A LONG-TERM PROJECTION OF CONDITIONS FOR ECONOMIC GROWTH OF THE CZECHOSLOVAK ECONOMY BASED ON AN AGGREGATE GROWTH MODEL

Agregated growth models are beginning to play a specific role in the complex of methodological instruments, which are being created and applied in ČSSR to the working out of long-term forecasts and on their basis even of long-term plans.*) These models are important mainly because they enable the study of some of the very long-term macroeconomic aspects of economic growth, especially the growth of national income, gross product, investments, fixed assets, consumption, empolyment, capital accumulation, etc.

In the course of the preparations for working out a long-term forecast of growth of the Czechoslovak economy the aggregated growth model was worked out in order to conform to the economic conditions of Czechoslovakia and was applied by the Research Institute for Economic Planning. The aim of the application of this model was to ascertain the objective relations or, as they are sometimes called, the technical-economical relations, which are the inner content of a certain type of economic growth characterized by the volume and rate of growth of the national income and of the final consumption as well as by further economic indicators.

The aggregated growth model was therefore able to give us important data for a disaggregated structural model based on input-output methodology.

At the present level of knowledge of various economic growth models it is quite obvious that there are, and for some time still will be, many aggregated growth models, each reflecting more or less at least the basic relations of the actual economy. Of course they differ also by the number of basic relations, which they reflect.

In spite of that, however, we can say today that there are certain *types of models* suitable to solve specific types of problems. Within the scope of a particular type of model a process of integration or at least of the expression of mutual interrelations may be observed, offering thus an opportunity to evaluate their ability to analyse or predict.

After extensive computations we decided to use, as a ground for our aggregated model, a substitutional production function of the Cobb-Douglas type

$$Y_t = aK_t^{\beta} \; L_t^{1-\beta} \qquad (1)$$

*) Research reports of the Research Institute of Economic Planning of the Czechoslovak State Planning Commision NO 77/1965 and No 108,1967. See also the author's articles in Politická ekonomie No 10/1965, No 6/1966 and in Plánované hospodarství No 8/1967.

where Y_t = net national income (material product definition ($Y' = Y +$ + depreciation, i.e. gross national income)

K_t = volume of fixed assets

L_t = labour force, for example number of employees

β = function parameter (according to our conception, as explained further, the coefficient of elasticity of the relative increase of labour productivity to the relative increase of capital-labour ratio).

We would like to stress the economic meaning of the parameters β and $(1 - \beta)$ as distinct from the common interpretation.

The CD production function was dynamized by special approach to the estimation of its parameters.

Parameter β can be ascertained mathematically as the elasticity coefficient of the relative increase of the productivity of labour to the relative increase of the capital-labour ratio:

$$\beta = \frac{dw}{w_0} : \frac{dv}{v_0} \qquad (2)$$

where w_0 = productivity of labour (dw — its absolute increment) in the basic period

v_0 = capital-labour ratio in the basic period (dv — its absolute increment);

and parameter $(1 - \beta)$ either as a supplement of β to one or separately as an elasticity coefficient of the relative increase of the capital-output ratio (m) to the relative increase of the capital-labour ratio (v)[*]

$$1 - \beta = \frac{dm}{m_0} : \frac{dv}{v_0} \qquad (3)$$

The economic significance of both elasticity coefficients is very clear. The parameter β indicates the percentage of increase in the productioity of labour if the capital-labour ratio grows by 1%. The parameter $(1 - \beta)$ indicates the percentage by which the capital-output ratio will increase (or decrease), if the capital-labour ratio likewise grows by 1%. The increment of the national income also corresponds to these relations, as can be seen from the original substitutional production function (4), because simultaneously it is true that the parameter β is, at the same time the elasicity coefficient of the national income to the capital assets and parameter $(1 - \beta)$ the elasticity coefficient of the national income to the labour force. It is obvious that the technical progress in the magnitude of the parameters is reflected.

There is a possible method of determining both parameters by one of the econometric methods, for example by the method of the least squares of the deviations from the data of the time series on the national income, capital assets and the labour force. As shown by extensive calculations, this method is contradictory to the preceding method, as it usually does not yield satisfactory results.[*]) This may be caused by frequent multicolinearity of time series data characterizing economic development.

*) cf. quoted Paper p. 884

We shall now explain briefly the procedure, which we have used in the first step of determining the rate of growth of the national income and of the relations and proportions connected with it.

Let us start from the production function defined by equation (1). After differentization we obtained

$$\frac{dY}{Y_t} = \beta \frac{dK}{K_t} + (1 - \beta) \frac{dL}{L_t} \qquad (4)$$

The relative increment of the national income *(y)* depends on the relative increase of fixed assets weighted by parameter β and on the relative increment of employment weighted by parameter $(1 - \beta)$.[2])

From the relation (4) we can also derive the corresponding share of the increment of the fixed assets in the national income, which we shall treat later as the share of net investments:

$$\frac{dK}{Y_t} = \frac{1}{\beta} \quad \frac{K_0}{Y_0 (1 + y)\, t} \quad \left[\frac{dY}{Y_0} - (1 - \beta)\right] \frac{dL}{L_t} \qquad (5)$$

The share of the net increment of the fixed assets in the national income is inversely proportional to the parameter »β« and the rate of growth of national income and directly proportional to the capital-output ratio and to the rate of growth of the national income decreased by the rate of growth of employment weighted by parameter $(1 - \beta)$.

If we do not take into account the increase of employment in the production sphere, which is an assumption acceptable with a high degree of probability for the perspective of the Czechoslovak economy, then the relation (4) expressing the relative increment of the national income will be simplified $\left(\text{for} \frac{\alpha L}{L_{(t)}} = 0\right)$ as follows:

$$\frac{dY}{Y_0} = \beta \frac{dK}{K_0} \qquad (6)$$

After a certain modification (integration of eq. 6 according to time), we can get the production function for the growth of national income in the form

$$Y_t = Y_0 e^{\beta k t} \qquad (7)$$

where $k = \frac{d k}{K_t}$ is the average annual rate of growth of the fixed assets, or in the form (for discrete values of the time periods)

$$Y_t = Y_0 (1 + \beta k)^t \qquad (7a)$$

Due to the fact that changes of employment are not expected, the relative increment of fixed assets is in this case identical with the relative increment of the capital-labour ratio. This simultaneously formulates a demand on the technical progress. The new fixed assets should have proportionately lower demands on labour. In other words it determines the qualitative requirements on the level of the capital-labour ratio no matter whether it concerns fixed assets for replacement or for development.

[2]) In relation (14) a simplification has been made with respect to the assumption about the continuity of the volume of fixed assets K(t) with respect to time even though current statistics record their volume only in discrete time intervals.

The ratio of the net capital accumulation in the national income $i^{(c)}_{(t)}$ can be further adjusted with regard to (5) as follows:[3])

$$i^{(c)}_{(t)} = n_0 k \left(\frac{(1 + k)^{t-1}}{(1 + \beta k)^t}\right) \qquad (8)$$

where n_0 is the capital-output ratio in the basic period.

Provided we know the ratio of the gross investments $I^{h4)}_t$ to the net increment of the fixed assets dK_t :

$$\frac{I^h_t}{dK_t} = h \qquad (9)$$

The share of the gross investments in the national income is equal to:

$$i^h_t = n_0 \cdot k h \frac{(1 + k)^{t-1}}{(1 + \beta k)^t} \qquad (10)$$

The symbol h is partly influenced by the degree of retirement of fixed assets, partly by the volume of capital construction in progress. Three and five year moving averages of the symbol h for the total productive fixed assets of the production sphere demonstrate a relatively constant ratio of about 1. 2.

We can further derive the absolute investment volume from the share of the investments in the national income.

The following relation holds true for net investments:[5])

$$I^c_t = K_0 k (1 + k)^{t-1} \qquad (11)$$

The relation for gross investments can be written as follows:

$$I^h_t = K_0 k h (1 + k)^{t-1} \qquad (12)$$

Since we assume a division of the national income only into two components:

$$Y_t = I^c_t + C_t. \qquad (13)$$

[3]) And due to the fact that $K_{(t)} = K_{(0)} (1+k)^t$

[4]) $h = 1 + \frac{1}{\alpha}$ where α is the so-called reproduction ratio, i.e. the ratio of expansion to replacement investments. Cf. Politická ekonomie, No. 1 (1966), p. 36. It is possible to demonstrate under the condition of a constant labour force that this rate of growth (k) depends on the quantity $k\ v$ or under the following conditions: $k = kv$ or $k = \frac{1}{h-1}$, where (kv) is a percentage of fixed assets replacement.

[5]) Let us assume that the increment of the fixed assets corresponds to the volume of net investments: As far as dynamics are concerned, we can introduce to a certain extent, the assumption of an invariable time-lag between the investments and the increment of the fixed assets. Since we operate with the increment of the fixed assets and since we define the net investments on this basis, the time-lag between the expediture of the investment funds and the increment of the national income will become quite negligible.

we can ascertain the volume of consumption as:

$$C_t = Y_t - I_t^c \qquad (14)$$

Besides the usual concept of the national income as a sum of the volume of net investments and consumption, we can use, as a basis for the projections, the so-called gross national income (Y_t^h)

$$Y_t^h \equiv I_t^h + C_t \qquad (15)$$

On the basis of the gross national income it is possible to ascertain the share of gross or net investments, which is more comparable with the highly developed capitalist economies.

For the verification calculations however, we have used a variant of the model, which is based on the active, decisive component of the fixed assets — on machinery and equipment — instead of the total fixed assets.

The reason for this procedure is above all a closer tie between the development of the capital-labour ratio, and thus of the capital-output ratio, and the productivity of labour.

In this case the parameter β' has to be ascertained in relation only to the machinery and equipment capital assets-labour ratio. In Czechoslovakia, in the years 1950 — 1961, this parameter displays a considerably lower variability then the parameter ascertained on the basis of the total fixed assets and thus has a tendency to an absolute decrease.

The other symbols of the model are identical with the preceding explanations with the exception that the bars above the symbols indicate that they concern machinery and equipment.

Preliminary calculations based on the machinery and equipment have only demonstrated that this model reflects a more realistic growth of the volume of productive investments and their share in the national income than a model based on the total fixed assets. The reason for this difference is above all, a higher stability of the parameter β'.

The projection based on machinery and equipment fixed assets stipulates »harder« conditions for the maintaining of a certain rate of growth of the national income in future in our conditions due to the level of parameter β', than the model based on the total fixed assets. In addition to this it seems to be more advantageous to base the projection on fixed assets of the machinery and equipment also in view of the development trends mentioned above. The change of the share of fixed assets of a non-machinery character (buildings, structures) influences the parameter β', but does not have such a close relation to the growth of the productivity of labour as in connection with fixed assets of the machinery and equipment.

However, it is necessary to stress another aspect of the long-term development, which is very important for the Czechoslovak economy. Besides the improvement of the system of planing and management of the national economy at all levels, which is evidently the most important prerequisite not only for our effective economic growth but of any growth in general.

We have shown a preference for the substitutional type of model where the problem of the substitution of labour by fixed assets does not disappear by the assumption of constant employment, which may appear to have eliminated the labour force from the model. The substitutional character of the model is maintained even in this case. As we have already shown, it is partly included in the parameter β', even though it can be interpreted in that case as an elasticity coefficient of production and fixed assets. It is also however, indirectly included in the choice of the rate of growth of the fixed assets.

In order to correctly evaluate the possibilities of our further economic development, it is necessary to realize, that the rate of growth of the fixed assets cannot be an arbitrarily planned quantity. *It follows from the substitution principles in the economy and from the model for Czechoslovakia that the rate of growth of the fixed assets in the production sphere can be only as high as the rate of growth of the capital-labour ratio.*[6])

The rate of growth of the fixed assets is, from a methodological point of view, an exogenous variable in the model; but owing to the lack of the labour in ČSSR, it is a *limited quantity.*

The rate of growth of the fixed assets will be effective under the condition when its productive utilization by labour can be ensured. Generally speaking the labour savings achieved by the liquidation and reconstruction of the existing capacities have to meet the needs not only of reconstructed fixed assets but also of the new fixed assets. The logical conclusion is that the rate of growth, i.e. the net increment of the fixed assets will have to be curtailed within the scope of their gross incrment in favour of an increase in the retirement of obsolete fixed assets. On a high technical level the resulting decrease of the rate of growth of the national income should be compensated by an increase of the parameter β.

In the revised version of the model we therefore use the rate of growth of fixed assets as an endogeneous variable. It depends on the ration of the scrapped fixed assets k^e to the stock of fixed assets and on the so-called reproduction ratio α (which is the ratio of net increase of fixed assets to the volume of the scrapped fixed assets).

The reproduction ratio should not be greater than the capital-labour ratio of new fixed assets to the capital-labour ratio of scrapped assets. Under these conditions and under the presumption of constant level of employment, the new fixed assets could be sufficiently supplied by labour force. The admissible rate of growth of fixed assets gives us the relation

$$k = f^e \cdot \alpha$$

The parameter β can be also expressed as endogenous. It may be proved (under the assumption of $k = v$) that parameter β is a function of the rate of growth of the output-capital ratio (u) and capital-labour ratio (v)

$$\beta = \frac{u + v}{v}, \text{ or}$$

$$\beta = 1 + \frac{u}{v}.$$

[6]) It is perhaps clear that it cannot be just a matter of a simple mechanical commensurability of the volume of fixed assets and of the number of employees. Objectively determined technological and organisational relations always exist.

The production function in the revised enlarged version of the aggregated model (under conditions of the constant level of employment in the material sphere of production) could therefore be written as follows:

$$Y_t = [1 + k_t^I \alpha_t + u_t] \cdot Y_{(t-1)}$$

The rate of growth of national income as

$$y = k^I \cdot \alpha \cdot u$$

which is identical with

$$y = \beta k$$

from the first version of the model. Because $k = k^I \alpha$ and $\beta = 1 + \frac{u}{\nu}$.

The enlarged aggregated model considers ten exogeneous parameteres and ten equations for the endogenous variables; the initial values for them have to be known.

The exogeneous parameters are as follows:

a) the share of scrapped fixed assets, both for machinery and equipment and for buildings

b) the reproduction ratio for both groups of fixed assets

c) the average duration of construction for both groups of fixed assets

d) the rate of growth of capital-output ratio

e) the ratio of amortization of productive fixed assets

f) parameter of the increment of circulating assets and stock (ratio to the increment of fixed assets)

g) the parameter for the value of overhauls.

The last three parameters are relevant for the prjection of the structure of the national income.

The endogenenous variables are as follows:

a) national income

b) fixed assets and machinery, equipment, buildings

c) the volume of net investments of machinery and equipment and building

d) the increment of fixed funds in construction both for machinery and buildings

e) the increment of circulating assets and stocks

f) ammortization fund

g) the value of overhauls.

There are also definition equations, such as those for gross national income, capital accumulation, consumption, gross investment, etc. There was

a possibility to calculate a great number of such indicators like labour productivity, capital-labour ratio, capital-consumption ratio etc. All these calculations were accomplished step by step for every year of the projection.

The determination of the objective possibilities for the rate of growth of the national income and even for the rate of growth of fixed assets supposes the working out of a separate analysis of the problem of the reproduction of fixed assets in a long-term development. This has not been available so far, and we have therefore started in this stage of the work to investigate the alternative variants of the rate of growth of the fixed assets and the variants of the changes of the parameters β or u respectively.

We consider however, the method based on various alternative analyses at the present stage of our investigations as an advantage. The approach towards a prognosis of the long-term growth cannot be based, in our opinion, on the endeavour to »guess« as exactly as possible, the future. Today we can only indicate the conditions leading towards the growth, which we are striving to attain. In this case it is rather a project than a prognosis of the future development.

The first approach is however a simple case of extrapolation. This extrapolation, though usually a priori rejected, can have a certain importance. In connection with an analysis of the development of economic relations and proportions, which can also be deduced from an aggregated model, a mechanical extrapolation can show the objective possibility, or on the contrary, the impossibility of a continuation of the past trends of the growth. In the case of a national economy, where its growth is governed by a plan, the mechanical extrapolation implicitly means, that it can detect in time the danger of certain trends in the national economy, as for instance in the investment policy which objectively leads to such a development.

The second approach consists in using the aggregated growth model to analyse the future growth while keeping certain initial conditions. In stipulating these conditions, or better still, the limits of the future development, by using the »mechanism« of the production model, we will not be entirely dependent on »pure« extrapolation as would appear at first glance. We shall apply the methodical principles of dynamic programming and we shall consider every admissible set of values of the parameters to be one of the possible variants of the future growth. Unlike the simple extrapolation this method requires unusually exacting calculations, but it permits a direct confrontation of economic contemplations with their impact on the main indicators of the national economy.

The model should therefore provide the answer to the question »what should be done« and from that we can derive the answer to the question »what will happen« or more accurately »what can result under certain specific conditions«. Once we know what can be attained under certain specific conditions, it will be possible to try and find out from the given set of possible and admissible solutions an optimum solution for the economy.

However, even in this case it will not be possible to accept a deterministic solution but a probability interval solution.

The Most Significant Results of the Application of the Growth Model

It is quite obvious from the preceding discussion that as long as certain specific conditions exist, the model reliably reflects the long-term rate of growth of the national income, consumption, the net and gross increment of fixed assets, and the corresponding volume of productive investments as well as the qualitative indicators derived from them such as the share of the net and gross productive investments in the national income, the capital and investments output ratios, the productivity of labour, the capital-labour ratio etc.

In the first stage of the calculation of the individual projection variants, we have assumed average parameters and exogenous variables of the model for the entire investigated period. It would have just as well been possible to assume their acceleration (rise) or slowing down (decrease) in individual intervals of the entire period of time. However, the method applied was in conformity with the questions to which an answer was sought.

The first question was, what are the long-term requirements imposed by a certain rate of growth of the national income, employment remaining constant on investments, on the rate of growth of the fixed assets and thus also on the investment and capital-output ratios, on the capital accumulation, characterized in this case by the share of the net or gross productive investments in the national income.

The next question was, how should the character of the future type of technical, or better still of economic progress (i.e. also in relation to the non-investment factors) be changed in comparison with the present development in order to increase the effectiveness of accumulation and thus decrease the investment intensity of the growth or, on the contrary, to increase the rate of growth of the national income at a given investment and capital.

The firts question can of course also be formulated in such a way as to ask: »what is the maximum long-term rate of growth of the national income attainable under the condition that the share of the gross productive investments in the national income, does not exceed for example he selected range od 14% — 21%. How high should the rate of growth of the fixed assets be in this case, how big the volume of the gross productive investments and what values should parameter β have, i.e. the elasticity coefficient of the relative capital-labour ratio increase to the relative labour productivity increase. All this of course holds good under the condition that the ascertained rates of growth of the national income, will not be limited by a foreign trade barrier, including an acceptable degree of its effectiveness.

An analysis of the past development by means of the model is not difficult. In the course of such an analysis the model has tested its own ability to give evidence. The correctness of the picture of the real relations in the long-term average is quite natural because it is derived, as has already been shown, from the way the model has been designed. The search for variants of the future relations is of course more difficult because a knowledge of at least some of the quantities constituting the model are assumed.

We have already said that there is no sense in trying to »guess« complexely the future economic development, that the realistic method is to ascertain the conditions and prerequisites required to achieve a certain type of future development. In this sense it is quite expedient to use the analyses of the consequences of past development as a springboard because a certain continuity can never be quite denied. On the other hand, the prerequisites, conditions and consequences of deviations from the past development, as long as they can contribute to more positive results, can not be overlooked.

The application of a growth model for the mechanical extrapolation of the development due to the apparently favourable economic conditions of the period from 1955 to 1961, proves that from a long-term point of view it would not only be a very ineffective but even an economically unrealizable development.

It is true that during the above mentioned period relatively high rates of growth of the national income exceeding 7 per cent on the average were achieved. A high rate of growth of the capital-labour ratio average almost 10 per cent annually, in the production sphere, also corresponded with this development. At the same time however, this development was also accompanied by a high rate of growth of machinery and equipment which exceeded 10 per cent and a substantially lower rate of growth of the overall productivity of labour so that parameter β amounted to 0,68 on the average. Its magnitude (less than one) demonstrates capital absorbing technical progress from the points of view of machinery and equipment. This technical progress was labour-saving but had considerable demands on the active component of the fixed assets.

If we further assume the other variables of the model on the level of the average for the same period, then twenty years later, i.e. in 1985, the share of gross investments in the national income would attain approximately 42 per cent, and 23 per cent in machinery and equipment. The machinery and equipment capital-output ratio would also rise substantially, from the calculated 0,985 in 1955 to 1.327 in 1975 and to 1.788 in 1985.

An unfavourable development of the capital, as well as investment intensity of the growth of the national income, would have been accompanied by a substantial decrease of the share of consumption in the national income as well as by a decrease of its rate of growth. The consumption would decrease from 80 per cent in 1965 to 63 per cent in 1985. Even if we were to consider a correction (according to the previous explanation) of the usual conception of consumption (with a difference between the depreciation fund and the value of the retire fixed assets), only the percentages in the above mentioned years would change; but the decreasing trend of the percentage and of the rate of growth which interested us more in this projection than its absolute level, would not undergo very big change.

It appears therefore that a pronounced increase of the effectiveness of capital accumulation, which will lead to a growth of parameter β is a necessary condition for maintaining the past rates of growth of the national income. To maintain the rates of growth of the fixed assets under an otherwise unchanged situation i.e. to increase the share of the gross productive investments in the national income above the level of 1960 and 1961, when it already exceeded 18 per cent (11,8 per cent in 1955), is evidently beyond the material, labour and organisational possibilities of the Czechoslovak capital construction activities. Here the effectiveness of such a procedure is not taken into consideration. The new system of planning and management

of the national economy should certainly be able to expand our possibilities in this respect, but increasing the volume of investments without a corresponding increase of their effectiveness is definitely not an acceptable way of long-term development.

The only possible way to achieve a solution lies in decreasing the rate of growth of fixed assets, which should permit to spend a greater part of the gross productive investments on renovation and reconstruction, particularly of machinery and equipment. At the same time it will, of course, require the strict fulfillment of the condition that the new machinery and equipment will have a much higher technical standard both with regard to labour requirements as well as to production efficiency. We have already shown that the objectively possible and effective rates of growth of the fixed assets in the future decades depend, after all, on the technical standard of the fixed assets.

The most concentrated and at the same time the most synthetic criterion of the effectiveness of technical progress, and in general, of the effectiveness of the internal reproduction process is, in this case, the magnitude of parameter β. When the rates of growth of the fixed assets will be reduced then the rates of growth of the national income can be maintained only by a substantial increase of parameter β. In the long run there must be a substantial increase of the effectiveness of the technical and organisational progress in comparison with both the average of 1955 — 1961 and particularly with the one achieved in the recent years.

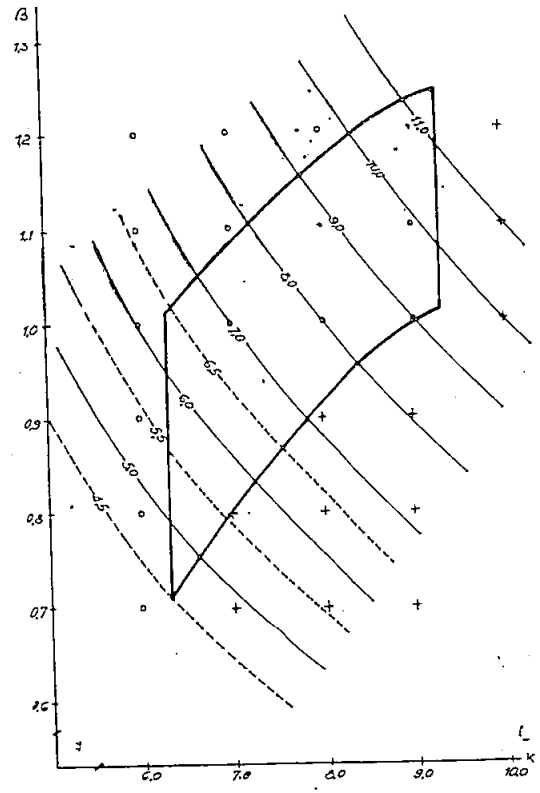
The graph of isotemps of the national income is a very clear illustration of the conditions required to reach a certain rate of growth of the national income, which does not, exceed the ratio of gross productive investments within a period of twenty years to the national income within the limits of 14 — 21 per cent.

The average annual rate of growth of the fixed assets (machinery and equipment) k is represented by the horizontal x-axis. Parameter β is given on the vertical y-axis. The oblique lines in the graph — the individual istotemps — indicate the same average annual rate of growth of the national income. This rate of growth is due to individual combinations of parameter β and the rate of growth of machinery and equipment fixed assets k during the twenty years under investigation. For better orientation the more important rates of growth of the national income (4%, 4,5%, 5%, 5,5%, 6% 6,5% 7% etc) are drawn in separate lines.

The area bounded by heavy lines within the graph defines the area of those long-term rates of growth of the national income, which corresponds to the given combination of technical progress (parameter β) and an average annual rate of growth of machinery and equipment. It is assumed that the ratio of the overall net investments to the volume of net investments into machinery and equipment is $q = 1.83$, and a ratio of gross and net investments into machinery and equipment is $h = 1.3$ (i.e. $\alpha = 3.33$). These data are based on the year 1965 (for Y_0, K_0) and during the entire twenty years period do not require a lower ratio of 14 per cent, of the gross productive investments to the national income nor a ratio higher than 21 per cent. The points marked with a small circle indicate that either during the first years or during the entire twenty years period the given rate of growth of the

national income does not reach the 14 per cent ratio. The points marked with a cross indicate that the 21 per cent limit has been exceeded towards the end or throughout the entire period. Those variants, which are marked with a full circle remain throughout the entire period within the 14 — 21 per cent limits. For the sake of simplicity the interpolation between the various points has been done linearly.

The Graph of Isotemps of the National Income



For example it can be seen from the graph that for the chosen range of parameter β and the rate of growth of the fixed assets k, the quantities q and h defined above, and on the basis of the initial year 1965. The variants of the rates of growth of the national income which lie within the area of adminissible solutions are approximately 5 per cent and higher. The manimum value of parameter is $\beta = 0.73$ and a minimum rate of growth of machinery and equipment is $k = 6.6$ per cent annually. The »area of admissible solution« expands gradually with the growing rate of growth of fixed assets k at a minimum value of parameter $\beta = 0.73$. All this holds good for $k = 9$

per cent. From the magnitude of $\beta = 1.04$, which represents the type of capital saving technical progress, the area of admissible solutions begins to narrow from the left because even at relatively higher rates of growth of machinery and equipment these variants do not reach the given 14 per cent limit on the ratio of gross productive investments. With a growing parameter β and therefore with the intensification of the type of technical progress with economical requirements on fixed assets, the »area of admissible solutions« would close completely from the left. All variants would lie below the chosen ratio of 21 per cent.

It is quite evident of course, that all the variants to the left of the »area of admissible solutions« which lie below the lower limit of 14 per cent are acceptable. Those variants, which represent comparatively the same long-term rate of growth of the national income (the same isotemps), are even relatively more effective. The same rate of growth of the national income is attained with a lower capital accumulation.

On the other hand these variants do not exhaust the entire volume of investment funds, which could be devoted to the development and under identical conditions could ensure an acceleration of the rate of growth.

The »area of admissible solutions« should therefore be understood as one including those variants, which fulfill the limiting conditions of a ratio of the gross productive investments to the national income which is not less than 14 per cent and not more than 21 per cent.

The graph clearly shows that rates of growth of the national income of approximately 7 per cent corresponding to the values of parameter $\beta = 0.65 - 0.70$ and the growth rates k about 10 per cent are all very far from the so-called area of admissible growth, because they highly exceed the 21 per cent limit of the ratio of gross productive investments in the national income. And that is a situation which is characteristic for the already mentioned extrapolation variant based on the conditions of the period 1955 — 1961.

The variant of the plan drafted for 1970 (worked out in March 1965) is closser to the area of admissible solutions because it assumes that parameter β will be approximately 0.71. The rate of growth of machinery and equipment will be 5.4 per cent and an average rate of growth of the national income will be below 4 per cent.

However with regared to the lower planned rate of growth of the national income to 1970 the average 5 per cent rate of growth planned for the investigated twenty years period necessitates its increase during the period 1970 — 1985 to about 5.5 per cent. With $k = 6.6$ per cent this would require the increase of parameter β to approximately 0.85.

The projection of various variant has also brought interesting results concerning the analysis of the fixed assets and investment of a capital-output ratio, capital-labour ratio, labour productivity, the development of the consumption, and other indicators.

The results provided by this relatively simple model contribute certain useful considerations about the possibilities of our further economic growth. Of course it is not a quite comprehensive survey, not even on a macroeconomic level. The model itself will have to be improved further by new relations. In spite of this, even in the present form, which reflects the consequences of the pressing need to know the answers as soon as possible, it had provided data for important economic conclusions. For us it represents a certain key point of the methodology of long-term projections on an aggregated level, which has to be followed by a disaggregation by means of a structural model.

Research Institute of Economic Planning of the Czechoslovak State Planning Commission, Prague

Zdeněk TLUSTÝ

Vladimír STRNAD

ANALIZA SMJEŠTAJNIH PREDNOSTI U GRANSKOM PROSTORNOM MODELU POMOĆU DUALA TRANSPORTNOG PROBLEMA LINEARNOG PROGRAMIRANJA

1. Uvod. Uvjeti ekonomske ravnoteže granskog prostornog modela svode se na takvo stanje danog ekonomskog kompleksa za kojeg bi ma koja druga prostorna realokacija resursa (proizvođačkih kapaciteta) dovela do povišenja smještajno zavisnih troškova proizvodnje, transporta i cijena realizacije. U našoj analizi ovo općenito načelo primijenit ćemo u okvirima jedne grane zanemarivši utjecaj internih ekonomija razmjera, eksternih ekonomija lokalizacije industrije i aglomeracije ekonomskih aktivnosti općenito. Bavit ćemo se, prema tome, jednim parcijalnim ekonomskim modelom, za koji su sve ostale okolnosti dane.

Uz ove pretpostavke problem se svodi na jedan oblik linearnog ekonomskog modela u kojem su nepoznate: optimalne lokacije, optimalne veličine kapaciteta na pojedinim lokacijama, putevi opskrbljivanja odnosno gravitaciona područja pojedinih lokacija i cijene realizacije finalnog proizvoda u mjestu potrošnje.

U ovom radu ćemo poći od optimalnog rješenja otvorenog transportnog problema, dvoetapnog i troetapnog, te odgovarajućom interpretacijom dualnih varijabli ukazati na mogućnost njegove primjene u ekonomskoj analizi granskih prostornih modela.

Značajniji radovi u ovoj oblasti javljaju se već sredinom 50-ih godina XX vijeka.

P. A. Samuelson [1] polazi od problema optimalnog izbora isporučilaca odnosno potrošača uz dane lokalne funkcije ponude i potražnje, a funkcija ekonomskog cilja je ukupna društvena »neto korisnost« zasnovana na relacijama viška funkcije ponude, viška funkcije potražnje i transportnih troškova. Radi se, dakle, o tržnim relacijama među pojedinim regijama i optimalnoj kombinaciji izvoza i uvoza među regijama.

Lefeberov [2] linearni prostorni model polazi od pretpostavke da su dani resursi sa troškovima proizvodnje na pojedinim lokacijama u određe-