5}$	1.4185	1.3718	0.360	5.828	1.473
$\widehat{\overline{Y}}_{p6}$	1.0167	0.9329	0.353	5.687	1.708
$\widehat{\overline{Y}}_{p7}$	2.147	2.233	0.708	9.699	2.392
$\widehat{\overline{Y}}_{p8}$	1.812	1.930	0.712	9.459	2.063
$\widehat{\overline{Y}}_{p9}$	1.355	1.398	0.703	9.296	2.322

Table 5: The biases of proposed ratio estimators.

Estimator	Bias				
	Population 1	Population 2	Population 3	Population 4	Population 5
$\widehat{\overline{Y}}_{p1}$	2.900	2.986	1.332	58.70	32.81
$\widehat{\overline{Y}}_{p2}$	2.120	2.263	1.346	56.11	24.79
$\widehat{\overline{Y}}_{p3}$	1.229	1.219	1.312	54.37	31.03
$\widehat{\overline{Y}}_{p4}$	1.641	1.475	0.322	19.30	16.23
$\widehat{\overline{Y}}_{p5}$	1.096	1.021	0.328	17.97	11.23
$\widehat{\overline{Y}}_{p6}$	0.563	0.472	0.314	17.11	15.10
$\widehat{\overline{Y}}_{p7}$	2.510	2.706	1.267	49.77	29.59
$\widehat{\overline{Y}}_{p8}$	1.788	2.021	1.280	47.34	22.01
$\widehat{\overline{Y}}_{p9}$	1.000	1.060	1.247	45.72	27.90

Table 6: Mean square errors of existing ratio estimators.

Estimator	Mean Square Errors				
	Population 1	Population 2	Population 3	Population 4	Population 5
$\widehat{\overline{Y}}_r$	10539.27	10960.76	18.36202	413243.6	189775.1
$\widehat{\overline{Y}}_1$	16673.45	17437.65	511.42	926660.70	581994.20
$\widehat{\overline{Y}}_2$	16619.64	17373.31	501.84	920662.50	581238.50
$\widehat{\overline{Y}}_3$	16666.14	17348.62	465.51	922242.50	582058.10
$\widehat{\overline{Y}}_4$	16146.61	17376.04	509.53	918082.10	594119.80
$\widehat{\overline{Y}}_5$	16663.31	17319.75	466.02	922003.40	582079.30
$\widehat{\overline{Y}}_6$	16639.85	17399.52	502.12	920873.20	581046.80
$\widehat{\overline{Y}}_7$	16626.87	17387.08	502.23	920560.30	580732.70
$\widehat{\overline{Y}}_8$	16554.4	17294.19	501.66	920108.20	581191.40
$\widehat{\overline{Y}}_9$	16338.65	17401.14	509.59	918382.80	597260.90
$\widehat{\overline{Y}}_{10}$	16657.19	17239.66	464.72	921833.60	582062.10
$\widehat{\overline{Y}}_{11}$	16600.54	17336.98	490.45	918450.90	580937.60
$\widehat{\overline{Y}}_{12}$	16665.98	17362.26	490.23	923260.70	582055.10
$\widehat{\overline{Y}}_{13}$	11489.70	11785.70	204.80	413230.8	217319.80
$\widehat{\overline{Y}}_{14}$	10800.4	11127.47	206.55	399044.9	172323.8
$\widehat{\overline{Y}}_{15}$	11440.8	12199.76	320.78	483450.4	225319.5
$\widehat{\overline{Y}}_{16}$	8945.9	11892.07	410.50	318486.7	20545.47
$\widehat{\overline{Y}}_{17}$	12875.36	13376.04	380.77	432164.60	267595.20

Table 7: Mean square errors of proposed modified ratio estimators.

Estimator	Mean Square Errors				
	Population 1	Population 2	Population 3	Population 4	Population 5
$\widehat{\overline{Y}}_{p1}$	11317.28	11429.08	186.51	324801.70	184446.20
$\widehat{\overline{Y}}_{p2}$	10649.40	10809.99	188.22	311358.90	142903.20
$\widehat{\overline{Y}}_{p3}$	9886.21	9915.81	183.92	302349.40	175238.70
$\widehat{\overline{Y}}_{p4}$	10239.11	10134.39	57.42	120605.10	98755.61
$\widehat{\overline{Y}}_{p5}$	9772.39	9745.79	58.13	113722.40	72582.52
$\widehat{\overline{Y}}_{p6}$	9316.02	9275.87	56.35	109258.60	92644.60
$\widehat{\overline{Y}}_{p7}$	10983.77	11189.04	178.12	278530.60	167778.60
$\widehat{\overline{Y}}_{p8}$	10365.55	10602.02	179.81	265934.80	128487.60
$\widehat{\overline{Y}}_{p9}$	9690.50	9779.43	175.57	257544.70	158990.70

TABLE 8: Percentage relative efficiency of existing estimators with respect to the proposed estimators of population 1.

Existing	Proposed								
	$\widehat{\overline{Y}}_{p1}$	$\widehat{\overline{Y}}_{p2}$	$\widehat{\overline{Y}}_{p3}$	$\widehat{\overline{Y}}_{p4}$	$\widehat{\overline{Y}}_{p5}$	$\widehat{\overline{Y}}_{p6}$	$\widehat{\overline{Y}}_{p7}$	$\widehat{\overline{Y}}_{p8}$	$\widehat{\overline{Y}}_{p9}$
$\widehat{\overline{Y}}_r$	93.1	99.0	106.6	102.9	107.8	113.1	96.0	101.7	108.8
$\widehat{\overline{Y}}_1$	147.3	156.6	168.7	162.8	170.6	179.0	151.8	160.9	172.1
$\widehat{\overline{Y}}_2$	146.9	156.1	168.1	162.3	170.1	178.4	151.3	160.3	171.5
$\widehat{\overline{Y}}_3$	147.3	156.5	168.6	162.8	170.5	178.9	151.7	160.8	172.0
$\widehat{\overline{Y}}_4$	142.7	151.6	163.3	157.7	165.2	173.3	147.0	155.8	166.6
$\widehat{\overline{Y}}_5$	147.2	156.5	168.6	162.7	170.5	178.9	151.7	160.8	172.0
$\widehat{\overline{Y}}_6$	147.0	156.3	168.3	162.5	170.3	178.6	151.5	160.5	171.7
$\widehat{\overline{Y}}_7$	146.9	156.1	168.2	162.4	170.1	178.5	151.4	160.4	171.6
$\widehat{\overline{Y}}_8$	146.3	155.4	167.4	161.7	169.4	177.7	150.7	159.7	170.8
$\widehat{\overline{Y}}_9$	144.4	153.4	165.3	159.6	167.2	175.4	148.8	157.6	168.6
$\widehat{\overline{Y}}_{10}$	147.2	156.4	168.5	162.7	170.5	178.8	151.7	160.7	171.9
$\widehat{\overline{Y}}_{11}$	146.7	155.9	167.9	162.1	169.9	178.2	151.1	160.2	171.3
$\widehat{\overline{Y}}_{12}$	147.3	156.5	168.6	162.8	170.5	178.9	151.7	160.8	172.0
$\widehat{\overline{Y}}_{13}$	101.5	107.9	116.2	112.2	117.6	123.3	104.6	110.8	118.6
$\widehat{\overline{Y}}_{14}$	95.4	101.4	109.2	105.5	110.5	115.9	98.3	104.2	111.5
$\widehat{\overline{Y}}_{15}$	101.1	107.4	115.7	111.7	117.1	122.8	104.2	110.4	118.1
$\widehat{\overline{Y}}_{16}$	79.0	84.0	90.5	87.4	91.5	96.0	81.4	86.3	92.3
$\widehat{\overline{Y}}_{17}$	113.8	120.9	130.2	125.7	131.8	138.2	117.2	124.2	132.9

TABLE 9: Percentage relative efficiency of existing estimators with respect to the proposed estimators of population 2.

Existing	Proposed								
	$\widehat{\overline{Y}}_{p1}$	$\widehat{\overline{Y}}_{p2}$	$\widehat{\overline{Y}}_{p3}$	$\widehat{\overline{Y}}_{p4}$	$\widehat{\overline{Y}}_{p5}$	$\widehat{\overline{Y}}_{p6}$	$\widehat{\overline{Y}}_{p7}$	$\widehat{\overline{Y}}_{p8}$	$\widehat{\overline{Y}}_{p9}$
$\widehat{\overline{Y}}_r$	95.9	101.4	110.5	108.2	112.5	118.2	98.0	103.4	112.1
$\widehat{\overline{Y}}_1$	152.6	161.3	175.9	172.1	178.9	188.0	155.8	164.5	178.3
$\widehat{\overline{Y}}_2$	152.0	160.7	175.2	171.4	178.3	187.3	155.3	163.9	177.7
$\widehat{\overline{Y}}_3$	151.8	160.5	175.0	171.2	178.0	187.0	155.1	163.6	177.4
$\widehat{\overline{Y}}_4$	152.0	160.7	175.2	171.5	178.3	187.3	155.3	163.9	177.7
$\widehat{\overline{Y}}_5$	151.5	160.2	174.7	170.9	177.7	186.7	154.8	163.4	177.1
$\widehat{\overline{Y}}_6$	152.2	161.0	175.5	171.7	178.5	187.6	155.5	164.1	177.9
$\widehat{\overline{Y}}_7$	152.1	160.8	175.3	171.6	178.4	187.4	155.4	164.0	177.8
$\widehat{\overline{Y}}_8$	151.3	160.0	174.4	170.6	177.5	186.4	154.6	163.1	176.8
$\widehat{\overline{Y}}_9$	152.3	161.0	175.5	171.7	178.6	187.6	155.5	164.1	177.9
$\widehat{\overline{Y}}_{10}$	150.8	159.5	173.9	170.1	176.9	185.9	154.1	162.6	176.3
$\widehat{\overline{Y}}_{11}$	151.7	160.4	174.8	171.1	177.9	186.9	154.9	163.5	177.3
$\widehat{\overline{Y}}_{12}$	151.9	160.6	175.1	171.3	178.2	187.2	155.2	163.8	177.5
$\widehat{\overline{Y}}_{13}$	103.1	109.0	118.9	116.3	120.9	127.1	105.3	111.2	120.5
$\widehat{\overline{Y}}_{14}$	97.4	102.9	112.2	109.8	114.2	120.0	99.4	105.0	113.8
$\widehat{\overline{Y}}_{15}$	106.7	112.9	123.0	120.4	125.2	131.5	109.0	115.1	124.7
$\widehat{\overline{Y}}_{16}$	104.1	110.0	119.9	117.3	122.0	128.2	106.3	112.2	121.6
$\widehat{\overline{Y}}_{17}$	117.0	123.7	134.9	132.0	137.2	144.2	119.5	126.2	136.8

Table 10: Percentage relative efficiency of existing estimators with respect to the proposed estimators of population 3.

Existing	Proposed								
	$\widehat{\overline{Y}}_{p1}$	$\widehat{\overline{Y}}_{p2}$	$\widehat{\overline{Y}}_{p3}$	$\widehat{\overline{Y}}_{p4}$	$\widehat{\overline{Y}}_{p5}$	$\widehat{\overline{Y}}_{p6}$	$\widehat{\overline{Y}}_{p7}$	$\widehat{\overline{Y}}_{p8}$	$\widehat{\overline{Y}}_{p9}$
$\widehat{\overline{Y}}_r$	9.8	9.8	10.0	32.0	31.6	32.6	10.3	10.2	10.5
$\widehat{\overline{Y}}_1$	274.2	271.7	278.1	890.7	879.8	907.6	287.1	284.4	291.3
$\widehat{\overline{Y}}_2$	269.1	266.6	272.9	874.0	863.3	890.6	281.7	279.1	285.8
$\widehat{\overline{Y}}_3$	249.6	247.3	253.1	810.7	800.8	826.1	261.3	258.9	265.1
$\widehat{\overline{Y}}_4$	273.2	270.7	277.0	887.4	876.5	904.2	286.1	283.4	290.2
$\widehat{\overline{Y}}_5$	249.9	247.6	253.4	811.6	801.7	827.0	261.6	259.2	265.4
$\widehat{\overline{Y}}_6$	269.2	266.8	273.0	874.5	863.8	891.1	281.9	279.3	286.0
$\widehat{\overline{Y}}_7$	269.3	266.8	273.1	874.7	864.0	891.3	282.0	279.3	286.1
$\widehat{\overline{Y}}_8$	269.0	266.5	272.8	873.7	863.0	890.3	281.6	279.0	285.7
$\widehat{\overline{Y}}_9$	273.2	270.7	277.1	887.5	876.6	904.3	286.1	283.4	290.2
$\widehat{\overline{Y}}_{10}$	249.2	246.9	252.7	809.3	799.4	824.7	260.9	258.5	264.7
$\widehat{\overline{Y}}_{11}$	263.0	260.6	266.7	854.1	843.7	870.4	275.3	272.8	279.3
$\widehat{\overline{Y}}_{12}$	262.8	260.5	266.5	853.8	843.3	870.0	275.2	272.6	279.2
$\widehat{\overline{Y}}_{13}$	109.8	108.8	111.4	356.7	352.3	363.4	115.0	113.9	116.6
$\widehat{\overline{Y}}_{14}$	110.7	109.7	112.3	359.7	355.3	366.5	116.0	114.9	117.6
$\widehat{\overline{Y}}_{15}$	172.0	170.4	174.4	558.7	551.8	569.3	180.1	178.4	182.7
$\widehat{\overline{Y}}_{16}$	220.1	218.1	223.2	714.9	706.2	728.5	230.5	228.3	233.8
$\widehat{\overline{Y}}_{17}$	204.2	202.3	207.0	663.1	655.0	675.7	213.8	211.8	216.9

Table 11: Percentage relative efficiency of existing estimators with respect to the proposed estimators of population 4.

Existing	Proposed								
	$\widehat{\overline{Y}}_{p1}$	$\widehat{\overline{Y}}_{p2}$	$\widehat{\overline{Y}}_{p3}$	$\widehat{\overline{Y}}_{p4}$	$\widehat{\overline{Y}}_{p5}$	$\widehat{\overline{Y}}_{p6}$	$\widehat{\overline{Y}}_{p7}$	$\widehat{\overline{Y}}_{p8}$	$\widehat{\overline{Y}}_{p9}$
$\widehat{\overline{Y}}_r$	127.2	132.7	136.7	342.6	363.4	378.2	148.4	155.4	160.5
$\widehat{\overline{Y}}_1$	285.3	297.6	306.5	768.3	814.8	848.1	332.7	348.5	359.8
$\widehat{\overline{Y}}_2$	283.5	295.7	304.5	763.4	809.6	842.6	330.5	346.2	357.5
$\widehat{\overline{Y}}_3$	283.9	296.2	305.0	764.7	811.0	844.1	331.1	346.8	358.1
$\widehat{\overline{Y}}_4$	282.7	294.9	303.6	761.2	807.3	840.3	329.6	345.2	356.5
$\widehat{\overline{Y}}_5$	283.9	296.1	304.9	764.5	810.7	843.9	331.0	346.7	358.0
$\widehat{\overline{Y}}_6$	283.5	295.8	304.6	763.5	809.8	842.8	330.6	346.3	357.6
$\widehat{\overline{Y}}_7$	283.4	295.7	304.5	763.3	809.5	842.6	330.5	346.2	357.4
$\widehat{\overline{Y}}_8$	283.3	295.5	304.3	762.9	809.1	842.1	330.3	346.0	357.3
$\widehat{\overline{Y}}_9$	282.8	295.0	303.7	761.5	807.6	840.6	329.7	345.3	356.6
$\widehat{\overline{Y}}_{10}$	283.8	296.1	304.9	764.3	810.6	843.7	331.0	346.6	357.9
$\widehat{\overline{Y}}_{11}$	282.8	295.0	303.8	761.5	807.6	840.6	329.7	345.4	356.6
$\widehat{\overline{Y}}_{12}$	284.3	296.5	305.4	765.5	811.9	845.0	331.5	347.2	358.5
$\widehat{\overline{Y}}_{13}$	127.2	132.7	136.7	342.6	363.4	378.2	148.4	155.4	160.5
$\widehat{\overline{Y}}_{14}$	122.9	128.2	132.0	330.9	350.9	365.2	143.3	150.1	154.9
$\widehat{\overline{Y}}_{15}$	148.8	155.3	159.9	400.9	425.1	442.5	173.6	181.8	187.7
$\widehat{\overline{Y}}_{16}$	98.1	102.3	105.3	264.1	280.1	291.5	114.3	119.8	123.7
$\widehat{\overline{Y}}_{17}$	133.1	138.8	142.9	358.3	380.0	395.5	155.2	162.5	167.8

Table 12: Percentage relative efficiency of existing estimators with respect to the proposed estimators of population 5.

Existing	Proposed								
	$\widehat{\overline{Y}}_{p1}$	$\widehat{\overline{Y}}_{p2}$	$\widehat{\overline{Y}}_{p3}$	$\widehat{\overline{Y}}_{p4}$	$\widehat{\overline{Y}}_{p5}$	$\widehat{\overline{Y}}_{p6}$	$\widehat{\overline{Y}}_{p7}$	$\widehat{\overline{Y}}_{p8}$	$\widehat{\overline{Y}}_{p9}$
$\widehat{\overline{Y}}_{r}$	102.9	132.8	108.3	192.2	261.5	204.8	113.1	147.7	119.4
$\widehat{\overline{Y}}_{1}$	315.5	407.3	332.1	589.3	801.8	628.2	346.9	453.0	366.1
$\widehat{\overline{Y}}_{2}$	315.1	406.7	331.7	588.6	800.8	627.4	346.4	452.4	365.6
$\widehat{\overline{Y}}_{3}$	315.6	407.3	332.2	589.4	801.9	628.3	346.9	453.0	366.1
$\widehat{\overline{Y}}_{4}$	322.1	415.7	339.0	601.6	818.5	641.3	354.1	462.4	373.7
$\widehat{\overline{Y}}_{5}$	315.6	407.3	332.2	589.4	802.0	628.3	346.9	453.0	366.1
$\widehat{\overline{Y}}_{6}$	315.0	406.6	331.6	588.4	800.5	627.2	346.3	452.2	365.5
$\widehat{\overline{Y}}_{7}$	314.9	406.4	331.4	588.1	800.1	626.8	346.1	452.0	365.3
$\widehat{\overline{Y}}_{8}$	315.1	406.7	331.7	588.5	800.7	627.3	346.4	452.3	365.6
$\widehat{\overline{Y}}_{9}$	323.8	417.9	340.8	604.8	822.9	644.7	356.0	464.8	375.7
$\widehat{\overline{Y}}_{10}$	315.6	407.3	332.2	589.4	801.9	628.3	346.9	453.0	366.1
$\widehat{\overline{Y}}_{11}$	315.0	406.5	331.5	588.3	800.4	627.1	346.3	452.1	365.4
$\widehat{\overline{Y}}_{12}$	315.6	407.3	332.1	589.4	801.9	628.3	346.9	453.0	366.1
$\widehat{\overline{Y}}_{13}$	117.8	152.1	124.0	220.1	299.4	234.6	129.5	169.1	136.7
$\widehat{\overline{Y}}_{14}$	93.4	120.6	98.3	174.5	237.4	186.0	102.7	134.1	108.4
$\widehat{\overline{Y}}_{15}$	122.2	157.7	128.6	228.2	310.4	243.2	134.3	175.4	141.7
$\widehat{\overline{Y}}_{16}$	11.1	14.4	11.7	20.8	28.3	22.2	12.2	16.0	12.9
$\widehat{\overline{Y}}_{17}$	145.1	187.3	152.7	271.0	368.7	288.8	159.5	208.3	168.3

6. Conclusion

Sampling plays a vital role in all kinds of disciplines. The availability of auxiliary information enhances the efficiency of the estimators. Mean ratio estimators have been proposed using known values of population tri-mean, mid-range, Hodges-Lehmann estimator, coefficient of variation and population correlation coefficient by using the study variable and auxiliary variable information. It is observed that the mean squared errors of the suggested estimators based on the tri-mean, mid-range, Hodges-Lehmann, coefficient of variation and population correlation coefficient of the auxiliary variable are smaller than those for the existing modified ratio estimators for all the five known populations considered in the numerical study. Also, it is pertinent to note that the parameters such as the mean, coefficient of skewness and coefficient of kurtosis are affected by the extreme values in the population, whereas the tri-mean, mid-range, Hodges-Lehmann are robust to extreme values. Hence, the modified ratio estimators proposed in this study may be used for better and more stable results, and are preferred to the existing modified ratio estimators for practical applications. Moreover, empirical studies reveal that the bias and mean square error for the proposed estimators are lower than that of the existing methods in terms of various natural populations. For the given populations alone, the proposed estimators perform better than the exiting estimators.

[Received: June 2014 — Accepted: March 2015]

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