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Interval velocity determination by downward continuation of the traveltime function: Paraxial ray approximation

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RESUMEN

Presentamos un método para estimar las velocidades de intervalo, la profundidad y la geometría de los reflectores en modelos 3D que constan de un apilado de capas homogéneas e isotrópicas con velocidades y densidades arbitrarias, separadas por interfaces suaves.

El tiempo de tránsito de cualquier rayo reflejado en una interfaz particular y registrado en la vecindad de un rayo zero-offset se expresa mediante una función referida a un sistema de coordenadas centrado en el rayo; tal función se estima en la superficie superior del modelo. La función tiempo de tránsito de reflexión asociada a cada superficie reflectora se determina en la superficie superior en la vecindad del rayo central.

La geometría de la superficie limitante superior de una capa particular y el tiempo de tránsito estimado sobre la misma permite calcular la velocidad de intervalo de la capa en cuestión y la geometría de la interfaz limitante inferior. Con la velocidad de intervalo y la geometría de las interfaces limitantes, se estima la función de tiempo de tránsito del siguiente reflector sobre la interfaz limitante inferior. En este paso se simula el posicionamiento de las fuentes y los detectores sobre la superficie anterior de la próxima capa subyacente.

El proceso se repite recursivamente en las capas más profundas hasta obtener la solución completa sin conocimiento previo, excepto el obtenido en las capas superiores y la función tiempos de tránsito de cada superficie reflectora.

Se desarrollaron programas de computador que expresan el algoritmo del método y, posteriormente, se probaron con datos sintéticos, suministrando velocidades de intervalo y profundidades de los reflectores con errores considerados aceptables.

PALABRAS CLAVE: TRAZAMIENTO DE RAYOS, TEORÍA DEL RAYO, PROBLEMA INVERSO, CONTINUACIÓN HACIA ABAJO, VELOCIDAD DE INTERVALO.

ABSTRACT

We present a method to estimate interval velocities, reflector depths and geometries in 3D models consisting of a pile of isotropic and homogeneous layers of any velocities and densities separated by smooth interfaces.

The travel time of a ray reflecting on a particular interface and registered in the vicinity of a zero-offset ray is expressed by a function referred to a ray-centered coordinated system, function which is estimated at the uppermost surface of the model. The reflection travel time function associated to each reflecting surface is determined at the superior surface in the neighborhood of the reference ray.

The geometry of the upper limiting surface of a particular layer and the travel time function estimated on this interface allow to calculate the interval velocity of the layer and the geometry of the bottom limiting interface. With the interval velocity and geometry of the two limiting interfaces of the layer, the travel time function of the following reflector is estimated at the bottom interface. This step simulates positioning the sources and detectors on the anterior surface of the next subjacent layer.

The procedure is repeated recursively at deeper layers getting the complete solution without a priori knowledge but the upper determined layers and the estimated travel time functions of each reflecting surface.

Computer's programs expressing the algorithm of the method were developed and tested with synthetic data, providing the interval velocities and reflector´s depths with errors considered acceptable.

KEY WORDS: RAY TRACING, RAY THEORY, INVERSE PROBLEM, DOWNWARD CONTINUATION, INTERVAL VELOCITY

INTRODUCTION

The interval velocity is a basic parameter in seismic data processing, specially in depth migration and interpretation of seismic sections. In the high-frequency range the propagation of seismic wave is well described by paraxial ray theory. In order to estimate the interval velocity using a travel time inversion method we recourse to the mathematical formalism developed by Bortfeld (1989). The Bortfelds´s originally ideas were presented for earth models consisting of homogeneous layers. Using dynamic ray tracing, Hubral et al. (1992) extended posteriorly the validity of the formalism to layered media with lateral inhomogeneity. The parameter describing a paraxial ray are its distance to the central ray and the deviation of its slowness vector from that of the central ray. At any point of a paraxial ray, the parameters are linearly dependent on those at its initial point. The dependency is described by the propagator matrix, which is written as a product of many ray-segment propagator matrices, each one associated to each layer. After Kahn (1987) the ray-segment propagator matrix through one layer is decomposed in a product of matrix operators containing the parameters of the layer, and to determine the parameters of that layer is necessary to know the travel time function on its anterior surface. At the uppermost surface the travel time function of each reflecting interface can be estimated measuring the travel times of several rays in the neighborhood of the central ray. Based on the anterior ideas a method to estimate the interval velocities and reflector´s depth was developed. Test on synthetic data showed that the algorithm representing the method provides results with errors considered acceptables.

The solutions obtained are valid only in the second order approximation of the travel times function and its range of applicability corresponds to the hyperbolic dynamic correction.

TRAVELTIME FUNCTION

According the formalism developed by Bortfeld (1989), the earth model consists of a pile of homogeneous layers of any velocities and densities. The layers are separated by arbitrarily curved but smooth interfaces. The uppermost interface, called anterior surface, is assumed to be plane and contains sources and detectors. The reflecting interface is called the posterior surface. One normal ray to the reflecting interface, called central ray, intersects the anterior surface at the origin of the xyz coordinate system and the posterior surface at the origin of the $x'y'z'$ coordinate system. In the vicinity of the central ray, any transmitted ray is described by its initial position $\vec{x}$ and slowness $\vec{p}$ vectors, and by its final position $\vec{x}'$ and slowness $\vec{p}'$ vectors. Representation is by two- component vectors. The two-component position and slowness vectors are obtained by projecting the three-component vectors into the tangent plane to the anterior and posterior surface at the intersection point with the central ray. The vectors $\vec{x}$ and $\vec{p}$ of any transmitted ray determine the vectors $\vec{x}'$ and $\vec{p}'$. The first-order approximation of $\vec{x}$ and $\vec{p}$ sets a linear relationship:

$$\begin{bmatrix} \vec{x}' \\ \vec{p}' \end{bmatrix} = \begin{bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{bmatrix} \begin{bmatrix} \vec{x} \\ \vec{p} - \vec{p}_0 \end{bmatrix} \tag{1}$$

$\underline{A}$, $\underline{B}$, $\underline{C}$ and $\underline{D}$ are $2x2$ Jacobian matrices and $\vec{p}_0$ is the slowness vector of the central ray. Similar equations apply for reflected events. According Bortfeld, in the second-order approximation, the travel time of a transmitted ray from any source position at the anterior surface $\vec{x}$ to any receiver position $\vec{x}'$ at the reflecting interface is given by:

$$\begin{aligned} t(\vec{x},\vec{x}') = t_0 - \vec{p}_0 \cdot \vec{x} + \tfrac{1}{2}\vec{x}' \cdot \underline{D}\underline{B}^{-1}\vec{x}' \\ + \tfrac{1}{2}\vec{x} \cdot \underline{B}^{-1}\underline{A}\vec{x} - \vec{x} \cdot \underline{B}^{-1}\vec{x}' \end{aligned} \tag{2}$$

Two transmitted paraxial rays (one starting at $\vec{x}$ and $\vec{x}''$ other at) combine into a reflected one, with the travel time function $t(\vec{x},\vec{x}'')$ given by the sum of the travel times of the two transmitted rays:

$$\begin{aligned} t(\vec{x},\vec{x}'') = t_0 - 2\vec{p}_0 \tfrac{1}{2}(\vec{x}+\vec{x}'') + \tfrac{1}{2}(\vec{x}+\vec{x}'') \cdot \\ \underline{D}^{-1}\underline{C}\tfrac{1}{2}(\vec{x}+\vec{x}'') + \tfrac{1}{2}(\vec{x}''-\vec{x}) \cdot \underline{B}^{-1}\underline{A}\tfrac{1}{2}(\vec{x}''-\vec{x}) \end{aligned} \tag{3}$$

where t_0 is the two way travel time of the central ray.

Expression (3) is called the parabolic approximation of the travel time along the paraxial ray, and is not the best approximation. It is known a long time ago that in simple layered media, seismic near-vertical reflections are better approximated by hyperbolic rather than parabolic travel times curves.

If we square (3) and retain only its terms up to the second-order in $\vec{x}$ and $\vec{x}'$, the hyperbolic approximation of the travel time for the paraxial ray is obtained (Ursin, 1982).

$$\begin{aligned} t(x,x'')^2 = (t_0 - 2p_0 \cdot \tfrac{1}{2}(x+x''))^2 \\ + 2t_0(\tfrac{1}{2}(x+x'') \cdot \underline{D}^{-1}\underline{C}\tfrac{1}{2}(x+x'') \\ + \tfrac{1}{2}(x''-x) \cdot \underline{B}^{-1}\underline{A}\tfrac{1}{2}(x''-x))^2 \end{aligned} \tag{4}$$

Equations (3) and (4) contains the same nine unknown parameters: one in t_0, two in $\vec{p}_0$, three in $\underline{B}^{-1}\underline{A}$ and three in $\underline{D}^{-1}\underline{C}$ (because the matrices are symmetric), in consequence (3) and (4) can be determined registering at least nine travel times for rays in the vicinity of a central ray at uppermost surface. For each reflector interface in the model, a travel time function like equation (4) will be estimated at the uppermost surface.

INTERVAL VELOCITY

Considering the travel time function determined at the anterior surface of a layer and known the geometry of that interface ($\underline{K}_{N-1}$), we will show how the interval velocity of the layer can be determined, uniquely. The total transfer matrix, which describes the ray propagation in the N^{th} layer, can be decomposed after Kahn (1987) in a product of matrices:

$$\begin{aligned} \begin{bmatrix} \underline{A}_N & \underline{B}_N \\ \underline{C}_N & \underline{D}_N \end{bmatrix} = \begin{bmatrix} I & 0 \\ V_N^{-1}\underline{K}_N & I \end{bmatrix} \begin{bmatrix} I & V_N S_N I \\ 0 & I \end{bmatrix} \\ \begin{bmatrix} T & 0 \\ 0 & T^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ -cos(\beta_N)V_N^{-1}\underline{K}_{N-1}^{-1} & I \end{bmatrix} \end{aligned} \tag{5}$$

$$\text{with} \quad \underline{T} = T(\beta_N) = \begin{bmatrix} cos(\beta_N) & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

The capital letters in the matrix relation (6) are 2x2 matrices: e.g., $\underline{I}$ is the unity matrix, 0 is the zero matrix and $\underline{K}_N$ is the matrix of curvature of the N^{th} interface. The N^{th} interval velocity is V_N, and S_N the path length of the central ray in the layer. The angle between the normal vector of the $(N-1)^{th}$ interface at the intersection point with the central ray and the slowness vector of the central ray is β_N. The matrices on the right side of the transfer matrix are responsible for the following transfers of positions and slowness vectors, from right to left: (1) from the $(N-1)^{th}$ interface into the tangent plane, (2) from the tangent plane into the plane perpendicular to the central ray, (3) from the plane perpendicular to the central ray at the $(N-1)^{th}$ interface into a perpendicular plane to the central ray at the N^{th} interface, and finally from the tangent plane into the N^{th} interface, i.e. into the reflector. Δt_N is the one way travel time of the central ray through the N^{th} layer. In this step, besides $\underline{K}_{N-1}$, we assume to know the interval velocity of the superior layer V_{N-1} and the angle of incidence of the paraxial ray α_{N-1} at the anterior surface.

Multiplying the matrices in expression (5), we obtain the following equations:

$$\underline{A}_N = \underline{T}(\beta_N) - cos(\beta_N) S_N \underline{T}^{-1}(\beta_N) \underline{K}_{N-1} \tag{7}$$

$$\underline{B}_N = V_N S_N \underline{T}^{-1}(\beta_N) \tag{8}$$

$$\underline{C}_N = V_N^{-1} \underline{K}_N \underline{T}(\beta_N) - V_N^{-1} cos(\beta_N) [S_N \underline{K}_N + \underline{I}] \underline{T}^{-1}(\beta_N) \underline{K}_{N-1} \tag{9}$$

$$\underline{D}_N = [S_N \underline{K}_N + \underline{I}] \underline{T}^{-1}(\beta_N) \tag{10}$$

Determining the inverse of $\underline{B}_N$ from (8) and multiplying by (7) we obtain $\underline{B}_N^{-1} \underline{A}_N$, that we recalled $\underline{U}$:

$$\begin{bmatrix} u_{11} & u_{12} \\ u_{12} & u_{22} \end{bmatrix} = \frac{1}{V_N S_N} \underline{T}^2(\beta_N) - \frac{cos(\beta_N)}{V_N} \underline{K}_{N-1} \tag{12}$$

equation (12) is equivalent to the following three equations:

$$u_{11} = \frac{1}{V_N S_N} cos^2(\beta_N) - \frac{1}{V_N} cos(\beta_N) K_{N-1,11} \tag{13}$$

$$u_{12} = \frac{1}{V_N S_N} cos(\beta_N) K_{N-1,12} \tag{14}$$

$$u_{22} = \frac{1}{V_N S_N} - \frac{1}{V_N} cos(\beta_N) K_{N-1,22} \tag{15}$$

At the anterior surface of the layer the Snell's law is used:

$$cos^2(\beta_N) = 1 - sin^2(\alpha_{N-1}) \left\{ \frac{V_N}{V_{N-1}} \right\}^2 \tag{16}$$

where α_{N-1} is the angle of incidence and β_N is the angle of transmission of the central ray, see figure 1.

By definition the interval velocity is:

$$V_N = \frac{2 S_N}{\Delta t_N} \tag{17}$$

Solving simultaneously the equations (13), (14), (15), (16) and (17), see Appendix A, the interval velocity of the layer V_N is determined, except in case of a spherical reflector as was pointed by Krey (1989).This step permits to know also the three others unknowns S_N, Δt_N, β_N, and in consequence to determine $\underline{A}_N$ and $\underline{B}_N$.

GEOMETRY OF POSTERIOR SURFACE

To determine the geometry of the posterior surface is necessary to know S_N, Δt_N, β_N and V_N, i.e., which were solved the anterior step. To calculate the inverse of $\underline{D}_N$ Equation (10) is used:

$$\underline{D}_N^{-1} = \underline{T}(\beta_N) [S_N \underline{K}_N + \underline{I}]^{-1} \tag{18}$$

the product of (18) by (9) gives the matrix

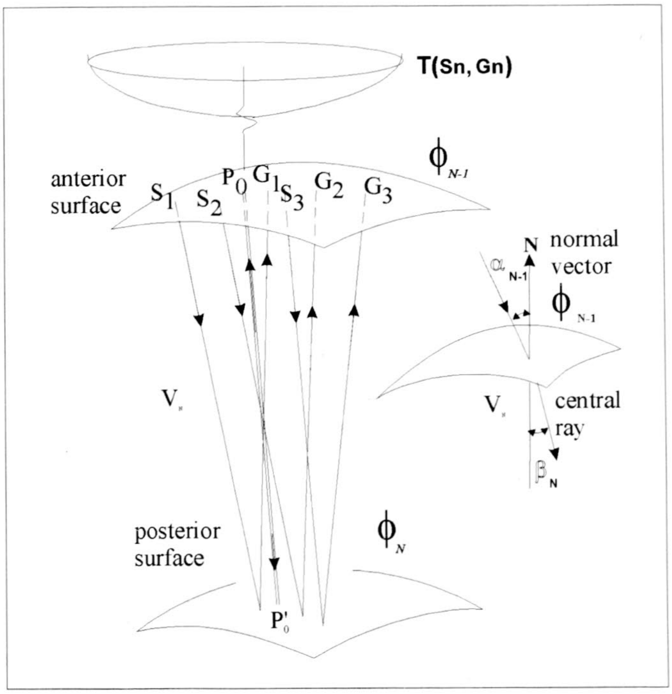

Figure 1. Left: A seismic system and the travel time function of rays reflected on the posterior surface, sources and detectors on the anterior surface. Right: The central ray cross the anterior surface with angles of incidence and transmission.

$$\underline{D}_N^{-1} \underline{C}_N = \underline{T}(\beta_N) [S_N \underline{K}_N + \underline{I}]^{-1} \{ \frac{1}{V_N} \underline{K}_N \underline{T}(\beta_N) - \frac{cos(\beta_N)}{V_N} [S_N \underline{K}_N + \underline{I}] \underline{T}^{-1}(\beta_N) \underline{K}_{N-1} \} \tag{19}$$

Reorganizing terms in (19)

$$V_N \underline{D}_N^{-1} \underline{C}_N = \underline{T}(\beta_N) [S_N \underline{K}_N + \underline{I}]^{-1} \{ \underline{K}_N \underline{T}(\beta_N) - cos(\beta_N) [S_N \underline{K}_N + \underline{I}] \underline{T}^{-1}(\beta_N) \underline{K}_{N-1} \} \tag{20}$$

then

$$\underline{T}(\beta_N) [S_N \underline{K}_N + \underline{I}]^{-1} \underline{K}_N \underline{T}(\beta_N) = V_N \underline{D}_N^{-1} \underline{C}_N + cos(\beta_N) \underline{K}_{N-1} \tag{21}$$

to find:

$$[S_N \underline{K}_N + \underline{I}]^{-1} \underline{K}_N = \underline{T}^{-1} \{ (\beta_N) V_N \underline{D}_N^{-1} \underline{C}_N + cos(\beta_N) \underline{K}_{N-1} \} \underline{T}^{-1}(\beta_N) \tag{22}$$

term simplified by recalling

$$\underline{Q} = \underline{T}^{-1}(\beta_N) \{ V_N \underline{D}_N^{-1} \underline{C}_N + cos(\beta_N) \underline{K}_{N-1} \} \underline{T}^{-1}(\beta_N)$$

to get:

$$\underline{K}_N = [S_N \underline{K}_N + \underline{I}] \underline{Q} \tag{23}$$

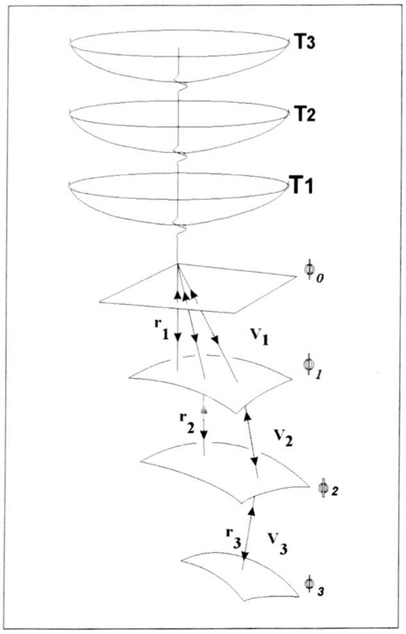

Figure 2. The model is analyzed like three seismic systems, each one with its own central ray and each reflector like its posterior surface. There are travel time functions associated to each system and estimated on the anterior surface (flat).

Finally the matrix of curvature of the posterior surface in the vicinity of the central ray $\underline{K}_N$ is obtained:

$$\underline{K}_N = \underline{Q}\{\underline{I} - S_N \underline{Q}\}^{-1} \tag{24}$$

Now, we can know numerically the transfer matrix which describes the ray propagation through the N^{th} layer, substituting the terms $V_N,\ S_N,\ \Delta t_N,\ \beta_N,\ \underline{K}_N$ in equation (5).

TRAVELTIME FUNCTION IN DEPTH

It will be necessary to introduce different central rays in the model, a central ray for each "piece" of seismic system. The first piece of seismic system is defined between the uppermost surface (zero interface) and the first reflecting surface, the second between the first one and the second one, and so on. Each piece is considered a seismic system with a central ray perpendicular to the corresponding reflecting surface at the midpoint position (see figure 2).

The determination of the travel time function in depth is interpreted like if sources and detectors were moved from the uppermost surface, following the raypaths, to the anterior surface of the deepest layer of the seismic system (see figure 3). We will show how to determine the travel time function at the anterior surface of the deepest layer from the knowledge of the travel time functions at the upper interfaces of the seismic system.

The propagator matrix describes the ray propagation of the paraxial rays through the system, and can be decomposed in a product of many-ray segment propagator matrices. In a system with N layers the propagator matrix is written as the product of the N individual propagator matrices of each "piece":

$$\begin{bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{bmatrix} = \begin{bmatrix} \underline{A}_N & \underline{B}_N \\ \underline{C}_N & \underline{D}_N \end{bmatrix} \prod_{j=1}^{k=N-1} \begin{bmatrix} \underline{A}_j & \underline{B}_j \\ \underline{C}_j & \underline{D}_j \end{bmatrix} \tag{25}$$

If we have solved the system composed by the upper N - 1 layers, i.e., the transfer matrix for each superimposed layer is known, then (25) is rewritten:

$$\begin{bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{bmatrix} = \begin{bmatrix} \underline{A}_N & \underline{B}_N \\ \underline{C}_N & \underline{D}_N \end{bmatrix} \begin{bmatrix} \underline{A}_{\Pi k} & \underline{B}_{\Pi k} \\ \underline{C}_{\Pi k} & \underline{D}_{\Pi k} \end{bmatrix} \tag{26}$$

where the matrix in the middle of (26) propagates the ray in the Nth layer (assumed unknown), and the left most matrix in (26) corresponds to the product of the N - 1 propagator matrices of the upper N - 1 solved layers, i.e., it has been assumed that the transfer matrix for each superimposed layer is numerically known. The transfer matrix posses the property of simplecticity (Borfeld & Kempert, 1990), this property means that the inverse of the propagator matrix of the system composed of N - 1 upper layers is:

$$\begin{bmatrix} \underline{A}_{\Pi k} & \underline{B}_{\Pi k} \\ \underline{C}_{\Pi k} & \underline{D}_{\Pi k} \end{bmatrix}^{-1} = \begin{bmatrix} \underline{D}^t_{\Pi k} & -\underline{B}^t_{\Pi k} \\ -\underline{C}^t_{\Pi k} & \underline{A}^t_{\Pi k} \end{bmatrix} \tag{27}$$

the super index means the matrix transposed, and by the same argument the inverse of the propagator matrix of the N^{th} layer is:

$$\begin{bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{bmatrix}^{-1} = \begin{bmatrix} \underline{D}^t_N & -\underline{B}_N \\ -\underline{C}_N & \underline{A}^t_N \end{bmatrix} \tag{28}$$

using (27) and (28) we transform (26) in:

$$\begin{bmatrix} \underline{D}^t_N & -\underline{B}^t_N \\ -\underline{C}^t_N & \underline{A}^t_N \end{bmatrix} = \begin{bmatrix} \underline{A}_{\Pi k} & \underline{B}_{\Pi k} \\ \underline{C}_{\Pi k} & \underline{D}_{\Pi k} \end{bmatrix} \begin{bmatrix} \underline{D}^t & -\underline{B}^t \\ -\underline{C}^t & \underline{A}^t \end{bmatrix} \tag{29}$$

$$\begin{bmatrix} \underline{D}^t_N & -\underline{B}^t_N \\ -\underline{C}^t_N & \underline{D}^t_N \end{bmatrix} = \begin{bmatrix} \underline{A}_{\Pi k}\underline{D}^t - \underline{B}_{\Pi k}\underline{C}^t & \underline{B}_{\Pi k}\underline{A}^t - \underline{A}_{\Pi k}\underline{B}^t \\ \underline{C}_{\Pi k}\underline{D}^t - \underline{D}_{\Pi k}\underline{C}^t & \underline{D}_{\Pi k}\underline{A}^t - \underline{C}_{\Pi k}\underline{B}^t \end{bmatrix} \tag{30}$$

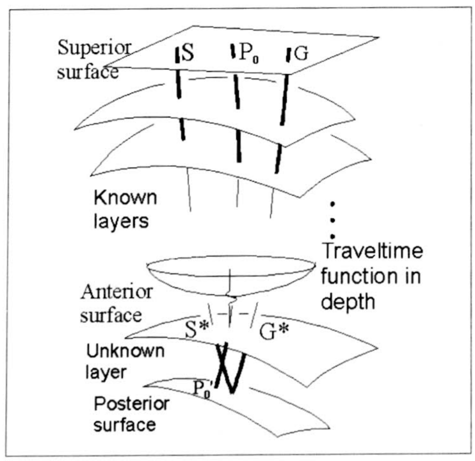

Figure 3. If the sources and detectors were moved from the superior surface to the anterior surface following the ray paths, then the travel time function is determined at the top of the deeper system.

By simmetry $\underline{D}_N^{-1}\underline{C}_N = \underline{C}_N^t \underline{D}_N^{-t}$ and from (30):

$$\underline{D}_N^{-1}\underline{C}_N = [\underline{D}_{\Pi k}\underline{C}^t - \underline{C}_{\Pi k}\underline{D}^t][\underline{A}_{\Pi k}\underline{D}^t - \underline{B}_{\Pi k}\underline{C}^t]^{-1} \quad (31)$$

factorizing $\underline{D}^t$ at left of (31)

$$D_N^{-1}C_N = [D_{\Pi k}C^t - C_{\Pi k}D^t][(A_{\Pi k} - B_{\Pi k}C^t D^{-t})D^t]^{-1}$$

(32)

The product of symetric matrices is conmutative, then

$$D_N^{-1}C_N = [D_{\Pi k}C^t - C_{\Pi k}D^t]D^t[(A_{\Pi k} - B_{\Pi k}C^t D^{-t})D^t]^{-1}$$

(C33)

$$D_N^{-1}C_N = [D_{\Pi k}(C^t D^{-t}) - C_{\Pi k}][A_{\Pi k} - B_{\Pi k}(C^t D^{-t})]^{-1}$$

(34)

by symmetry $\underline{D}^{-1}\underline{C} = \underline{C}^t\underline{D}^{-t}$ then

$$\underline{D}_N^{-1}\underline{C}_N = [\underline{D}_{\Pi k}(\underline{D}^{-1}\underline{C}) - \underline{C}_{\Pi k}][\underline{A}_{\Pi k} - \underline{B}_{\Pi k}(\underline{D}^{-1}\underline{C})]^{-1}$$

(35)

Following a similar process we demonstrate that

$$\underline{B}_N^{-1}\underline{A}_N = \{\underline{D}_{\Pi k}[\underline{B}^{-1}\underline{A}] - \underline{C}_{\Pi k}\}\{\underline{A}_{\Pi k} - \underline{B}_{\Pi k}[\underline{B}^{-1}\underline{A}]\} \quad (36)$$

This step makes possible to know the matrices $\underline{B}_N^{-1}\underline{A}_N$ and $\underline{D}_N^{-1}\underline{C}_N$ in depth. The two-way time T_{oN} and the slowness vector $\vec{p}_{oN}$ complete the knowledge of the travel time function in depth, these last two parameter are calculated by ray tracing through the model, as it is shown in the next procedure.

RAY TRACING

In this step, the interfaces limiting the N upper layers and their interval velocities are known. The paths of rays traveling across those layers can be calculated (see figure 4).

In the vicinity of the central ray, the N^{th} interface is approximated by the second order polynom $z = \frac{1}{2}(x, y)\cdot \underline{K}_N(x, y)$, where $K_N = (k_{11}, k_{12}, k_{22})$ is the matrix of curvature.

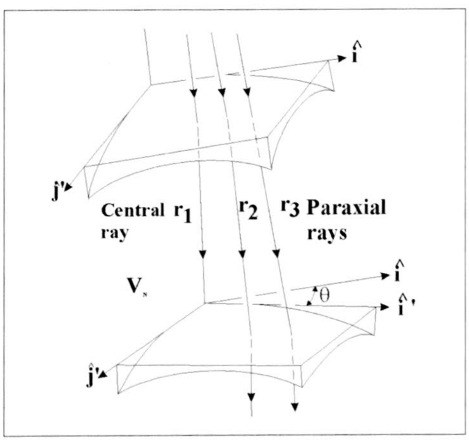

Figure 4. Ray tracing of the central and the paraxial rays through the last solved layer. The new coordinate system $(\hat{i}', \hat{j}')$ has rotated an angle θ.

According figure 5, the normal ray to the $N + 1$ interface is refracted at the N surface in the point:

$$(x, y, z) = -\vec{r}_1 + \Delta\vec{r} + \vec{r}_2 \quad (37)$$

where S_1, S_2 are the respective path lengths of the paraxial and central ray in the N^{th} layer, $\vec{r}_1 = (0, 0, S_1)$ and $\Delta\vec{r} = (\Delta r_1, \Delta r_2, \Delta r_3)$ are known. Therefore (37) is equivalent to equation (38).

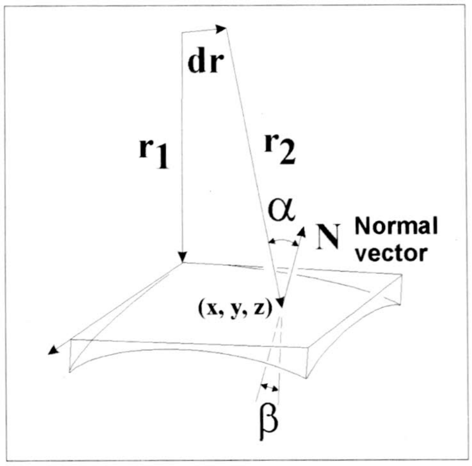

Figure 5. The ray $\vec{r}_2$ is normal to the next interface in depth, been refracted in (x, y, z). α, β: incidence and transmission angles.

Substituing x, y in the third expression of (38) we obtain:

$$S_2^2[\sin^2\beta(\tfrac{1}{2}k_{11}\cos^2\theta + \tfrac{1}{2}k_{22}\sin^2\theta + k_{12}\sin\theta\cos\theta)] + S_2[\sin\beta\{k_{11}\cos\theta\Delta r_1 + k_{22}\sin\theta\Delta r_2 + k_{12}(\sin\theta\Delta r_1 + \cos\theta\Delta r_2)\} - \cos\beta] + [S_1 - \Delta r_3 + \tfrac{1}{2}(k_{11}\Delta r_1^2 + k_{22}\Delta r_2^2 + k_{12}\Delta r_1\Delta r_2)] = 0$$

(39)

which has the form $AS_2^2 + BS_2 + C = 0$. With S_2 known, we calculated: the position (x, y, z), the normal vector in this point by $\vec{N} = \frac{\nabla\phi_N}{|\nabla\phi_N|}$ and the incidence angle in the N^{th} interface by $\cos\alpha_N = -\frac{\vec{N}\cdot\vec{r}}{|\vec{r}|}$. Using the Snell´s law, the refracted angle β_{N+1} of the paraxial ray is determined. Now is possible to estimate the time used by the paraxial ray to travel through the upper known layers and the remaining time to reach the posterior interface of the next unknown layer, i.e, we estimate the travel time function at the anterior surface of the unknown layer, to be solved.

APPLICATION TO SYNTHETIC DATA

Synthetic data were generated in several models using the ray tracing program Anis_ray_3D, developed by Costa et al, at the UFPa in Brazil (1993), but only two of them are shown here. Two programs were developed, the first one to estimate the travel time function at the uppermost interface of the seismic system, and the second one to determine the interval velocities by the estimation of the travel time in depth as was explained in before.

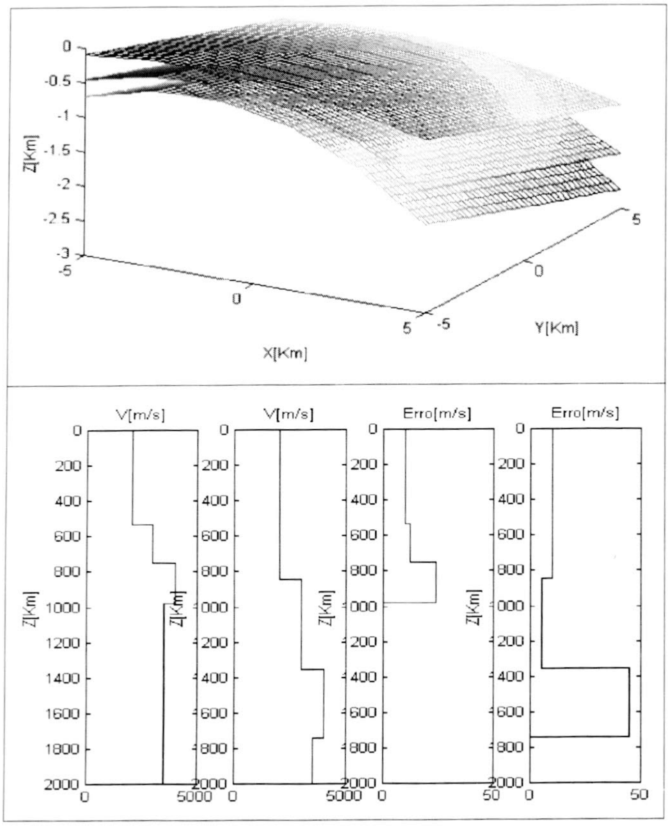

Figure 6. On top the model used to test the algorithm. On bottom interval velocities and erros obtained in two points at the superior surface.

Besides, the second program also calculates the matrices of curvature of the interfaces an the points where the rays intercept them.

Figure 6 shows on top the first model used to generate the travel time and in bottom the interval velocity model obtained by the program, in two different positions fo the model. The two leftmost graphics in figure 6 shows the errors in estimating the interval velocities, being fewer than 50 m/s.

A second model was used to show the program estimating the points where the rays reach the interfaces; the model is shown in top, figure 7, and in bottom the final position reached by the rays calculated by the program. In the third interface the program smoothes it, satisfying the restriction impossed on the surfaces of the models. The others points show a well fitted with the interfaces.

CONCLUSIONS

The second order approximation of the travel time and the paraxial ray theory make possible to develop an inversion method to estimate the interval velocities in models with no structural complexity or strong dipping interfaces. The method is a tool to estimate interval velocity and reflector depths, based on post stacked seismic data. Due to the paraxial ray theory difficulties could occur when the positions of sources and detectors exceed the validity of the approximation, being valid only for small values of distance to the zero-offset ray position. Its range of applicability correspond to the hyperbolic dynamic correction. Because

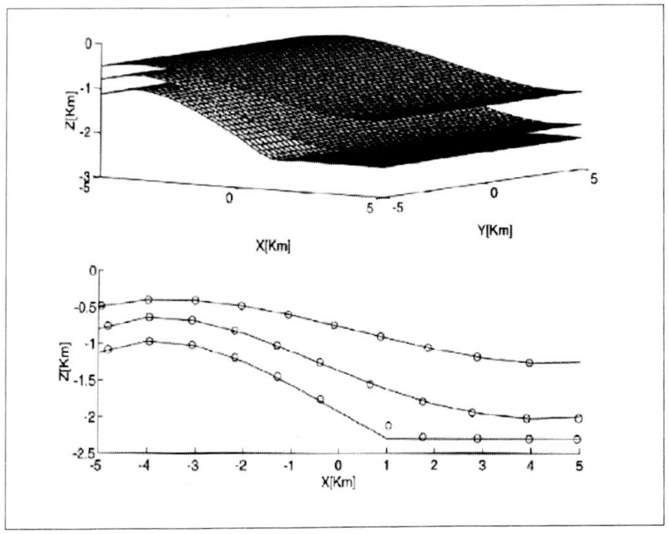

Figure 7. On top the second model and on bottom the interfaces and the points where the algorithm estimates final positions for normal rays.

the input data are travel time measurements on seismic traces, a good quality on seismic data is impossed. Nevertheless the method provides the inverse solution without a priori knowledge of the model, just using travel time measurements on the uppermost surface.

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APPENDIX

DETERMINING THE INTERVAL VELOCITY

To determine the interval velocity in the N^{th} layer is necessary to solve the set of simultaneous equations, represented by the matrix equation $\underline{B}^{-1}\underline{A}$:

$$u_{11} = \frac{1}{V_N S_N} cos^2(\beta_N) - \frac{1}{V_N} cos(\beta_N) K_{N-1,11} \quad \text{(A1)}$$

$$u_{12} = \frac{1}{V_N S_N} cos(\beta_N) K_{N-1,12} \quad \text{(A2)}$$

$$u_{12} = \frac{1}{V_N S_N} cos(\beta_N) K_{N-1,12} \quad \text{(A3)}$$

the Snell's law at the anterior surface of the N[th] layer

$$cos^2(\beta_N) = 1 - sin^2(\alpha_{N-1})\left\{\frac{V_N}{V_{N-1}}\right\}^2 \quad \text{(A4)}$$

with angles α_{N-1} of incidence and β_N of transmission, and the interval velocity

$$V_N = \frac{2S_N}{\Delta t_N} \quad \text{(A5)}$$

In order to solve (A1), (A2), (A3), (A4) and (A5) we will consider five different cases:

First: when $k_{N-1,11} = k_{N-1,12} = k_{N-1,22} = 0$, i.e., the anterior surface is a flat one, or when $k_{N-1,12} = k_{N-1,22} = 0$ and $k_{N-1,11} \neq 0$, substituting (A5) in the reduced equation (A3)

$$V_N = \sqrt{\frac{2}{u_{22}\Delta t_N}} \quad \text{(A6)}$$

Second: when $k_{N-1,12} \neq 0$ we use equations (A2) and (A4) to get

$$V_N^2 = \left(\frac{k_{N-1,12}}{u_{12}}\right)^2 \left(1 - sin^2(\alpha_{N-1})\frac{V_N^2}{V_{N-1}^2}\right) \quad \text{(A7)}$$

and finally

$$V_N = \left(\frac{u_{12}}{k_{N-1,12}}\right)^2 + \left(\frac{sin^2(\alpha_{N-1})}{V_{N-1}^2}\right) \quad \text{(A8)}$$

Third: when $k_{N-1,12} = k_{N-1,11} = 0$ and $k_{N-1,22} \neq 0$ we substitute (A5) in (A1)

$$u_{11} = \frac{1}{V_N S_N}\left(1 - 1 - sin^2(\alpha_{N-1})\frac{V_N^2}{V_{N-1}^2}\right) \quad \text{to get (A9)}$$

$$V_N = \sqrt{\frac{\Delta t_N u_{11}}{2} + \frac{sin^2(\alpha_{N-1})}{V_{N-1}^2}} \quad \text{(A10)}$$

Fourth: when $k_{N-1,22} \neq k_{N-1,11} \neq 0$ and $k_{N-1,12} = 0$ we rewrite equation (A1)

$$\frac{u_{11}}{K_{N-1,11}} = \frac{cos^2(\beta_N)}{V_N S_N K_{N-1,11}} - \frac{cos(\beta_N)}{V_N} \quad \text{(A11)}$$

and equation (A3)

$$\frac{u_{22}}{K_{N-1,22}} = \frac{1}{V_N S_N K_{N-1,22}} - \frac{cos(\beta_N)}{V_N} \quad \text{(A12)}$$

Subtracting (A12) from (A11) and factorizing the factor $V_N S_N$ we obtain

$$V_N S_N\left(u_{11} - u_{12}\frac{k_{N-1,11}}{k_{N-1,12}}\right) + \frac{k_{N-1,11}}{k_{N-1,12}} = 1 - sin^2(\alpha_{N-1})\frac{V_N}{V_{N-1}} \quad \text{(A13)}$$

including (A5) in (A13) and reorganizing terms,the interval velocity is

$$V_N = \sqrt{\frac{k_{N-1}^{-1} - k_{N-1,22}^{-1}}{\frac{sin^2(\alpha_{N-1})}{k_{N-1}V_{N-1}^2} + \frac{\Delta t_N}{2}\left(\frac{u_{11}}{k_{N-1}} - \frac{u_{22}}{k_{N-1,22}}\right)}} \quad \text{(A14)}$$

Fifth: when $k_{N-1,22} = k_{N-1,11} \neq 0$ and $k_{N-1,12} = 0$, i.e., the anterior surface is a spherical one. From (A3)

$$cos(\beta_N) = \frac{1}{S_N K_{N-1,22}} - \frac{V_N u_{22}}{K_{N-1,22}} \quad \text{(A15)}$$

and using (A4) in (A15) and squaring the resulting expression we have

$$1 - sin^2(\alpha_{N-1})\left(\frac{V_N}{V_{N-1}}\right)^2 = \left(\frac{1}{k_{N-1,22}S_N} - \frac{V_N u_{22}}{k_{N-1,22}}\right)^2 \quad \text{(A16)}$$

and substituting (A5) in (A16)

$$k_{N-1,22}^2 - k_{N-1,22}^2 sin^2(\alpha_{N-1})\left(\frac{V_N}{V_{N-1}}\right)^2 = \frac{4}{V_N^2\Delta t_N^2} - 4\frac{u_{22}}{\Delta t_N} + V_N^2 u_{22} \quad \text{(A17)}$$

The expression (A17) is reorganized in terms of V_N

$$\left(u_{22} + k_{N-1,22}\frac{sin^2(\alpha_{N-1})}{V_{N-1}^2}\right)V_N^4 + \left(-k_{N-1,22} - 4\frac{u_{22}}{\Delta t_N}\right)V_N^2 + \frac{4}{\Delta t_N^2} = 0 \quad \text{(A18)}$$

The expression (A18) has the form $a(V_N^2)^2 + b(V_N^2) + c = 0$ and its solution is given by $V_N^2 = (2a)^{-1}(-b \pm \sqrt{b^2 - 4ac})$, in consequence the square of the interval velocity can be known but there is an ambiguity to determine the interval, i.e., the unique solution does not exist for this particular situation. The result shown here was first reported by Kahn (1987) and later by Krey (1989).