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ON GRANTS AND LENDING

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1. INTRODUCTION

In a recent article, Danny M. Leipziger (1983) makes a useful distinction between the grant element of a loan to the lender, and to the borrower. Simply restated, the difference between the two arises because of a difference in the discount rate (or the opportunity cost of capital, or the cost of funds) to the lender and the borrower. If we suppose, taking Leipziger's example, that the rate of interest charged by the lending country is 4% p.a., while the alternative rate of return available to it is 10% p.a. (equal to the marginal product of capital in the lending country), the grant element to the lender of a 10 years loan of $ 100 is equal to $ 21.87. The grant element to the borrower (e.g. an LDC) will be greater if we suppose that the marginal product of capital in the borrowing country is greater, or equivalently that the marginal cost of funds to the borrower exceeds 10%. If it is, say, 15% p.a., the grant element of the same loan to the borrower will be equal to $ 34.93.[1] Once the difference between the two grant components is acknowledged, it can be easily seen that for a given grant element say, to the lender, one may determine the most advantageous point for the borrower. Leipziger presents the analysis in terms of indifference curves for the lender and the borrower, and also gives a numerical example showing how a *given* amount of the grant element to the donor involves different amounts of assistance to the borrower. As Leipzinger writes, a change in credit terms (in his example, in maturity) enables »(t)he borrower ... to gain at no additional cost to the lender« (p. 333). We propose in this note to extend Leipziger's approach into the following direction. We shall consider in general what is the optimal repayment schedule for the borrowing country when the cost (i.e. grant element) to the lender is given.

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[1] The difference between a 10% rate of interest at which the industrialized country (the lender) may invest or borrow and 15% interest charged to an LDC borrower may be explained by a superior risk element in the latter loan.

Adoption of such a strategy also obviously implies the maximization of the total lender's and borrower's utility.

2. THE OPTIMAL REPAYMENT SCHEDULE (WHEN THE COST TO THE LENDER IS GIVEN)

Our problem is to find the optimal repayment stream which, with the given grant element to the lender, maximizes the assistance to the borrower. We shall work in terms of grant elements which serve as proxies for utility: we assume that there is a monotonically increasing (decreasing) relationship between the level of utility and the grant element to the borrower (lender). Let us suppose, first, that there is absolutely no requirement as to the pattern of the repayment stream; that is, once the grant element to the lender is given, there is no requirement for the borrower to pay any predetermined amount (say, amortization plus interest) each year. We shall, however, relax later this »no requirement« condition in order to consider more common examples of debt repayment.

Consider the problem first intuitively. So long as the rate of return in LDC (= borrower) is greater than the rate of interest the LDC pays *(i)*, the optimal strategy for LDC is to continue borrowing. We may visualize the country as either borrowing, and repaying, and again borrowing the same amount *each year*, or more simply, on the example of a single loan, as borrowing once and then deferring *all* payments so long as its MPk > i. Essentially, it means that the LDC's optimal strategy would be to extend the grace period until such a moment when MPk = i. When the two become equal, the country would repay all of its debt (including the interest) *at once*. In a world of full certainty, no intergenerational utility problems[2] and perfectly known MPk's, this would be also the optimal strategy for the world as a whole, and thus for the lending country *if its loan involved no grant element*. To take the previous example: if the lender loaned out its money at 10% p.a. (its opportunity cost of capital) the optimal strategy for the *world as a whole* would be that the borrower *pays no single cent* until its marginal product of capital becomes equal to *i*. Once it happens, the borrower pays all. A somewhat similar idea underlies the credit circle theory whereby the borrowing country slowly evolves into the lending one.

But the above analysis obviously does not hold once *i* is less than the opportunity cost of capital to the lender (r). For such a strategy would involve an increasing grant cost with each year, and eventually the actual grant element to the lender may exceed the amount he is willing to give. The grant element to the lender therefore is not fixed. Yet the example is useful in that it illuminates the point which we shall prove rigorously below, namely that the optimal strategy for the borrowing country is to defer payments as long as possible (i.e. as it is compatible with the *a priori* fixed grant cost to the lender)

[2] Or, equivalently, zero rate of pure time preference.

and to repay all of the debt at once. We may dub this strategy »the point repayment strategy«.

Let i = interest rate paid by the LDC, r_l = opportunity cost of borrowing to the LDC = MPk in LDC, r = marginal product of capital in developed country, L = amount of the loan disbursed in totality in time t = 0, and P(t) = stream of repayments (including amortization of the debt *and* interest) expressed as a function of time. Then, the lender's grant element (g_l) is equal to

$$g_l = L - \int_0^T \frac{P(t)}{e^{rt}} dt \tag{1}$$

where T is the moment when all of the loan is definitely paid back. The borrower's grant element (g_b) is equal to

$$g_b = L - \int_0^T \frac{P(t)}{e^{-r_l t}} dt. \tag{2}$$

Now we assume that L, r, r_l, i and g_l are given.[3] (Note that T is free.) We can write the problem as

$$\underset{P(t)}{Max} - \int_0^T P(t)\, e^{-r_l t} dt$$

under

$$\int_0^T P(t)\, e^{-rt} dt = L - g_l = given. \tag{3}$$

The first part of expression (3) is clearly the equivalent of maximization of eq. (2). The second part of (3) gives the constraint. Note that since L and g_l are fixed, the equation (3) can also be interpreted as maximization of $p_b = g_b/L$ under a given $p_l = g_l/L$. The problem is thus set in more general terms.

The relation (3) may be solved by application of Hestenes' theorem (see e.g. Takayama, p. 651). For the optimal repayment schedule, $(\hat{P})(t)$, the relation H ($\hat{P}$(t),t) must be greater than any other H ($\hat{P}$(t),t):

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[3] Note that *i* enters into equations (1) and (2) in the following way: for any given pattern of repayment schedule P (t) rate of interest will determine the actual amounts of payments.

$$H(\hat{P}(t), t) = -p_0\,[\hat{P}(t)\,e^{-r_1 t}] + p_1\left[\hat{P}(t)\,e^{-rt} - \frac{L - \bar{g}_1}{T}\right]$$

$$H(P(t), t) = -p_0\,[P(t)\,e^{-r_1 t}] + p_1\left[P(t)\,e^{-rt} - \frac{L - \bar{g}_1}{T}\right] \tag{4}$$

where p_0 and p_1 are non-negative parameters. The expression following parameter p_0 is the rewriting of the objective function; the expression following p_1 is the rewrite of the constraint.[4] Let, without any loss in generality, $p_0 = 1$.

By rearranging (4) we have

$$-e^{-r_1 t}\;[\hat{P}(t) - P(t)] > p_1\,e^{-rt}\,[P(t) - \hat{P}(t)].$$

Now, if $\hat{P}(t) > P(t)$ we must have $e^{-r_1 t} < p_1 e^{-rt}$. If, on the contray, $\hat{P}(t) < P(t)$ then $e^{-r_1 t} > p_1 e^{-rt}$. Let us define $t = t^*$ such that $e^{-r_1 t} = p_1 e^{-rt^*}$. Consequently, for all $t < t^*$, we shall have $e^{-r_1 t} > p_1 e^{-rt}$, and the optimal repayment schedule $\hat{P}(t)$ will be smaller, at any instant, than all other possible repayment schedules $P(t)$ belonging to the admissible region $P(t) \in P$.[5] For all $t > t^*$, on the contrary, we have $e^{-r_1 t} < p_1 e^{-rt}$ and the optimal repayment schedule $\hat{P}(t)$ will at any moment be greater than all the other possible repayment schedules.

We have assumed above that there is no requirement about the absolute amount to be repaid at any moment (i.e. each year in the discrete case). It is then evident that $\hat{P}(t)$ must be such that up to the point $t = t^*$ no payment whatsoever is made (that is, $\hat{P}(t) = 0$), while at the moment $t = t^*$ all the loan plus interest is paid out. The latter is obviously the highest value $P(t)$ stream belonging to the admissible region that can be taken, so that the $\hat{P}(t) > P(t)$ condition is satisfied. We thus have

$$P(t) = \begin{cases} 0 & \text{for } t < t^* \\ Le^{it} & \text{at } t = t^* \end{cases} \tag{5}$$

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[4] The constraint $\int_0^T P(t)\,e^{-rt}\,dt = L - \bar{g}l$ may be rewritten as $\int_0^T [P(t)\,e^{-rt} - L - \bar{g}_1)/T]\;dt = 0$ to satisfy the form in which the constraint must be expressed to apply Hestenes' theorem. Note that $\bar{g}_1$ denotes that g_1 is fixed.

[5] The admissible region comprises all schedules such that the loan (including interest) is paid out.

At moment $t = t^*$, total payment in absolute amount is Le^{it}. It is equal to the original loan augmented by the interest accrued continuosly for t^* periods on the totality of the loan (i.e. it includes interest on interest).

We have thus established the optimal repayment schedule for the borrowing country. It can be now easily seen that, from (2),

$$g_b = L - L\,e^{it^*}/e^{r_1 t^*} = L\,[1 - e^{t^*(i-r_1)}]. \tag{6}$$

On the other hand, the lender's grant element is equal to

$$g_b = L - Le^{it^*}/e^{rt^*} = L\,[1 - e^{t^*(i-r)}]. \tag{7}$$

So long as $r_1 > r$, we must have $g_b > g_1$.

Note also that if $i = r$, g_1 becomes nil (no grant element to the lender), but $g_b > 0$ provided $r_1 > i$. Accordingly, even if LDC borrows at commercial terms, it will enjoy a grant element. Yet it may be worthwhile pointing out that this grant element (flowing from $r_1 > i = r$ inequality) is equivalent to the Marshallian producer surplus, and is, thus, conceptually different from the grant element *in proprio* (due to $i < r$) which the lender is willingly transferring to the borrower.

We must now determine the value of t^*, i.e. the optimal maturity. This can be easily found by making the lender's grant element under the optimal repayment conditions (equation (7)) equal to some specified amount $\overline{g_1}$. Then, we obtain

$$t^* = \frac{\ln\left[1 - \dfrac{\overline{g_1}}{L}\right]}{(i - r)} = \frac{-\ln(1 - \overline{p_1})}{r - i} \tag{8}$$

From equation (8) we can readily find the optimal maturity as a function of the percentage of the loan the lender is willing to transfer as grant ($\overline{p_1}$), the rate of interest (i), and the opportunity cost of capital to the lender (r). Accordingly, the optimal maturity does not depend on the marginal product of capital in the borrowing country. Value of t^* increases as a function of p_1 and i, and decreases as a function of r. One can readily see that the sensitivity of t^* and of the grant element to the borrower, p_b, can be studied as a function of these three parameters and of the marginal product of capital in LDC (upon which p_b depends).

Table 1 shows the maximum grant element to the borrower of a loan on which the lender is willing to give 20% grant, and the optimal maturity. Both are expressed as a function of the rate of interest. We assume $r = 0.1$ and $r_1 = 0.15$ as above.

Table 1.

The Optimal Maturity and Grant to the Borrower When Grant Element to the Lender is 20%

i	t*	pb
0.04	3.719	0.336
0.05	4.463	0.360
0.07	7.438	0.448
0.08	11.157	0.542
0.09	22.314	0.738
0.095	44.629	0.914

Table 1 shows that if lender is willing to give a grant equal to 20% of the loan, and the rate of interest is 4%, the borrower's optimal strategy is to pay out the loan in entirety after 3.7 years (assuming r and r_1 as given). Following such a strategy the borrower will be able to maximize his grant element: it will equal 33.6%. However, what Table 1 also shows is that — given the 20% grant element to the lender — a *higher rate of interest is advantageous to the borrower.* This apparently paradoxical conclusion can be explained as follows. With an increase in *i*, the optimal maturity recedes (t^* becomes greater). The lender is still transferring the same proportion of its loan, but to the borrower the extension of the maturity (e.g. from 3.7 to 11.2 years when the rate of interest increases from 4 to 8 percent p.a.) more than compensates for the increased rate of interest. *Grant element to the borrower accordingly augments.*[6] As Table 1 shows, with $i = 0.08$ and $t^* = 11.157$, the assistance to the borrower exceeds 54 percent of the loan.

Finally, when the rate of interest approaches the opportunity cost of capital to the lender, the optimal maturity becomes infinite, and the grant to the borrower approaches 1.[7] However, the extension of the maturity implies that no problems of uncertainty, or inter-generational transfers arise. This thus reveals the *limits* of increased

[6] Combining (6) and (8) we can write the grant element to the borrower, if he follows the optimal strategy, as

$$p_b = 1 - (1 - \bar{p}_l)\frac{r_l - i}{r - 1}. \qquad (9)$$

If we differentiate p_b with respect to the rate of interest we obtain

$$\frac{d\,p_b}{di} = -(1 - p_l)\frac{r_l - i}{r - i}\cdot \ln(1 - p_l)\cdot\frac{r_l - r}{(r - i)^2} > 0.$$

[7] From (9), $p_b = 1 - (1 - p_l)\frac{r_l - i}{r - i}$. Now, if $i \to r$, the exponent tends toward infinity, and since $1 - p_l < 1$, we must have $p_b \to 1$.

assistance to the borrowing country, since maturity of the loan may become unrealistically remote. For example with $i = 9.9\%$, $r = 10\%$ and $r_1 = 15\%$ p.a. loan matures after more than 200 years, the grant element to the borrower is almost equal to 1, and the grant element to the lender remains 0.2.[8]

3. ANNUAL REPAYMENT REQUIREMENTS FOR THE BORROWER

Heretofore we have considered the repayment optimization problem when there is no requirement upon the amount of money the lender pays each year, and when the time horizon by which the overall debt must be reimbursed is unbounded. We shall briefly show that no major difference in conclusions emerges when these two assumptions are abandoned.

Suppose first that the lender wants the loan to be paid back in entirety by the year T. No requirement upon the annual amounts of repayments exists. The grant element p_1 is fixed. We must again solve equation (8): if the optimal maturity t^* is less than T, the borrower will be able to maximize his grant element. If, for example, $T = 4$ when $t^* = 3.7$ (see Table 1), no problem arises.[9] If, however, $T < 3.7$ the borrower will again optimize his position by deferring his payments until the last moment, i.e. by paying all of its debt $t = T$. If we suppose that the lender requires to be repaid in entirety after 3 years, it can be easily seen from equation (6) that the borrower's grant element (when $i = 0.04$, $r = 0.1$ and $r_1 = 0.15$) will drop from 0.336 to 0.281. On the other hand, the lender's grant element will be less than the one he is willing to give: instead of 20% it will equal only 16.5 percent (from (7)). We thus see that here we have two constraints: p_1 and T. If the former is the binding constraint, the borrower optimizes by choosing the optimal t^*. If the latter is the binding constraint, the borrower optimizes by repaying his debt in entirety at $t = T$. In either case, the optimal repayment schedule (»the point repayment strategy«) is the same.

Similar conclusions are obtained if there are annual repayment requirements. If the lender asks that the accrued interest be paid

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[8] It is within the same framework that we can now situate the problem we solved intuitively above. If $i = r$, the grant element to the lender becomes nil, as can be checked from (7). Whenever the lender is paid back, he will be paid entirely according to his opportunity cost of capital, so that the exact time of repayment for him becomes immaterial (assuming, of course, zero rate of pure time preference). The grant element to the borrower (see eq. (6)) increases with time. Consequently, he will have all reasons to pay his debt as late as possible. The optimal maturity thus approaches infinity, although, of course, it must be a real number if debt is ever to be repaid. In that sense, the »gain function« for the borrower is not bounded from below. However, if MPk of the debtor country declines, the problem is considerably simpler: the debt will be repaid when r_1 becomes equal to r.

[9] Note, however, that the borrower would not be allowed to repay his loan at $t = T$, since it would involve a greater grant element to the lender than he is willing to consent.

each year, the borrower will optimize again by paying out this minimum only, and repaying all of the debt either at the optimal maturity date, or at T. Alike in the previous case, he will pay all of the debt at T only if the lender requires to be repaid in totality by that time, and if so only if T is the binding constraint, i.e. smaller than the optimal maturity. The formula for the optimal maturity, however, now changes. Instead of the repayment stream as given by (5), the optimal repayment stream is now

$$\hat{P}(t) = \begin{cases} iL & for\ t \leqslant t_1^* \\ L & for\ t = t_1^* \end{cases} \tag{5a}$$

and the lender's grant element is equal to

$$g_1 = L - L/e^{r t_1^*} - \int_0^{t_1^*} i L e^{-rt} = L \left[1 - e^{-r t_1^*} - \right.$$

$$\left. - \frac{i}{r} (1 - e^{-r t_1^*}) \right] = L \left[(1 - e^{-r t_1^*}) \left(1 - \frac{i}{r} \right) \right]. \tag{7a}$$

Setting this last expression equal to some predetermined grant element the lender is willing to give ($\overline{p_1} = g_1/L$) we obtain the value for t_1^*:

$$t_1^* = \frac{-\ln \left[1 - \frac{\overline{p_1}}{1 - (i/r)} \right]}{r}. \tag{8a}$$

The borrower's grant element is now

$$P_b = \frac{gb}{L} = 1 - e^{-r_1 t_1^*} - \frac{i}{r} (1 - e^{-r_1 t_1^*}). \tag{6a}$$

The grant element to the borrower — with a given p_1 — is now reduced comparatively to what it was when no annual repayment requirements were present. For example, when $p_1 = 0.2$, we saw that for $i = 0.04$, we had $t^* = 3.719$ and $p_b = 0.336$. Now, introduction of the requirement that the borrower pay all of its interest annually will lengthen the optimal maturity ($t_1^* = 4.055$), but will reduce the assistance he receives ($p_b = 0.334$). Lengthening of the optimal maturity will not compensate for the interest payments the borrower must pay annually. If, in addition, we assume that the lender requires to be wholly repaid by, say $T = 4$ (i.e. a T smaller than t_1^*), the grant element to the borrower wolud decrease further.[10]

A special, and in actual practice the most frequent, case arises when the borrower is required to pay both the interest and amortization of debt annually. The borrower thus has no control over the repayment stream. For instance, if the borrower must pay each year an amount equal to 10 percent of the debt plus interest on the outstanding part, it is obvious that both the repayment stream and the maturity are uniquely determined. The payment function then becomes

$$P(t) = \hat{P}(t) = \frac{L}{T} + i\left(L - \int_0^t \frac{L}{T}\,dt\right) = \frac{L}{T}(1 - it) + iL. \qquad (5b)$$

The grant element to the borrower is

$$p_b = \frac{g_b}{L} = 1 - \frac{\int_0^T \left(\frac{L}{T}(1 - it) + iL\right) e^{-r_1 t}\,dt}{L} =$$

$$= 1 - \int_0^T \left[\frac{1}{T}(1 - it) + iL\right]^{-r_1 t} dt =$$

$$= 1 + \frac{1}{T r_1}\left[\left(1 - \frac{i}{r_1}\right)(e^{-r_1 T} - 1) - iT\right]. \qquad (6b)$$

The grant element to the lender has, of course, the same form as (6b) except that r is substituted for r_1. Note also that in this case, once p_1 is fixed, the final repayment time, T, is uniquely determined. This is unlike the previous cases (no annual requirement case, and only interest paid out annually) where a fixing of p_1 only still allowed the borrower to determine his optimal maturity (t^* and t_1^* respectively).

Table 3 compares assistance to the borrower under various repayment schedules. The grant element to the lender is put throughout equal to 0.2. The first column is calculated under the assumption that both the repayment schedule and the maturity are free. This is the case considered in Section 2, and the values reported here taken from Table 3. For the second column we assume that the borrower must

[10] The lender's actual grant element will now fall short of the amount (20%) he is willing to consent.

pay interest annually but is allowed to determine himself (subject to the $p_1 = 0.2$ constraint) the maturity. For the third column we assume that the borrower must take both T and the annual repayment schedule (1/T of the loan plus interest) as given.[11]

Table 3.

Grant to the Borrower and the Maturity o fa Loan with 20% Grant Element to the Lender Under Various Repayment Schedules

($r_1 = 0.15$, $r = 0.1$)

	(1)	(2)	(3)
$i = 0.04$			
T	3.719	4.055	8.742
p_b	0.336	0.334	0.325
$i = 0.05$			
T	4.463	5.108	11.263
p_b	0.360	0.357	0.349
$i = 0.07$			
T	7.438	10.986	28.215
p_b	0.448	0.431	0.409

The results show that a given grant element to the lender involves different assistance to the borrowing country according to the repayment schedule selected (or imposed). As expected, p_b is greater the less constrained is the borrower in his repayment schedule. Although the loss of the assistance to the borrower is not very significant in Table 3, it is achieved only at the cost of substantial increase in maturity.

Obviously, if the lender is unwilling to accept such an increase in maturity, the level of assistance to the borrower will be accordingly reduced under the alternatives (2) and (3). A more realistic view of the grant element under various repayment schedules is therefore obtained if we fix the maturity T, and observe how the grant element is then distributed between the lender and the borrower. In Table 4 we let $T = 10$, $r_1 = 0.15$, $r = 0.10$.

[11] T is calculated so as to make the grant element to the lender be exactly 0.2. The three cases are thus comparable.

Table 3.

Grant to the Borrower and to the Lender When the Maturity is 10 years Under Various Repayment Schedules

	(1)	(2)	(3)
$i = 0.04$			
p_l	0.451	0.379	0.221
p_b	0.667	0.570	0.354
$i = 0.05$			
p_l	0.393	0.316	0.184
p_b	0.632	0.518	0.321
$i = 0.07$			
p_l	0.259	0.151	0.110
p_b	0.551	0.414	0.257

As can be seen, the most restrictive repayment schedule, in which both debt amortization and interest are paid annually, cuts significantly into the grant element. The decline is more severe for the borrower than for the lender. For example, a concessional loan at 4% p.a. which would be given for 10 years without any constraint on repayment schedule would yield an assistance to the borrower equal to 2/3 of the loan. Once the lender exacts that both 1/10 of the loan and interest be paid out annually, the assistance falls to almost 1/3 of the loan. The grant element to the lender is simultaneously reduced from 45 to 22 percent of the loan.

Consequently, when the exact repayment schedule is predetermined, the only factor whereby the borrower may seek to increase his grant element is the grace period. The addition of the grace period which leaves the period during which the actual repayment is made unchanged (e.g. two years grace plus 10 years of repayment) obviously involves a postponement of the maturity date.

However, when both duration of the grace period and maturity are subject to negotiation, the well-known trade-off between the two arises. The borrower is able to compare different alternatives and to choose the most favorable one. For a given grant element to the lender, the lender's indifference curve may look like LL in Figure 1. Then, to each point on the lender's indiference curve will correspond a point on *different* borrower's indifference curves. As the grace period along the LL curve becomes greater relatively to the maturity (i.e. as one moves north-west along the LL curve), ever higher borrowers's indifference curves are reached.[12] Finally, the highest borrower's indifference curve — for a given grant element to the lender — will be attained at the intersection of the LL curve and the 45° line. Along the latter, grace period and maturity are exactly equal: all the loan (plus interest) is repaid at the moment of maturity and nothing is paid before. Consequently, the optimal point for the borrower will always lie along the 45° line. The 45° line is the locus of optimal repayment time points (t*'s). Where exactly the borrower will operate depends on the grant element the lender is willing to transfer.[13] The borrower will settle for an »interior solution«, to the right of the 45° line, only if requirements upon the annual payments are imposed. For example, if there is an annual repayment requirement with no grace period and a given p_1 as depicted by the repayment stream (3) in Tables 3 and 4, the selected points will lie at the intersection of LL indifference curves and the horizontal axis. It can be thus seen, graphically, that when there are annual repayment requirements it

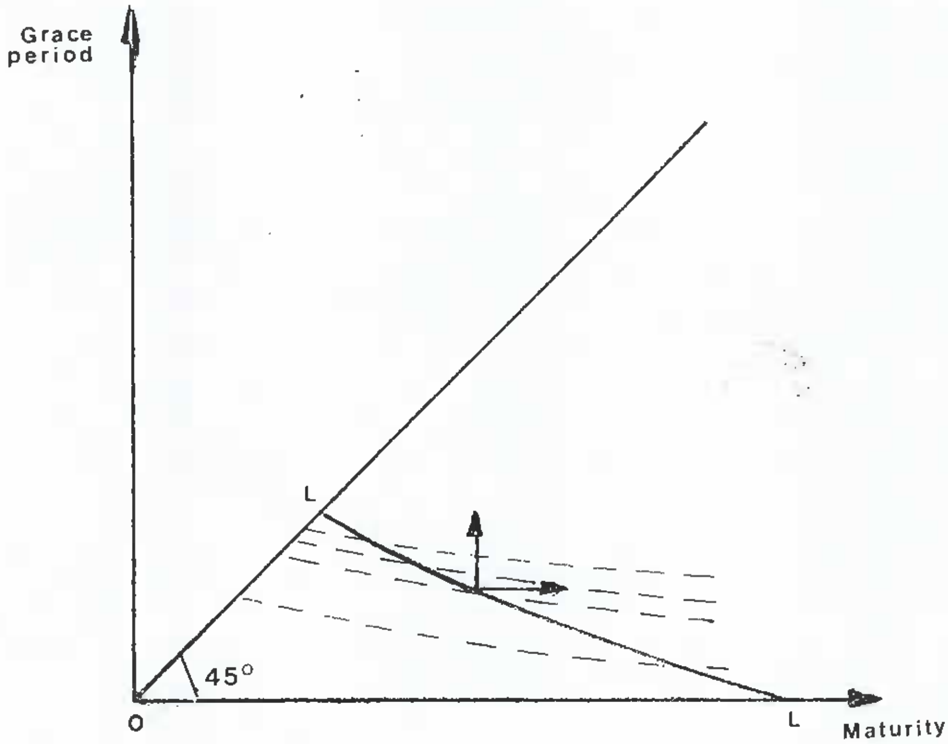

Figure 1

[12] The borrower's indifference curves must be flatter than the lender's because of the greater discount rate: $r_1 > r$. Thus, a one year extension of the grace period will increase by more the grant element (assistance) to the borrower than the cost to the lender. The borrower will be accordingly ready to consent to a greater reduction in the maturity date than the lender, and still remain on the same indifference curve. (The lender will for that earlier maturity date move to a higher indifference curve.) Note also that the Figure must be drawn so as to contain all indifference curves to the right-hand side of the 45° line, since the total duration of the loan (between the point of disbursement and final repayment of the loan) must be equal to, or greater than, the grace period.

[13] As this becomes greater, the optimal point will, of course, slide upwards, in the north-easterly direction.

suffices to fix the grant element to the lender in order to determine uniquely the maturity date. Finally, if the grant element the lender is willing to transfer is not fixed, the borrower will obviously attempt to increase the grace period for a given maturity (or to expand maturity for a given grace period). The two moves are depicted by arrows in Figure 1.

4. CONCLUSIONS

The grant element to the lending and to the borrowing country differ because of differences in the opportunity cost of capital. The latter is generally greater in the borrowing (developing) country. The same repayment stream is accordingly discounted by a greater rate in the developing country, than in the developed one, and the grant element to the borrower exceeds the grant element to the lender. Then, for a given grant element to the lender, one might try to find such a repayment schedule which would maximize the grant element to the borrower.

We have shown that the optimal schedule requires that the borrower pay the totality of its debt plus interest at once. We term this the »point repayment strategy«. It is equivalent to a relative extension in the grace period until the grace period and the maturity of the loan become equal. In order to determine the optimal maturity of the loan, one needs to know (a) percentage of the total loan the lender is willing to transmit as grant, (b) rate of interest charged on the loan, one needs to know (a) percentage of the total loan the lender ments (a) and (b) should be explicitly stated when the loan is made. Element (c) is readily available. It is important to point out that the optimal maturity does not depend on the marginal product of capital in the developing country, which may be relatively more difficult to assess.

Other than maximizing the assistance to the borrowing country — at a given cost to the lender (donor) — this approach has a further advantage of necessitating an explicit statement by the lender of the portion of the loan he is willing to consent as grant. Concurrently, it would enable concessional finance institutions to classify borrowing countries according to the percentage of the loan the institution is willing to transmit as grant. Several categories may thus be formed such that the grant element given by the lender decreases with the level of development. This would lead to a greater differentiation among the countries than the present essentially two-tier system followed by international development institutions. Another policy lever which the lending institution retains is the rate of interest. One could envisage a situation where the discrimination among a given group of projects and countries, which all »cost« the lending insitution the same (i.e. the grant element to the lender is the same), would be made through difference in rates of interest, and consequently in maturities. Longer maturity would require a higher rate of interest if the grant element to the lender is to be the same. The borrowing country may be allowed to select, among a given range of interest rates and maturities, the preferred combination. A possible drawback of the point repayment strategy may be the fear that, since all of the debt must be repaid at a single date, the borrower may, due to fluctuations in his income, be unable to meet the deadline. While it is a legitimate concern for a single borrower and a single loan (although a more prudent financial management should be able to take care of it), it is less so for a country which faces many loans. It is, in effect, then equivalent whether one repays a portion of all the loans annually, or pays every year one loan in entirety. So long as no bunching of loans occurs, and all loans are of approximately similar size, the two repayment streams will be identical. Yet, the point repayment strategy will yield a greater benefit to the borrower.

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O BESPOVRATNOJ POMOĆI I ZAJMOVIMA

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Rezime

Cilj članka je da se utvrdi optimalni program otplate duga i optimalno dospeće za zemlju—dužnika kada zajmodavac odobrava elemenat bespovratne pomoći. Optimalna strategija se sastoji u odlaganju otplate (ili otplaćivanju isključivo minimalne svote) do optimalnog dospeća, kada zemlja—dužnik otplaćuje ceo dug i akumuliranu kamatu. Tvrdi se da usvajanje programa jednokratne celokupne otplate može biti obostrano korisno: on bi zahtevao od zajmodavca jasnu obavezu u pogledu iznosa bespovratne pomoći koju je spreman da odobri; takvo maksimiranje pomoći dužniku takođe bi maksimiralo korisnost zajmodavca i zajmoprimaoca.