EARTH SCIENCES
RESEARCH JOURNAL

Earth Sci. Res. J. Vol. 23, No. 4 (December, 2019): 379-383

UNIVERSIDAD
NACIONAL
DE COLOMBIA

Advance of research on the numerical simulation of sediment transport in the Yellow River estuary

Jinfeng Jian*, Huanzhen Chen
[1]School of Mathematics and Statistics, Shandong Normal University, Jinan 250014, China
* Corresponding author: jjf_sdnu@163.com

ABSTRACT

The numerical simulation of sediment transport in the Yellow River estuary is an important method to quantitative analyze the problems by water and sediment movement, including sediment deposition and river bed evolution, beach deposition and development, the formation and development of the delta, and so on. Making an intensive study of the problems is useful to study the principles of sediment movement and the evolution law of the estuary bed, and it is also important significance to scientific demonstrate of the estuary sediment deposition, sea water intrusion prevention treatment scheme and solve the problems of engineering. Based on the mathematical modeling of sediment transport and our research work, this paper analyzes the research statement, the unsolved issues and the developed trend of this kind problem, to provide the theoretical reference on more accurate numerical simulation of sediment transport in the Yellow River estuary and the scientific reference on the demonstration and decision of estuarine regulation scheme.

Keywords: The Yellow River estuary; Sediment transport; Numerical simulation; Summary and Prospect.

Avance de la investigación sobre la simulación numérica del transporte de sedimentos en el estuario del río Amarillo

RESUMEN

La simulación numérica del transporte de sedimentos en el estuario del río Amarillo es un método importante para analizar cuantitativamente los problemas por el movimiento del agua y los sedimentos, incluida la deposición de sedimentos y la evolución del lecho del río, la deposición y el desarrollo de las playas, la formación y el desarrollo del delta, y demás. Hacer un estudio intensivo de los problemas es útil para estudiar los principios del movimiento de sedimentos y la ley de evolución del lecho del estuario, y también es importante demostrar científicamente la deposición de sedimentos del estuario, el esquema de tratamiento de prevención de intrusiones de agua de mar y resolver los problemas de ingeniería. Basado en el modelo matemático del transporte de sedimentos y en nuestro trabajo de investigación, este estudio analiza la declaración de la investigación, los problemas no resueltos y la tendencia desarrollada de este tipo de problemas, para proporcionar la referencia teórica sobre una simulación numérica más precisa del transporte de sedimentos en el estuario del río Amarillo y la referencia científica sobre la demostración y decisión del esquema de regulación estuarina.

Palabras clave: Estuario del río Amarillo; transporte de sedimentos; simulación numérica; resumen y perspectiva.

Record
Manuscript received: 27/04/2019
Accepted for publication: 25/09/2019

How to cite item
Jian, J., & Chen, H. (2019). Advance of research on the numerical simulation of sediment transport in the Yellow River estuary. *Earth Sciences Research Journal*, 23(4), 379-383 DOI: https://doi.org/10.15446/esrj.v23n4.84100

ISSN 1794-6190 e-ISSN 2339-3459
https://doi.org/10.15446/esrj.v23n4.84100

Introduction

The mathematical model of sediment transport of estuary is a branch of river simulation. It is an important method to quantitatively predict the problems caused by water and sediment movement, including river bed evolution, formation and development of deltas, expansion of siltation areas, transport and deposition of silt and sediment, and so on. The Yellow River estuary is a strong accumulative estuary with frequent fluctuation of weak tides and much sediment. Its evolution is influenced not only by the little water and much sediment of upper reaches, but also by the main factors such as tidals and ocean dynamic characteristics in coastal areas, including tides, currents and waves. At the same time, the influence of interaction between cohesive sediment and turbulence, the flocculation of cohesive sediment and the ocean chemistry should be considered. The problem is complex, the simulation is difficult and the cost is high. With the rapid development of modern computing technology, especially parallel computing technology, compared with traditional simulation methods such as measured model and physical similarity model, the numerical simulation of sediment transport by using mathematical model has the advantages of low investment, short cycle, repeatability, and easy improvement, which has become an important method of estuary simulation. The thorough study of this kind of problem can help people to more clearly grasp the mechanism and form of the sediment transport in the estuary. It will also be helpful for the scientific demonstration and engineering solution of the estuary control schemes such as governing estuarine siltation, slowing down estuarine extension velocity, preventing seawater intrusion, which has a positive and important significance for the effective implementation of the national strategies of "Development Plan of High Efficiency Ecological Economic Zone in Yellow River Delta" and "Development Plan of Shandong Peninsula Blue Economic Zone".

There are many mathematical models describing the sediment transport problem of the Yellow River. Different hydrodynamic conditions, different demands and different boundary conditions can lead to subtle differences in the models. The one-dimensional model can simulate the siltation and deformation of riverbed sediment in long-term and non-uniform sand, and it is often used in the study of river channel simulation in the long reach. The two-dimensional model can describe the horizontal or vertical distribution of velocity field and sediment field in water area and it is often used in the study of riverbed deformation in short-term river sections such as estuaries and ports. However, the sediment content in the practical problem is generally non-uniformly distributed along the water depth, and the sediment transport presents a typical three-dimensional property. At this time, the three-dimensional mathematical model can simulate the sediment transport and sedimentation law more accurately and completely. However, the development of three-dimensional sediment mathematical model is slow, the main reasons are: (1) The study of the three-dimensional sediment movement law is still not perfect, the sediment transport rate, resistance and other sediment parameters still adopt one-dimensional sediment transport calculation formula. (2) The existing numerical calculation format has a complex structure, large calculation amount, and it is not easy to be visualized and softwareized. (3) The boundary conditions are not easy to be obtained. Because of the complexity of riverbed terrain and the complex and variable dynamic conditions of rivers and oceans, the real initial boundary conditions are not easy to be obtained.

In recent years, hydraulic engineers and computational mathematicians at home and abroad have proposed one-dimensional and two-dimensional mathematical models of sediment transport in estuaries under certain special simplifying assumptions, and realized the numerical simulation of finite difference method, finite element method and finite volume method, which have been applied to solve large-scale engineering sediment problems. However, in the general sense, the mathematical model and numerical simulation of sediment transport in the Yellow River estuary, especially the three-dimensional problem, are still rare, and the systematic and rigorous theory of numerical analysis has not been reported.

On the basis of referring to relevant literature and combining with our research work, this paper attempts to make a brief analysis and comment on the current situation of numerical simulation of sediment transport problem in estuaries, and put forward its own views and opinions on the future development, with a view to benefiting.

Two-dimensional mathematical model of sediment transport

In the water era of the Yellow River estuary, the sediment movement is mainly suspended load, and the planar flow velocity is significantly higher than the vertical flow velocity. Considering the influence of turbulence, tidal current, wave and the assumption of hydrostatic pressure distribution, By performing Reynolds average, wave period average and water depth average on the three-dimensional sediment transport model, a planar two-dimensional sediment transport model composed of hydrodynamic model (water flow continuous equation, water flow movement equation) and sediment transport model (sediment continuous equation, riverbed deformation equation) can be obtained (Yang, 1993; Dou, Dong, Dou & Li, 1995; Cao & Wang, 1993).

Water flow continuous equation

Van Rijin, Tan (1)

In the equation, H represents the water depth, $(u,\ v)$ represents the component of the vertical average velocity along the direction of x and y, and it is understood as different composite velocity fields in different water areas (for example, estuary water area is understood as a composite field of wave, runoff and tidal current velocity).

Water flow movement equation

$$\begin{aligned}&\frac{\partial Hu}{\partial t}+\frac{\partial\left(Huu\right)}{\partial x}+\frac{\partial\left(Hvu\right)}{\partial y}+gH\frac{\partial\eta}{\partial x}+\frac{\tau_{bx}}{\rho_m}-\frac{\tau_{wx}}{\rho_m}-fv+\frac{1}{\rho_m}\\&\left(\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}\right)=\varepsilon\mathrm{V}Hu\\&\frac{\partial Hu}{\partial t}+\frac{\partial\left(Huu\right)}{\partial x}+\frac{\partial\left(Hvv\right)}{\partial y}+gH\frac{\partial\eta}{\partial y}+\frac{\tau_{by}}{\rho_m}-\frac{\tau_{wx}}{\rho_m}-fu+\frac{1}{\rho_m}\\&\left(\frac{\partial S_{yx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}\right)=\varepsilon\mathrm{V}Hv\end{aligned}\tag{2}$$

In this equation, η represents the water level, $\eta = H + Z_b$, $Z_b$represents the riverbed elevation, ρ_m represents the fluid density, τ_{hx}, τ_{by} represents the component of the shear stress on the bottom of the riverbed in the direction of x and y, τ_{wx}, τ_{wy} represents the component of the surface wind shear stress in the direction of x and y, $f = 2\phi \sin \psi$ represents the Coriolis force coefficient reflecting the effect of the earth's rotation, ϕ represents the angular velocity of the earth's rotation, ψ represents the local latitude, S_{xx}, S_{xy}, S_{yx}, S_{yy} represents the wave radiation stress term, i.e. the residual momentum flux caused by fluctuations, and ε is the turbulent viscous coefficient.

Sediment continuous equation

$$\frac{\partial S}{\partial t}+v\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}-D\mathrm{V}S=\frac{F_s}{H}\tag{3}$$

In this equation, S represents the sediment concentration, F_s represents the function of sediment scouring and silting on the riverbed, generally taken as $F_s = \alpha\omega(S_* - S)$, ω represents the sediment settling velocity, α represents the recovery saturation coefficient, S_* represents the sediment transport capacity, and D represents the sediment diffusion coefficient. This is a uniform sediment model. If the non-uniform sediment is taken into account, the sediment concentration of each group of sediment gradation satisfies this equation.

Suspended load riverbed deformation equation

$$\gamma'\frac{\partial Z_b}{\partial t}=-F_s\tag{4}$$

In this equation, γ' is the dry bulk density of sediment.

The conditions of initial boundary values. Initial water level (water depth) usually starts from downstream water level, and the initial value of the water level at each point is given by linear interpolation according to the average river bed gradient. The initial flow velocity and sediment concentration are given if there are measured values. Otherwise, they can be taken as 0. With the process of numerical calculation, the influence of initial values will gradually disappear. The initial siltation elevation is usually taken as zero. On this basis, the calculated positive value is the siltation thickness, while the negative value is the scouring thickness. The inflow boundary generally gives the flow or water level, and for the suspended load, the sediment concentration should also be given. At the outflow boundary, the sediment flux is generally considered to be zero. For the boundary conditions of the fixed shore, the condition of no slip of flow is generally adopted, and the velocity field is taken as 0. When studying the sea area, due to the periodicity of the tide level, the edge beach changes with the water level, which leads to the change of boundary. At this time, the boundary "freezing" technology can be used to process the complex moving boundary into the fixed boundary.

The determination of parameters. In the two-dimensional sediment transport model, the determination of the parameters describing sediment characteristics and hydrodynamic forces, such as shear stress at the bottom of the riverbed, shear stress of surface wind, wave radiation stress, sediment transport capacity of current, recovery saturation coefficient, sedimentation velocity and dry bulk density of sediment, have not been effectively expressed in the two-dimensional motion theory. Most of them directly use the corresponding data or regular formulas of one-dimensional sediment transport.

Numerical simulation method

The numerical simulation mainly includes the discrete format construction, the mathematical demonstration of the format (the uniqueness, stability, and convergence of the existence of the solution), the program implementation of the format, and the practical application of the format. In terms of format construction methods, there are mainly finite difference, finite volume element, finite element, mixed finite element, characteristic finite element, discontinuous finite element and other methods. At the same time, in order to better deal with the problem of discontinuous (shock) flow, sometimes the technique of windward and adding artificial dissipative items is combined.

Finite difference method (Lu & Peng, 1985; Zheng, 2003; Spasojevic & Holly, 1990; Chang, 1998)

The finite difference method which is widely used in the planar two-dimensional sediment transport model is the alternating direction implicit format method (*ADI*). The traditional difference method has the advantages of simple structure and easy implementation. However, it is inflexible and the accuracy is difficult to improve when dealing with irregular boundary and complex boundary conditions. Although it can basically adapt to irregular boundary after using the curve coordinate technology, there are still some shortcomings. In order to solve the numerical oscillation that occurs when the shock current is generated, the windward *ADI* method is formed in combination with the windward.

At present, the main model used in engineering is *MOBED*2 model established by Spasojevic and Holly in curvilinear coordinate system, which can simulate sediment deposition in waterways, estuaries and coastal areas; the FLUVIAL12 model established by Chang in rectangular coordinate system, which can simulate the changes of riverbed section and sediment gradation caused by river bending; the DELFT-2D model used to simulate wave and current movement can describe the influence of wave motion, but these models do not give strict mathematical proof.

Finite difference method (Li & Zhang, 1999; Zhang & Yin, 2002; Chippada, Dawson, Martinez & Wheeler, 1998; Chippada, Dawson, Martinez & Wheeler, 1998; Dawson & Martinez, 2000)

The finite element method can deal with irregular boundaries and complex boundary conditions better, and it can improve the calculation accuracy more conveniently, but the storage amount and calculation amount are larger than the finite difference method. Both the finite difference method and the finite element method can generate numerical dispersion due to the existence of convection terms. Combined with the windward technology, the windward finite element, mass concentrated windward finite element and other methods are produced. These methods are effective, but due to the introduction of excessive artificial viscosity, the discrete accuracy is reduced and the discontinuous screeding phenomenon is caused.

Petrov-Galerkin and streamline diffusion methods can better avoid numerical dispersion and non-physical numerical oscillations, but they still have problems when dealing with interrupted flow. High-resolution format finite element method can better avoid the high frequency oscillation of the interrupted flow and the false oscillation caused by the convection dominant by introducing minimum and sufficient consumption in the stiffness matrix, but no strict mathematical proof has been given. At present, the models used in the engineering are mainly TABS-2 developed by the US Army Corps of Engineers Waterway Experimental Station, which can be used for the simulation of rivers, reservoirs and estuaries, and the CCHE2D established by Jia,Wang ,which can simulate the secondary flow effect formed in the curve.

Finite volume method (Guo, Han & He, 1996; Pan, Lu & Yu, 2009; Hu & Tan, 1995; Wu & Wang, 2012; Benkhaldoun, Elmahi & Seaid, 2010)

The finite volume method is more suitable for complex regions than the finite difference method, and its calculation amount is lower than that of the finite element, so it has been widely used. There are two key problems to be solved in the finite volume method, one is the arrangement of variable nodes in the integral unit, the other is the calculation method of boundary flux of the integral unit. The traditional finite volume method can not deal with shock wave problem, but it has the ability to deal with discontinuous problem when using Riemann solution operator, which can accurately capture the shock wave. The finite volume method can use unstructured grid at the same time. In the volume method of second-order and higher accuracy, the limiter function should be introduced to eliminate numerical oscillation in the process of flux reconstruction.

At present, the finite volume method is the mainstream method to solve the problem of sediment transport. It mainly uses the body-fitted grid to establish the *FAST*2*D* model in the curvilinear coordinate system to simulate the sediment transport and topographic dynamics the alluvial channels; the SUTRENCH-2D model established by VanRijin,Tan is used to simulate the sediment transport at the bottom of the sedimentary river bed and the change of the riverbed elevation under the effect of quasi-steady-state water flow and wind-induced waves, and there is also no strict mathematical proof.

Characteristic finite element method (Li & Chen, 2011; Wang & Liu, 2003; Liu, Li & Chen, 2013; Zhou & Chen, 2007; Luo, Zhu, Zeng & Xie, 2007; Dawson & Martinez, 2000)

Since the convection terms of the first three equations of the sediment transport model can lead to numerical dispersion and non-physical oscillation, and have hyperbolic characteristics, numerical dispersion along the characteristic line can improve the stability of the format. At the same time, the truncation error about time is smaller, so the larger time step can be adopted. The characteristic line method is suitable for the strong estuary areas where the hydrodynamic changes are severe. Firstly, the characteristic finite element scheme of shallow water equation is proposed by, Dawson,Monica, and the suboptimal error estimation is obtained. Wang Jiwen and Liu Ruxun get the CN-Galerkin method based on the characteristic direction of the shallow water wave equation, and obtain the suboptimal error estimation; Zhou Zhaojie and Chen Huanzhen apply it to the planar two-dimensional water and sediment model, and a suboptimal error estimation is obtained.

Mixed finite element method (Luo, Zhu, Zeng & Xie, 2004; Lu, 2007; Xia, 2013)

The mixed finite element can approximate the function and its gradient with high accuracy at the same time, which overcomes the shortcomings of the standard finite element method in calculating the gradient by solving itself, and the requirement for the solution of space smoothness becomes lower. Luo Zhendong and others give a mixed finite element method for two-dimensional sediment transport model, give the existence and uniqueness of generalized solution and the mixed finite element solution, and obtain the optimal order error estimation by increasing the number of interpolations in the test function space of flow velocity field and sediment field. Lu Xiuying and Xia Kaifeng propose a characteristic mixed finite element method for shallow water equation and sediment transport equation, and obtain suboptimal error estimation.

Discontinuous finite element method (Shi, 2013; Dawson & Proft, 2004)

Discontinuous finite element not only retains the advantages of the finite element which can be applied to irregular boundary and easy to improve accuracy, but also has the advantages of easy processing discontinuous problems, easy carrying out parallel computation, good stability and entropy compatibility. Dawson,Proft use the methods of discontinuous finite element and finite element coupling to study the shallow water equations, gives the calculation format, and obtains the suboptimal error estimation of the L^2 model. Shi Baohai proposes a semi-discrete format for the two-dimensional sediment transport model by using discontinuous finite element method, estimates the errors, and obtains suboptimal error estimation of the L^2 model.

Further research and outlook

At present, although the research on the establishment of mathematical models and the technology of numerical simulation for sediment transport problems have made great progress, some major problems such as the construction and simulation of mathematical models in three dimensions need to be further studied and explored.

(1) Research on the basic laws of sediment movement. The factors that strongly influence the law of sediment diffusion and transport include riverbed resistance, sedimentation, sedimentation velocity and recovery saturation coefficient in the sediment transport model, and flocculation caused by seawater temperature and salinity changes. These laws do not necessarily conform to the classical Gauss distribution, and may conform to some algebraic distribution. This needs to be further explored in mathematical theory and physical experiments to form a constitutive equation that reflects the mathematical physical nature of practical problems, which can more accurately describe the process of sediment transport.

(2) Research on the basic laws of hydrodynamics. Sediment deposition or seawater erosion in the waters of the estuary coast is affected by a variety of hydrodynamic factors. Therefore, we should consider the wave-current coupling model formed by runoff, tidal current and wind, and pay attention to the shape, function and development law of shear peak and pinnacle peak, which are caused by runoff and ocean dynamics, and the mechanism of sediment accumulation and capture, so as to form an accurate hydrodynamic system.

(3) Research on numerical simulation technology. Based on the hydrodynamics and laws of sediment movement in the estuary, we should consider factors such as strong convection, nonlinearity, strong coupling, and data roughness in practical problems to establish computable models and mathematical theories, which provide a solid theoretical basis for a more accurate simulation of sediment transport in the Yellow River estuary, and provide a reliable scientific basis for the demonstration and decision-making of the control scheme of the Yellow River estuary.

Acknowledgement

The paper is supported by the National Natural Science Foundation of China (No. 11471196, 11301311, 10971254) and the Natural Science Foundation of Shandong Province (No. 2014ZRB01849).

References

Benkhaldoun, F., Elmahi, I., & Seaid, M. (2010). A New Finite Volume Method for Flux-gradient and Source-term Balancing in Shallow Water Equations. *Computer Methods in Applied Mechanics and Engineering*, 199, 49-52.

Cao, Z. & Wang, G. (1993). Numerical Simulation of Wave-induced Sediment Lifting and Tidal Sediment Transport. *Acta Oceanologica Sinica*, 15(1), 107-118.

Chang, H. H. (1998). Generalized computer program: Users' manual for FLUVIAL-12: Mathematical model for erodible channels. San Diego.

Chippada, S., Dawson, C. N., Martinez, M. L., & Wheeler, M. F. (1998). Finite Element Approximations to the System of Shallow Water Equations I: Continuous Time a Priori Error Sstimates. *SIAM Journal on Numerical Analysis*, 35(2), 692-711.

Chippada, S., Dawson, C. N., Martinez, M. L., & Wheeler, M. F. (1998). Finite Element Approximations to the System of Shallow Water Equations II: Discrete-Time a Priori Error Estimates. *SIAM Journal on Numerical Analysis*, 36(1), 226-250.

Dawson, C. N., & Proft, J. (2004). Coupled Discontinuous and Continuous Galerkin Finite Element Methods for the Depth-Integrated shallow Water Equations. *Computer Methods in Applied Mechanics and Engineering*, 193(1), 289-318.

Dawson, C. N. & Martinez, M. L. (2000). A Characteristic-Galerkin Approximation to a System of Shallow Water Equations. *Numerische Mathematik*, 86, 239-256.

Dawson, C. N. & Martinez, M. L. (2000). Finite Element Approximations to the System of Shallow Water Equations, Part III: On the Treatment of Boundary Conditions. *SIAM Journal on Numerical Analysis*, 38(1), 149-159.

Dou, G., Dong, F., Dou, X., & Li, T. (1995). Study on Mathematical Model of Coastal Sediment in Estuary. *Science In China* (Series A), 25(9), 995-1001.

Guo, Q., Han, Q., & He, M. (1996). Mathematical Model of Two-Dimensional Tidal Current and Sediment. *Journal of Sediment research*, 1, 48-55.

Hu, S. & Tan, W. (1995). Numerical Modeling of Two-Dimensional Shallow Water Flow on Unstructured Grids. *Advances in Water Science*, 6(1), 1-9.

Li, D. & Zhang, H. (1999). Finite Element Method for Simulation of Two-Dimensional Flow and Sediment Movement in the Lower Yellow River. *Journal of Sediment Research*, 4, 59-63.

Li, W. & Chen, H. (2011). A Finite Element Numerical Simulation for 2-D Non-Homogeneous Current and Silt Sedimentation Model. *Applied Mathematics A Journal of Chinese Universities*, 26(2), 169-178.

Liu, M., Li, W., & Chen, H. (2013). Multistep Characteristic Finite Element Method for Flat Non-homogeneous Current and Silt Sedimentation Model. *Acta Mathematicae Applicatae Sinica*, 36(5), 870-880.

Lu, X. (2007). *Numerical Simulation of Two-Dimensional Shallow Water Equation*. Jinan: School of Mathematical Science, Shandong Normal University.

Lu, X. & Peng, R. (1985). Calculation of Suspended Sediment Scouring and Silting -- Solution of Implicit Finite Difference Chasing Method. *Journal of Sediment Research*, 1, 32-43.

Luo, Z., Zhu, J., Zeng, Q., & Xie, Z. (2004). Mixed Finite Element Methods for the Shallow Water Equations Including Current and Silt Sedimentation - The Continuous-Time Case. *Applied Mathematics and Mechanics*, 25(1), 74-84.

Luo, Z., Zhu, J., Zeng, Q., & Xie, Z. (2004). Mixed Finite Element Methods for the Shallow Water Equations Including Current and Silt Sedimentation - The Discrete-Time Case Along Characteristics. *Applied Mathematics and Mechanics*, 25(2), 166-180.

Pan, C., Lu, H., & Yu, P. (2009). Numerical Simulation of Sediment Transport in Discontinuous Shallow Water Flows with Triangle Grids. *Journal of Hydrodynamics*, 24(6), 778-785.

Shi, B. (2013). *An Up-Wind Discontinuous Galerkin Method for Current and Silt Sedimentation in Two Space Dimensions*. Jinan: School of Mathematical Sciences, Shandong Normal University.

Spasojevic, M. & Holly, F. M. (1990). *MOBED2: Numerical simulation of two-dimensional mobile-bed processes*. Technical Report No.344, Iowa Institute of Hydraulic Research, University of Iowa.

Wang, J. & Liu, R. (2003). A Characteristics Based on Galerkin Method for the System of Shallow Water Equations. *Acta Mathematicae Applicatae Sinica*, 26(3), 458-466.

Wu, H. & Wang, J. (2012). A New High Resolution Finite Volume Method for Solving Two-Dimensional Shallow Water Equation. *Computer Technology and Development*, 22(10), 55-58.

Xia, K. (2013). *A Characteristic-Mixed Finite Element Method for the Simulation of Two-Dimensional Water and Sediment Transport Model*. Jinan: School of Mathematical Science, Shandong Normal University.

Yang, G. (1993). *River Mathematical Model*. Beijing: *Ocean Press*.

Zhang, X. B. & Yin, R. L. (2002). Planar 2-D Flow and Sediment Mathematical Modeling. *Advances in Water Science*, 12 (6): 665-669.

Zheng, J. (2003). Development and Application of Numerical Models for Flow Motion and Sediment Transport in an Orthogonal Body-fitted Coordinate System. *Marine Science Bulletin*, 22(1), 1-8.

Zhou, Z. & Chen, H. (2007). A Characteristic Finite Element Simulation for Current and Silt Sedimentation Model in Two Space Dimensions. *Numerical Mathematics A Journal of Chinese Universities*, 29(3), 245-256.