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LEARNING BY DOING IN CAPITALIST AND ILLYRIAN FIRMS: A CONTROL THEORETIC EXPLORATION

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I

In a pathbreaking paper of 1962, Keneth Arrow[1] introduced the concept of »learning by doing« to systematic economic theory. The central insight of that paper — that the practical experience of ongoing production is a major source of productivity increases — has yet to be fully integrated into the theory of the worker-managed enterprise or, indeed, of the capitalist firm. This paper uses optimal control theory to explore the implications of learning by doing for the »Illyrian« model of worker management[2] and, for comparison, for a capitalist firm which maximizes the discounted present value of the firm. This section discusses an approach to modelling learning by doing. Section II demonstrates the significance of learning by doing for the profit-maximizing firm, Section III for the Illyrian firm, Section IV compares the two cases, and Section V summarizes and concludes.

We assume that the firm produces a homogenous output, using two tangible inputs: undifferentiated labor and an undifferentiated[3]

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[1] Note also Levhari, Sheshinsky. The literature on learning by doing is modest, but a complete review is nevertheless beyond the scope of this paper.

[2] The founding paper, of course, was Ward. Domar's paper and Vanek's book are also among the essential sources. The literature of extensions and criticism of this model is very large and beyond the scope of this paper.

[3] It is now well-known that aggregate capital cannot in general be defined. However, the simplifying assumption of a single, homogenous (non-human) capital remains a common one in current economic work. In the context of this paper, the assumption is made purely for simplicity, and there is no reason to think that the results would be affected if, instead, the model

nonhuman means of production which (following the tradition of bourgeois economics) we shall call »capital«. Production possibilities at a particular moment of time are represented by a production function, so that

$$Q \leq f(K, N, X) \qquad 1.1.$$

where Q is the instantaneous flow of output, N the flow of labor input, K is the »capital« stock[4] and X is a (hypothetical) measure of an intangible fund of experience of use to the enterprise at the moment in time. In the terms of recent bourgeois economics, X is a fund of largely firm-specific »human capital«, encompassing (at least) the »organizational capital« of Prescott and Visscher. I adopt this terminology.

What X has in common with K (and what justifies the reference to it as a form of »capital« in the language of bourgeois economics) is that it is a fund which is augmented only over time, and thus an »irreducibly dynamic concept« (to use a phrase of Horvat's). It differs, however, in that increments of X are not the result of diverting resources from consumption to investment but are a by-product of the ongoing work of production. This is the hypothesis of learning by doing: the routine work of production induces learning which increases the magnitude of the intangible fund of experience X and thus increases the productivity of K and N. Reference here is both to average and to marginal productivity.

It may be worthwhile to digress a bit on the material nature of X and its relation to learning. Obviously X is embodied in material objects, namely the nervous systems of the workers (including, but not exclusively, managerial workers). Certainly X includes the ordinary technical skills of production which the workers have acquired through experience, often in the form of »tacit knowledge« (Polanyi). If that were all that X encompassed, then each worker would be able to depart from the enterprise, taking with him his own aliquot part of X, which would have its proportionate value to another enterprise which would consequently be repaid to him in wages in an idealized capitalist market economy. That is, in the language of recent bourgeois economics, the human capital represented by X would then be possibly industry-specific but not firm-specific.[5]

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were to allow for an indefinite number of distinct kinds of capital goods. The analysis would be very much complicated.

[4] As Nicholas Geogescu-Roegen argues, even if aggregate capital may be defined, it may be impossible to define a production function in such a way that the rate of output is a simple function of the rates of input. Instead, the output may have to be a functional of input rates; e. g. an integral equation over the history of input rates. Here, again, (as in the previous footnote) a general analysis would be inordinately complicated. Thus the neoclassical production function is, again, adopted purely as a simplifying assumption.

[5] This concept (like so many in the extensive literatrue on »human capital«) is due to Becker.

However, X encompasses more than ordinary technical skills. It encompasses also the »organizational capital« of Prescott and Visscher — that is, some knowledge of the unique capabilities and incapabilities of each worker (on the part of supervisers and managers) and of the unique needs and routines of the enterprise (on the part of the worker). This component of human capital is firm-specific — for if the worker leaves the enterprise, that knowledge is no longer of use and has no value to any other enterprise.

I would argue that organizational capital is a subset of a broader category which is of fundamental importance for our understanding of the determinants of productivity. I propose to call this broader category »role learning« where the reference is to social roles within the microsociety which is the enterprise. When human beings engage in coordinated work they usually assume differentiated, complementary roles in production activity. These roles may be expressed by such phrases as »I'll lift while you shove,« »you take the front end and I'll take the back'« or »you hit 'im high and I'll hit 'im low.« As the phrases themselves suggest, the allocation of roles among persons may be settled by discussion (i. e., planning), but when a group of persons engage repetitively in the same tasks the allocation of roles may become a matter of habit and tacit knowledge. This tacit knowledge can imply considerable savings of the time which would be required for discussion, and of other resources which would go to waste while the discussion is carried on.

In this context, »learning by doing« has two quite different meanings. On the one hand, the structure of social roles in the group may be subject to gradual change and refinement through experience. A less effective habitual allocation of roles may be substituted by a more effective one. As both are tacit, the substitution may well go unnoticed. In this way, the group acquires a distinct identity, and its unique, unstated role structure may be an important determinant of its productivity. This is what I mean by role learning. It is literally learning by the group as a whole, which takes the form of change in the tacit, habitual social role structure of the group and which raises group productivity. I suggest that role learning, by the group as a whole, is the principle source of productivity improvement through learning by doing.

Now suppose that one of the members of the work group departs and is replaced by a new member. Initially, that member does not know his social role, and he must learn it by experience (since it is largely tacit). This, too, is learning by doing, but it is likely that the »new 'un« will learn his social role in the existing group much more rapidly and easily than that social role was originally learned by the group as a whole. Thus the replacement of a single worker, or a small proportion of the members of the work force, will dilute the fund of experience X but will dilute it less than proportionately. However, if the work force were to be entirely dispersed and replaced with a new force of unacquainted workers, then the fund of experience would be entirely dissipated, and the role learning process would have to begin again from start. (This point may have considerable significance for economic development). It seems that the dilution would increase nonlinearly with the rate of replacement of the work force, so that replacement of (say) 80 or 90% of the work force would have the same effect as its entire dispersion, but this is a conjecture and the contrary case might be argued.

These reflections lead me to posit the following »law of motion« for X[6].

$$\dot{X} = g(N, I, H) \qquad 1.2.$$

where $\dot{X}$ denotes the derivative of X with respect to time, I is the gross investment of the enterprise, and H is the rate of new hires or recruitment of new members in the case of a worker-managed enterprise. It is assumed that g (. , . , .) increases with increasing N, i. e., that the greater the work force exposed to learning, the greater is the learning which occurs. It is assumed that g (.,.,.) increases with increasing I, on the grounds that new capital goods are of more recent vintages and generally superior to old capital goods (reflecting learning by doing in the capital goods industries: *vide* Arrow) and because the new capital goods create new opportunities for further role learning which leads to further increases in X. Finally, it is assumed that g (.,.,.) decreases with increasing H, perhaps at an increasing rate reflecting the nonlinear impact of replacement on role learning.

Arrow had assumed that the increase of productivity depended *only* on I, i. e. that learning by doing took place only in capital goods industries, but this was a simplifying assumption to produce a tractible aggregate growth model in the vintage capital tradition. Arrow did observe informally that output might be used as an index of learning. Given his aggregative viewpoint, the size of the work force is given, so the rate of hires or of labor turnover at the enterprise level did not arise in his model.[7]

In the sections to follow, the methods of control theory will be used (Takayama, Hestenes) and X will be treated as a state variable. As usual, K will also be treated as a state variable, with a law of motion which reflects the »radioactive decay« hypothesis of depreciation:

$$\dot{K} = I - \sigma K \qquad 1.3.$$

and σ is the rate of depreciation. Finally, N, the labor input, will also be treated as a state variable, i. e., as being determined by the work force, which may not be instantaneously flexible. Since we must in any case treat H separately, it is natural to define a law of motion for N as

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[6] The dot notation for derivatives with respect to time is adopted in what follows, so that the notation x means the derivative of the variable x with respect to time.

[7] Sheshinsky also made learning a function of gross investment in his optimal growth model, presumably for similar reasons, but allowed productivity improvement to be disembodied.

$$\dot{N} = H - S \qquad 1.4.$$

where S is separations. Supply conditions imply that both H and S may be constrained. For H,

$$H \leq \phi(w) \qquad 1.5.$$

where w is the wage. The reasoning here is that (first) the enterprise will be unable to recruit new workers unless it pays enough so that they receive at least their best alternative ,and (second) that inducing workers to transfer from other sectors will impose on them a transactional cost or »job market toll« (Okun) and moreover the transactional cost will rise with the rate of recruitment. Finally, with respect to S,

$$S \geq \Psi(w) \qquad 1.6.$$

since workers cannot be retained unless they get at least their best alternative, net of transactional costs, but can be laid off.

II

By hypothesis, a capitalist firm seeks to maximize the present value of the flow of net revenue from the firm. That is, defining

$$\pi = pQ - wN - I \qquad \text{II.1.}$$

where p is the price of the output at a moment of time and w the wage, the firm would strive to

$$max \int_0^{\infty} e^{-rt}\pi(t)dt \qquad \text{II.2.}$$

subject to the constraints and laws of motion outlined in Section I. Assuming that the integral converges to a finite value (as it must if $r > 0$ and π is bounded) and taking

$$F(K(T), N(T), X(T)) = \int_T^{\infty} e^{-rt}\pi(t)dt \qquad \text{II.3.}$$

that is

$$\int_0^{\infty} {}^{-rt}\pi(t)dt = \int_0^{T} {}^{-rt}\pi(t)dt + F(K(T), N(T), X(T)) \qquad \text{II.4.}$$

where the horizon, T, is chosen purely for computational convenience and F(K(T), N(T), X(T)) is the discounted value of the firm at time T.

It may be observed that in assuming an infinite horizon for the capitalist firm we are making a rather strong assumption about the bequest motives of capitalists, but exploration of bequest motives is beyond the scope of this paper. The problem thus becomes

$$max \int_0^T e^{-rt} \pi(t)dt + F(K(T), N(T), X(T)) \qquad \text{II.5}$$

subject to the constraints and laws of motion already explained.

The Hamiltonian-Lagrangean function and some necessary conditions are shown in Table 1. This model does not admit of a stationary state with increasing experience and consequent increasing productivity, and since increasing productivity due to increasing experience is precisely the phenomenon we wish to explore, examining stationary states will do us no good (Steven Smith and Meng-hua Lee). However, we may consider the following special case, which may be called a »quasi-stationary state«, in which p, N, H, S, and f_K[8], the marginal productivity of capital, and the derivatives of $\phi(.)$ and $\Psi(.)$ are constants over time. In such a state, shifts in $\phi(.)$ $\Psi(.)$ over time, and the consequent induced increases in w, just offset the increasing productivity of labor due to rising X, and so the profit-maximizing N does not change. The shifts in $\phi(.)$ and $\Psi(.)$ must reflect changing opportunities for labor due to rising productivity in the economy as a whole, so that we are saying in effect that an enterprise will occupy a quasi-stationary state only if its experience of rising productivity reproduces in the small that of the whole society. In that sense, the enterprise in a quasi-stationary state is a representative enterprise. Nevertheless, this quasi-stationary state probably will not be instantiated by real enterprises with any very high probability. It may serve as a road-sign on the way to a more »realistic« model, and does admit of reasonably transparent comparison of the cases of capitalist and Illyrian firms with and without learning by doing.

Furthermore, for the special case we assume a linear specification on g so that

$$\dot{X} = \alpha N - \beta H + \gamma I \qquad \text{II.6.}$$

The »classical« special case of no learning by doing is then defined by

$$\partial f / \partial X \equiv 0 \qquad \text{II.7.}$$

$$g(.,.,.) \equiv 0 \qquad \text{II.8.}$$

From T. 1. 2. and T. 1. 9. we have

$$\dot{y}_3 = y_3 (r + \delta) - pf_K \qquad \text{II.9.}$$

[8] The subscript notation for partial derivatives is adopted, so that, for example the notation f_x means the partial derivative of the function f with respect to the variable x.

where f_K denotes the partial derivative of f with respect to K, i.e., the marginal physical productivity of capital. With pf_K nonzero and constant, the solution to this differential equation requires y_3 = constant, i.e. $\dot{y}_3 = 0$. Thus we have the familiar condition

$$pf_K = y_3 (r + \delta) \qquad \text{II.10.}$$

i.e. that the capital stock should at every moment be large enough so that the value of the marginal product of capital is equal to its »cost,« the rate of discount plus the rate of depreciation times the present value of one unit of »capita«l. Equivalently,

$$pf_K = (1 - \gamma y_1) (r + \delta) \qquad \text{II.11.}$$

From T.1.4, the constancy of y_3 just demonstrated, and the constancy of g_1, y_1 is also constant in a quasi-stationary state. On the assumption that there are always some separations (due to death and exit from the labor force, if for no other reasons), $S > 0$ regardless of w and from T. 1. 6, $y_2 = \lambda_3$. Then, from T. 1. 3, 5, and 6, the constancy of y_1 and g_H, and the constancy of N, Φ_W, and Ψ_W in a quasi-stationary state, y_2 also is constant. The constancy of λ_2 and λ_3 follows from all of this, while the constancy of λ_1 would follow from that of p. All of this seems to correspond with one's intuition of a state which is economically stationary: all of the »shadow prices«, both intertemporal and istantaneous, are constant.

It follows then that in quasi-stationary state,

$$ry_1 = pf_x \qquad \text{II.12.}$$

and

$$w + ry_2 = pf_N + y_1 g_N \qquad \text{II.13.}$$

Equation II.12. may be interpreted as follows: the unit »interest« cost of the »capital« asset ,experience, should be set equal to its marginal value product. Equation II.13. is interpreted as prescribing that the marginal whole cost of labour be equal to its marginal whole product. The marginal whole cost comprises the wage plus the »interest cost« of future wages which will have to be paid to retain a work force larger by one unit of labor. The marginal whole product comprises the market value of direct product plus the asset value of the indirect product of experience of the additional worker.

Several other points may be extracted from this model. One point which is of interest is that the capitalist firm, as here described, will never reduce its work force by layoffs, but will instead simply reduce the wage until any force reductions occur through voluntary separations (that is, the constraint $S \geq \Psi(w)$ is always a binding constraint). This does not depend on the assumption of a quasi-stationary state but is true in general. Real capitalist firms do not behave in this fashion. This counterfactural prediction of the model probably reflects the simplifying assumption that everything occurs with certainty, since recent theories of layoffs suggest that uncertainty or opportunism, or both, play important roles in the determination of the rate of layoffs. This point, however, merits explicit exploration which is beyond the limits of this paper.

III

This section explores the implications of learning-by-doing in a modified Ward model of the labor-managed enterprise. The modified Ward model is, in some ways, a »worst case« for labor management. It is chosen for that reason, and for comparability with some previous literature, and not as a complete model of the labor-managed enterprise. The point is that, even in this unfavorable case, the comparative efficiency of labor-managed and capitalist-managed enterprises depends on the treatment of learning-by-doing.

Nevertheless, something should be said about the shortcomings of this model. Two pathologies have been attributed to the modified Ward model considered here: a tendency to under-recruit in some circumstances, sometimes known as the »Ward-Vanek effect«, and a tendency to underinvest, sometimes known as the «Furubotn-Pejovich effect.«[9] These tendencies are, however, quite sensitive to a variety of institutional and other assumptions in the modified Ward model. The Ward-Vanek effect is eliminated if the cooperative has zero net worth (as a capitalist corporation does) and accrues no rents. Proposals such a partership deep market (Sertel) or similar proposals of Bonin and others serve to eliminate the Furubotn-Pejovich effect and at least to mitigate the Ward-Wanek effect, as does an option of external financing without increasing risk (McCain 1977). The Ward-Wanek effect is also eliminated in a short run model assuming expected value maximization[10] or in a long run model assuming entry,

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[9] Furubotn and Pejovich, 1970, seems to be the earliest statement of this position. Note also Furubotn 1976, 1978, 1980, 1985; Jensen and Meckling.

[10] Note e. g. R. McCain (1985), which draws on the implicit contracts literature. McCain assumes that each worker has an equal chance of being laid off and establishes that risk-averse workers will not lay off to the point at which income per worker is equal to the ex post social opportunity cost of labor, which may be the dole, while risk-neutral workers will lay off exatly up to that point. In an Illyrian model layoffs would go beond that point whenever the »capitalist twin« would be profitable. The implicit contracts theory had been offered as a theory of rigid wages and involuntary (layoff) unemployment. The founding papers in the implicit contracts school were Gordon, Azariadis (1975), and Baily. For a survey of the earlier work see Azariadis (1979). For a key critique see Akerlof and Miyazaki. For quantitative evidence of the existence of such a distinct school see K. McCain. It is now generally accepted that this early generation of work did not supply a satisfactory theory of layoffs. More recent work in the school has generally assumed »asymmetrical information«, i. e. assumed that the employment contract must somehow protect the employees from misrepresentation by the employers. For examples see Azariadis (1983), Grossman and Hart, Hart. Misrepresentation by employers certainly plays no part in the idealized model of section II, which, if we accept the newer work in the impli-

even when entry is costly (Martin). Moreover, endogenous bias in technical change will, in some cases, offset the two pathological effects (McCain 1988). Thus the modified Ward model captures, at most, one set of tendencies among many operating on the labor-managed enterprise.

In Ward's Illyria, the pay per head of the work force is an endogenous variable to be decided by the work force, subject only to the constraint of a given budget determined *(inter alia)* by the market value of sales. With effort, hours and other determinants of worker well-being taken as givens, Ward assumed that the work force would maximize pay per head. In the intertemporal world of this paper, presumably instead they would maximize the discounted present value of pay per head over some period of future time. Thus we have

$$z \varsigma \frac{pQ - I}{N} \qquad \text{III.1.}$$

where z is instantaneous income per worker, and

$$max \int_0^T e^{-rt} z(t)dt + F^*(K(T)\,,\ (N(T)\,,\ X(T) \qquad \text{III.2.}$$

Notice that z is constrained revenue net of *investment*. That is, we make the strict, non-Illyrian ,and unrealistic assumption of 100% internal finance in the worker-managed firms as we did in the analysis of the capitalist firm. In III.2, F*(K)(T) , N(T) , X(T) is the value of worker incomes from the firm from T for all time thereafter, discounted to present value, as before. This function has been denoted by F* (contrast F in the previous section) to point up the fact that it is likely to be different in the labor-managed case than in the capitalist case. We shall assume that the horizon, T, is less than the remaining work lifetime of the median worker, so that F*(K(T) , N(T) , X(T)) > 0.

On this basis, and with the constraints and laws of motion explained in Section I, the Lagrangian-Hamiltonian function and some of the necessary conditions for a maximum are given in Table 2. Notice that z, the net revenue per head of the work force, replaces w, the wage in these computations.

The strategy of analysis is similar to that of the last section, and we may thus proceed to a discussion of the key equations descriptive of a quasi-stationary state. However, one preliminary remark

cit contracts school, might account for the absence of layoffs in the model. It is, however, not clear that the newer work is satisfactory as a theory of layoffs either: note Azariadis and Stiglitz. McCain (1978) proposes a related theory of layoffs which stresses the necessity that workers be protected againts opportunistic revisions of wages and hours by the employers. This phenomenon, too, is missing in the model of Section II and again its absence might explain the absence of layoffs.

needs to be made. In the case that F*(.,.,.) is identically zero (we might call that the Furubotn case), there can be no quasi-stationary state with positive investment in either tangible or intangible capital. In such a case, the costate variables y_1, y_2 and y_3 must take a value of zero at T; therefore zero is the only constant value they may take over (0, T) inclusive. It follows that in the Furubotn case, positive investment can be only a transitional phenomenon. This applies equally to the capitalist firm which F(.,.,.) is zero. In what follows, however, F*(.,.,.) > 0 is assumed.

The condition for optimal investment in physical capital then is

$$pf_K = [1 - \gamma y_1 \frac{N}{\lambda_4}] (r + \delta) \qquad \text{III.3.}$$

The condition for optimal investment in experience is

$$pf_x = ry_1 N/\lambda_4 \qquad \text{III.4.}$$

and the condition for optimal recruitment of the labor force is

$$z + [N/\lambda_4]\, ry_2 = pf_N + y_1 g_N [N/\lambda_4) \qquad \text{III.5.}$$

These equations bear a clear parallel to II.11, II.12, and II.13 respectively, and the same explanation of their meanings applies. In particular, in the case of no learning by doing i.e. g(.,.,.) = 0, equation III.4. becomes

$$pf_K = r + \delta \qquad \text{III.6.}$$

the familiar neoclassical condition for optimal internal investment, and III.5. becomes

$$z = pf_N \qquad \text{III.7.}$$

Now, III.6. and III.N. reproduce the central »Illyrian« results for this model: capital use is optimal but he labor force needs not be optimal. Payment per head is equal to the marginal product of labor but may in turn differ from the social opportunity cost of labor; in circumstances in which the »capitalist twin« would experience profits, the work force is rescricted so that the value marginal product of labor is above is social opportunity cost — »inefficiently« restricted. This is the Ward-Vanek pathology. These results justify the claim the model of this section is an extension of the Illyrian model.

Now let us see what difference the introduction of learning by doing makes. In particular, what effect has is it on the »Illyrian pathology?« There would seem to be two possibilities. On the one hand (as Horvat argues) a greater labor force will generate more learning, thus learning by doing may tend to offset the »Illyrian pathology«. On the other hand, experience is a form of capital and has an imputed rent, which may exacerbate the »Illyrian pathology«. From III.5. we obtain

$$z - pf_N = [N/\lambda_4]\,(y_1 g_N - r y_2) \qquad \text{III.8.}$$

Here, y_1 is the discounted present value of a marginal unit of experience, and y_2 is the capital cost of a one-unit addition to the work force, in future recruitment costs to maintain the larger work force. The term $[N/\lambda_4]$ is positive, so everything depends on the sign of $(y_1 g_N - r y_2)$. If it is positive — if, that is, the value of learning induced by a larger work force is greater than the cost of recruitment to maintain the larger work force — then a positive gap is induced between z and the value marginal product of labor, which tends to displace the worker-managed enterprise toward a larger work force. This is illustrated by Figure 1: for the Illyrian firm without learning by doing, the work force is at N_0 while with learning by doing and similar productivity curves, it would be at N_1. Thus Horvat's conjecture is verified and the contrary conjecture falsified.

The case of $(y_1 g_N\, r y_2)$ negative corresponds to Domar's case in which the size of the labor force is determined by a constraint of labor supply; the details will not be explored here. Examination of the impact of learning by doing on the accumulation of physical capital (equations III.4. and III.7) will be deferred until the next section, since this is central to the comparison of capitalist and worker-managed firms when both experience learning by doing.

The intertemporal optimum for the Illyrian firm, unlike that for the untrammelled capitalist firm, can sometimes require members of the enterprise to be laid off. Here, again, it seems appropriate to consider this result as an artifact of the simplifying assumptions, and in particular of the choice of the maximand together with the assumption of certainty. It is now known (McCain 1985) that an enterprise which maximizes the expected value of pay in a milieu of state-contingent uncertainty does not lay off under circumstances in which an Illyrian enterprise does.

What we may say at this point is that learning by doing does make an important difference to the behavior of the worker-managed firm. We have introduced only two modifications into the Illyrian model: learning by doing and an explicity intertemporal or dynamic environment. We have identified a »quasi-stationary state«, which is the nearest approximation to the neoclassical stationary state which this environment seems to allow and which arguably could describe a »representative firm« in an economy experiencing growth as a result of learning by doing in many sectors. In this »quasi-stationary state«, the behavior of the worker-managed firm deviates from what it would be in the absence of learning by doing and on plausible assumptions about the benefits of learning and the costs of recruitment, deviates in a way which offsets the »Illyrian pathology« of restriction of the labor force.

IV

We now proceed to compare the capitalist firm with the worker-managed firm as these forms of enterprise have been described in the preceding sections. It should be stressed that the comparisons are tentative, dependent as they are on the simplifyng assumptions made, some of which may be both important and counterfactural. The comparisons may, nevertheless, serve to indicate the importance of learning by doing for an understanding of the performance of enterprises of different types, both as to ideal and actual cases.

The keys to the comparison are to be found in equations II.11, 12, and 13. (for the capitalist firm) and in equations III.3,4 and 5 (for the labor-managed enterprise), together with the transversality conditions (equations 10, 11, and 12. in Tables 1 and 2). Using these results and assuming infinite horizons in each case, and further assu ming that the work force of each sort of firm in constant for all time in a quasi-stationary state, we have the equations given in Table 3 For the two forms of capital, learning and tangible capital respectively, the necessary conditions for optima are respectively the first and second equations in the two columns. We observe that these differ only in the inclusion of a term $1/\lambda_4$ i the equations for the labor-managed enterprise. From Table 2, equation 2 we have

$$\lambda_4 = 1 + \lambda_2\Phi_z - \lambda_3\Psi_z \qquad \text{IV.1.}$$

We note that since Ψ_z is negative, this term is no less than one, so its inverse is no more than one. In the event that neither the hiring constraint, I.5, nor the separations constraint, I.6, is binding, $\lambda_2 = \lambda_3 = 0$ and λ_4 is exactly one. In that case the first two equations are identical across the two kinds of firms. In case one or more of the labor supply constraints is binding, pf_k and pf_x will be less in the labor-managed enterprise (ceteris paribus) than in the capitalist enterprise. This occurs because the wage payments required to maintain a larger work force and thus accumulate more experience, which are a cost to the capitalist enterprise, are not a cost but an income component to the self-managing workers. Thus experience is effectively cheaper to the labor-managed enterprise. This phenomenon also magnifies the effect of investment on learning and accounts for the tendency to higher intensity of tangible capital use for the labor-managed firm. It might, of course, be argued that the capitalist firm's behavior is »efficient« and that the labor-managed firm becomes »over-experienced« because it gives too much weight to the well-being of labor; but assessment of normative claims such as this is beyond the scope of this paper.

The third of equations, which refer to the optimal labor force, are less simple, in that the $1/\lambda_4$ terms occur on both sides of the equation. Moreover, z and w will generally differ. It does not seem possible to say which enterprise form will employ more labor, in absolute terms, without some further information; but clearly the tendency of the labour-managed firm to operate more intensively in both kinds of capital (when $\lambda_4 > 1$) implies that it will use less labor relative to a give output.

It seems apparent that, in any attempt to compare the behavior of capitalist and labor-managed firms in similar environments, the responses of the two forms to learning by doing are crucial determinants of the differences predictable by theory. This is so even in the highly stylized and simplified Illyrian context, and there is no reason to suppose that it will be less so in models which make other concessions to reality beside the introduction of learning by doing. This element has heretofore been altogether neglected in the Illyrian literature. Perhaps this neglect will contribute to the already considerable doubt that he Illyrian model describes anything actual, or can serve as anything more than a pedagogically convenient starting-place on the way to a predictive theory of labor-managed enterprises.

V

In this paper, two models of idealized enterprises operating under certainty and with learnling by doing over time have been explored. The first is a model of a capitalist firm, that is, one which maximizes the present value of revenue net of wage cost and reinvestment. The second is a model of an otherwise »Illyrian« labor-managed firm. Idealized and counterempirical as these models may be (and especially the latter) the results serve to establish that learning by doing is a critically important determinant of the behavior of both forms of the enterprise, and of their comparative properties, insofar as these can be determined from theoretical considerations.

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TABLE 1

$$L = e^{-rt}\{pQ - wN - I + y_1 g(N, H, I) + y_2(H - S) + y_3(I - \delta K) + \lambda_1[f(K,N,X) - Q] + \lambda_2[\phi(w) - H] + \lambda_3[S - \Psi(w)]\} \quad \text{T.1.1.}$$

$$L_Q = e^{-rt}(p - \lambda_1) \leq 0 \text{ ; and } = \text{ if } Q > 0 \quad \text{T.1.2.}$$

$$L_w = e^{-rt}(-N + \lambda_2\phi_w - \lambda_3\Psi_w) \leq 0 \text{ ; and } = \text{ if } w > 0 \quad \text{T.1.3.}$$

$$L_I = e^{-rt}(-1 + y_1 g_I + y_3) \leq 0 \text{ ; and } = \text{ if } I > 0 \quad \text{T.1.4}$$

$$L_H = e^{-rt}(y_1 g_H + y_2 - \lambda_2) \leq 0 \text{ ; and } = \text{ if } H > 0 \quad \text{T.1.5.}$$

$$L_S = e^{-rt}(-y_2 + \lambda_3) \leq 0 \text{ ; and } = \text{ if } S > 0 \quad \text{T.1.6}$$

$$\frac{d(e^{-rt}y_1)}{dt} = -L_X = -e^{-rt}\lambda_1 f_X \quad \text{T.1.7.}$$

$$\frac{d(e^{-rt}y_2)}{dt} = -L_N = -e^{-rt}(-w + y_1 g_N + \lambda_1 f_N) \quad \text{T.1.8.}$$

$$\frac{d(e^{-rt}y_3)}{dt} = -L_K = -e^{-rt}(\lambda_1 f_K + y_3\delta) \quad \text{T.1.9.}$$

$$y_1(T) = \frac{\partial F}{\partial X(T)} \quad \text{T.1.10.}$$

$$y_2(T) = \frac{\partial F}{\partial N(T)} \quad \text{T.1.11}$$

$$y_3 = \frac{\partial F}{\partial K(T)} \quad \text{T.1.12}$$

TABLE 2

$$L = e^{-rt}\{z + y_1 g(N,H,I) + y_2(H - S) + y_3(I - \delta K) + \lambda_1(f(K,N,X) - Q) + \lambda_2(\phi(z) - H) + \lambda_3(S - \Psi(z)) + \lambda_4\left[\frac{pQ - I}{N} - z\right]\} \quad \text{T.2.1}$$

$$L_z = e^{-rt}[1 - \lambda_4 + \lambda_2\Phi_z - \lambda_3\Psi_z] \leq 0 \text{ and } = 0 \text{ if } z > 0. \quad \text{T.2.2.}$$

$$L_Q = e^{-rt}\left[-\lambda_1 + \frac{\lambda_4 p}{N}\right] \leq 0 \text{ and } = 0 \text{ if } Q > 0. \quad \text{T.2.3.}$$

$$L_I = e^{-rt}\left[y_1 g_I + y_3 - \frac{\lambda_4}{N}\right] \leq 0 \text{ and } = 0 \text{ if } I > 0 \quad \text{T.2.4.}$$

$$L_H = e^{-rt}[y_1 g_H + y_2 - \lambda_2] \leq 0 \text{ and } = 0 \text{ if } H > 0 \qquad \text{T.2.5.}$$

$$L_S = e^{-rt}[-y_2 + \lambda_3] \leq 0 \text{ and } = 0 \text{ if } S > 0. \qquad \text{T.2.6.}$$

$$\frac{d(e^{-rt}y_1)}{dt} = -L_X = -\lambda_1 f_X e^{-rt} \qquad \text{T.2.7.}$$

$$\frac{d(e^{-rt}y_2)}{dt} = -L_N = -e^{-rt}[\lambda_1 f_N - \lambda_4 \frac{Z}{N} + y_1 g_N] \qquad \text{T.2.8.}$$

$$\frac{d(e^{-rt}y_3)}{dt} = -L_K = -e^{-rt}[y_3\delta + \lambda_1 f_K] \qquad \text{T.2.9.}$$

TABLE 3

Capitalist firm	Labor-Managed Enterprise
$pf_K = [1 + \gamma \int_T^\infty e^{-rt} pf_K dt]\,(r + \delta)$	$pfK = \left[\dfrac{1 - \gamma \int_T^\infty e^{-rt} pf_K dt}{\lambda_4}\right](r+\delta)$
$pf_X = r \int_T^\infty e^{-rt} pf_X dt$	$pf_X = \dfrac{r \int_T^\infty e^{-rt} pf_X dt}{\lambda_4}$
$w + r \int_T^\infty e^{-rt}(pf_N - w)dt =$	$\dfrac{w + r \int_T^\infty e^{-rt}(pf_N - z)dt}{\lambda_4} =$
$pf_N + g_N \int_T^\infty e^{-rt} pf_X dt$	$\dfrac{pf_N + g_N \int_T^\infty e^{-rt} pf_X dt}{\lambda_4}$

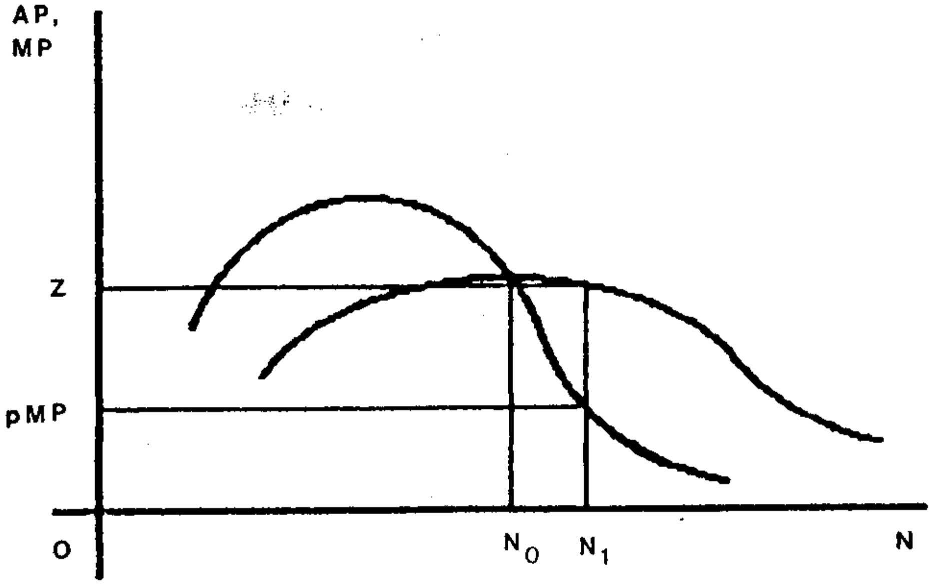

Figure 1

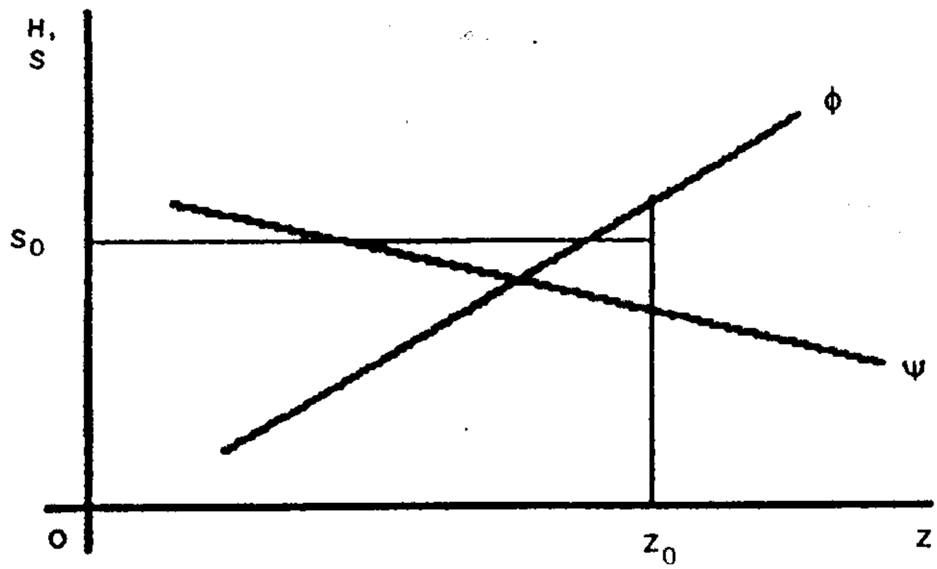

Figure 2