ON THE NOTION OF EQUILIBRIUM IN MARX'S ECONOMICS*

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The aim of this paper is to prove that, contrary to what is generally held among orthodox Marxists and some other economists, Marx was an equilibrium theorist in the sense that he used an equilibrium concept as a basic analytical tool.

The focus of the analysis in this paper is on those propositions of Marx's equilibrium analysis which refer to the general principles of the functioning of an economic system. Such propositions, based on the microeconomic type of equilibrium, can be derived from Marx's model of the formation of the prices of production whereas macroeconomic implications can be studied on the amalgam of his models of expanded reproduction and the circular flow of capital.

Drawing a parallel with the neoclassical equilibrium as regards the principle of achieving the allocative efficiency of an economic system shows that those two concepts are not exclusive of each other and that, indeed, "...general equilibrium theorists are much closer to Marx than many a Marxist".[1]

Non-existence of a generally accepted definition of equilibrium in economics still causes a lot of confusion and misunderstanding among economists. We take that, in its most general form, economic equilibrium means a spontaneous reconciliation of numerous interdependencies making up an economic system. Furthermore, it is certain that "...if economic theory has anything to offer on the interaction of market signals and agents' actions then it will need to formulate an equilibrium concept".[2]

However, equilibrium is not only a theoretical problem. In its practical counterpart is appears to be one of the most fundamental problems in the reality of any economic system. An efficient alloca-

    [1] Hahn (1973), pp. 33.
    [2] Hahn (1984), pp. 4.

tion of resources, as an equilibrium outcome of numerous interdepend-
encies of market signals and agents' actions, is a prerequisite for any
organization of social production whose aim is the maximum satisfac-
tion of needs of the members of a society. This problem is of a gen-
eral character and it poses itself in every society, regardless of the in-
stitutional framework.

Consequently, a certain notion of equilibrium is an essential or-
ganizing concept upon which all major strands of economic theory are
built. Different conceptual frameworks that emerge, among other
things, as a consequence of the adoption of different theories of value,
generate and determine the basic characteristics of specific notions of
equilibrium. Although most of economic theory is developed around
classical and neoclassical concepts of equilibrium, this rough classifi-
cation by no means exhausts all the variety in form and/or levels of
abstraction that modify basic concepts in accordance with the purpose
and assumptions of a specific model.

Marx's economics is no exception. Although following the struc-
ture of a classical model (taking technology and real wage as para-
meters and prices and the uniform rate of profit as variables), Marx's
equilibrium analysis is specific in that it is fundamentally related to the
labour theory of value. Taking the labour theory of value as its start-
ing point determines the initial assumptions and, to a certain extent,
the purpose of the model. Substituting surplus value for surplus
output in the analysis necessarily brings about the problem of trans-
formation of value into prices which is the main point of departure
from the classical duality between quantity relations and prices.

While in his early work very critical of equilibrium analysis in
economics,[3] Marx himself evidently accepted it in his mature work.
Although not always very clearly separated, it is possible to distinguish
Marx's analysis of a capitalist economy, as of a specific mode of pro-
duction, from his analysis of the general conditions of equilibrium re-
gardless of the institutional framework of an economic system. It is
in this latter area that a close parallel can be drawn with the neoclas-
sical equilibrium concept especially as regards the efficiency of the
allocation of resources.

Two main features of Marx's equilibrium analysis are the uni-
formity principle and the concept of reproducibility. The uniformity
principle implies the formation of the uniform rate of profit as the
consequence of the law of value which rules in the system of commod-
ity production with private ownership over the means of production
and with absolute mobility of capital and labour among alternative
uses.

The concept of reproducibility, on the other hand, implies the
conditions under which an economy can sustain or reproduce itself.
An economy is reproducible if the flow of commodities as outputs is
at least sufficient to produce the labour services and other commodi-
ties necessary to produce that same flow again.

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[3] See Morishima — Catephores (1973).

Marx's model is clearly an economy of perfect competition. In it, market prices, understood as the expression of value in terms of money, oscillate around value under the influence of supply and demand thus creating the proportions of the distribution of the total fund of social labour. This unequivalent exchange arising from unmatching the individual amount of value with the corresponding price, causes permanent movement of capital seeking the maximum rate of return. Such processes adjust the structure of social production with the social needs as expressed by means of the market demand.

Thus, market prices act as signals to the producers concerning the questions "what", "how" and "for whom" to produce. They also show the producers how successful they were in their predictions. This is the microeconomic aspect of the law of value whose essence is based on the formation of the uniform rate of profit.

The uniformity principle and, consequently, prices of production as well as their relation to the total output, can be analysed by a simple linear model. Consider an economy with n commodities. Let A_{ni} represent a row vector with n components and L_i a scalar denoting the amount of labour used in a process i.

Technology consisting of n processes can be represented by a square $(n \times n)$ matrix of fixed technical coefficients A' and by a column vector denoting used labour L (with n components). If the consumption of workers, d, is fixed on the subsistence level, we can write a so-called socio-technological matrix A:

$$A = A' + d \times L \tag{1}$$

Then, matrix A denotes total amounts of every good needed for production whether directly as an input or indirectly as a part of workers' consumption.

Matrix A is a non-negative, productive and indecomposable matrix. Productivity of a non-negative matrix A ensures that there exist vectors $x, y > 0$ so that $x > Ax$ or $y > Ay$. Since matrix A represents a matrix of enlarged technical coefficients, productivity implies the existence of a vector of total output which satisfies the reproduction demand and leaves positive quantities of goods for final consumption in all sectors. Productivity of the matrix of enlarged technical coefficients, thus, ensures the reproducibility of the system.

Let p^* be the price of production vector, w^* wage rate and π^* the uniform (equilibrium) rate of profit, the price of production relation can be written as

$$p^* = (1 + \pi^*)(p^*A' + Lw^*) \tag{2}$$

Taking wage as numéraire, we can write $w^* = pd = 1$ which enables us to rewrite (2) as

$$p^*A = \frac{1}{1 + \pi^*} p^* \tag{3}$$

From (3) it is evident that $1 + \pi^*$ is the reciprocal value of the eigenvalue ρ of the matrix A and that p^* is its associated eigenvector. A system can be reproduced only when $\rho < 1$ which gives a positive rate of profit π^*. This relation $(\rho < 1 \Rightarrow \pi^* > 0)$ is satisfied only when matrix A is a productive matrix.

Solutions at (ρ^*, p^*) must be real and non-negative because we interpret ρ as $1/1 +$ profit rate and p^* as prices of production. By the Perron-Frobenius Theorem every non-negative square matrix has a real non-negative eigenvalue with greatest modul to which a non-negative non-zero eigenvector is associated. For an indecomposable non-negative square matrix, as is the case with A, there is only one possible solution for (ρ^*, p^*) which means that equilibrium is uniquely determined.[4]

In the economic interpretation, indecomposability of a matrix means that all commodities enter into production of all other commodities. That is, it is not possible to split up the processes (activities) into two sets so that processes from one set do not use commodities produced by the processes from the other set.

According to the Perron-Frobenius Theorem, because of its indecomposability, the matrix A has a strictly positive left Frobenius eigenvector which can be interpreted as the price of the production vector, or

$$p^* = (1 + \pi^*)\, p^* \,(A' + dL) \tag{4}$$

In the same way its right eigenvector which is also strictly positive, can be interpreted as the output vector. It can be written as

$$x = (1 + \pi^*)\,(A' + dL)\, x \tag{5}$$

In both (4) and (5) the eigenvalue of A is $1/1 + \pi^*$. The solution for π^* which is not trivial, i. e. which ensures the reproducibility of the system is the one for which it is $\pi^* > 0$.

On the basis of what has been said, we can draw the conclusion that the general price of production system which is formed in this way depends entirely on technology and distribution (matrix A) and that it is independent of the conditions of consumers' demand. This type of equilibrium can be established and maintained only by that price system which generates the uniform (average) profit in all sectors of production and this conditions is ensured by the indecomposability of the matrix A.

Maximizing profit a capitalist will not invest in those processes which do not satisfy this condition which means that such processes would not operate. For reproducibility of the system it is necessary that all processes operate (which is the meaning of the indecomposability assumption) and that means that they all offer the maximum (uniform) rate of profit. Since this system of prices of production ensures the same profitability of capital in all alternative uses, it also denotes a state of rest in which a rational allocation of resources is achieved and where there is no motivation for further reallocation of capital.

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[4] For detailed proofs see Gjenero (1985), Roemer (1980 and 1981).

By proving that equilibrium values are determined by technological coefficients which are independent of market fluctuations and which reflect the structure and relations of production, Marx points out the technological essence of equilibrium and its independency of the institutional organization of a society. It follows, though, that markets and prices are important mechanisms of allocation of factors of production as well as automatic regulators of the total economic processes.

In its macroeconomic aspect the law of value acts as the law which governs the process of social reproduction. Enforcing the differentiation among the producers, as the consequence of competition, it also appears as the law of development of capitalism.

A distinguished feature of Marx's approach to the reproduction problem is its social aspect. His analysis of the reproduction of the relations of production in capitalism enabled Marx to draw his well-known conclusions as regards the historical relativity of this mode of production.

However, it is not only relations of production that form the results and the conditions of reproduction. The main problem in the theory of reproduction is the possibility of replenishing the elements of production through the realization of the produced value of the GMP. In this way, value and material structure of the GMP become the basic analytical framework for the analysis of the process of reproduction.

Value aspect af the GMP means that it consists of the "carried-over" value and of the newly added value. The first one represents constant capital whereas the latter one refers to the fund of wages plus the surplus value. Although the value composition depends upon the specific relations of production, every society must dispose with the means of production and the corresponding labour. Output composition of the GMP implies a certain structure of the use values contained in it. In this way, Marx treats the GMP as a composite commodity which, the same as any other commodity, has its value and its use value.

For the smooth and continuous reproduction the total output must qualitatively and quantitatively match the total social needs. It means that an equilibrium is a prerequisite and a condition for the continuous social reproduction. In this light, the central problem that Marx developed in his theory of reproduction was the question how and in what relations the capital invested in production replenishes itself and how this replenishment intertwines with consumption. In order to distinguish the segments of the GMP that are aimed for production or for consumers' consumption, he constructed a two sector model.

Marx's schemes of simple and expanded reproduction represent economic models which enable an analysis of the stationary state and of the dynamic process of development. They are general in the sense that they can serve as the basis for the analysis of the process of reproduction of any society because every element of the schemes represents both supply and demand and it is possible to analyse their output structure as well as their value magnitude.

Schemes of reproduction as a two sector model (Sector I produc-
ing means of production and Sector II consumers' goods, "e" repre-
senting rate of exploitation) in matrix notation can be stated as follows:

$$P_I A'_I X_I + (1 + e) (P_{II} dL_I X_I) = P_I X_I \tag{6}$$

$$P_I A'_{II} X_{II} + (1 + e) (P_{II} dL_{II} X_{II}) = P_{II} X_{II}$$

In an economy with surplus there always arises the question of
how to allocate it among the accumulation and the luxury consump-
tion. This is a typically classical general equilibrium problem.

In macroeconomic analysis it is rather obvious that an increase
of production and the reproduction of a sector on an expanded scale
must be accompanied by the proportional increases of production in
other sectors. It follows that there exist certain proportionalities
among the sectors of production that must be satisfied if an economy
is to grow and develop. Such fundamental proportionalities in the
case of an expanded reproduction are as follows:

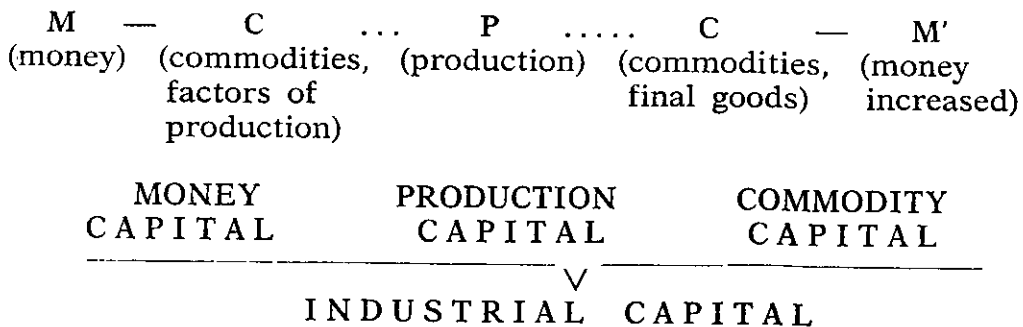$$(1 + e) (P_{II} dL_I X_I) > P_I A'_{II} X_{II} \tag{7}$$

$$(1 + e) (P_{II} dL_I X_I) + (1 + e) (P_{II} dL_{II} X_{II}) > P_{II} X_{II} \tag{8}$$

$$P_I X_I > P_I A'_I X_I + P_I A'_{II} X_{II} \tag{9}$$

In this analysis of the process of reproduction, as the schemes of
reproduction show, Marx was concerned primarily with the material
conditions of a stable and balanced economic growth. This is verified
by recognizing Marx's influence in some contemporary models that
take up the same problem such as those of the Leontief or von Neumann
type. The introduction of the specificities of the capitalist relations of
production enabled Marx to discover the causes of the unstable path
of the capitalist system of production.

However, schemes of reproduction alone do not exhaust Marx's
macroequilibrium analysis. They are completed as a dynamic theory
when the model of the circular flow of capital is added to them. The
analysis of the circular flow of total capital that is involved in the
process of reproduction enables the investigation of the quantitative
relations which must be satisfied in order to achieve the necessary
proportionalities in the economy which will reproduce it continuously.

Industrial capital, as Marx divided it, takes on three main forms
as is shown schematically:

M   —   C   ...   P   C   —   M'
(money) (commodities, (production) (commodities, (money
         factors of   final goods)   increased)
         production)

MONEY   PRODUCTION   COMMODITY
CAPITAL   CAPITAL   CAPITAL
————————————————————————————ⱽ————————————————————
              INDUSTRIAL   CAPITAL

Although comparatively independent in their movement, after a certain period of time they undergo a metamorphosis changing from one form into another. If their time coordination is disrupted i. e. if an economy suffers from quantitative disproportions, then there occur disturbances and discontinuities in the process of total reproduction. Owing to this model it is possible to conclude that general equilibrium can be disrupted at any point of metamorphosis of one form of capital into another and not only by disproportionalities of total output as schemes of reproduction alone may suggest.

All basic elements of the quantitative approach to Marx's concept of macro-equilibrium can be synthesized in an amalgam of the schemes of expanded reproduction and the circular flow of capital as follows:

$$M - C < {MP \atop LP} \dots P \dots C' \begin{pmatrix} C_I + AC_I \\ C_{II} + AC_{II} \end{pmatrix} - M' \begin{pmatrix} R + A \\ Q + AQ \end{pmatrix}$$

M = money capital
P = production capital
C = commodity capital
MP = means of production
LP = labour power
R = replenishment of the means of production used up in the process of production
A = accumulation
Q = total (personal and public) consumption

The distribution is an equilibrium one if the following quantitive proportions are satisfied

$$C_I = R + A$$
$$C_{II} = Q + AQ \tag{10}$$

or, more specific,

$$AC_I = A$$
$$AC_{II} = AQ \tag{10 a}$$

However, whether or not an equilibrium is possible as regards the inner, qualitative structure, it remains to be seen at the next metamorphosis of money capital into the factors of production.[5] In this way Marx's theory of reproduction and of the circular flow of capital offer not only conditions of a dynamic equilibrium but they also highlight the causes as well as the possible loci of disproportionalities, quantitative and qualitative, that disrupt the continuous process of social reproduction.

In the end, as regards the efficiency of Marx's equilibrium, a brief parallel can be drawn with the neoclassical equilibrium analysis. It is possible to prove that the uniformity principle and the principle of optimization, the latter being very important for the neoclassical gen-

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[5] See Stojanović (1967).

eral equilibrium analysis, are but two different approaches to basically the same phenomenon. The criterion of equalization of marginal (and average) rates of return on capital among its alternative uses under perfect competition, known in literature as the equimarginal principle[6], is the fundamental presupposition of the allocative efficiency of an economic system. An efficient social production is generally desirable and, in a suitable institutional framework, it ensures the satisfaction of the constantly rising social needs which is its very purpose.

Although among the component parts of Marx's equilibrium model one finds such "unequilibrium" elements as exploitation and unemployment, it should be noted that they belong to the specificities of the capitalist mode of production and, thus, should be viewed in the context of their historical relativity. On the other hand, the concept of the formation of the uniform rate of return on capital in all sectors of social production as well as the concept of reproducibility with its dynamic implications enter into that minimum of conditions of equilibrium of an economic system regardless of its socio-economic character. These concepts, namely, stem from understanding equilibrium as a reconciliation of numerous interdependencies making up an economic system qualifying Marx as an equilibrium theorist and, at the same time, giving his equilibrium concept a special "flavour".

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[6] The term used in Blaug (1980).

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