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METEOROLOGY

Estimation of IDF Curves of Extreme Rainfall by Simple Scaling in Northern Oueme Valley, Benin Republic (West Africa)

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ABSTRACT

Rainfall intensity-duration-frequency (IDF) curves are of particular importance in water resources management, for example, in urban hydrology, for the design of hydraulic structures and the estimation of the flash flood risk in small catchments. IDF curves describe rainfall intensity as a function of duration and return period, and they are significant for water resources planning, as well as for the design of hydraulic constructions and structures. In this study, scaling properties of extreme rainfall are examined to establish the scaling behavior of statistical non-central moment over different durations. IDF curves and equations are set up for all stations by using the parameter obtained from scaling behavior, the location and scale parameters μ24 and σ24 of the Gumbel distribution (EVI) sample of annual maximum 1440 min rainfall data. In another hand, we have established the IDF curves for ten selected rain gauge stations in the Northern (Oueme Valley) parts of Benin Republic, West Africa by using the simple scaling approach. Analysis of rainfall intensities (5 min and 1440 min rainfall data) from the ten rainfall stations shows that rainfall in north-Benin displays scales invariance property from 5 min to 1440 min. For time scaling, the statistical properties of rainfall follow the hypothesis of simple scaling. Therefore, the simple scaling model applies to the rainfall in (Oueme Valley). Hence, the simple scaling model is thought to be a viable approach to estimate IDF curves of hourly and sub-hourly rainfall form rainfall projections. The obtained scaling exponents are less than 1 and range from 0.23 to 0.59. The empirical model shows that the scaling procedure is a good estimator as it is more efficient and gives more accurate estimates compared with the observed rainfall than the traditional method which only consists the Gumbel model in all stations for lower return periods (T<5 years) but not for higher return periods.

Keywords: Simple scaling, IDF curves, annual maximum rainfall, frequency analysis, Oueme Valley.

Estimación de las Curvas IDF de Extrema Precipitación por Escala Simple en el Valle Oueme, al Norte de la República de Benín (Africa occidental)

RESUMEN

Las curvas de precipitación Intensidad-Duración-Frecuencia (IDF) son de particular importancia en el manejo de los recursos hídricos, como es el caso de la hidrología urbana o para el diseño de estructuras hidráulicas y la estimación del riesgo de crecidas en pequeñas captaciones. Las curvas IDF describen la intensidad de las precipitaciones como una función con períodos de duración y recurrencia, lo que las hace significativas en la planeación de recursos hídricos así como en el diseño de construcciones y estructuras hidráulicas. Este estudio examina las propiedades de escala en precipitaciones extremas para establecer un comportamiento en momentos estadísticos marginales en diferentes períodos de duración. Se establecieron las curvas IDF y las ecuaciones para todas las estaciones a partir del parámetro obtenido del comportamiento de escala, la ubicación y los parámetros de escala μ24 and σ24 de la muestra de información de precipitación máxima anual de 1440 minutos de la distribución de Gumbel (EVI). Por otro lado, se establecieron las curvas IDF para 10 estaciones pluviométricas seleccionadas en el Valle Oueme, al norte de la República de Benín (África occidental), con el uso de aproximación simple de escala. El análisis de las intensidades de precipitación en las diez estaciones pluviométricas muestra que la precipitación en el norte de Benín expone propiedades de poca variación en la escala 5 min y 1440. En el tiempo de escala, las propiedades estadísticas de precipitación confirman la hipótesis de escala simple; además, este modelo so corresponde a la precipitación del Valle Oueme. Por lo tanto, el modelo de escala simple se considera una aproximación viable para estimar las curvas IDF en las proyecciones de precipitación de cada hora y sub-hora. Los exponentes de escala obtenidos son menores a 1 y oscilan de 0,23 a 0,59. El modelo empírico muestra que el procedimiento de escala es un buen estimativo, más eficiente y con cálculos más exactos que el método tradicional, el cual consiste solamente en el modelo Gumbel aplicado en todas las estaciones pluviométricas en períodos de menor recurrencia (T<5 años) pero no en lapsos de mayor recurrencia.

Palabras clave: Escala simple, curvas de Intensidad-Duración-Frecuencia, precipitación máxima anual, análisis de frecuencia, Valle Oueme.

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1. Introduction

West African countries have been struck these last years by floods and inundations of unprecedented magnitude (Danida, 2008; Zannou, 2011; G. Panthou, 2014). For example we can cite the inundations that occurred from 2007-2009 in Benin, Niger and Burkina-Faso (World bank, 2009) which caused drastic consequences in more field of development agriculture (destroyed cultivation), aquatic ecosystems, economy, urban drainage systems, energy distribution and social security by displacing people (Danida, 2008, Descroix et al., 2009). Likewise, the emblematic Niger River floods of 2010 and 2012 are the two most important floods ever recorded at the Niamey station since the beginning of the observation in 1929, causing heavy casualties to people living close to the river (Panthou, 2014). Projection in the northwest of Benin, one of the high-risk regions, predicts trends including lower average annual rainfall (15% up to 2025); more variation in precipitation; more frequent extreme climatic elements such as heavier rains and more powerful storms; shorter, delayed monsoons with irregular precipitation; and higher long-term average temperatures (Danida, 2008). This illustrates that extreme events can appear in dry or moderately wet years anywhere in West Africa where Benin Republic is located and in which wet years can occur without any extreme rainy event being recorded. In urban drainage systems, short duration rainfall events are one of the most critical elements for hydrological investigation (Bougadis and Adamoski, 2006), useful for the control of storm run-off such as flood detention, reservoirs, sewer systems etc. In particular, rainfall extremes with high temporal resolution (one hour or shorter) are necessary for the design of drainage systems, usually characterized by fast response. The temporal resolution of rainfall data usually available for practical applications are often lower than the data requested for the design procedures or mathematical models application that greatly affect their reliability (Nhat et al, 2008) because they depend on many assumptions such as the distribution selection for durations. Rainfall data generally require a large number of parameters and are time dependent. Such information is often expressed in Intensity-Duration-Frequency (IDF). The establishment of such relationships goes back to the 1930s (Bernard, 1932). Since then, different forms of IDFs have been constructed for several regions of the world. In Africa we can cite the studies which utilized statistical method to construct IDF curves: in Congo (Mohymont et al., 2004; Mohymont & Demarée, 2006; Van de Vyver & Demarée, 2010), in Ghana (Endreny & Imbeah, 2009), in Cote d'Ivoire (Soro et al., 2008; Soro et al., 2010) and in Nigéria (Oyegoke & Oyebande, 2008; Okonkwo & Mbajiorgu, 2010). Traditionally the IDF curves are constructed by performing statistical analysis on Annual Maxima Series (AMS) or Partial Duration Series (PDS) by fitting probability distribution for several pre-selected rainfall durations (Bougadis and Adamowski, 2006). Such a procedure has several disadvantages including fitting distribution and parameters estimations for each duration, and requirements to extrapolate result to different time-scales. The usefulness and accuracy of such procedures are limited because of fitting uncertainties and lack of ability to adequately describe rainfall properties at different time scales (Van Nguyen and Wang, 1996). In addition, this traditional method has certain limitations. For example, a high number of parameters are involved, which makes it none-parsimonious from the statistical point of view (Baghirathan and Shaw, 1978; Bell, 1969; Sherman, 1931; Bernard, 1932; Chen, 1983; Garcia-Bartual and Schneider, 2001; Takara, 2005; Nhat et al., 2008). Traditional method cannot take into consideration characteristics of rainfall for different durations; and it is based on the annual extreme data available at local site only. It is therefore advantageous to develop a model, which could adequately describe rainfall characteristics through a continuum of times scales including inferences at time resolutions that may not have been observed and to reduce the number of parameters to be estimated in order to increase their reliability.

Consequently, alternative approaches to construct IDF relationships are used. For example the fractal properties of rainfall which implies scaling invariance were developed (e.g., Menabde, et al., 1999) where the scaling properties of the statistical moments of rainfall in South Africa over a range of durations from 30 min to 1440 min were examined. Yu, et al., (2004) used regional IDF formulas for estimating scaling exponents for 46 non-recording rain gauge stations in Taiwan. Molnar and Burlando (2005) investigated the variability of the scaling properties from 62 rain gauge stations in Switzerland and Özger, et al. (2010) focused on the scaling properties of rainfall at 43 rain gauge stations in Texas. Recently, scaling formulas are proposed to extend the IDF relationships from daily time scale to shorter time intervals based on scaling properties. Gupta and Waymire (1990) studied the concepts of simple and multiple scaling to characterize the probabilistic structure of the precipitation process, Koutsoyiannis and Foulfoula-Georgiu (1993) used a scaling model to predict storm hyetographs. Menabde et al. (1999) showed that based on the empirically observed scaling properties of rainfall and some general assumptions about the cumulative distribution function for the annual maxima of mean rainfall intensity, it is possible to derive simple IDF relationships. Bendjoudi, et al. (1997) used a multifractal point of view on rainfall IDF curves. Rosso and Burlando (1990) and later Burlando and Rosso (1996) used this concept to study traditional forms of depth-duration-frequency relationships. De Michele et al. (2002) developed IDF curves design for storms in Milan (Italia); Yu, et al. (2004) developed regional IDF formulas for non-recording sites in Taiwan, Molnar and Burlando (2005) examined the variability of scaling exponents in Alpine mountainous region of Switzerland; Nhat, et al. (2007) developed regional relationship for ungauged locations based on the scaling theory in Japan. Molnar and Burlando (2008) investigated the variability of the scaling properties from 62 rain gauge stations in Switzerland, Acar, et al. (2008) used a multilayer perception artificial neural network model to assess IDF relationships for short duration rainfalls. Recently, Özger, et al. (2010) focused on the scaling properties of rainfall at 43 rain gauge stations in Texas; Ceresseti (2011) used a multifractal point of view on rainfall IDF curves in France to characterize IDF curves; Ghanmi (2014) developed regional relationship for ungauged locations based on the scaling theory in Tunisia. Scaling properties of extreme rainfall has never been investigated in Benin Republic. The objectives of the paper in this study area are: (i) to examine the scaling properties of extreme rainfall in the Oueme valley (Benin), (ii) to establish scaling behavior of statistical non central moment over different durations of rainfall intensity and (iii) to derive the IDF relationships for short-duration rainfall from daily data using the scaling methodology and to compare with empirical method.

2. Methodology

The scaling or scale-invariant models enable transforming hydrologic information from one temporal or spatial to another one, and thus, help overcome the difficulty of inadequate hydrologic data. A natural process fulfills the simple scaling property if the underlying probability distribution of some physical measurements at one scale is identical to the distribution at another scale. The basic theoretical development of scaling has been investigated by many authors and considerable amount of studies were devoted to extreme rainfall and its scaling properties, including Waymire and Gupta (1981); Waymire, et al.(1984); Rodriguez-Iturbe, et al. (1984); Marien and Vandewiele (1986); Sivapalan and Wood (1987); Gupta and Waymire(1990); Rosso and Burlando (1990); Smith (1992); Koutsoyiannis and Foulfoula-Georgiu (1993); Burlando and Rosso (1996); Veneziano et al. (1996), Bendjoudi et al. (1997); Willems (2000); Hubert et al. (2002) ; De Michele et al. (2002); Molnar and Burlando (2005); Ceresetti (2011); Panthou (2013); Ghanmi (2014). In this work the simple scaling hypothesis is adopted to test the scaling behavior of rainfall in northern of Benin, following the methods described in Menabde et al. (1999); Yu et al. (2004); Kuzuha et al. (2002); Nhat et al. (2007); Bara et al. (2009) and Ghanmi (2014) and are briefly outlined in the ensuing sections.

2.1 Scaling characteristics of extreme rainfall

Let $\boldsymbol{I_d}$ and $\boldsymbol{I_D}$ denote the annual maximum rainfall intensity series for time durations $\boldsymbol{d}$ and $\boldsymbol{D}$, respectively. In the model proposed by Menabde et al. (1999); Yu et al. (2004); Veneziano and Furcolo (2002); Bara et al.

(2009) rainfall intensity I_d of duration d is said to exhibit simple scale invariance behavior if the following equation holds true.

$$I_d \overset{dist}{=} \lambda^k I_D \quad \text{(1) Hold true}$$

The equality "$\overset{dist}{=}$" refers to identical propability distributions in both sides of the equations; $\lambda = d/_D$ denotes a scale factor and k is a scaling exponent. It follows that by raising both sides of equation (1) to the power of q and taking the ensemble average that is, taking the q^{th} moment of both distributions, we obtain

$$\langle I_d^q \rangle = (\lambda)^{k(q)} \langle I_D^q \rangle \quad (2)$$

Where $k\ (q)$ represents the scaling exponent of order q. In order to obtain the value of $k\ (q)$, we simply take logarithm of both sides of equation (2), which transforms it into:

$$log\langle I_d^q \rangle = log\langle I_D^q \rangle + k(q)\ log\lambda \quad (3)$$

When the moments $\langle I_d^q \rangle$ are plotted on a logarithmic chart versus the scale λ for various moments order q, each slope will represent $k\ (q)$ for the respective q. The various slope $k(q)$ are then plotted against q: if the graph is linear, it implies that k is a constant and $I(d)$ exhibits characteristics for simple scaling, while in other cases, the multi-scaling approach has to be considered.

2.2 Estimation of IDF rainfall extremes

The IDF curves are often fitted to the extreme value type I, EVI developed by Gumbel and it is still the most often used distribution by many national meteorological services in the world to describe rainfall extremes. It's also used in this study along with the method of moments. The annual maximum rainfall intensity $I(d)$ has a cumulative probability distribution (CDF) (Gumbel, 1958). According to the scaling theory, the IDF formula can be derived (Menabde et al, 1999; Nhat et al., 2007 and Ghanmi, 2014) as:

$$I_d^T = \lambda^k I_D^T = \left(\frac{d}{D}\right)^k * \left[\mu_D - б_D * ln\left(-ln\left(1 - \frac{1}{T}\right)\right)\right] \quad (4)$$

Where and are respectively the location and the scale parameter for Gumbel model at D hour data series; I_D^T and I_d^T represent the rainfall intensity for a duration d and D for a return period T. λ denotes a scale factor and k is a scaling exponent.

It was also shown in Menabde et al. (1999); Nhat et al. (2007) and Ghanmi (2014) that:

$$\mu_d = \lambda^k \mu_D \quad (5)$$

and

$$б_d = \lambda^k б_D \quad (6)$$

Where the location parameter μ_D and scale parameter σ_D of Gumbel model are calculated from daily data series by using the equations of Landwehr et al. (1979) given as:

$$б_D = {(M_D^0 - 2M_D^1)}/{ln2} \quad (7a)$$

$$\mu D = M_D^0 - 0.5772\ б_D \quad (7b)$$

M_D^0 and M_D^1 are probability weight moment (PWM) for order 0 and 1, and are estimated by using the daily annual maximum rainfall intensity. The IDF relationships can be derived from longer duration data series based on three parameters: scale exponent, the location and scale parameter of EVI.

The location and scale parameters (μ_{24} and σ_{24}) of the EVI (Gumbel model) sample of annual maximum 1440 min rainfall data are calculated using equation (7a) and (7b). The results are recorded in Table 2.

2.3 Model performance

As an indication of goodness of fit between the observed and predicted values, the coefficient of determination (R^2), the root mean square error (RMSE), the mean absolute percent error (MAPE) are calculated. The RMSE and MAPE are defined as:

-The Root Mean Square Error (RMSE):

$$RSME = \sum_{i=1}^n \sqrt{\left(\frac{1}{n}\sum_{i=1}^n \left(X_{obs,i} - X_{Est,i}\right)^2\right)} \quad (8)$$

And

-The Mean Absolute Percent Error (MAPE):

$$MAPE = 100 * \sqrt{\left(\frac{1}{n}\sum_{i=1}^n \left(\frac{X_{obs,i} - X_{Est,i}}{X_{obs,i}}\right)^2\right)} \quad (9)$$

$X_{obs,i}$ is the i_{th} observed data point, $X_{Est,i}$ is the i_{th} estimated value, n is the number of data point. IDF curves are plotted together with the actual observed data, and shown in **Fig.4**. To evaluate the goodness-of-fit graphically, IDF curves derived by fitting Gumbel extreme value distribution to actual observed data (of various durations d) are also plotted on the same graphs (**Fig.4**).

The statistical Root Mean Square Error (RMSE) and Mean Absolute Percent Error (MAPE) are also used to evaluate the performance of the simple scaling model. The results are presented in Table 3 are for 5 min, 10 min, 15 min, 20 min, 30 min and 60 min IDF curves simple scaling.

2.4 Study sites and data analysis

The Hydro meteorological Observatory of upper Oueme Valley (OHHVO) has an area of about 14,000 km2 and is located in northern of Benin Republic, an area of humid tropical climate of Sudanese type and delimited by latitude 9°N-10°N and longitude 1.5°E-3°E. This area was instrumented during the CATCH (Coupling of Tropical Atmosphere and Hydrological Cycle) and AMMA (Multidisciplinary Analyses of African Monsoon) projects. OHHVO is an area characterized by a single rainy season from mid-March to late October, the heart of the rainy season is between July and August with an interannual average rainfall of about 1200 mm (Lawin, 2007). The study area has a subdued topography with an elevation of about 200 m (Le Barbé et al., 1993). The location of OHHVO in Benin is shown in **Fig. 1**.

For a good representativeness of the samples, ten rain stations of OHHVO which operating rate (the ratio of the total number of days of operation to the total number of days in the year) during the year is greater than 90% have been chosen. The locations of these stations are shown in **Fig. 1**.

The data used are time series of rainfall intensity 5 min and 1440 min, measured from 1999 to 2012. MATLAB (V10.2) programming code is used to accomplish the tasks, allowing the aggregation of data over periods of 5 min and more and longer (i.e. 10 min, 20 min, 30 min, 60 min, 90 min, 120 min, 180 min, 240 min, 1440 min), using disjoint windows. Missing in these raw data in the sets are provided by interpolating the neighboring values using the proposed formula of Hoang, (2008), given as:

$$P = n * \frac{\sum_{i=1}^n P'_i}{n'} \quad (10)$$

P is the intensity of the calculated length, n is the total number of time steps of 5 min from the calculated time, n' is the number of time steps of 5 min measured from the calculated time and P'_i is the i^{th} measured intensity pitch 5 min i.

Table1: Name of stations

N°	1	2	3	4	5	6	7	8	9	10
Stations	Affon	Copargo	Djougou	Donga	Koko	Pelebina	Sarmanga	Sonoumon	Tobre	Wewe

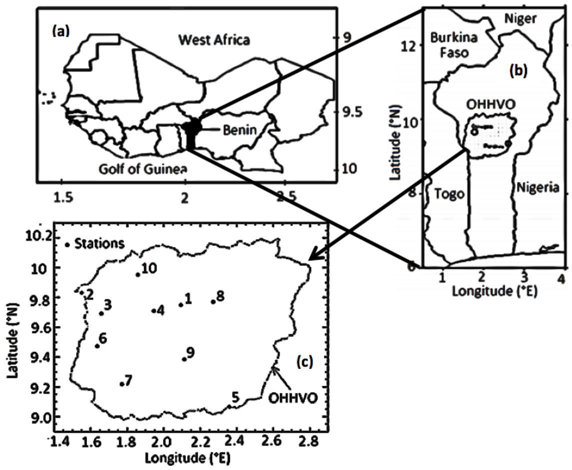


Fig. 1. Study site: (a) Benin Republic location in west Africa, (b) OHHVO location in Benin Republic and (c) Rain gauge location in OHHVO (1 = Affon, 2 = Corpargo, 3 = Djougou, 4=Wewe, 5= Koko, 6= Pélébina, 7= Sarmanga, 8= Sonoumon, 9= Tobre and 10= Wewe).

3. Results and discussions

3.1 Scale invariance properties of rainfall

The property of simple scaling of rainfall intensities in the wide sense is demonstrated in the ten selected sample stations. The scaling exponents are derived by including all the analyzed durations of rainfall (5 min, 10 min, 15 min, 20 min, 30 min, 60 min, 90 min, 120 min, 180 min and 1440 min) to the analysis. Fig. 2 displays the relationships between the log-transformed values of moments of various orders q (q values of 0.5 to 5 at increments) against various rainfall durations at Copargo station. It can be observed that, for all the considered moment orders, log (moment) and log (duration) have a linear relationship for durations between 5 min and 1440 min. It's not broken between the short and long duration data. Linear regression performed for the regime confirmed the linear relationship, the correlation is consistently strong with coefficient of determination R^2, greater than 0.94. The behavior (linearity) found from Copargo station is similar to the other examined stations in the study area.

The linearity of the moment from this study is also observed in others region of world. For example Ghanmi (2014) in Tunisia (Northern of Africa) find that data from (Tunis) present linearity, Ceresseti (2011) in France; Bara, et al. (2009) in Slovakia; Nhat (2006) in Japan, Bougadis and Admowski (2006) in Canada have found linearity in such study.

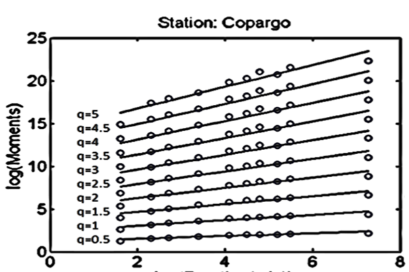


Fig. 2. Relationships between the log-transformed values of moments of various orders q and various rainfall durations at the station Copargo. The orders $q_1 - q_{10}$ have values from 0.5 – 5 in steps of 0.5 respectively.

The scaling exponents of the moments of various orders are estimated as the slopes of the linear regression between these moments and the rainfall duration. Fig. 3 shows the relationships between the scaling exponents of the moments and the orders of the moments at Copargo station.

It's obvious that the scaling exponents decrease with the order of moments and a linear relationship exists between scaling exponents and the moment orders. This implies that the property of wide sense simple scaling of rainfall intensity exists in this station. The scaling exponents and order of moments were linearly related with R^2 value of 0.9932 for Copargo station. Data from other gauges of the 10 stations showed similar scaling relationships, indicating that scaling may be applicable in all the stations.

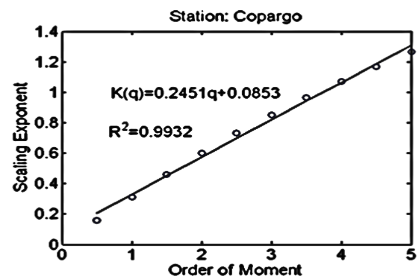


Fig.3 Relationship between $k(q)$ and order of moment q (q= [0.5:0.5:5]) for range, 5 min to 1440 min at Copargo station.

Table 1 presents the characteristics of the examined stations, showing the stations and results of the scaling exponent factor k of the 10 stations in OHHVO. Catchments are shown with high coefficients of determination ranging from 0.98 to 1.

Table 2: Characteristics of examined stations: the values scale exponent k, and Gumbel's parameters of statistical analysis for 1440 min (σ_{24}, μ_{24})

N°	Stations	scale exponents	μ_{24}	σ_{24}
1	Affon	0.3492	70.76	14.44
2	**Copargo**	**0.2451**	**65.23**	**10.040**
3	Djougou	0.3887	76.65	23.19
4	Donga	0.3427	74.0487	19.76
5	Koko	0.3682	64.23	15.68
6	Pelebina	0.2365	71.25	9.625
7	Sarmanga	0.5209	60.21	12.57
8	Sonoumon	0. 5869	52.59	9.54
9	Tobre	0.4829	40.26	13.47
10	Wewe	0.5521	58.18	11.38

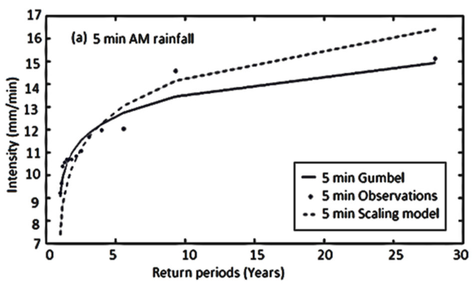
The result indicates that for all the stations, the scaling exponents are less than 1 and in ranging from 0.23 to 0.59 in all the stations, showing considerable variability. The highest (the lowest) scaling exponent was found at Sonoumon (Pelebina) station. It is likely that the variability of the scaling exponents is influenced by the geographical location of the stations and/or the different mechanisms of rainfall generation. However, the results of Olsson et al. (1992) which identified [0.6 - 0.8] as an adequate interval of scaling exponents at 45 min are not coherent with ours. These differences in the number of regime of scaling can be attributed to the differences of rainfall threshold detection of different used devices. Comparing the value of scaling exponents k to those found by Bara et al. (2009) in Slovakia and Chang (2013) in China, our estimation for all the 10 stations are different to other authors results, because these workers found the scaling exponents k higher than 6.2; but for few stations (Sarmanga, Wewe and Sonoumon) where the value of k is approximately equal to those obtained by Nhat (2006) in Japan, Bougadis et al. (2006) in Canada and Ghanmi (2014) in Tunisia.

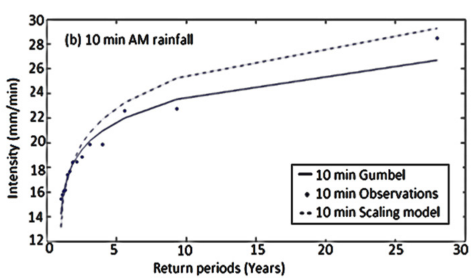
The results indicate a strong validity of the simple scaling properties of the extremes rainfall in time series for the stations.

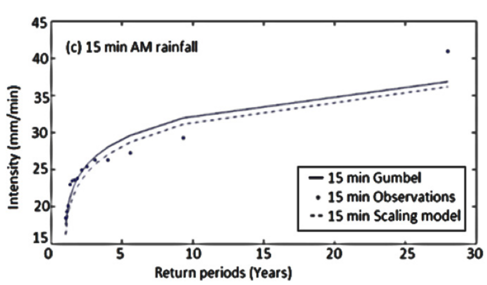
3.2 Estimation of IDF curves and evaluation of model performance

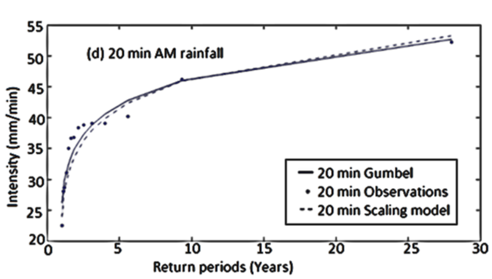
The figures (Fig.4) show the IDF curves estimated by simple scaling and by fitting EVI to the observations. It can be observed that the estimated IDF curves of the short-duration rainfall by simple scaling fit the observed data point well at lower return periods (*T<5* years) but not as well at higher return periods. This result may be due to the short observation period (only 13 years) of qualitative available rainfall data.

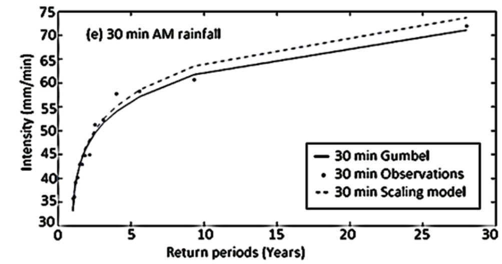
From table 2, the location parameter μ_{24} and scale parameter σ_{24} calculated using equation (7) for the 10 stations is shown. Subsequently, the mean μ_d and standard deviation σ_d for various sub-daily durations *d* are calculated from μ_{24} and σ_{24} using equations (5) and (6).











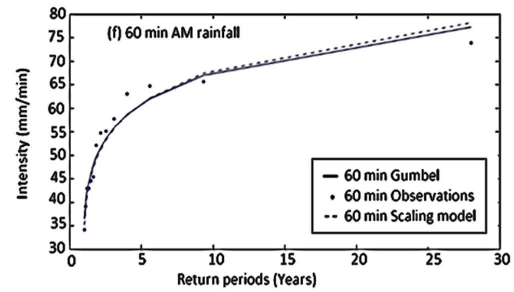


Fig. 4 IDF curves estimated by simple scaling and by fitting EVI to the observations for: (a) 5 min, (b) 10 min, (c) 15 min, (d) 20 min), (e) 30 min and (f) 1440 min rainfall.

Table 3. RMSE and MAPE for 5 min, 10 min, 15 min, 20 min, 30 min, and 1440 min IDF curves estimate using (a) simple scaling assuming EVI and (b) fitting EVI to observed data.

	(a) Simple Scaling assuming EVI		(b) Fitting EVI to observed data	
	RMSE	MAPE (%)	RMSE	MAPE (%)
5 min	1.04	10	0.44	3.59
10 min	1.23	6.56	0.76	3.7
15 min	2.04	7.63	1.67	5.5
20min	2.4	6.7	1.93	6.3
30min	1.8	3.7	1.62	3.4
1440 min	2.5	4.23	2.4	4.42

The table 3 shows that MAPE values (Scaling assuming EVI) are in the range of 3.7-10%, which is slightly higher than the MAPE values when fitting EVI to the observed data, ranging from 3.4-6.3%. The RMSE values are in the range of 1-2.5 which is approximately higher than the RMSE values when fitting EVI to the observed data, ranging from 0.4-2.4. These results validate the performance of the simple scaling model in estimating the IDF curves of short-duration rainfall in OHHVO valley (Northern Benin).

4. Conclusion

Properties of the time scale invariance of selected rainfall data at 10 rainfall station in OHHVO valley (northern Benin Republic) are studied. A simple analytical formulation for rainfall IDF relationship, which utilizes the scaling behavior, is presented. The benefit of using the principals of scaling reduces the amount of parameters required to compute the quantiles. The main result of the study can be summarized as follows:

(i) rainfall displays scale invariance property in one scaling regime from 5 min to 1440 min,

(ii) the properties of rainfall follow the hypothesis of simple scaling, following the result of Menabde et al. (1999),

(iii) it is possible to derive the rainfall IDF curves with only three parameters (k, μ_{24} and σ_{24}) for duration shorter than a day. From only the location and scale parameters μ_{24} and σ_{24} of the EVI sample of annual maximum 1440 min rainfall data and the scaling exponents, the IDF curves of short-duration rainfall for low return periods (T<5years) could be estimated but may not be a reliable estimate for higher return period. In future studies, we should consider to exclude higher return periods,

(iv) for the all stations the scaling exponents are less than 1 and range from 0.23 to 0.59, showing a considerable variability in values for the all stations. Since it was not possible to find any clear dependence of the variability of the scaling exponents, it is believed that this variability may be accounted for by either for the geographical location of the stations or it the result of it is the sampling variability of the data (different mechanisms of rainfall generation),

(v) IDF relationships show acceptable result in comparison with the IDF curves obtained from at-site short duration rainfall data. The key advantage is that IDF curves of shorter rainfall duration for low return periods (T<5years) that are not measured or unavailable can be derived solely based on the statistical properties of the annual maximum 1440 min rainfall.

This is relevant for Benin Republic daily data, available for long duration period, but data for short duration are often not available for the required site. The finding from this study may have significant bearing beyond the study area.

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