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A MODEL OF MANY CAPITAL GOODS*

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A. CHANGING LABOUR

Quantity equations

Consider an economy with one consumer good (X_1) and two machines (X_2, X_3). Let the life-span of the two machines be the same ($n_1 = n_2 = n$). The number of quantity equations matches the number of variables:

$$\begin{aligned} &\lambda_1 X_1 + \lambda_2 X_2 + \lambda_3 X_3 = L \\ &r\kappa^1{}_1 X_1 + r\kappa^1{}_2 X_2 + r\kappa^1{}_3 X_3 = rK^1 = X_2, \qquad \kappa^1{}_i = \frac{K^1{}_i}{X_i} \\ &r\kappa^2{}_1 X_1 + r\kappa^2{}_2 X_2 + r\kappa^2{}_3 X_3 = rK^2 = X_3 \qquad \kappa^2{}_i = \frac{K^2{}_i}{X_i} \end{aligned} \tag{1}$$

where K^1 and K^2 are stocks of machines of type one and two, and X_2 and X_3 are per period outputs of the same machines.

The last two equations in (1) represent a homogeneous system. If X_1 is eliminated from these two equations, X_2 bears a fixed proportion to X_3 as given by

$$\frac{X_2}{X_3} = \frac{K^1}{K^2} = \frac{rb_2 + \kappa^1{}_1}{rb_1 + \kappa^2{}_1}, \qquad \begin{aligned} b_1 &= \kappa^1{}_1\kappa^2{}_2 - \kappa^2{}_1\kappa^1{}_2 \\ b_2 &= \kappa^2{}_1\kappa^1{}_3 - \kappa^1{}_1\kappa^2{}_3 \end{aligned} \tag{2}$$

If either $r = 0$ or $b_1 = b_2 = 0$, the proportion in which the two types of machines are produced is equal to the ratio of capital coefficients of those machines in the consumer-good industry. Now the role of machine-worker ratio (the degree of mechanization) is replaced by machine-output ratio (capital coefficient); $b_1 = b_2 = 0$ implies .

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$$\frac{\kappa^1_1}{\kappa^2_1} = \frac{\kappa^1_2}{\kappa^2_2} = \frac{\kappa^1_3}{\kappa^2_3} = \frac{K^1}{K^2} = \frac{X_3}{X_3}$$

In other words, it is implied that in each industry machines of type 1 and 2 are used in the same proportion in which they are produced.

From $K^1/K^2 = X_2/X_3$ it follows that $X_3/K^2 = X_2/K^1$, i. e., gross investment rates are the same in both industries. Generally, the ratio X_2/X_3 will be a function of the investment rate and all capital coefficients (changing under TP).

The elimination of X_1 from the first two equations gives

$$(rm^1_3 + \lambda_1)\, X_2 + rm^1_1\, X_3 = r\kappa^1_1 L \quad ,$$

$$m^1_1 = \kappa^1_1 \lambda_3 - \kappa^2_3 \lambda_1 \quad , \qquad m^1_3 = \kappa^1_1 \lambda_2 - \kappa^1_2 \lambda_1$$

Use (2) to obtain

$$X_1 = \left(\frac{1}{\lambda_1} - \frac{r^2 C + rD}{\lambda_1 (r^2 A + rB + \lambda_1)} \right) L \; , \qquad A = \frac{b_2 m^1_3 + b_1 m^1_1}{\kappa^1_1}$$

$$X_2 = \frac{r\,(rb_2 + \kappa^1_1)}{r^2 A + rB + \lambda_1} L \; , \qquad B = \frac{m^1_3 \kappa^1_1 - m^1_1 \kappa^2_1 + \lambda_1 b_2}{\kappa^1_1} \qquad (3)$$

$$X_3 = \frac{r\,(rb_2 + \kappa^2_1)}{r^2 A + rB + \lambda_1} L \; , \qquad C = \lambda_2 b_2 + \lambda_3 b_1 \; , \quad D = \lambda_2 \kappa^1_1 + \lambda_3 \kappa^2_1$$

The introduction of just one additional machine makes life quite complicated. That explains the popularity of one-machine economies, or of a triangularity of input matrix, or of machines being produced by labour alone.

Price equations

Price equations are also fully determined and also more complicated because each equation has two rental terms corresponding to two capital goods.

$$\begin{aligned} rp_2\kappa^1_1 + rp_3\kappa^2_1 + \lambda_1 &= p_1 \\ rp_2\kappa^1_2 + rp_3\kappa^2_2 + \lambda_2 &= p_2 \qquad (4) \\ rp_2\kappa^1_3 + rp_3\kappa^2_3 + \lambda_3 &= p_3 \end{aligned}$$

Machine prices are determined exclusively by the coefficients of their own industries. Once determined, they can be used to find out the consumer good price.

$$p_1 = \frac{r^2(\lambda_1 b_3 + \lambda_2 b_2 + \lambda_3 b_1) + r(m^1_3 + m^2_1) + \lambda_1}{r^2 b_3 - r(\kappa^1_2 + \kappa^2_3) + 1} \qquad b_3 = \kappa^2_3\kappa^1_2 - \kappa^1_3\kappa^2_2$$

$$p_2 = \frac{\lambda_2 + rm^2_2}{r^2 b_3 - r(\kappa^1_2 + \kappa^2_3) + 1}, \qquad m^1_2 = \kappa^1_2\lambda_3 - \kappa^1_3\lambda_2$$

$$m^2_2 = \kappa^2_2\lambda_3 - \kappa^2_3\lambda_2$$

$$p_3 = \frac{\lambda_3 - rm^1_2}{r^2 b_3 - r(\kappa^1_2 + \kappa^2_3) + 1}, \qquad m^2_1 = \kappa^2_1\lambda_3 - \kappa^2_3\lambda_1$$

$$m^1_3 = \kappa^1_1\lambda_2 - \kappa^1_2\lambda_1 \tag{5}$$

For $r = 0$, all prices reduce to their labour coefficients, $p_1 = \lambda_1$, $p_2 = \lambda_2$, $p_3 = \lambda_3$. If $m^1_2 = m^2_2 = 0$, the ratio of machine prices is equal to the ratio of labour coefficients, $p_2/p_3 = \lambda_2/\lambda_3$. Note that m^i_j are not the same mechanization indices as before. For instance, $m^1_2 = 0$ implies

$$\frac{\kappa^1_2}{\lambda_2} = \frac{K^1_2}{X_2} \bigg/ \frac{L_2}{X_2} = \frac{\kappa^1_3}{\lambda_3} = \frac{K^1_3}{X_3} \bigg/ \frac{L_3}{X_3}$$

and

$$\frac{K^1_2}{L_2} \neq \frac{K^1_2}{L^1_2}, \qquad \frac{K^1_3}{L_3} \neq \frac{K^1_3}{L^1_3}$$

L^1_2 is the number of workers who operate K^1_2 machines of type one in industry two. L_2 is the total number of workers employed in industry two, $L_2 = L^1_2 + L^2_2 + L^3_2$. Thus, K^1_2/L^1_2 is a proper degree of mechanization, and K_2^1/L_2 may be called quasi intensity of mechanization. But equal quasi mechanization in machine industries is not a sufficient condition for the two ratios to be equal when consumer good price is considered. For that five additional conditions are required:

$m^1_3 = m^2_1 = 0$, and $b_1 = b_2 = b_3 = 0$. Note, however, that $b_1 = b_2 = 0$ implies $b_3 = 0$. In other words, apart from equal quasi mechanization ratios in all industries, the proportions of machines produced and used in each industry must also be equal.

The wage curve

Per capita consumption of a single consumer good represents the real wage. It follows from the first equation of (3)

$$\bar{w} = x_1 = \frac{1}{\lambda_1} - \frac{r^2 C + rD}{\lambda_1 (r^2 A + rB + \lambda_1)} \tag{6}$$

The curve is now considerably more complicated than before because r^2 appears in the numerator and denominator.

If w (r) is to be made linear, some coefficients must be zero: C = A = B = 0. That implies uniform quasi mechanization ($m^1_1 = m^1_3 = 0$) and also uniform capital-output intensity ($b_1 = b_2 = 0$). Under these conditions the wage curve reduces to a straight line

$$\bar{w} = \frac{1}{\lambda_1} - r \frac{D}{\lambda^2_1} \tag{7}$$

Since there is only one consumer good, maximum per capita consumption remains the same as in the standard two-sector model and is determined by the labour coefficient of the first industry, $W(r = 0) = \frac{1}{\lambda_1}$. That also follows from the first equation in (1) when we put $X_2 = X_3 = 0$. But maximum rental depends apparently on all capital and labour coefficients and is determined by a quadratic equation

$$R\,(w = 0) = R^2 (A - C) + R\,(B - D) + \lambda_1 = 0$$

This equation can be somewhat simplified if it is recalled that in the labour system the real wage is the reciprocal of the price of the consumer good. Using (5), we obtain

$$\bar{w} = \frac{1}{p_1} = \frac{r^2 b_3 - r\,(\kappa^1_2 + \kappa^2_3) + 1}{r^2 (\lambda_1 b_3 + \lambda_2 b_2 + \lambda_3 b_1) + r\,(m^1_3 + m^2_1) + \lambda_1} \tag{8}$$

As a ratio of two second degree polynominals, the wage curve may have three inflection points [since $d^2w/dr^2 = f\,(r^3)$].

For $r = 0$ or $b_1 = b_2 = b_3 = m^1_3 = m^2_1 = 0$, the wage curve reduces to a straight line

$$\bar{w} = \frac{1}{\lambda_1} - r \frac{\kappa^1_2 + \kappa^2_3}{\lambda_1} \tag{7a}$$

which is indentical with (7) because

$$\frac{D}{\lambda^2_1} = \frac{\kappa^1_2 + \kappa^2_3}{\lambda_1}$$

if $m^2_1 = m^1_3 = 0$.

The maximum wage remains, of course, the same, $\overline{W}(r = 0) = \frac{1}{\lambda_1}$, but the maximum R (w = 0) results from a simpler quadratic equation

$$r^2 b_3 - r(\kappa^1{}_2 + \kappa^2{}_3) + 1 = 0 \quad (9)$$

and depends *exclusively* on capital coefficients in machine industries.

An analysis of equation (9) and of its solutions

$$R_{1,2} = - \frac{(\kappa^1{}_2 + \kappa^2{}_3) \pm \sqrt{(\kappa^1{}_2 + \kappa^2{}_3)^2 - 4b_3}}{2b_3}$$

reveals the classifying role of b_3, i.e., of the interrelations among four capital coefficients of the two machine industries:

$$b_3 > 0 \qquad : R_1 > 0, \quad R_2 < 0$$

$$b_3 = 0 \qquad : R = \frac{1}{\kappa^1{}_2 + \kappa^2{}_3}$$

$$0 < b_3 < \left(\frac{\kappa^1{}_2 + \kappa^2{}_3}{2}\right)^2 \qquad : R_1, R_2 > 0$$

$$0 < b_3 = \left(\frac{\kappa^1{}_2 + \kappa^2{}_3}{2}\right)^2 \qquad : R = \frac{2}{\kappa^1{}_2 + \kappa^2{}_3}$$

$$b_3 > \left(\frac{\kappa^1{}_2 + \kappa^2{}_3}{2}\right)^2 \qquad : R_1, R_2 \text{ do not exist}$$

R is unique in two cases: for $b_3 = 0$ and $b_3 = \left(\frac{\kappa^1{}_2 + \kappa^2{}_3}{2}\right)^2$.

For negative b_3, only one R is positive and that one is admissible. Both R's are positive for positive b_3 smaller than $\left(\frac{\kappa^1{}_2 + \kappa^2{}_3}{2}\right)^2$. In this case only the smaller of the two is admissible because $\overline{w}(r) \geqslant 0$ and decreasing. Finally, b_3 cannot be larger than $\left(\frac{\kappa^1{}_2 + \kappa^2{}_3}{2}\right)^2$ because in that case the wage function does not exist. Note that

$\frac{\kappa^1_2 + \kappa^2_3}{2}$ is an average of the own capital coefficients in two machine industries and $b_3 = \kappa^2_3 \kappa^1_2 - \kappa^1_3 \kappa^2_2 = 0$ implies $\frac{\kappa^2_3}{\kappa^1_3} = \frac{\kappa^2_2}{\kappa^1_2}$, i. e., equal ratios of machine-output intensities in the two machine industries.

B. TECHNOLOGICAL PROGRESS

Capital coefficients unchanged

Let us again consider labour augmenting unembodied technological progress. As two different machines are produced, we now have two investment equations:

$$\text{Labour equation:} \qquad \sum_{i=1}^{3} (\lambda_i \Gamma_i^{-1}) X_i H_i = (\Gamma^{-1} L) \Gamma = L$$

$$\text{Investment equations:} \qquad \sum_{i=1}^{3} \kappa^1_i X_i (h^1_i + \delta^1_i) = (h^1 + \delta^1) K^1 = X_2 \qquad (9)$$

$$\sum_{i=1}^{3} \kappa^2_i X_i (h^2_i + \delta^2_i) = (h^2 + \delta^2) K^2 = X_3$$

Three equations determine three H_i.

The vertical summation of the components of quantity equations evaluated and the corresponding prices produces the Pasinetti system of price equations

$$p_2 \kappa^1_i r_i + p_3 \kappa^2_i r_i + w\lambda_i = p_i, \; i = 1, 2, 3, \quad r_i = h^1_i + \delta^1_i = h^2_i + \delta^2_i \qquad (10)$$

In each industry $\underline{i}$, K^1_i and K^2_i expand at the same rate h_i determined by the expansion of effective labour, which is different for each industry. This implies three different profit rates h_i, one for each industry.

Gross investment rates $r^i = h^i + \delta^i$, determined by two investment equations in (9), represent averages for two types of machines and they are equal, $r^1 = r^2$. This is the required uniform rental which implies a uniform profit rate.

With uniform $\underline{r}$, prices are labour prices constructed in such a way that individual stationary costs are augmented by the (average) investment costs (corrected for replacement reduction) incurred by the economy in order to employ workers rendered redundant by technological progress.

If working hours are also changing, the (positive or negative) rate of change must be included in the rental rate

$$r = \delta + \pi, \qquad \pi = GH - 1 \tag{11}$$

If the durability of machines differs, $n_1 \neq n_2$, there will be two different δ's.

They will be related as follows

$$\frac{\delta^1}{\delta^2} = \frac{\pi}{\Pi^{n_1} - 1} \bigg/ \frac{\pi}{\Pi^{n_2} - 1} = \frac{\Pi^{n_2} - 1}{\Pi^{n_1} - 1}, \qquad \Pi = 1 + \pi \tag{12}$$

For $\pi = \text{const.}$, (12) holds strictly; if π changes in time, (12) is an approximation.

Now all rental rates are different

$$r^j = \delta^j + \pi. \qquad j = 1, 2 \tag{11a}$$

where the superscript j indicates the type of the machine. The profit rate is uniform

$$\pi = GH - 1 \doteq g + h \tag{12}$$

and it may be positive, zero or even negative for a sufficiently large contraction of labour ($g < 0$).

Capital coefficients change

If both labour productivity and capital efficiency improve, the situation becomes somewhat messy. Let L^s_j represent the number of workers in industry j operating machines of type s. Productivity of that group of workers increases by Γ^s_j and output by H^s_j. The output of corresponding machines must also increase by H^s_j. Their efficiency increases by $^*\Gamma^s_j$. The labour and machine balances amount to

$$(\Gamma^1_1)^{-1}L^1_1H^1_1 + (\Gamma^1_2)^{-1}L^1_2H^1_2 + (\Gamma^1_3)^{-1}L^1_3H^1_3 = (\Gamma^1)^{-1}L^1H^1$$

$$(\Gamma^2_1)^{-1}L^2_1H^2_1 + (\Gamma^2_2)^{-1}L^2_2H^2_2 + (\Gamma^2_3)^{-1}L^2_3H^2_3 = (\Gamma^2)^{-1}L^2H^2$$

$$(\Gamma^1)^{-1}L^1H^1 + (\Gamma^2)^{-1}L^2H^2 = L \tag{13}$$

$$(^*\Gamma^1_1)^{-1}K^1_1H^1_1 + (^*\Gamma^1_2)^{-1}K^1_2H^1_2 + (^*\Gamma^1_3)^{-1}K^1_3H^1_3 = (^*\Gamma^1)^{-1}K^1H^1$$

$$(^*\Gamma^2_1)^{-1}K^2_1H^2_1 + (^*\Gamma^2_2)^{-1}K^2_2H^2_2 + (^*\Gamma^2_3)^{-1}K^2_3H^2_3 = (^*\Gamma^2)^{-1}K^2H^2$$

These balances may be simplified. We do not know separate outputs for the two machines. They jointly contribute to the same output and so $H^1_j = H^2_j$. Next, we may assume that the efficiency of a machine, and the corresponding productivity of labour, increase about equally in all uses, $^*\Gamma^s_j = {}^*\Gamma^s_i = {}^*\Gamma^s$, $\Gamma^s_j = \Gamma^s_i = \Gamma^s$. Consequently,

$$(\Gamma^1)^{-1} \; (L^1_1H_1 + L^1_2H_2 + L^1_3H_3) \; = (\Gamma^1)^{-1}L^1H^1$$

$$(\Gamma^2)^{-1} \; (L^2_1H_1 + L^2_2H_2 + L^2_3H_3) \; = (\Gamma^2)^{-1}L^2H^2$$

$$(\Gamma^1)^{-1}L^1H^1 \; + (\Gamma^2)^{-1}L_2H^2 = L$$

$$({}^*\Gamma^1)^{-1} \; (K^1_1H_1 + K^1_2H_2 + K^1_3H_3) \; = ({}^*\Gamma^1)^{-1}K^1H^1 \qquad (13a)$$

$$({}^*\Gamma^2)^{-1} \; (K^2_1H_1 + K^2_2H_2 + K^2_3H_3) \; = ({}^*\Gamma^2)^{-1}K^2H^2$$

where $H^1 = H^2$.

Transformed into quantity equations, the first three equations may be simplified further into

$$\lambda_1X_1H_1 + \lambda_2X_2H_2 + \lambda_3X_3H_3 = L$$

where the new labour coefficients are

$$\lambda_j = \frac{L^1_j\,(\Gamma^1)^{-1} + L^2_j\,(\Gamma^2)^{-1}}{X_1}, \qquad j = 1, 2, 3 \qquad (14)$$

As in (9), (13a) reduces to three equations which determine three H_i. The average productivity factor for labour employed in industry j is

$$\Gamma_j^{-1} = \frac{L^1_j\,(\Gamma^1)^{-1} + L^2_j\,(\Gamma^2)^{-1}}{L_j}, \qquad j = 1, 2, 3 \qquad (15)$$

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