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Developing Algebraic Thinking Skills among Grade Three Pupils through Pictorial Models

ABSTRACT*: During the primary grades, young children work with patterns. At an early age, children have a natural love for Mathematics, and their curiosity is a strong motivator as they try to describe and extend patterns of shapes, colors, sounds, and eventually letters and numbers. At a young age, children can begin to make generalizations about patterns that seem to be the same or different. This kind of categorizing and generalizing is an important developmental step on the journey toward Algebraic thinking. Algebra instruction has traditionally been postponed until adolescence, because of the assumptions about psychological development and developmental readiness. Concrete operational children tend to be capable of mental operations as long as they relate to real objects, events, and situations. As they mature, they are able to work with more abstract concepts without the aid of concrete objects. The study attempted to determine the effect of using pictorial models on the Algebraic thinking skills of grade three pupils. The one-group pre-test – post-test experimental research design was used in this study. Twenty-eight grade three pupils participated in the study. To determine the effect of using pictorial models, an Algebraic thinking skills test was given to the pupils before and after using pictorial models. Results showed that the use of pictorial models significantly improved the Algebraic thinking skills of the pupils. Interviews from the pupils revealed that pictorial models helped them to solve problems easier. The findings suggest that Algebraic thinking can be taught even at the early age.*
KEY WORDS*: Algebraic Thinking; Young Children; Patterns; Letters and Numbers; Pictorial Model.*

INTRODUCTION

During the primary grades, young children work with patterns. At an early age, children have a natural love for Mathematics, and their curiosity is a strong motivator as they try to describe and extend patterns of shapes, colors, sounds, and eventually letters and numbers. At a young age, children can begin to make generalizations about patterns that seem to be the same or different. This kind of categorizing and generalizing is an important developmental step on the journey toward algebraic thinking (Seeley, 2004; Warren, 2007; and Stump, 2011).

Algebra instruction has traditionally been postponed until adolescence, because of the assumptions about psychological development and developmental readiness. Concrete operational children, according to J. Piaget (1952 and 1994) and others, tend to be capable of mental operations as long as they relate to real objects, events, and situations. As they mature, they are able to work with more abstract concepts without the aid of concrete objects (Piaget, 1952 and 1994; Blair, 2003; Simatwa, 2010; Bautista & Francisco, 2011; and Vorpal, 2012).

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Some researches provide evidence that young students, aged 9-10 years, can make use of Algebraic ideas and representations typically absent from the early Mathematics curriculum, and thought to be beyond students' reach (Bednarz & Janvier, 1996; Blanton & Kaput, 2004; and Badger & Velatini, 2010). One of these researches comes from a 30-month longitudinal classroom study of four classrooms in a public school in Massachusetts, with students from grades two to four. The data help clarify the conditions under which young students can integrate algebraic concepts and representations into their thinking (Carraher *et al*., 2001).

An increasing number of Mathematics educators, policy makers, and researchers believe that Algebra should become part of the elementary education curriculum. The NCTM (National Council of Teachers of Mathematics), in 2000, and a special commission of the RAND (Research And Development) Corporation, in 2003, have welcomed the integration of Algebra into the early Mathematics curricula. These endorsements, however, do not diminish the need for research. On the contrary, they highlight the need for a solid research base for guiding the Mathematics education community along this new venture (NCTM, 2000; and RAND Corporation, 2003).

Within the past decade, discussions and opinions pertaining to school Algebra have changed dramatically. Professional organizations, policy makers, Mathematicians, Mathematics educators, administrators, and teachers, who once considered Algebra as a course just for university-bound students, now espouse the notion of Algebra for all students. Overarching policy statements from these groups and individuals indicate a widening of the range of topics that constitute Algebraic thinking to encompass now more than just the structural aspects of Algebra (Ferrucci, 2004). This broader description of thinking Algebraically has led to the introduction of Algebraic ideas into the curriculum at much earlier grade levels.

It is now widely understood that preparing elementary students for the increasingly complex Mathematics of this century requires an approach different from the traditional methods of teaching Arithmetic in the early grades, specifically an approach that cultivates habits of mind that attend to the deeper, underlying structure of Mathematics (Booker *et al*., 2004; Blanton & Kaput, 2008; and Brizuela & Schliemann, 2003). Hence, it explains the purpose of this study on pictorial models in developing Algebraic thinking of primary pupils was made with the hope that it could help improve the basic Mathematics foundation of our learners.

Students and teachers need to appreciate that there can be a number of ways to visualize a problem, as well as number of ways to solve a problem non-visually. Some students might benefit from visualization more than others. Sometimes, students resist using visual models, when a solution is readily apparent to them. Mathematics teachers should always be open to various approaches that can be introduced to the pupils for them to develop higher-order thinking skills.

Young children today need a different kind of Mathematics from what their parents learned. This study was intended to introduce the use of pictorial models that can be of great help in helping the primary pupils formulate and manipulate Algebraic expressions, and understand relationship between quantities in the problem and leading them to a strategy in solving it.

This study is indeed important to the Mathematics teachers and the learners as well, because it established a basis on what must be given more focus on the elementary curriculum, and how the Arithmetic thinking can be developed into Algebraic thinking, thus promoting a deeper understanding of the Mathematical concepts.

CONCEPTUAL FRAMEWORK

This study was based on the following theories: Jerome S. Bruner's Constructivism and Development Learning and Visual Learning Theory, in 1960. Jerome S. Bruner's theoretical framework is based on the notion that learners construct new ideas or concepts based upon existing knowledge. Learning is an active process. Facets of the process include selection and transformation of information,

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decision making, generating hypotheses, and making meaning from information and experiences (Bruner, 1960).

Jerome S. Bruner (1960) postulated three stages of intellectual development. The first stage, he termed "Enactive" transpires, when a person learns about the world through actions on physical objects and the outcomes of these actions. He calls second stage "Iconic", when learning takes place through the use of models and pictures. The final stage is termed "Symbolic", which occurs when the learner develops the capacity to think in abstract terms. Based on this three-stage notion about the development of the intellect, Jerome S. Bruner recommended using a combination of concrete, pictorial, and symbolic activities believing that this will lead to more effective learning (Bruner, 1960).

Jerome S. Bruner (1960) inspired an approach in teaching Mathematics, which is called the CPA (Concrete-Pictorial-Abstract) approach. *Concrete* components include manipulatives, measuring tools, or other objects that the students can handle during the lesson. *Pictorial* representations include drawings, diagrams, charts, or graphs that are drawn by the students or are provided for the students to read and interpret. *Abstract* refers to symbolic representations, such as numbers or letters that the student writes or interprets to demonstrate understanding of a task (Bruner, 1960).

Research has shown that visual learning theory is especially appropriate for the attainment of Mathematics skills for a wide range of learners. Understanding abstract Math concepts is reliant on the ability to "see" how they work; and children naturally use visual models to solve Mathematical problems. They are often able to visualize a problem as a set of images. By creating models, they interact with Mathematical concepts, process information, observe changes, reflect on their experiences, modify their thinking, and draw conclusions (Warren & Cooper, 2005; and Murphy, 2006).

One example of a pictorial model is

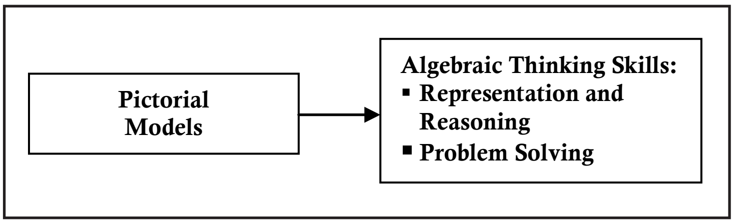

Figure 1:
Conceptual Paradigm

a structure comprised of rectangles and numerical values that represent all the information and relationships presented in a given problem. The rectangles replace the unknown represented by letters in equations. The rectangle, known as a unit, becomes the "generator" of the model about which other relations are constructed. The model method can be used to solve an Arithmetic problem, where pupils work with known values to solve the unknown. As pupils progress, the model method is used to solve Algebraic problems involving unknowns, part-whole concept, and proportional reasoning.

In view of the theoretical bases of this study, the conceptual paradigm is shown in the figure 1.

Algebraic thinking skills are organized into two general categories: (1) problem solving skills; and (2) representation and reasoning skills. Young children are capable of making generalizations and constructing ways of representing them (Kaput, 2004; and Windsor, 2007). These generalizations make powerful Mathematical ideas accessible to students to solve problems and to deepen understanding. Generalization and formalization involve the articulation and representation of unifying ideas that make explicit important Mathematical relationships. Thus, these forms of thinking build directly on conceptions of understanding as constructing relationships and reflecting on and articulating those relationships (Carpenter & Lehrer, 1999).

Problem solving is knowing what to do, when one doesn't know what to do. Students who have a tool kit of problem solving strategies, e.g. guess and check, make a list, work backwards, use a model, solve a simpler problem, etc., are better able to get started on a problem, attack the problem, and figure out what to do. Giving students opportunities to explore Math problems, by using multiple approaches or devising Math problems that have multiple solutions, allows students to not only develop good problem solving skills, but also to experience the utility of Mathematics. Children initially solve problems by modeling the problem situations using physical materials. By reflecting on the modeling strategies, children abstract these strategies so that they no longer need the actual materials to solve the problem (Yeap, 1997; and Carpenter & Lehrer, 1999).

The ability to use and make connections among multiple representations of Mathematical information gives us quantitative communication tools. Mathematical relationships can be displayed in many forms: *visually*, i.e. diagrams, pictures, or graphs; *numerically,* i.e. tables and lists; *symbolically*; and *verbally*. Often a good Mathematical explanation includes several of these representations, because each one contributes to the understanding of the ideas presented. The ability to create, interpret, and translate among representations gives students powerful tools for Mathematical thinking (Carpenter & Levi, 2000; Carpenter, Franke & Levi, 2003; and Cai & Knuth, 2005).

Shelley Kriegler (2008) pointed out that the ability to think and reason is fundamental in Mathematics. Understanding of the core ideas can influence one's success in solving word problems, the strategies they use in their solution processes, and the justifications they provide for the solutions. Inductive reasoning involves examining particular cases, identifying patterns and relationships among those cases, and extending the patterns and relationships. Deductive reasoning involves drawing conclusions by examining a problem's structure (Kriegler, 2008).

The development of Algebraic thinking does not emphasize manipulating symbols, but rather encourage children to make explicit ideas and to construct ways to represent those ideas for thinking about them and for communicating them. In this context, E. Yackel (1997) posited that non numerical reasoning about quantities is foundational to Algebraic reasoning. Algebraic reasoning in the elementary level can be accomplished through activities that encourage children to move beyond numerical reasoning to more general reasoning about relationships, quantities, and ways of notating and symbolizing (Yackel, 1997).

In the teaching of Mathematics, words, numbers, and pictures should come together to clearly demonstrate what is taking place (Murphy, 2006). Using pictorial models, pupils can help make sense of complex data. Through the children's image-making, they have the capacity to internalize and make connections to other areas of learning.

The main purpose of this study was to determine the effect of the use of pictorial models in developing Algebraic thinking among primary pupils. Specifically, this study sought answers to the following questions: (1) What are the pupils' Algebraic thinking skills before and after using pictorial models in terms of the following: representation and reasoning, and problem solving?; (2) Is there a significant difference between the pre-test and post-test mean scores of the participants?; and (3) What are pupils' experiences on the use of pictorial models?

METHODS

Research Design. This study utilized the experimental method of research, specifically the one-group pre-test – post-test design. The experimental research design is appropriate to this study, because it is the only design that can truly test a hypothesis concerning cause-and-effect relationship (Sevilla *et al.*, 1992).

Participants of the Study. There were 28 grade three pupils at the La Salle Green Hills in the Philippines, who participated in this study. These pupils were heterogeneously grouped according to their mental ability: nine pupils belong to the Above Average group, 13 from the Average group, and six from the Below Average group. The classification of the pupils was based from the result of the SAT (School Ability Test) given by the Guidance Office, when these pupils were in grade two.

Grade three pupils were chosen as the participants of the study, since they belong to the primary level and they are being trained to analyze word problems using pictorial models. LSGH (La Salle Green

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Hills) Mathematics Area, in the Grade School Department, started introducing the use of pictorial models in solving word problems to grades one to seven pupils in the beginning of school year 2011-2012.

Though the use of pictorial method is not fully implemented yet, the participants have an idea on how to apply the method. However, there is still a need to explore other means of applying this method, and how useful the method is in developing the Algebraic thinking of the pupils.

RESEARCH INSTRUMENTS

The following research instruments were used to gather the needed data and information relevant to the study.

Algebraic Thinking Skills Test. The test was developed to measure the following Algebraic thinking skills adapted from Shelley Kriegler (2008), namely: problem solving skills, representation skills, and reasoning skills (Kriegler, 2008). The test was divided into two parts, which are as follows:

Part I (Multiple Choice). This part consists of 20 multiple-choice items. These items aim to test the pupils' ability to generate, represent, and justify generalizations about fundamental properties of Arithmetic.

Part II (Problem Solving). The second part consists of five open-ended word problems, which are Algebraic in nature. This part tested how the pupils apply pictorial models in arriving at their answers (Kriegler, 2008).

In developing the test instrument, a table of specifications was prepared. This instrument underwent content validation through the help of three teacher-experts: a high school Algebra teacher, an elementary Algebra teacher, and a grade school Mathematics coordinator. The solution and the answers to problems were checked according to the rubric provided.

Scoring Rubric. The scoring rubric was used to determine how the participants performed in problem solving. A *rubric* is a set of criteria used to determine scoring for an assignment, performance, or product. Analytical rubric is an example of a scoring designed to assess students' work based on specified criteria and different degrees of quality of the assignment.

In problem solving, the pupils were given scores according to the following criteria: accuracy, understanding, communication, and Mathematical reasoning and strategies. Again, the three teacher-experts helped in preparing this scoring rubric.

Interview Guide. This was used to identify how the pupils answered the test questions, particularly the problem solving part. Hence, this was helpful in verifying answers of the participants. Informal interviews were also conducted during the teaching-learning process, wherein the pupils explained how the pictorial models helped them arrived at their answers.

Lesson Plans/Work Plans. The work plans served as guide in conducting the lesson. A seven-week semi-detailed plan was prepared before the experiment took place. Topics covered were fractions and decimals. The learning activities were categorized into two parts: (1) the preliminary activities, which are composed of drills, mental problem, and review; and (2) the developmental activities, which include motivation, presentation of the lesson, comparison and abstraction, generalization, and application. The preliminary activities present mental problem that can be solved using the pictorial models. Each lesson focused mainly on developing algebraic thinking of the pupils.

Data Gathering Procedure. To gather data relevant to the study, permission to administer the tests was sought from the grade school principal, assistant principal, student activities coordinator, and Mathematics coordinator. With the approval of the request, the list of materials needed for the study were immediately prepared.

The process of data gathering involved three stages: Pre-Experimental Stage, Experimental Stage, and Post-Experimental Stage.

Stage 1: *Pre-Experimental Stage.* A pre-test was administered to the pupils a week before the actual implementation of the study. This was done to identify how the pupils think prior to the discussion of the topics.

Stage 2: *Experimental Stage.* The lessons on fractions and decimals were taught for seven weeks, following the work plans prepared. The pupils was exposed to drills that involve finding the value of unknown and problem solving that involve addition and subtraction of whole

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Table 1:
Results of the Pre-Test and Post-Test of the Participants
in Terms of Representation and Reasoning (Multiple-Choice Type)

	Mean	Standard Deviation
Pre-Test	7.11	2.53
Post-Test	14.29	2.54

Table 2:
Results of the Pre-Test and Post-Test of the Participants
in Terms of Problem Solving (Open-Ended Type)

	Mean	Standard Deviation
Pre-Test	3.26	1.81
Post-Test	6.23	2.51

numbers, fractions and decimals, using the pictorial method.

The lessons on fractions and decimals were introduced using the CPA (Concrete-Pictorial-Abstract) approach from Jerome S. Bruner (1960). The use of manipulations like food/bacon shared with the class, which the pupils represented through drawings and blocks greatly helped the pupils in understanding the concepts. During this phase, interviews were conducted as follow-ups to the pupils' answers during the discussion.

Stage 3: *Post-Experimental Phase.* The post-test was given after seven weeks of exposure to the pictorial models to find out if there are improvements in the Algebraic thinking of the respondents. Afterwards, the pupils were asked to summarize how the use of pictorial models helped them in their Mathematics class.

***Data Analysis Procedure*.** The following statistical tools were used in this study. Firstly, Mean and Standard Deviation. These were used to describe the pupils' scores in the Algebraic thinking skills test before and after using the pictorial models.

Secondly, t-test for dependent samples. This was utilized to determine if a significant difference between the pre-test and post-test mean scores of the pupils.

RESULTS AND DISCUSSION

Table 1 presents the Mean and Standard Deviation of the pupils' scores in part I of the test, which measures their representation and reasoning skills. The scores in the reasoning and representation skills show a marked improvement. This result may be attributed to their conceptual understanding of the Mathematical concepts. Conceptual understanding refers to their integrated and functional grasp of Mathematical ideas.

It can be deduced that the pupils developed a deep understanding of the concepts learned, and they were able to manipulate numbers and represent value of the unknown after using pictorial models.

M. Burns (2004) said that teachers in the lower grades routinely focus on teaching procedures, rather than on conceptual understanding. Pupils are able to perform tasks, but they do not understand why they work (Burns, 2004). Some researchers say that the difficulty to transfer from rote Arithmetic operations of quantities to Algebraic thinking is due to the lack of conceptual understanding (*cf* Desforges, Hughes & Mitchell, 2000; Burns, 2004; Kaput, Carracher & Blanton eds., 2008; and Clark, 2009). For the primary pupils to understand the concepts of fractions and decimals better, pictorial models should be used.

As J. Kaput, D. Carracher & M. Blanton eds. (2008) suggest, the primary goal of early Algebra is that pupils learn to see and to express generalization in Mathematics. Algebraic thinking cannot be developed without the pupils' understanding of the numerical relationships, operations, and properties (Kaput, Carracher & Blanton eds., 2008:60). The NCTM (National Council of Teachers of Mathematics), in 2000, emphasized that "*Algebra is more than the manipulation of symbols*"; and that its study should start from the earliest years of schooling (NCTM, 2000).

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Table 3:
Results of the t-Test for the Comparison of the Pupils' Pre-Test and Post-Test Mean Scores

	Mean	Standard Deviation	Mean Difference	t-value	Critical Value	Interpretation
Pre-Test	7.11	2.54	7.18	16.22	2.77	Significant
Post-Test	14.29	2.53				

Table 2 presents the pupils' performance in problem solving. Though there is an increase in the scores of the pupils, there is still a need to focus more in developing their problem solving skills. Mathematical problem solving has been instrumental in achieving a variety of goals within the Mathematics curriculum. A classroom environment that values and promotes problem solving can facilitate Algebraic thinking.

In using pictorial method, a block may represent one-fourth and four blocks may represent one whole. In this regard, the pupils will be able to understand the relation between 1/4 and 1 whole. Pupils will also learn to represent values using blocks, as two blocks as to 2/4 and three blocks are equal to 3/4.

Illustrations, in figures 2 to 5, show some excerpts from the pupils' work on how they applied the pictorial model in answering word problems that are Algebraic in nature.

Figure 2 shows that the pupil used a circle to represent a set of apples. Each half was labeled as four. Two halves equal eight. The picture shows that the concept of half in relation to a whole must be fully understood by the pupils. The pupils had to utilize their knowledge of fractions to draw the model properly. In teaching fractions, the CPA (Concrete-Pictorial-Abstract) approach is helpful in developing the pupils' conceptual understanding (cf Bruner, 1960; Walle, 2004; and Diezzman & McCosker, 2011).

The answer proved that the pupil was able to show visual representation of the concrete objects. Thus, this helped him/her visualized Mathematical operations during problem solving.

Problem A:
Mark gave 4 apples to Lorna. If what he shared represents 1/2 of his apples, how many apples did he have at first?

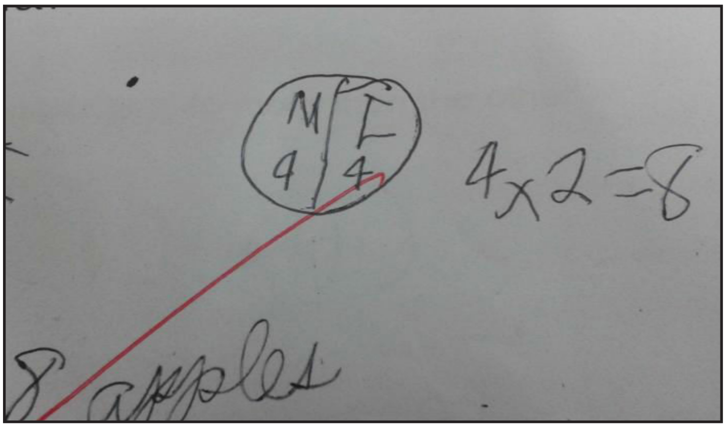

Figure 2:
Pupil's Solution to Problem A

Problem B:
Marko and Anna have P 24.00 together. If Marko has twice as much as Anna's, how much does Anna have?

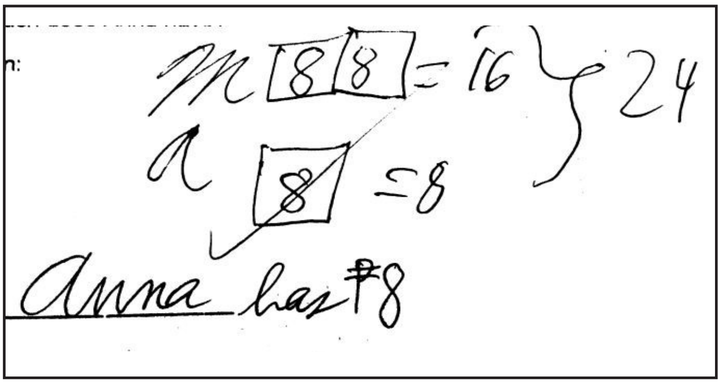

Figure 3:
Pupil's Solution to Problem B

Using the pictorial model, the pupil used two blocks to represent Marko's money. Two blocks represent Marko's since he has twice as much as Anna's. The pupils should be able to understand the concept of sum, in relation to its parts/addends to be able to solve this problem. As suggested by Carolyn Kieran (2004), pictorial equation solving can help students to focus on both representing and solving the problem rather than on merely solving it (cf Lee et al., 2004; Kieran, 2004; and Ronda, 2004). See figure 3.

The solution in figure 4 shows that the pupil drew 3 blocks for Jeremy's number of stickers, since Janna has only one-third of Jeremy's. In this kind of problem, if the pupil doesn't fully understand the concept of fraction in relation to a whole, wrong representation could be made, as three units for Janna instead of the other way around. This only proves that teaching Algebraic thinking should start from developing the conceptual understanding of the pupils.

Problem C:
Janna and Jeremy have 36 stickers together. Janna's stickers' are 1/3 of Jeremy's. How many stickers does Janna have?

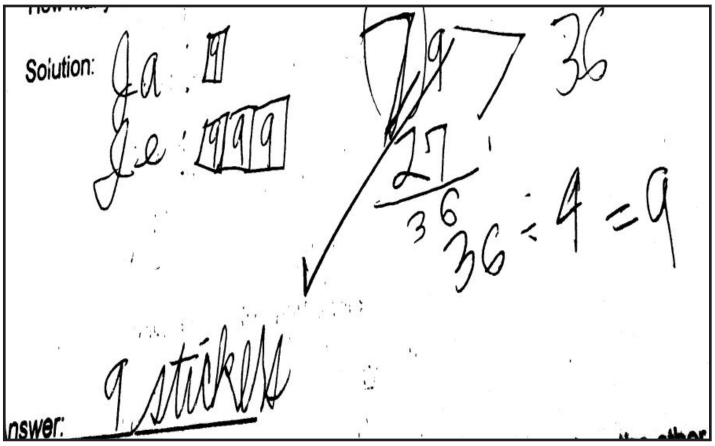

Figure 4:
Pupil's Solution to Problem C

It can be seen from figure 5 that the pupil subtracted the difference from the total (12.40 – 3.40) to get the remaining parts. Since there were 2 blocks left, he equally divided 9.00 into two equal parts, hence each number has 4.5. The first number is 3.4 more than the second number, which is 4.5. In this solution, three operations were used: addition, subtraction, and division. Though the pupil was able to arrive at the correct answer, the way he represented the numbers must be corrected. That is a bigger number should represent a bigger block.

Problem D:
The sum of two numbers is 12.40. The first number is 3.40 more than the other number. What are the two numbers?

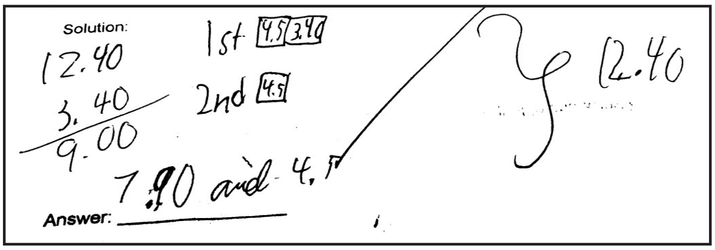

Figure 5:
Pupil's Solution to Problem D

***Comparison of the Pupils' Pre-Test and Post-Test Mean Scores*.** Table 3 shows that there is a significant difference between the pre-test and post-test of the participants. This implies that the use of pictorial models had a positive effect on the development of the pupils' Mathematical thinking. This result is consistent with positive statements on the effectiveness of pictorial models in Mathematics learning.

It seems that the use of pictorial models provides the pupil opportunities to deepen their understanding of Mathematical concepts, apply their knowledge of the four basic operations, and model the problem situation through representation. As cited by Stuart Murphy (2006), visual learning strategies can make a profound difference in a student's depth of understanding about Mathematics (Murphy, 2006).

It is a powerful teaching tool for kids who are natural visual/spatial learners, for children who are English language learners, and for students of all learning modalities (*cf* Katz, 2006; Murphy, 2006; and Richarson, Sherman & Yard, 2009). By using visual learning strategies in the teaching of Mathematics, teachers can increase the learning potential of children.

Supporting the result of this study is found in a case study made by Swee Fong Ng (2004). Swee Fong Ng and others said that through the model method (pictorial models), pupils with no knowledge of formal Algebra are provided with a tool to construct pictorial equations to

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solve increasingly challenging word problems, involving simple part-whole relationships as far as those that require proportional reasoning (*cf* Fong Ng, 2004; Ernest, 2006; and Krulik & Posamentier, 2009).

Pupils' Experiences on the Use of Pictorial Models. Pupils' experiences on the use of pictorial models were obtained through formal interviews. Based on the conducted oral interviews, most of the pupils said that they use the pictorial model only "sometimes". That is when it is applicable to the problem presented. The pupils were one in saying that not only have the pictorial models helped them in improving their Mathematical thinking skills, but they have also developed their Algebraic thinking skills unconsciously (interview with Respondents A, B, C, and D, 19/12/2011).

Generally, the pupils find the use of pictures, "blocks" or rectangles helpful in understanding concepts and word problems deeply. Some of the pupils' comments were as follows:

> "*Pictorial model helps me compute better*" (interview with Respondent A, 19/12/2011).

> "*It makes the word problem easier*" (interview with Respondent B, 19/12/2011).

> "*It helps me analyze word problems well*" (interview with Respondent C, 19/12/2011).

> "*It is easy to understand numbers through drawings, that is through the use of rectangles*" (interview with Respondent D, 19/12/2011).

The comments of the pupils only proved that the use of pictorial models aided them in solving word problems. These clearly show that through the use of pictorial models, pupils were able to represent numbers using bars or blocks and learned how to model problem situations. Similar findings were found in the study of N. De Guzman (2009) and others, where the grade five pupils perceived the block model as a useful tool in solving word problems (*cf* Charlesworth & Radeloff, 1978; Carruthers & Worthington, 2003; and Guzman, 2009).

Since equations were introduced through pictures, pupils were able to use this to represent quantitative relationships. This relates with J. Cai (2005) and othres' contention that "pictorial equations" do not only provide a tool for students to solve Mathematical problems, but they also provide a means for developing pupils' Algebraic ideas (*cf* Lew, 2004; Cai, 2005; and Jackson, 2009).

Based on the data gathered, the following are the findings of this study. *Firstly*, there is a significant difference between the pre-test and post-test mean scores of the pupils on representation and reasoning skill and problem solving skill. *Secondly*, the pupils were able to answer word problems, which are Algebraic in nature through the use of pictorial models. *Thirdly*, the pupils perceived pictorial models as helpful tools in analyzing word problems as the models make them understand word problems better. Finally, *fourthly*, teaching Algebraic thinking can start at an early age (Maletsky & Sobel, 1988; Nebres, 2006; Sousa, 2007; Lee & Lee, 2009; and Ptylak, 2010).

CONCLUSION

Based on the findings of the study, the following conclusions were drawn. The use of pictorial models has a positive effect in developing Algebraic thinking of primary pupils. Primary pupils can engage in powerful Mathematics structures if given appropriate learning activities. One of these activities is engagement in problem solving through the use pictorial models to deepen understanding of word problems. In the pictorial models, the use of blocks as units representing the unknowns provides a link to more abstract ideas, like letters representing the unknowns.

From the findings and conclusions, the following recommendations are given:

The use of pictorial models should be introduced to the pupils, when solving word problems. Pictorial representations, drawings, and models may lead children to understand the symbols which seem abstract to them initially.

Mathematics teachers should allow pupils to think out of the box or to find creative ways in solving word problems. Mathematics teachers should be provided with frequent opportunities for high quality training. Teachers should be exposed to a wide variety of approaches, such as the use of pictorial models, which they can introduce to the pupils to develop their higher-order thinking skills.

Further studies should be done also in the public schools to determine whether this approach has a positive effect on the performance of the pupils in Mathematics. Future researchers are encouraged to conduct similar studies that may help the pupils develop their Algebraic thinking skills.[1]

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Interview with Respondent B, grade three pupil at the La Salle Green Hills, in the Philippines, on December 19, 2011.

Interview with Respondent C, grade three pupil at

[1]***Statement:*** Herewith, we have declared that this paper is our original work; so, it is not product of plagiarism and not yet be reviewed as well as be published by other scholarly journals.

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Pupils at the La Salle Green Hills in the Philippines
(Source: http://www.interaksyon.com, 20/5/2015)

The use of pictorial models has a positive effect in developing Algebraic thinking of primary pupils. Primary pupils can engage in powerful Mathematics structures if given appropriate learning activities. One of these activities is engagement in problem solving through the use pictorial models to deepen understanding of word problems. In the pictorial models, the use of blocks as units representing the unknowns provides a link to more abstract ideas like letters representing the unknowns.

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