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XL4C4D – Adding the Graph Transformation Language XL to CINEMA 4D

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Abstract: A plug-in for the 3D modeling application CINEMA 4D is presented which allows to use the graph transformation language XL to transform the 3D scene graph of CINEMA 4D. XL extends Java by graph query and rewrite facilities via a data model interface, the default rewrite mechanism is that of relational growth grammars which are based on parallel single-pushout derivations. We illustrate the plug-in at several examples, some of which make use of advanced 3D features.

Keywords: Scene graph, L-system, relational growth grammar, CINEMA 4D

1 Introduction

Most 3D modeling systems represent their 3D content as a scene graph. In general, this is a directed acyclic graph or even just a tree, where nodes contain geometry data and further properties, while edges define spatial and logical relations between nodes. E.g., the coordinate system of a node is typically inherited to its children, and often this also holds for properties like the color.

To create and modify the scene graph, 3D modeling applications typically do not only provide direct user interaction, but also built-in (textual or visual) programming languages. But none of these languages makes use of graph transformation techniques, although they suggest themselves for such a language, given the underlying scene graph.

What has been used for 3D scene graph creation are L(indenmayer)-systems [PL90], starting with the successful specification of 3D plant models, and nowadays this parallel string-rewriting formalism is directly supported by many 3D modeling applications. But as it is based on strings, it necessitates an additional interpretation step from strings to graphs. In previous research, we developed relational growth grammars as a rewriting mechanism for graphs which incorporates both the possibilities and ease of use of L-systems (but applied to graphs) and of true graph transformations based on single-pushout derivations [KBHK08, Kni09].

We also developed a textual programming language XL which extends Java by graph transformation syntax and semantics. This can be used to implement relational growth grammars, and there is a ”reference implementation” within the Java-based open-source 3D platform GroIMP [KHK]. This software has been developed with the needs of relational growth grammars and plant modeling in mind, so it provides strong support for this field of application, while it has fewer general 3D features than traditional all-purpose 3D modeling systems.

Through the data model interface of XL, it is possible to let XL operate on any kind of graph. We will present an implementation for the scene graph of CINEMA 4D [MAX] as a plug-in, and we will show some examples of its application.

2 Relational Growth Grammars

In this section, we will sketch *relational growth grammars* (RGG for short) to have a formal basis for the following applications. We do not give complete definitions here as this would become too lengthy, they can be found in [Kni09].

At first, the *graph model* has to be specified. To be able to use type hierarchies known from object-oriented programming for nodes and edges, and to store attributes at nodes, we are using *typed attributed graphs with inheritance* [EEPT06]. Since the semantics of edges within the RGG formalism is to stand for plain relationships between their incident nodes, we exclude the possibility of edge attributes, and parallel edges of the same type are not allowed. The latter means that the edges of a graph G are simply represented as a subset of $G_V \times \Lambda_E \times G_V$, where G_V denotes the nodes of G and Λ_E is the set of edge types.

The rewriting mechanism of relational growth grammars is based on the algebraic single-pushout (SPO) approach [EHK+97]. The SPO approach has the nice properties that dangling edges are removed automatically, and that deletion is prefered over preservation in case of a conflict. This is very useful for the domain of models we are interested in: E.g., think of an animal being represented as a node, then there might be a rule describing death because of ageing which simply deletes the animal node. Without the automatic deletion of dangling edges, we would have to take into account all possible edges of the animal in the death rule, which would be cumbersome in practice. If there is a second rule for the animal which models its movement, and if both rules are applied in parallel, there is a conflict between deletion and preservation, but it makes perfect sense to prefer deletion as the death rule models a deliberate and rather drastic event.

In order to instantiate the SPO approach for RGG graphs, we have to define a corresponding category **RGGGraph** of graphs and their homomorphisms. The single-pushout approach works with *partial homomorphisms* $\mathrm{Hom}_P(G,H)$, i. e., graph homomorphisms $G \rightarrow H$ which are defined on some subgraph of their domain G. To integrate the inheritance relation in the notion of a graph homomorphism, we use a technique based on [PEM87]: A homomorphism $f : G \rightarrow H$ in **RGGGraph** is a structure-preserving mapping such that for every object (node or edge) $x \in G$ the type of its image $f(x)$ is a subtype of the type of x.

To be convenient for the modeling of scene graphs of plants and other structures, *RGG rules* have to be sufficiently expressive. Basically, the application of a rule shall remove the match of its left-hand side (LHS, the *predecessor*) from the *host graph* and insert its right-hand side (RHS, the *successor*) into the *derived graph*. The exact definition of a rule is based on SPO productions [EHK+97], but we allow the right-hand side to dynamically depend on the match. As an example, consider a reproduction rule of an animal where the number of children depends on some properties of the animal. If we were constrained to conventional productions, we would have to either specify several productions (one for each possible number of children) or to use several derivation steps. Both solutions are not feasible from a practical point of view. The dependence of the right-hand side on the match provides a solution to this problem (cf. [Kur94, PKL07, CJ92]). As a result, an RGG rule consists of a left-hand side L, an application condition c and a mapping p which maps a match m for L to the actual SPO production $p(m)$ to be used for the derivation:

Definition 1 (Application Condition) Let L be a graph. An *application condition* c on the set $\mathrm{Hom}(L,\cdot)$ of all total graph homomorphisms with domain L is a boolean-valued function on $\mathrm{Hom}(L,\cdot)$.

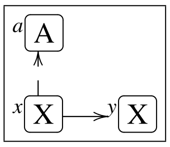
Definition 2 (Simple RGG Rule) Let L be a graph. A simple *RGG rule* $r = (L,c,p)$ is given by a graph L, an application condition c, and a mapping $p : \mathrm{Hom}(L,\cdot) \to \mathrm{Mon}_P(\cdot,\cdot)$, where $\mathrm{Mon}_P(\cdot,\cdot)$ denotes the set of all partial, injective homomorphisms, such that for some $m : L \to G \in \mathrm{Hom}(L,\cdot)$ the image $p(m)$ is an SPO production $M(m) \xrightarrow{p(m)} R(m)$ whose domain (in the categorical sense) $M(m)$ is a subgraph of G and a supergraph of $m(L)$.

Definition 3 (Match) A *match* for a simple rule $r = (L,c,p)$ in a host graph G is a total graph homomorphism $m : L \to G$ such that the application condition c is fulfilled ($c(m) = \mathrm{true}$).

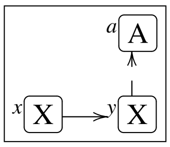
Definition 4 (Simple RGG Derivation) Let $m : L \to G$ be a match for a simple rule $r = (L,c,p)$. A *direct derivation* using r via m, denoted as $G \overset{r,m}{\Longrightarrow} H$, is given by a direct SPO derivation using $M(m) \xrightarrow{p(m)} R(m)$ via the inclusion $M(m) \hookrightarrow G$, i. e., by the following commutative diagram in the category **RGGGraph**, where the square is a pushout:

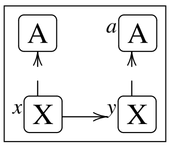
$$\begin{tikzcd}
L \arrow[r, dashed] \arrow[rrd, "m"'] & m(L) \arrow[r, hook] & M(m) \arrow[r, "p(m)"] \arrow[d, hook] & R(m) \arrow[d] \\
& & G \arrow[r] & H
\end{tikzcd}$$

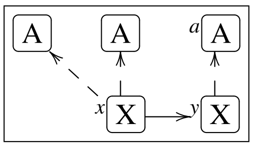
The following is an example for a simple RGG rule which moves some sort of animal (A-typed node) along a path of X-typed nodes if its energy is above the threshold 1 and creates some children at the left location, the number depending on the energy.

$L =$ 

$, c(m) = (m(a).\mathrm{energy} > 1)\,,\ p(m) = m(L) \to R_{\lfloor m(a).\mathrm{energy} \rfloor}$

with $R_1 =$ 

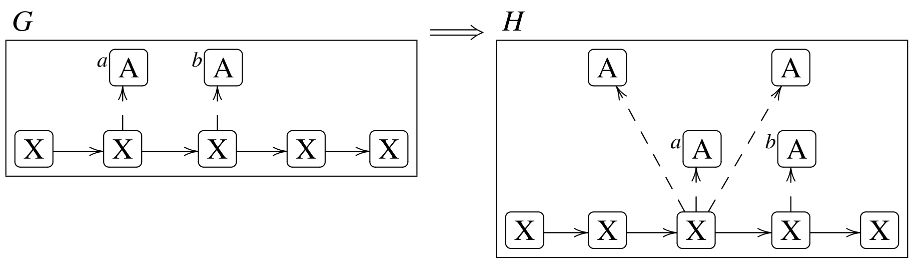
$, R_2 =$ 

$, R_3 =$ 

$, \ldots$

Nodes are represented as oval boxes around their type, node identifiers are placed before the upper left corner. $m(a)$.energy refers to the value of the energy attribute of the match for the A-typed node a, and $\lfloor x \rfloor$ denotes the integral part of x (floor function). The mapping $p(m)$ is indicated by the reuse of node identifiers of the LHS for the RHS.

Because processes in a living system happen in parallel, there has to be the possibility to apply RGG rules in parallel. For a family of simple RGG rules with corresponding matches, we can create a *parallel derivation* from the generated SPO productions $p_i(m_i)$ as defined in [EHK+97]. The following shows such a parallel RGG derivation for the single example rule from above using the two obvious matches, if we assume that the energy of node a is 1.5 and that of b is 3.5.



Unfortunately, this straightforward parallelism fails if rule application successors of neighbouring parts shall be connected by edges, which is a very important case and needed by the embedding of L-systems in the RGG formalism. Several *connection mechanisms* were studied in [LR76] to address this problem, of which the *operator approach* [Nag76, Nag79] turns out to be a suitable technique for relational growth grammars. The application of a rule to a match also establishes *connection transformations* which are given by a quintuple $(s,(A,d,\gamma),t)$ with a node s from the match, a node t from the right-hand side, an *operator* A, a direction flag d (either "source" or "target") and an edge type γ. An operator A yields for every node $n \in G$ of the match a set $A_G(n)$ of related nodes (e. g., its neighbours).

We may think of such a connection transformation as an arrow which points from a node s in the host graph G to a node t in the derived graph H resulting from the simple RGG derivation. The operator mechanism then shifts connections within the host graph along a pair of matching arrows to the derived graph: For a matching arrow pair $(s_1,(A_1,\text{“source”},\gamma),t_1)$, $(s_2,(A_2,\text{“target”},\gamma),t_2)$ where the host graph nodes s_1,s_2 are related according to the operators, i. e., $s_2 \in A_1(s_1)$ and $s_1 \in A_2(s_2)$, an additional γ-typed edge from t_1 to t_2 is created within the derived graph H.

Definition 5 (RGG Rule) An *RGG rule* $r = (L,c,p,z)$ is a simple RGG rule (L,c,p) together with a mapping z which assigns to each match $m : L \to G$ for the simple rule a set of connection transformations $(s,(A,d,\gamma),t)$ with $s \in M(m)$ (domain of $p(m)$), $t \in R(m)$ (codomain of $p(m)$), an operator A, a direction d (either "source" or "target") and an edge type γ.

3 The XL Programming Language

XL is a proper extension of Java. The main new features are *rule blocks* and *graph queries*. A rule block can be used everywhere where Java allows a normal code block, it is distinguished by square brackets instead of the usual curly braces. The following example shows the translation of the animal movement and reproduction RGG rule from above:

```
[
    x:X [a:A] y:X, (a.energy > 1) ==>>
        x for(int i = 2; i <= a.energy; ++i) ( [A] ) y [a];
]
```

`x:X` is a pattern which matches a node of Java class `X` and assigns the identifier `x` to the match. Within left- and right-hand sides, square brackets are used to enclose branches, so that the left-hand side of the example has to be read as: Find an `X`-typed node `x` which has an `A`-typed child `a` and an `X`-typed successor `y`. Child and successor relationships are represented by distinct edge types. The left-hand side also contains an application condition in round parentheses.

The right-hand side depends on the match because the for-loop expands into a variable number of nodes, depending on `a.energy`: If the energy is less than 2, we have `x y [a]` which just moves `a` to `y`. If it reaches 2, we get `x [A] y [a]`. This creates a new `A` node (because there is no node identifier `A`) as a child of `x`. For higher energies, we get even more new `A` nodes.

Cycles of non-tree-like structures are obtained by repeating a node identifier. E.g., the pattern `a:A B C a` represents a circular graph of three nodes of types `A`, `B`, `C`.

Graph queries are expressions which use the syntax of left-hand sides within `(* *)`. E.g., the expression `count((* x [A] *))` contains a query for all `A`-typed children of a given node `x`, and counts the number of matches.

All graph operations of XL are defined on top of an abstract data model interface. By implementation of this interface, XL can operate on any kind of graph. The most sophisticated implementation exists for GroIMP, but there are also implementations for XML documents or minimalistic Sierpinski graphs [Kni09].

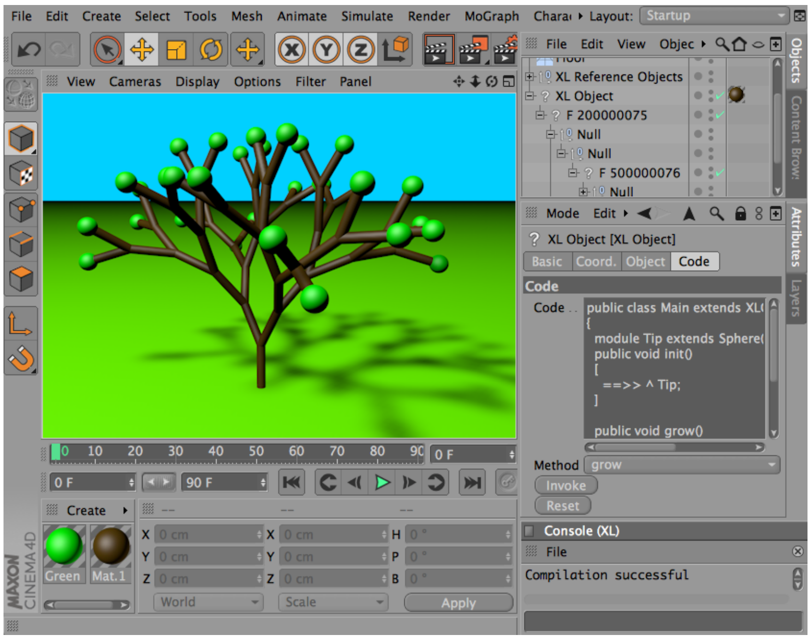
In the following, we will show some more examples of XL. An in-depth description can be found in [Kni09].

4 The Plug-In XL4C4D and Its Application

The 3D modeling system CINEMA 4D [MAX] uses a scene graph to represent spatial and logical relations between objects. In fact it is rather a tree than a graph, but objects can have additional link attributes which may point to arbitrary objects in the scene tree, thus yielding a true graph.

Each object has its own local 3D coordinate system. As usual for scene graphs, this is defined by the coordinate system of the parent object, multiplied with a transformation attribute of the object itself, so that an object moves together with its descendants when one modifies the transformation of the object. There are objects with a geometry such as Cube, Sphere, or PolygonObject for a general polygon mesh, but also a Null object which has no geometry itself, but can be used to group its children and to inherit its transformation to them. Instance objects also have no geometry themselves, but a link to another object. They show the geometry of the linked object within their own coordinate system. This saves a lot of memory when one wants to place the same geometry at several locations.

CINEMA 4D provides a C++ SDK which allows to write custom plug-ins. With the help of the Java Native Interface as a bridge between Java and C++, we implemented XL's data model

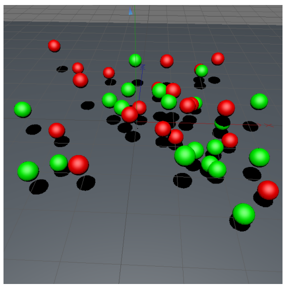


```
public class Main extends XL(
{
  module Tip extends Sphere(
  public void init()
  [
    ==>> ^ Tip;
  ]

  public void grow()
```

```
Compilation successful
```

(a)


(b)

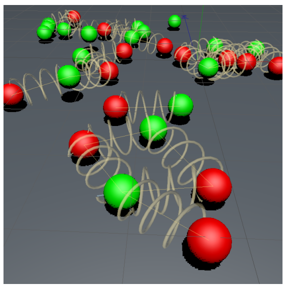

(c)

Figure 1: **(a)** Screenshot showing the plug-in with a binary tree; **(b,c)** polymerization model

interface for CINEMA 4D's scene graph. The plug-in (available at [KHK]) adds the following features to CINEMA 4D:

- The XL Console shows output from XL execution, and it allows to directly type in XL code which is then executed.
- There is a new scene graph object XL Object. It has a code attribute which allows to enter a complete XL program. Methods of the program may be invoked interactively, and parameters defined in the program are accessible in the attribute manager.
- The XL Instance object behaves similar to a normal Instance object. It has additional attributes for scaling and translation which are more convenient for XL models than the normal transformation attributes.

Figure 1(a) shows a screenshot of CINEMA 4D at the example of a simple 3D binary tree. The code properties of the single XL Object are opened. The complete code for the model is

```
public class Main extends XLObject {
  module Tip extends Sphere(20).(setMaterial("Green"));
  public void init() [ ==>> ^ Tip; ]
  public void grow()
    [ Tip ==> F(100) [RU(30) RH(90) Tip] [RU(-30) RH(90) Tip]; ]
}
```

The code shows the basic structure of an XL model within XL4C4D: The actual code has to be surrounded by a class extending `XLObject`. Parameterless **public void** methods of that class can be invoked interactively by choosing them in a drop-down menu and clicking on an Invoke-button, so one typically puts the actual rules within such methods. In the example there is an `init`-method with a simple RGG rule with empty LHS: Out of nothing, the rule creates a two-node graph starting at the XL Object itself, which is represented by ^, and followed by an instance of `Tip`. The Java class `Tip` itself is specified in a **module** declaration which is a simplified **class** declaration and in this case corresponds to a Sphere object in the scene graph with a radius of 20 units and the material named "Green".

A `Tip`-node represents a leaf (in the mathematical sense) of the tree, therefore for a growth of a binary tree such a node has to be replaced by an inner node bearing two new `Tip`-nodes. This is implemented by the `grow`-method which uses a non-simple RGG rule to replace (in parallel) each `Tip`-node by an `F`-node with two branches. The branches do not consist of `Tip`-nodes only, but also of 3D rotation nodes of the types `RU` and `RH` put in front. An `F`-node corresponds to an XL Instance object in the scene graph which instantiates a Cylinder object, the parameter 100 to the `F`-constructor scales this cylinder to a height of 100 units, and it also shifts the coordinate system by 100 units along the cylinder axis (the local y-axis). Together with the rotations around the local x- and y-axes (`RU` and `RH`, respectively), this places the base of the cylinder at the location of the previous `Tip`, and adds the two new tips at the top of the cylinder, but with a rotated coordinate system such that the new branches emerge in different directions in 3D space.

The `grow`-rule requires connection transformations (there are no common nodes on LHS and RHS for gluing). They can be specified for each pair of LHS/RHS-nodes, but for convenience two such transformations are automatically added if one uses the rule arrow ==>, namely one from the leftmost (unbracketed) LHS node to the leftmost (unbracketed) RHS node and one for the rightmost nodes. This is exactly what is needed for an L-system rule when it shall be applied not to a string but to a graph.

Besides `Sphere`, `F`, `RU` and `RH`, the plug-in also provides `RL` for rotation around the z-axis, `M` for y-translation, `Scale` for uniform scaling, `Null` for general transformations, `Cube`, `Box`, `Instance`, and even `Spring` for physics simulations. Each node of such a Java class corresponds to a CINEMA 4D object of suitable type, e.g., pure transformation nodes are represented as Null objects in the scene graph. The general `BaseObject` class corresponds to CINEMA 4D's BaseObject C++ class. With its methods and the numerical IDs from CINEMA 4D's resource files it is possible to create objects of any type, and to set most of their attributes. Attributes of the CINEMA 4D types Bool, LONG, Real, Vector, Matrix, String and BaseLink are supported, the same holds for additional user data attributes.

One of CINEMA 4D's advanced features is the Dynamics module which provides a rigid body simulation. With its help we can create a toy model of polymerization: A lot of monomers of two types (`MonomerA` and `MonomerB`) move around according to the laws of rigid body physics. At each frame (i.e., each simulation step), it is checked if two monomers of different type come close to each other. If so, the rule triggers and creates a bond in the form of a Spring object for the rigid body simulation:

```
public class Main extends XLObject {
  module MonomerA extends Sphere(20).(setColor(1, 0, 0));
  module MonomerB extends Sphere(20).(setColor(0, 1, 0));
  public void init() [ ... ]
  public void step()
  [
    a:MonomerA, b:MonomerB,
    ((distance(a, b) <= 50) && empty((* Spring(a, b) *)))
    ==>> a, b, ^ Spring(a, b).(setBoolean(FORCE_ALWAYS_VISIBLE, true));
  ]
}
```

The bonding rule `step` looks for two monomers `a`, `b` which, due to the comma inbetween, need not have any relationship in the graph. Its application condition tests if `a`, `b` are close to each other (at most 50 units in 3D space) and makes sure by a graph query that no `Spring` node already exists. The RHS repeats `a`, `b` so that they don't get deleted, and it creates and initializes a new `Spring` node linked to them.

The setup of the simulation environment for the example is done manually. Figure 1(b) shows the initial situation with monomers randomly placed in 3D space above a Floor object. The result after some simulation steps is shown in Figure 1(c). For the simulation it is necessary to put the bonding rule in a method named `step` because a method of this name is automatically invoked by the plug-in during the simulation whenever the document time changes.

Figure 2 shows the result of a more advanced example which combines XL with Dynamics and collision detection via CINEMA 4D's built-in visual programming language XPresso: The large sphere rolls along the floor, and whenever it collides with one of the small grid points, this is detected by XPresso which then sets a user attribute on the grid point object. This in turn can be evaluated by XL to let a simple binary tree grow at such a point until it is eventually dissolved by XL into a set of light particles which fall down according to Dynamics.

5 Discussion

The structure of CINEMA 4D's scene graph is well-suited for the application of XL. The shown examples could be adapted from their original GroIMP implementations [Kni09] without major changes, and they could easily be combined with advanced features like Dynamics. In fact, the GroIMP implementation of the polymerization has to include some simple simulation rules for monomer movement which are superfluous for XL4C4D. There are a lot more features from animation to sophisticated rendering from which one can benefit.

On the other hand, the Java Native Interface introduces a performance bottleneck, and CINEMA 4D isn't able to handle very large amounts of objects or deep structures efficiently. So the system is not suitable for graphs of million nodes, which may appear in detailed plant models and which can be handled by GroIMP. E.g., for the classical example of the Koch snowflake curve, the plug-in allows only up to three iterations, reaching a graph depth of 384. Within GroIMP, nine iterations resulting in a depth of more than 1.5 million are possible without any problems.

Therefore, in its current state, XL4C4D is a useful addition for CINEMA 4D if one wants to create and modify the graph of scenes within typical domains of CINEMA 4D such as motion graphics. In motion graphics, visually interesting effects have to be achieved, but they need not

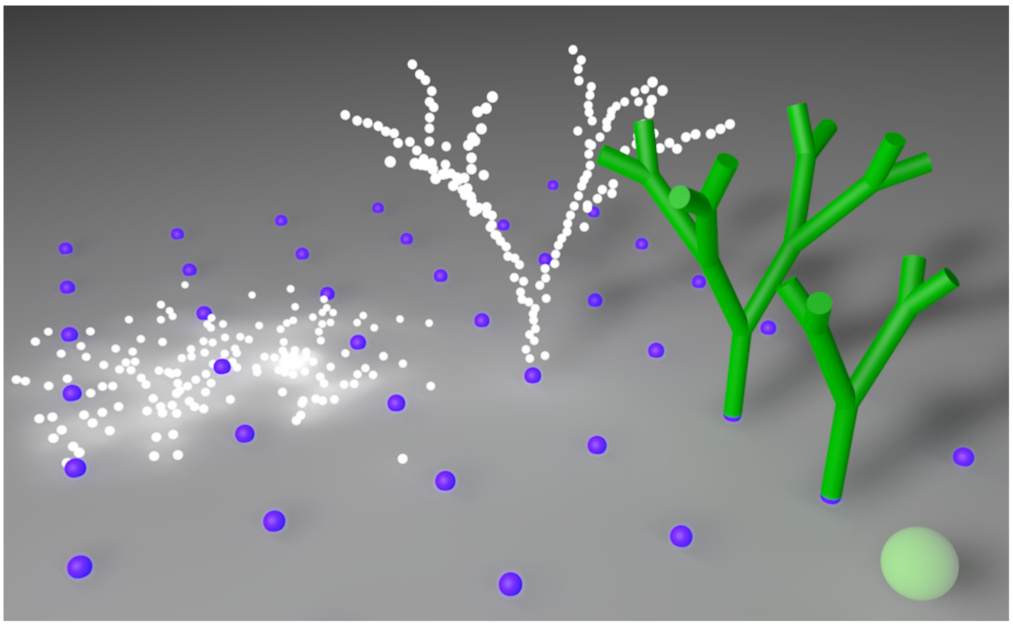


Figure 2: The sphere triggers tree growth, eventually trees dissolve into light particles.

be realistic. Also the graph structures are typically relatively simple, but still too extensive to be modeled by hand. The last example can be seen as a (somewhat unskilled) representative of this kind of computer graphics.

XL4C4D operates on the scene graph level. A much more complex kind of 3D modeling happens at the deeper level of actual geometry. This is usually represented as a polygon mesh, i.e., another kind of graph. Typical operations on a mesh like extrusion [BPA$^+$10] or subdivision [SMG10, BPA$^+$10] can be modelled as graph transformations. We haven't implemented such facilities for the plug-in, but it could be done based on the vv approach [SPS03, Kni09].

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