

# A MIXED-INTEGER LINEAR PROGRAMMING MODEL FOR AGGREGATING MULTI-CRITERIA DECISION MAKING METHODS

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Received: 2 February 2022;

Accepted: 18 June 2022;

Available online: 18 June 2022.

*Original scientific paper*

**Abstract:** *Selecting an MCDM method to use in any decision-making problem is always a difficult issue regarding that there is no agreement generally on which method is the most appropriate one. This paper addressed a proposal of a hybrid approach for this problem. Under the assumption that there is no superiority among well-established and accepted MCDM methods, we defined a minimax strategy based on the fact that the highest total rank deviation between MCDMs and the proposed hybrid approach in terms of alternative rankings should be as low as possible. Even though MCDM methods often rank the alternatives differently, many methods perform similar ranking due to sharing alike mathematical operations. To avoid positive bias towards these methods in an integrated approach, we focused on a prioritizing scheme that supports differentiated rankings from others. This prioritizing scheme also contributed to hindering the problem of selecting MCDMs with constraining the compound effect of similar rankings. We developed a hybrid decision-making model combining different MCDM methods with prioritizing them by using a mixed-integer linear programming model. We compared the proposed approach with some well-known prioritizing methods and the results revealed that the proposed approach produced better outcomes in obtaining the desired outputs.*

**Key words:** *Multiple Criteria Analysis; Aggregating MCDMs; Comparative Analysis; Minimax Strategy.*

## 1. Introduction

Decision-making has always been an important part of everyone's life. While some decisions need to be made daily, some others should be taken with long-term strategic considerations. If someone needs to address the problem through decision-making, then it is called a decision problem. These problems become Multi-Criteria Decision-

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A mixed-integer linear programming model for aggregating multi-criteria decision making... Making (MCDM) problems when it is necessary to evaluate them according to more than one criterion. Solving MCDM problems require a process that involves determining the most appropriate alternative among several options with considering the perspective of decision-makers and all criteria. To use in this process, the MCDM methods that have their mathematical basis were developed. Some of the most common MCDM methods are Weighted Sum Model (WSM), Weighted Product Model (WPM), ELimination Et Choice Translating Reality (ELECTRE) by Roy (1968), Decision-Making Trial and Evaluation Laboratory (DEMATEL) by Gabus & Fontela (1972), Analytic Hierarchy Process (AHP) by Saaty (1980), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) by Hwang & Yoon (1981), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) by Brans (1982), Compromise Ranking Method (VIKOR) by Opricovic (1998).

Numerous MCDM methods have been proposed to assist decision-makers in solving MCDM problems. However, as Baydas et al. (2022) stated it is very complicated and difficult to choose an MCDM method for practitioners in their decision-making problems. These MCDM methods differ from each other in different aspects, such as using continuous or discrete data, qualitative or quantitative criteria, and in purpose like choosing, ranking, sorting the alternatives (Zavadskas et al. 2014). Moreover, MCDM methods employ different techniques to normalize decision matrix and use those outcomes to calculate utility scores of alternatives by varied mathematical operations, such as addition, multiplication, exponentiation, or logarithm. Therefore, distinct MCDM methods often yield conflicting results. For example, Pamucar et al. (2021) used six different types of MCDM methods in their study and found six different rankings. Overcoming this problem is not an easy task because there is no theoretical superiority between any two MCDM methods. Baydas & Elma (2021) stated that the elements in every MCDM problem can vary and these changes affect the outcomes of an MCDM in different ways, and added that there can be no absolute superiority among the MCDM methods. Kou et al. (2012) offered using various MCDM methods instead of one, to get more trustful results. If we agree with this opinion, then a new question arises. How can we aggregate a group of MCDM methods? This question has attracted quite the attention of researchers.

To answer this question, some combining methods have been proposed in which each of them used dissimilar perspectives, such as weighting MCDM methods with Spearman's rank correlation coefficient introduced by Kou et al. (2012) and Peng (2015). Using alternatives' utility scores, which were obtained by different MCDM methods, as input of response surface methodology to produce final rankings introduced by Wang et al. (2016). Employing Borda count method, in which rankings obtained by MCDMs are summed, by Barak & Mokfi (2019). Ranking alternatives with MULTIMOORA method that uses Delphi method and dominance theory to reach an agreement on the final ranking introduced by Brauers & Zavadskas (2010). Biswas (2020) used WSM to obtain a synthesized ranking of three equally weighted MCDM methods. Mohammadi & Rezaei (2020) used half-quadratic theory and consider an MCDM method that has a different ranking from the others as an exception and rated with a lower weight in their optimization model which evaluates the MCDM importance in the overall ranking. Pramanik et al. (2021) employed WSM to aggregate five MCDM methods that have been assessed of identical importance in the overall ranking.

Although many approaches have been developed to address the combining MCDM methods, these techniques have some issues to discuss. First, using only ranking without utility scores like in Borda count and dominance theory, the final rankings' precision and accuracy rates would be significantly decreased. Second, it is obvious

that MCDM methods with similar algorithms tend to produce more alike rankings. With that in mind, even assigning equal weights to all methods may prioritize the correlated ones and underrate the uncorrelated ones while achieving the final aggregate ranking. Under the assumption that there is no superiority among MCDM methods in terms of decision theory, we cannot let similar methods gain upper hand in the combined final rankings. Therefore, it will be a challenging issue to the determination of MCDM methods that will be chosen to form the appropriate combination that avoids the dominance of similar methods. At this point, the aggregation problem can be seen as an MCDM problem in which the best mixing ratio of the methods is the decision-making problem, and also the methods are the criteria. In this case, it is necessary to answer the question of how should the MCDM methods, which are expressed as criteria in the aggregation problem, be prioritized.

In MCDM theory, we can categorize the criteria weighting approaches into three groups: subjective, objective, or combination of both of them. Subjective approaches such as point allocation by Doyle et al. (1997), direct rating by Bottomley & Doyle (2001), swing by Von Winterfeldt & Edwards (1986), a ranking method by Kirkwood & Corner (1993), pairwise comparison by Saaty (1980) has been widely popular in the real-world applications. Subjective approaches assume that the criteria weights can be predetermined by the decision-maker's judgments which are based on their knowledge and expertise on the MCDM problem.

However, if we cannot predetermine superiority among MCDM methods, then both the subjective approaches and the mixed ones that are all greatly based on predetermination and decision-makers preferences will have important shortcomings to use in the aggregation problem. As mentioned before, we defined the criteria of the aggregation problem as MCDM methods. So, the utility scores of alternatives obtained by each MCDM method in the actual problem can be put together into a whole to form the decision matrix of the aggregation problem. In this context, weighting MCDM methods according to utility scores would be specific to the dataset of the actual problem, and the general or individual judgments would not play a role in the process. Hence, these subjective approaches are inappropriate to use, as they rely heavily upon the decision-maker's judgments. Meanwhile, there are significant numbers of objective methods which prioritize the criteria using only the decision matrix. Objective methods, such as the Entropy method by Deng et al. (2000), the Standard Deviation (SD) and the Criteria Importance through Intercriteria Correlation (CRITIC) methods by Diakoulaki et al. (1995), the Correlation Coefficient and Standard Deviation (CCSD) by Wang & Luo (2010), the Integrated Determination of Objective CRiteria Weights (IDOCRIW) by Zavadskas & Podvezko (2016) a modified Entropy used by Biswas et al. (2019), and the Entropy and Correlation Coefficients (EWM-Corr) by Mukhametzyanov (2021) stand out as the most notable ones.

Although objective weighting methods are considered appropriate to prioritize MCDM methods, in this study, we proposed a new mixed-integer linear programming model that produces the importance level of each method for the aggregation problem in a better approach, intending to minimize the maximum total rank reversals from each ranking of MCDM methods in the final ranking. This would also maximize the lowest rank correlation between the final ranking and the rankings of the methods used in the final ranking. The main motivation of this study is the lack of an appropriate approach that eliminates the necessity of excessive pre-examination for choosing a group of MCDM methods that would be used to solve MCDM problems. So, the proposed direct prioritization scheme will consider the final ranking in a better way with behaving fairly and equally for each MCDM method whether they have similar properties or not.

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The main purpose of this study is to aggregate the scores of MCDM methods with a unique perspective aiming to minimize the maximum total rank deviations between any single MCDM ranking and the final aggregated ranking. So, the proposed approach which is called Aggregation with Minimax Total Rank Deviation (AMTRD) was compared with some well-known objective weighting methods to reveal its performance on attaining the goals in two different MCDM problems. We selected a group of distinguished MCDM methods to illustrate the performance of the AMTRD approach in acquiring an aggregated ranking. There is no limitation to selecting another group of MCDM methods. Therefore the practitioners can form different groups of MCDM with the AMTRD approach.

The remainder of this paper is organized as follows. In section 2 a brief explanation of the novel reconciling process of the AMTRD and some MCDM methods that were used to illustrate the proposed approach. Two different illustrative examples are presented in section 3, followed by the comparative analysis and further analysis in sections 4 and 5 and the conclusion is the final part of the paper.

## 2. Methodology

MCDM methods are the specifically produced tools aiming to assess MCDM problems. Before using these methods, we should form a decision matrix. In this matrix, we generally present alternatives on rows and criteria on columns. First, all methods deal with the normalization of the decision matrix according to the type of criteria. Each method has its normalization procedure for the decision matrix and also handles cost and benefit criteria differently. Then, the methods assess the alternatives according to criteria. Finally, we obtain ranking/utility values/scores of the alternatives. In this study, we emphasize the AMTRD approach to aggregate a group of MCDM methods. The point to be underlined here is the proposed method focuses on the final scores of the MCDM methods and does not relies on specific MCDMs. However, to elucidate the AMTRD we used some well-known MCDM methods as an example. These methods and our proposed AMTRD approach are mentioned as follows.

### 2.1. CRITIC Method

In decision-making problems where there is more than one criterion, the importance of the criteria should be determined. A significant number of techniques have been developed to make this assessment, although most of them are subjective approaches. However, by using these subjective techniques, different criteria weights can be obtained from the same decision-maker. In addition, different decision-makers can make different evaluations with the same method (Diakoulaki et al. 1995).

Addressing these shortcomings of subjective methods, Diakoulaki et al. (1995) proposed an objective approach known as CRITIC. This method depends on both correlation coefficients between criteria and standard deviations of criteria. CRITIC can be applied with the following steps (Jahan et al. 2012):

First, a decision matrix  $X = \|x_{ij}\|_{(n \times m)}$  should be obtained for  $A_i$  ( $i = 1, \dots, n$ ) alternatives and  $C_j$  ( $j = 1, \dots, m$ ) criteria, and then we have to normalize criteria using Eqs. (1) and (2) where  $x_j^{\min}$  and  $x_j^{\max}$  are minimum and maximum elements of criterion  $j$ . Second, Eqs. (3) and (4) provided correlation coefficients and standard deviations,

respectively. Finally, with Eqs. (5) and (6), the importance levels of criteria  $w_j$  are obtained.

$$z_{ij} = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}} \quad \text{for benefit criteria} \tag{1}$$

$$z_{ij} = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}} \quad \text{for cost criteria} \tag{2}$$

$$\rho_{jk} = \frac{\sum_{i=1}^n (z_{ij} - \bar{z}_j)(z_{ik} - \bar{z}_k)}{\sqrt{\sum_{i=1}^n (z_{ij} - \bar{z}_j)^2 \cdot \sum_{i=1}^n (z_{ik} - \bar{z}_k)^2}}, \quad j, k = 1, \dots, m \tag{3}$$

$$\sigma_j = \sqrt{\frac{1}{n} \sum_{i=1}^n (z_{ij} - \bar{z}_j)^2}, \quad j = 1, \dots, m. \tag{4}$$

$$c_j = \sigma_j \sum_{k=1}^m (1 - \rho_{jk}), \quad j = 1, \dots, m. \tag{5}$$

$$w_j = \frac{c_j}{\sum_{k=1}^m c_k}, \quad j = 1, \dots, m. \tag{6}$$

### 2.2. ARAS Method

ARAS (A new additive ratio assessment) depends on comparing values of alternatives to optimum values which are added by decision-makers. The MCDM approach of ARAS was proposed by Zavadskas & Turskis (2010). They first formed the decision matrix  $X$  in Eq. (8) using Eq. (7) as follows;

$$x_{odj} = x_j^{\max} \text{ if } j \in \text{benefit criteria} \text{ and } x_j^{\min} \text{ if } j \in \text{cost criteria} \tag{7}$$

$$X = \begin{bmatrix} x_{od1} & x_{od2} & \dots & x_{odm} \\ x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \quad i = 1, \dots, n \quad j = 0, \dots, m \tag{8}$$

Afterward, they normalized the decision matrix using Eqs. (9) and (10) for benefit and cost criteria, respectively. They obtained a criteria weighted matrix by using  $w_j$  and Eq. (11). Lastly, the optimality function values and the utility degree of alternatives are calculated using Eqs. (12) and (13). The higher  $K_i$  scores represent more favorable results.

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$$r_{ij} = \frac{x_{ij}}{\sum_{i=0}^n x_{ij}} \text{ for benefit criteria} \quad (9)$$

$$x_{ij}^* = \frac{1}{x_{ij}}; \quad r_{ij} = \frac{x_{ij}^*}{\sum_{i=0}^n x_{ij}^*} \text{ for cost criteria} \quad (10)$$

$$v_{ij} = r_{ij} w_j \quad (11)$$

$$S_i = \sum_{j=1}^m v_{ij}, \quad i = 0, 1, \dots, n \quad (12)$$

$$K_i = \frac{S_i}{S_0} \quad (13)$$

### 2.3. COPRAS Method

The MCDM approach COPRAS (complex proportional assessment method) was firstly proposed by Zavadskas & Kaklauskas (1996). Later, the method was undergone some changes by Podvezko (2011). The decision matrix  $X = \|x_{ij}\|_{(n \times m)}$  can be normalized and weighted simultaneously by Eq. (14). The score values for the alternatives are obtained by using Eqs. (15) or (16) depends on whether the criterion is benefit ( $j=1, \dots, k$ ) or cost ( $j=g+1, \dots, m$ ) type, respectively. The relative importance levels of alternatives can be calculated by Eq. (17) and the performance index values of alternatives can be computed by Eq. (18), as well. The alternative with the highest performance index is determined as the best alternative.

$$d_{ij} = \frac{x_{ij} w_j}{\sum_{i=0}^n x_{ij}} \quad (14)$$

$$S_i^+ = \sum_{j=1}^k d_{ij}, \quad i = 0, 1, \dots, n \quad (15)$$

$$S_i^- = \sum_{j=k+1}^m d_{ij}, \quad i = 0, 1, \dots, n \quad (16)$$

$$Q_i = S_i^+ + \frac{\sum_{i=1}^n S_i^-}{S_i^- \sum_{i=1}^n \frac{1}{S_i^-}} \quad (17)$$

$$P_i = \frac{Q_i}{\max\{Q_i\}} \cdot 100\% \quad (18)$$

**2.4. EDAS Method**

Keshavarz Ghorabae et al. (2015) proposed the EDAS (Evaluation based on Distance from Average Solution), which depends on the average values of criteria. They first calculate the mean values for each criterion using the decision matrix  $X = \|x_{ij}\|_{(n \times m)}$  in Eq. (19). Then the positive and negative distance from average matrices PDA and NDA can be calculated for benefit criteria using Eqs. (20) and (21), for cost criteria employing Eqs. (22) and (23).

$$AV_j = \frac{\sum_{i=1}^n x_{ij}}{n} \quad j = 1, \dots, m \tag{19}$$

$$PDA_{ij} = \frac{\max(0, (x_{ij} - AV_j))}{AV_j} \tag{20}$$

$$NDA_{ij} = \frac{\max(0, (AV_j - x_{ij}))}{AV_j} \tag{21}$$

$$PDA_{ij} = \frac{\max(0, (AV_j - x_{ij}))}{AV_j} \tag{22}$$

$$NDA_{ij} = \frac{\max(0, (x_{ij} - AV_j))}{AV_j} \tag{23}$$

Positive sums and negative sums for the alternatives are obtained by using criteria weight  $w_j$  in Eqs. (24) and (25), respectively.

$$SP_i = \sum_{j=1}^m w_j PDA_{ij}, \quad i = 1, \dots, n \tag{24}$$

$$SN_i = \sum_{j=1}^m w_j NDA_{ij}, \quad i = 1, \dots, n \tag{25}$$

Thereafter, normalized SP and SN values can be acquired by utilizing Eqs. (26) and (27), respectively. Lastly, the assessment values are obtained by employing Eq. (28) and EDAS ends with ranking alternatives in a decrescent manner.

$$NSP_i = \frac{SP_i}{\max(SP_i)}, \quad i = 1, \dots, n \tag{26}$$

$$NSN_i = 1 - \frac{SN_i}{\max(SN_i)}, \quad i = 1, \dots, n \tag{27}$$

$$AS_i = \frac{1}{2}(NSP_i + NSN_i), \quad i = 1, \dots, n \tag{28}$$

**2.5. MOOSRA Method**

MOOSRA (multiobjective optimization based on simple ratio analysis) was developed by Das et al. (2012) to overcome the problems found in other MCDM techniques. Das et al. (2015) used MOOSRA for evaluating performance in the

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$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n x_{ij}^2}} \quad (29)$$

$$PS_i = \frac{\sum_{j=1}^g w_j r_{ij}}{\sum_{j=g+1}^m w_j r_{ij}} \quad (30)$$

## 2.6. WASPAS Method

Zavadskas et al. (2012) proposed WASPAS (The weighted aggregated sum product assessment) which consists of both weighted sum and weighted product model. The decision matrix  $X = \|x_{ij}\|_{(n \times m)}$  can be normalized by employing Eqs. (31) and (32). Weighted sum model and weighted product model can be acquired by using criteria weight  $w_j$  and calculating Eqs. (33) and (34) as follows:

$$z_{ij} = \frac{x_{ij}}{x_j^{\max}} \quad \text{for benefit criteria} \quad (31)$$

$$z_{ij} = \frac{x_j^{\min}}{x_{ij}} \quad \text{for cost criteria} \quad (32)$$

$$WSM_i = \sum_{j=1}^m w_j z_{ij}, \quad i = 1, \dots, n \quad (33)$$

$$WPM_i = \prod_{j=1}^m (z_{ij})^{w_j} \quad i = 1, \dots, n \quad (34)$$

Lastly, the alternatives are ranked in descending order according to performance scores acquired by Eq. (35).

$$Q_i = \frac{1}{2} (WSM_i + WPM_i) \quad i = 1, \dots, n \quad (35)$$

## 2.7. The Proposed AMTRD Method

Suppose there is a decision matrix  $X = \|x_{ij}\|_{(n \times m)}$  that includes MCDM methods as criteria  $C_1, \dots, C_m$  and also alternatives as  $A_1, \dots, A_n$  where  $x_{ij}$  denotes the scores of  $A_i$  in terms of  $C_j$ . The decision matrix that has only benefit criteria can be normalized by using Eq. (1). Afterward, the aggregated scores can be calculated from the normalized



score matrix  $Z = \left\| z_{ij} \right\|_{(n \times m)}$ . However, the weights of methods also have to be defined before obtaining the final scores of alternatives. Since the proposed AMTRD approach aimed to minimize the maximum rank reversals from each ranking of MCDM methods, we suggested using the most common and basic MCDM technique as WSM to acquire the final ranking.

The WSM obtain the  $S_i$ , the overall scores of alternative  $i$ , as follows (Fishburn, 1967):

$$S_i = \sum_{j=1}^m w_j z_{ij}, \quad i = 1, \dots, n \tag{36}$$

We can use a mixed-integer linear programming model to obtain criteria weights that would be used in Eq. (36) to minimize the deviations of final ranking from the ranking of each method as follows;

*Minimize Dev*

*Subject to;*

$$\sum_{j=1}^m w_j z'_{ij} \geq \sum_{j=1}^m w_j z'_{kj} - HY_{ikt} + N \quad (i, k = 1, \dots, n. \quad \forall t = 1, \dots, m. \quad \text{for } i < k) \tag{37}$$

$$\sum_{j=1}^m w_j z'_{ij} \leq \sum_{j=1}^m w_j z'_{kj} + HV_{ikt} - N \quad (i, k = 1, \dots, n. \quad \forall t = 1, \dots, m. \quad \text{for } i > k) \tag{38}$$

$$Dev \geq \sum_{i=1}^n \sum_{k=1}^n Y_{ikt} + V_{ikt} \quad (\forall t = 1, \dots, m) \tag{39}$$

$$\sum_{j=1}^m w_j = 1 \tag{40}$$

$$Y_{ikt} \in \{0, 1\} \quad (i, k = 1, \dots, n. \quad \forall t = 1, \dots, m. \quad \text{for } i < k) \tag{41}$$

$$V_{ikt} \in \{0, 1\} \quad (i, k = 1, \dots, n. \quad \forall t = 1, \dots, m. \quad \text{for } i > k) \tag{42}$$

$$w_j \geq 0 \quad (j = 1, \dots, m) \tag{43}$$

The decision variable called the *Dev* is used to ensure that the maximum sum of deviations between the ranking of any MCDM method and the aggregated AMTRD ranking is minimized.

Suppose that  $t$  is also an MCDM method as  $j$ . We sorted rows of  $Z = \left\| z_{ij} \right\|_{(n \times m)}$  according to scores of alternatives for each MCDM method ( $t = 1, \dots, m$ ) in a descended manner as  $Score'_1 > Score'_2 > Score'_3 > \dots > Score'_n$  before the optimization process and obtained  $m$  different  $z'_{ij}$  matrices.

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In Eq. (37), where if the condition that  $\sum_{j=1}^m w_j z'_{ij} \geq \sum_{j=1}^m w_j z'_{kj}$  is met, which refers to newly acquired scores are in order of  $Score^{AMTRD}_i > Score^{AMTRD}_k$  as with the method  $t$  that has also  $Score^t_i > Score^t_k$ . So there is no deviation for the rank of alternatives  $i$  and  $k$  between the MCDM method  $t$  and the AMTRD approach. But if the condition is not met then the binary variable  $Y_{ikt}$  which refers to whether there is a downward rank deviation or not for alternatives  $i$  and  $k$  according to MCDM method  $t$  would be equal to 1. Here  $w_j$  are the MCDMs weights that would be obtained in the optimization process. So, basically  $Score^{AMTRD}_i$  would be equal to  $\sum_{j=1}^m w_j z'_{ij}$  according to WSM that we used to obtain aggregated rankings.

In Eq. (38), where if the condition that  $\sum_{j=1}^m w_j z'_{ij} \leq \sum_{j=1}^m w_j z'_{kj}$  is met, which refers to newly acquired scores are in order of  $Score^{AMTRD}_i < Score^{AMTRD}_k$  as with the method  $t$  that has also  $Score^t_i < Score^t_k$ . So there is no deviation for the rank of alternatives  $i$  and  $k$  between the method  $t$  and the AMTRD approach. But if the condition is not met then the binary variable  $V_{ikt}$  which refers to whether there is an upward rank deviation or not for alternatives  $i$  and  $k$  according to method  $t$  would be equal to 1.

In Eq. (37) and Eq. (38), we expected both  $\sum_{k=1}^n Y_{ikt}$  and  $\sum_{k=1}^n V_{ikt}$  are equal to 0 when the AMTRD approach and the method  $t$  rank the alternative  $i$  in the same rank. For example, if AMTRD ranks alternative  $i$  3 levels lower than the method  $t$  then  $\sum_{k=1}^n Y_{ikt} = 3$  and a downward deviation of 3 points occurs for only alternative  $i$ . Conversely, if AMTRD ranks alternative  $i$  2 levels upper than the method  $t$  then  $\sum_{k=1}^n V_{ikt} = 2$  and an upward deviation of 2 points occurs for only alternative  $i$ .

In Eq. (37) and Eq. (38), using greater or equal and lesser or equal types of constraints can lead both  $Y_{ikt}$  and  $V_{ikt}$  are equal to 0, when  $Score^{AMTRD}_i \approx Score^{AMTRD}_k$  and  $Score^t_i \neq Score^t_k$ . To overcome this issue we can use a very small but also meaningful constant number  $N$  with the value of 0.0001 and turn the greater or equal and lesser or equal types of constraints into greater and lesser types, respectively. Here  $H$ , which is a sufficiently large number and used as 1000 to guarantee that the constraints hold. Since in some cases, the value of 1 for  $Y_{ikt}$  or  $V_{ikt}$  cannot be adequate to run the necessary restrictions correctly.

Eq. (39) would constraints that  $Dev$  will be greater than the each number of rank deviations between methods and the AMTRD approach. Since, the sum of rank deviation between the method  $t$  and the AMTRD approach is equal to  $\sum_{i=1}^n \sum_{k=1}^n Y_{ikt} + V_{ikt}$ .

While Eq. (40) limits the sum of  $w_j$  equal to 1, Eqs. (41), (42), and (43) are binary and nonnegative constraints of  $Y_{ikt}$ ,  $V_{ikt}$ , and  $w_j$ , respectively.

Let us consider an example of  $n=4$  alternatives and  $m=3$  MCDM methods. Then we have three different MCDM results for the same four alternatives. Assume that  $X = \|x_{ij}\|_{(4 \times 3)}$  is given as follows;

$$X = \begin{bmatrix} 12 & 0.9 & 100 \\ 10 & 0.5 & 80 \\ 5 & 1 & 45 \\ 9 & 0.7 & 15 \end{bmatrix}$$

We can obtain the normalized score matrix  $Z = \|z_{ij}\|_{(4 \times 3)}$  using Eq. (1) as follows;

$$Z = \begin{bmatrix} 1.000 & 0.800 & 1.000 \\ 0.714 & 0.000 & 0.765 \\ 0.000 & 1.000 & 0.353 \\ 0.571 & 0.400 & 0.000 \end{bmatrix}$$

After that, we can sort rows of  $Z = \|z_{ij}\|_{(n \times m)}$  according to scores of alternatives for each MCDM method ( $t = 1, 2, 3$ ) in a descended manner and obtained 3 different  $z_{ij}^t$  matrices as follows,

$$z_{ij}^1 = \begin{bmatrix} 1.000 & 0.800 & 1.000 \\ 0.714 & 0.000 & 0.765 \\ 0.571 & 0.400 & 0.000 \\ 0.000 & 1.000 & 0.353 \end{bmatrix} \quad z_{ij}^2 = \begin{bmatrix} 0.000 & 1.000 & 0.353 \\ 1.000 & 0.800 & 1.000 \\ 0.571 & 0.400 & 0.000 \\ 0.714 & 0.000 & 0.765 \end{bmatrix} \quad z_{ij}^3 = \begin{bmatrix} 1.000 & 0.800 & 1.000 \\ 0.714 & 0.000 & 0.765 \\ 0.000 & 1.000 & 0.353 \\ 0.571 & 0.400 & 0.000 \end{bmatrix}$$

So, the Eq. (37) in the AMTRD model can be defined with 18 constraints as follows;

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$$\begin{aligned}
 w_1 * z_{11}^1 + w_2 * z_{12}^1 + w_3 * z_{13}^1 &\geq w_1 * z_{21}^1 + w_2 * z_{22}^1 + w_3 * z_{23}^1 - HY_{121} + N \\
 w_1 * z_{11}^2 + w_2 * z_{12}^2 + w_3 * z_{13}^2 &\geq w_1 * z_{21}^2 + w_2 * z_{22}^2 + w_3 * z_{23}^2 - HY_{122} + N \\
 w_1 * z_{11}^3 + w_2 * z_{12}^3 + w_3 * z_{13}^3 &\geq w_1 * z_{21}^3 + w_2 * z_{22}^3 + w_3 * z_{23}^3 - HY_{123} + N \\
 w_1 * z_{11}^1 + w_2 * z_{12}^1 + w_3 * z_{13}^1 &\geq w_1 * z_{31}^1 + w_2 * z_{32}^1 + w_3 * z_{33}^1 - HY_{131} + N \\
 w_1 * z_{11}^2 + w_2 * z_{12}^2 + w_3 * z_{13}^2 &\geq w_1 * z_{31}^2 + w_2 * z_{32}^2 + w_3 * z_{33}^2 - HY_{132} + N \\
 w_1 * z_{11}^3 + w_2 * z_{12}^3 + w_3 * z_{13}^3 &\geq w_1 * z_{31}^3 + w_2 * z_{32}^3 + w_3 * z_{33}^3 - HY_{133} + N \\
 w_1 * z_{11}^1 + w_2 * z_{12}^1 + w_3 * z_{13}^1 &\geq w_1 * z_{41}^1 + w_2 * z_{42}^1 + w_3 * z_{43}^1 - HY_{141} + N \\
 w_1 * z_{11}^2 + w_2 * z_{12}^2 + w_3 * z_{13}^2 &\geq w_1 * z_{41}^2 + w_2 * z_{42}^2 + w_3 * z_{43}^2 - HY_{142} + N \\
 w_1 * z_{11}^3 + w_2 * z_{12}^3 + w_3 * z_{13}^3 &\geq w_1 * z_{41}^3 + w_2 * z_{42}^3 + w_3 * z_{43}^3 - HY_{143} + N \\
 w_1 * z_{21}^1 + w_2 * z_{22}^1 + w_3 * z_{23}^1 &\geq w_1 * z_{31}^1 + w_2 * z_{32}^1 + w_3 * z_{33}^1 - HY_{231} + N \\
 w_1 * z_{21}^2 + w_2 * z_{22}^2 + w_3 * z_{23}^2 &\geq w_1 * z_{31}^2 + w_2 * z_{32}^2 + w_3 * z_{33}^2 - HY_{232} + N \\
 w_1 * z_{21}^3 + w_2 * z_{22}^3 + w_3 * z_{23}^3 &\geq w_1 * z_{31}^3 + w_2 * z_{32}^3 + w_3 * z_{33}^3 - HY_{233} + N \\
 w_1 * z_{21}^1 + w_2 * z_{22}^1 + w_3 * z_{23}^1 &\geq w_1 * z_{41}^1 + w_2 * z_{42}^1 + w_3 * z_{43}^1 - HY_{241} + N \\
 w_1 * z_{21}^2 + w_2 * z_{22}^2 + w_3 * z_{23}^2 &\geq w_1 * z_{41}^2 + w_2 * z_{42}^2 + w_3 * z_{43}^2 - HY_{242} + N \\
 w_1 * z_{21}^3 + w_2 * z_{22}^3 + w_3 * z_{23}^3 &\geq w_1 * z_{41}^3 + w_2 * z_{42}^3 + w_3 * z_{43}^3 - HY_{243} + N \\
 w_1 * z_{31}^1 + w_2 * z_{32}^1 + w_3 * z_{33}^1 &\geq w_1 * z_{41}^1 + w_2 * z_{42}^1 + w_3 * z_{43}^1 - HY_{341} + N \\
 w_1 * z_{31}^2 + w_2 * z_{32}^2 + w_3 * z_{33}^2 &\geq w_1 * z_{41}^2 + w_2 * z_{42}^2 + w_3 * z_{43}^2 - HY_{342} + N \\
 w_1 * z_{31}^3 + w_2 * z_{32}^3 + w_3 * z_{33}^3 &\geq w_1 * z_{41}^3 + w_2 * z_{42}^3 + w_3 * z_{43}^3 - HY_{343} + N
 \end{aligned}$$

The Eq. (38) would also be defined with 18 constraints as follows;

$$\begin{aligned}
 w_1 * z_{21}^1 + w_2 * z_{22}^1 + w_3 * z_{23}^1 &\leq w_1 * z_{11}^1 + w_2 * z_{12}^1 + w_3 * z_{13}^1 + HV_{211} - N \\
 w_1 * z_{21}^2 + w_2 * z_{22}^2 + w_3 * z_{23}^2 &\leq w_1 * z_{11}^2 + w_2 * z_{12}^2 + w_3 * z_{13}^2 + HV_{212} - N \\
 w_1 * z_{21}^3 + w_2 * z_{22}^3 + w_3 * z_{23}^3 &\leq w_1 * z_{11}^3 + w_2 * z_{12}^3 + w_3 * z_{13}^3 + HV_{213} - N \\
 w_1 * z_{31}^1 + w_2 * z_{32}^1 + w_3 * z_{33}^1 &\leq w_1 * z_{11}^1 + w_2 * z_{12}^1 + w_3 * z_{13}^1 + HV_{311} - N \\
 w_1 * z_{31}^2 + w_2 * z_{32}^2 + w_3 * z_{33}^2 &\leq w_1 * z_{11}^2 + w_2 * z_{12}^2 + w_3 * z_{13}^2 + HV_{312} - N \\
 w_1 * z_{31}^3 + w_2 * z_{32}^3 + w_3 * z_{33}^3 &\leq w_1 * z_{11}^3 + w_2 * z_{12}^3 + w_3 * z_{13}^3 + HV_{313} - N \\
 w_1 * z_{31}^1 + w_2 * z_{32}^1 + w_3 * z_{33}^1 &\leq w_1 * z_{21}^1 + w_2 * z_{22}^1 + w_3 * z_{23}^1 + HV_{321} - N \\
 w_1 * z_{31}^2 + w_2 * z_{32}^2 + w_3 * z_{33}^2 &\leq w_1 * z_{21}^2 + w_2 * z_{22}^2 + w_3 * z_{23}^2 + HV_{322} - N \\
 w_1 * z_{31}^3 + w_2 * z_{32}^3 + w_3 * z_{33}^3 &\leq w_1 * z_{21}^3 + w_2 * z_{22}^3 + w_3 * z_{23}^3 + HV_{323} - N \\
 w_1 * z_{41}^1 + w_2 * z_{42}^1 + w_3 * z_{43}^1 &\leq w_1 * z_{11}^1 + w_2 * z_{12}^1 + w_3 * z_{13}^1 + HV_{411} - N \\
 w_1 * z_{41}^2 + w_2 * z_{42}^2 + w_3 * z_{43}^2 &\leq w_1 * z_{11}^2 + w_2 * z_{12}^2 + w_3 * z_{13}^2 + HV_{412} - N \\
 w_1 * z_{41}^3 + w_2 * z_{42}^3 + w_3 * z_{43}^3 &\leq w_1 * z_{11}^3 + w_2 * z_{12}^3 + w_3 * z_{13}^3 + HV_{413} - N \\
 w_1 * z_{41}^1 + w_2 * z_{42}^1 + w_3 * z_{43}^1 &\leq w_1 * z_{21}^1 + w_2 * z_{22}^1 + w_3 * z_{23}^1 + HV_{421} - N \\
 w_1 * z_{41}^2 + w_2 * z_{42}^2 + w_3 * z_{43}^2 &\leq w_1 * z_{21}^2 + w_2 * z_{22}^2 + w_3 * z_{23}^2 + HV_{422} - N \\
 w_1 * z_{41}^3 + w_2 * z_{42}^3 + w_3 * z_{43}^3 &\leq w_1 * z_{21}^3 + w_2 * z_{22}^3 + w_3 * z_{23}^3 + HV_{423} - N \\
 w_1 * z_{41}^1 + w_2 * z_{42}^1 + w_3 * z_{43}^1 &\leq w_1 * z_{31}^1 + w_2 * z_{32}^1 + w_3 * z_{33}^1 + HV_{431} - N \\
 w_1 * z_{41}^2 + w_2 * z_{42}^2 + w_3 * z_{43}^2 &\leq w_1 * z_{31}^2 + w_2 * z_{32}^2 + w_3 * z_{33}^2 + HV_{432} - N \\
 w_1 * z_{41}^3 + w_2 * z_{42}^3 + w_3 * z_{43}^3 &\leq w_1 * z_{31}^3 + w_2 * z_{32}^3 + w_3 * z_{33}^3 + HV_{433} - N
 \end{aligned}$$

So, the Eq. (39) would be defined with 3 constraints as follows;

$$\begin{aligned}
 Dev &\geq Y_{121} + Y_{131} + Y_{141} + Y_{231} + Y_{241} + Y_{341} + V_{211} + V_{311} + V_{411} + V_{321} + V_{421} + V_{431} \\
 Dev &\geq Y_{122} + Y_{132} + Y_{142} + Y_{232} + Y_{242} + Y_{342} + V_{212} + V_{312} + V_{412} + V_{322} + V_{422} + V_{432} \\
 Dev &\geq Y_{123} + Y_{133} + Y_{143} + Y_{233} + Y_{243} + Y_{343} + V_{213} + V_{313} + V_{413} + V_{323} + V_{423} + V_{433}
 \end{aligned}$$

The mixed-integer linear programming model of AMTRD is coded in the Matlab2021 environment and is presented in Appendix 1. After solving the optimization model and obtaining  $w_j$ , the final ranking would be acquired by using Eq. (36).

The proposed AMTRD method is neither dependent on any specific MCDM method nor relies on a certain number of methods. A decision-maker can consider and group different MCDM methods that are reliable on a specific problem in AMTRD. The AMTRD only needs scores of alternatives obtained by each MCDM method which they have conflicts about the alternative ranks between them, to aggregate the MCDMs. Considering and using the scores, the magnitude of the difference in scores in any MCDM method will also be included in the model. In this way, both ranking and ratings will collectively affect the final score and rankings.

### 3. Experimental Results

In this section, we used two different cases to analyze AMTRD in different circumstances. In Case 1, we consider five MCDM methods, and for each method, the criteria were weighted with the CRITIC method. To demonstrate the proposed method can be effective in any circumstances we formed a group of seven MCDMs for Case 2 and the criteria were equally weighted for all the methods in Case 2.

#### 3.1 Case 1

In Case 1, Better Life Index (BLI), a social index to compare well-being across the countries and being carried out by Organization for Economic Co-operation and Development (OECD), was used to demonstrate the capabilities of the proposed hybrid approach with aggregating five MCDMs namely ARAS, COPRAS, EDAS, MOOSRA, and WASPAS. In Case 1, the same data, which were examined by Depren & Kalkan (2018), were used. BLI data for the year 2017 with eleven main criteria and twenty-four sub-criteria were taken into consideration (OECD, 2017). Information about these criteria and also sub-criteria weights which were obtained by CRITIC were given in Table 1. In this study, these sub-criteria weights were evaluated under the main criteria due to the independence of the main criteria from each other, and the CRITIC method was used for obtaining these weights, while Depren & Kalkan (2018) considered all the sub-criteria at the same level and used entropy method for weighting them. With these two disparate perspectives and also using different weighting methods such as CRITIC and entropy, the results of criteria weights differ. For example, according to Depren & Kalkan (2018), the “Personal earnings” sub-criterion was found as the most important factor on the “Jobs” main criterion while this sub-criterion was followed by “Employment rate”, “Long-term unemployment rate” and “Labour market insecurity”, respectively. In comparison with Depren & Kalkan (2018) and our results, while the most important factor remained the same for the “Jobs” main criterion, the second most important factor was changed as “Labour market insecurity”, in our study.

**Table 1.** Sub Criteria Weights of BLI with CRITIC

Main Criteria	Main Criteria No	Sub Criteria	Sub Criteria Weight	Unit	Criteria Type
Housing	C1	Dwellings without basic facilities	0.25256	Percentage	min
		Housing expenditure	0.42091	Percentage	min
		Rooms per person	0.32652	Ratio	max
Income	C2	Household net adjusted disposable income	0.52068	US Dollar	max
		Household net financial wealth	0.47932	US Dollar	max
Jobs	C3	Labour market insecurity	0.17577	Percentage	min
		Employment rate	0.16890	Percentage	max
		Long-term unemployment rate	0.22764	Percentage	min
Community	C4	Personal earnings	0.42770	US Dollar	max
		Quality of support network	1.00000	Percentage	max
Education	C5	Educational attainment	0.36937	Percentage	max
		Student skills	0.25738	Average score	max
		Years in education	0.37325	Years	max
Environment	C6	Air pollution	0.49417	Micrograms /cubic metre	min
		Water quality	0.50583	Percentage	max
Civic engagement	C7	Stakeholder engagement for developing regulations	0.48124	Average score	max
		Voter turnout	0.51876	Percentage	max

Main Criteria	Main Criteria No	Sub Criteria	Sub Criteria Weight	Unit	Criteria Type
Health	C8	Life expectancy	0.41054	Years	max
		Self-reported health	0.58946	Percentage	max
Life Satisfaction	C9	Life satisfaction	1.00000	Average score	max
Safety	C10	Feeling safe walking alone at night	0.56169	Percentage	max
		Homicide rate	0.43831	Ratio	min
Work-Life Balance	C11	Employees working very long hours	0.53648	Percentage	min
		Time devoted to leisure and personal care	0.46352	Hours	max

After obtaining sub-criteria weights in Case 1, we used these weights and the same data with Depren & Kalkan (2018) in five different MCDMs to acquire scores of alternatives for all eleven main criteria. So, at this stage, we had a total of five different decision matrices, each with 11 criteria and 38 alternatives. These decision matrices are directly associated with the MCDM methods that were used to obtain them.

After that, using these decision matrices, main criteria weights were evaluated by using CRITIC for each related MCDM method. As expected the main criteria weights differ for each MCDM method since the evaluation process of MCDMs and its outcomes as the importance values of alternatives for main criteria differs among the MCDM methods in decision matrices. The only alternative importance values for the C4 and C9 main criteria have not changed according to each MCDM method. Due to having only one criterion as a sub-criterion, it was not necessary to make any calculations with MCDM methods and so OECD original survey results were used as alternative importance values for those criteria. Contrastingly, the importance values of alternatives for the main criteria other than C4 and C9 vary according to the MCDM approaches used.

Next, alternative final scores were obtained by each MCDM method using five different decision matrices and different criteria weights. The final rankings of the alternatives differed according to the MCDM methods.

According to the ARAS method, the countries in the top three were listed as the Netherlands, Russia, and Sweden, respectively. In the COPRAS rankings', Russia, Netherlands, and Sweden were the first three countries with the highest scores. According to EDAS rankings', the top three were the United States, Australia, and Canada. Iceland, Norway, and United Kingdom were listed as the top three countries while rankings' were made with MOOSRA. According to WASPAS rankings', Norway, Netherlands, and Sweden were the three leading countries, respectively.

Due to the differences in the results of alternative ranking obtained with the MCDM methods, there was not any common ranking or consensus among the methods. This situation, which poses a problem in decision-making, could be seen not only in the upper ranks but also in the lower ranks and overall MCDM rankings'. As it is known, although there is no superiority among MCDM methods, some methods may resemble each other more than others. Accordingly, it will be possible to overcome the problem of compromising these methods properly only by reconciling these methods and taking the methods' affinity issue into account.

Table 2 presents the normalized scores of alternatives for each MCDM method used in the optimization process of AMTRD to obtain weights of five MCDMs and the final scores and ranking with the proposed AMTRD approach. The weights of five individual MCDM in AMTRD were calculated according to their order in Table 2 as 0.00000, 0.53755, 0.45271, 0.00000, and 0.00974. So, the three MCDM namely COPRAS, EDAS, and WASPAS were adequate to cover all five MCDM and addressed the AMTRD rank in this problem.

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Depending on the scores of AMTRD, the United States, Canada, and Russia are the top three countries, respectively, for the BLI. While the United States was the best and Canada was the third for EDAS, Russia was the best for COPRAS. It seems that COPRAS and EDAS were the methods pulling the wire for AMTRD rankings', however, the AMTRD also established links with the other remaining methods via COPRAS and EDAS.

**Table 2.** Ranking Countries with Hybrid AMTRD for BLI (Case 1)

	ARAS	COPRAS	EDAS	MOOSRA	WASPAS	AMTRD
Australia	0.59734	0.52227	0.69634	0.76715	0.85289	0.60429(4)
Austria	0.46633	0.42622	0.05885	0.62707	0.66520	0.26223(21)
Belgium	0.51433	0.44537	0.29698	0.55582	0.72645	0.38093(14)
Canada	0.60660	0.54226	0.69187	0.75721	0.87328	0.61321(2)
Chile	0.14864	0.13189	0.00202	0.16510	0.23048	0.07405(37)
Czech Rep.	0.32313	0.31931	0.12586	0.40897	0.50713	0.23356(23)
Denmark	0.59260	0.54494	0.28247	0.76389	0.85193	0.42911(10)
Estonia	0.31503	0.28732	0.21735	0.35984	0.44045	0.25714(22)
Finland	0.48906	0.44424	0.29299	0.60795	0.72868	0.37854(15)
France	0.42203	0.38498	0.02009	0.49676	0.63121	0.22219(26)
Germany	0.54477	0.48736	0.15773	0.71426	0.79012	0.34108(17)
Greece	0.16962	0.18855	0.02762	0.20638	0.28895	0.11667(33)
Hungary	0.16738	0.18090	0.03669	0.20707	0.25660	0.11636(34)
Iceland	0.84244	0.58351	0.23304	1.00000	0.97091	0.42862(11)
Ireland	0.46890	0.40209	0.28109	0.56431	0.66030	0.34982(16)
Israel	0.30185	0.29721	0.22060	0.40888	0.44792	0.26400(20)
Italy	0.29474	0.29176	0.03698	0.33026	0.44098	0.17787(30)
Japan	0.43250	0.38012	0.26562	0.62355	0.51778	0.32962(18)
Korea	0.58025	0.30875	0.09216	0.46512	0.49997	0.21256(28)
Latvia	0.21258	0.20980	0.09867	0.22301	0.29009	0.16027(31)
Luxembourg	0.73999	0.49146	0.27524	0.67705	0.88233	0.39738(13)
Mexico	0.24120	0.17245	0.27349	0.21144	0.21911	0.21865(27)
Netherlands	1.00000	0.77352	0.27740	0.92919	0.99724	0.55110(5)
New Zealand	0.50919	0.45504	0.41408	0.65853	0.74625	0.43933(9)
Norway	0.82264	0.57891	0.29301	0.93433	1.00000	0.45358(8)
Poland	0.24120	0.24634	0.11963	0.30360	0.38357	0.19030129)
Portugal	0.18319	0.19539	0.02954	0.25636	0.28247	0.12116(32)
Slovak Rep.	0.25922	0.26098	0.19052	0.30120	0.40333	0.23047(24)
Slovenia	0.37037	0.35216	0.20659	0.46306	0.55809	0.28827(19)
Spain	0.37551	0.34023	0.07677	0.44454	0.56330	0.22313(25)
Sweden	0.86184	0.61733	0.45088	0.78653	0.99458	0.54565(7)
Switzerland	0.83133	0.55118	0.54173	0.78636	0.97193	0.55100(6)
Turkey	0.12706	0.12936	0.02241	0.15579	0.20709	0.08170(36)
United King.	0.63882	0.56885	0.23512	0.93236	0.82157	0.42023(12)
United States	0.68253	0.59228	1.00000	0.80599	0.89962	0.77985(1)
Brazil	0.14587	0.15131	0.00871	0.18589	0.20164	0.08724(35)
Russia	0.89040	1.00000	0.16019	0.90347	0.31252	0.61311(3)
South Africa	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000(38)



**3.2. Case 2**

In this study, we borrowed a material selection case from Shanian and Savadogo (2006). The decision matrix of the problem with eight alternatives and twelve criteria in which the first, fourth, fifth, and twelfth are the cost and the rest are beneficial criteria are presented in Table 3.

**Table 3.** Decision Matrix of Case 2

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
A1	8.25	560	940	0.78	15183	2916	380	560	138	465	105	18.64
A2	8.65	460	600	0.71	12472	2395	220	460	125	465	205	13.99
A3	8.94	50	210	0.08	1355	260	45	50	122	460	398	3
A4	8.95	340	380	0.48	9218	177	115	340	135	460	390	3.46
A5	2.67	190	295	0.25	20317	1966	87	191	73.59	741	152	2.81
A6	8.06	690	1030	1.55	5909	2174	350	800	190	189	17	5.99
A7	8.63	95	270	0.17	2711	520	63	100	116	174	185	3.32
A8	7.08	267	355	0.48	1957	720	110	265	205	329	50	1.04

Source: Shanian and Savadogo (2006).

As mentioned before, all criteria weights in Case 2 considered in each MCDM as equal with the value of 0.08333. First, we obtained alternatives' ranking for seven MCDMs, namely ARAS, COPRAS, EDAS, MOOSRA, WASPAS, WSM, and WPM. The Eqs. (33) and (34) present the calculations of the WSM and WPM which are basic ranking methods, respectively. Table 4 represents the scores and ranks of alternatives for each MCDM and also AMTRD. The weights of seven individual MCDM in AMTRD were obtained according to their order in Table 4 as 0.24290, 0.06798, 0, 0.37440, 0, 0, and 0.31473. So, the three MCDM namely ARAS, COPRAS, MOOSRA, and WPM were adequate to cover all seven MCDM and addressed the AMTRD rank in this problem. However, using AMTRD we found out the best alternative is A8 and second best is A6 while also it was considered as the top according to ARAS, COPRAS, EDAS, and WSM which two of them were not included in the final phase of AMTRD.

**Table 4.** Ranking Alternatives with MCDMs and Hybrid AMTRD for Case 2

	ARAS	COPRAS	EDAS	MOOSRA	WASPAS	WSM	WPM	AMTRD
SCORES of ALTERNATIVES								
A1	0.50271	97.03645	0.46362	1.80552	0.46416	0.54666	0.38165	0.59815
A2	0.42847	84.69563	0.42089	1.74793	0.41581	0.46510	0.36653	0.41990
A3	0.47157	84.63413	0.31088	2.67523	0.37096	0.45064	0.29127	0.59805
A4	0.41977	86.87727	0.57101	2.45927	0.41881	0.44734	0.39028	0.73403
A5	0.41959	72.21532	0.29427	1.99619	0.39069	0.43345	0.34793	0.41980
A6	0.51769	100.00000	0.60762	2.28269	0.45815	0.55642	0.35988	0.73414
A7	0.31515	69.23272	0.17624	1.80032	0.29286	0.31062	0.27509	0.01877
A8	0.44897	84.07422	0.39773	2.79296	0.41257	0.45042	0.37472	0.83989
RANK of ALTERNATIVES								
A1	2	2	3	6	1	2	2	4
A2	5	4	4	8	4	3	4	6
A3	3	5	6	2	7	4	7	5
A4	6	3	2	3	3	6	1	3
A5	7	7	7	5	6	7	6	7
A6	1	1	1	4	2	1	5	2
A7	8	8	8	7	8	8	8	8
A8	4	6	5	1	5	5	3	1

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Since the results of weighting with the proposed approach reveal that some methods can represent others with greater importance in the proposed AMTRD approach, it can be expected that the goal of the AMTRD which is the minimization of the maximum rank deviation from the MCDM methods achieved. With this regard, in order to demonstrate the effectiveness of the proposed method, the correlation between the methods in terms of ranking results was analyzed and also compared with different well-known methods and finally, the proposed approach was analyzed in terms of its validity.

#### 4. Comparative Analysis

To analyze and compare the effectiveness of the AMTRD regarding to the minimax total rank deviation strategy, the CRITIC method which favors the uncorrelated criteria in an MCDM problem was chosen. Additionally, with extracting the standard deviations from Eq. (5) the greatly enhanced version of CRITIC for focusing on correlation coefficients (CC) also compared with AMTRD. The results of the Equal Mean (EM) approach, which is the basic method in criteria weighting also represented to draw a wider perspective. All these models were formed using final scores which were obtained by five different MCDM methods as a decision matrix for Case 1. Appendix 2 presents the results of each weighting method and the rankings for the BLI problem with them obtained by using WSM to make fair comparisons. According to the results, both CC and CRITIC weigh the EDAS method most. However, AMTRD differs from them with prioritizing COPRAS as the highest of all MCDM methods. Apart from this; it is observed that the obtained results with CC were much closer to CRITIC than AMTRD's. So, AMTRD generally parted from them in the weighting scheme. While the United States was the leading alternative for the BLI in each approach including EM, the runner-up alternatives for BLI using AMTRD, CC, and CRITIC were Canada, Switzerland, and the Netherlands, respectively. EM approach and CRITIC ranked the same alternatives at the top four with a different order which can be seen as a sign of closer ranking results for them.

To compare the aggregating methods for Case 2, we used scores of single MCDMs in Table 4, we obtained weights of the seven method with the order in Table 4 and for CC as; 0.11148, 0.10853, 0.11246, 0.31352, 0.09357, 0.10049, 0.15995 and for CRITIC as; 0.09760, 0.10560, 0.10627, 0.35267, 0.08368, 0.08744, 0.16673. While CC and CRITIC have significantly close weight values they also ranked the alternatives identically with the AMTRD, the only exception is the ranking between A6 and A8 were reversed. AMTRD which differs a lot from both CC and CRITIC in terms of weightings revealed similar rankings with them. The EM presents the rank of alternatives as 2-5-6-3-7-1-8-4 which differs from all of them.

To identify the total rank deviation among both rankings with weighting methods and single MCDM rankings more approximately, the Total Rank Deviation (TRD) was defined as follows;

Suppose that, there is  $n$  number of alternatives that have to be ranked. While also there are two different rankings available as  $R = \|r_i\|_{(n)}$  and  $T = \|t_i\|_{(n)}$  then TRD between them would be;

$$TRD_{RT} = \sum_{i=1}^n |r_i - t_i| \quad (44)$$

Using Eq. (43) in Tables 5 and 6, the TRD values between individual MCDMs and each aggregated rankings are presented for Case 1 and 2, respectively.

**Table 5.** TRD values between individual MCDMs and Aggregated rankings' for Case 1

	CC	CRITIC	EM	AMTRD
ARAS	82	72	58	122
COPRAS	84	74	52	110
EDAS	170	178	192	132
MOOSRA	90	86	70	124
WASPAS	102	88	76	146
<b>Max TRD</b>	170	178	192	<b>146</b>

**Table 6.** TRD values between individual MCDMs and Aggregated rankings' for Case 2

	CC	CRITIC	EM	AMTRD
ARAS	10	10	6	12
COPRAS	8	8	4	10
EDAS	8	8	4	10
MOOSRA	14	14	20	12
WASPAS	12	12	6	12
WSM	12	12	8	14
WPM	14	14	10	14
<b>Max TRD</b>	<b>14</b>	<b>14</b>	20	<b>14</b>

According to Max TRD values in Table 5 and 6, for Case 1 the AMTRD has the minimum Max TRD with 146, while CC, CRITIC, and EM follow it with 170, 178, and 192, respectively. In Case 2 the three approaches AMTRD, CC, and CRITIC outperform EM in terms of Max TRD. The results indicated that the AMTRD is the better option for obtaining minimum Max TRD which is the main purpose of the AMTRD.

### 5. Further analysis

To analyze further effectiveness of AMTRD and also to demonstrate that it is not dependent on specific MCDM methods, Case 1 and 2 were investigated using different groups of MCDMs. In Table 7 the Max TRD between individual MCDMs and Aggregated rankings' were presented for Case 1. We sequentially discarded an MCDM from the original model for Case 1. So, this approach provides five different models with four MCDMs in each for Case 1. Using all these models with different weighting schemes we acquired final rankings via WSM, and then we calculated the TRD and Max TRD for each model to reveal which weighting schemes provide better results in line with the minimax strategy. While AMTRD outperformed the other weighting methods in all models, the runner-up was CC for four models and CRITIC only surpassed CC only in one model. EM could not compete with any of them concerning minimax strategy.

**Table 7.** Max TRD values between group of MCDMs and Aggregated rankings' for Case 1

The groups of MCDMs in the model	CC	CRITIC	EM	AMTRD
COPRAS, EDAS, MOOSRA, WASPAS	148	154	170	<b>124</b>
ARAS, EDAS, MOOSRA, WASPAS	154	174	186	<b>136</b>
ARAS, COPRAS, MOOSRA, WASPAS	70	62	78	<b>58</b>
ARAS, COPRAS, EDAS, WASPAS	168	170	180	<b>130</b>
ARAS, COPRAS, EDAS, MOOSRA	148	150	194	<b>128</b>

Considering different models for Case 2 we focused on different combinations of the MCDMs and obtained seven models and in Table 8 the Max TRD values between individual MCDMs and Aggregated rankings were listed for Case 2. In terms of minimax strategy, AMTRD and EM outperformed the other two models, while the proposed method slightly has a better mean of Max TRD than EM. But more importantly, AMTRD had the minimum Max TRD values in all models. Additionally, an important issue to be considered was the EM had better results than both CC and CRITIC in line with the minimax strategy for the first time. This consequence falsified our foresight that CC is the superior version of CRITIC for minimax strategy. Since CC even failed to dominate over the EM, it cannot be a valid option anymore. Similar to all other models, AMTRD has maintained its best performance in terms of Max TRD values in these models as well.

**Table 8.** Max TRD values between group of MCDMs and Aggregated rankings' for Case 2

The groups of MCDMs in the model	CC	CRITIC	EM	AMTRD
ARAS, WSM, WPM	12	12	12	<b>12</b>
COPRAS, WSM, WPM	12	12	10	<b>10</b>
EDAS, WSM, WPM	8	8	8	<b>8</b>
MOOSRA, WSM, WPM	18	18	14	<b>14</b>
WASPAS, WSM, WPM	12	12	10	<b>10</b>
ARAS, WASPAS, WSM, WPM	12	12	12	<b>10</b>
ARAS, COPRAS, WASPAS, WSM, WPM	12	12	10	<b>10</b>

Considering the inferences that can be made from all the results, with the AMTRD method, the best result was always obtained according to Max TRD. While the efficiency of AMTRD is more evident when the number of alternatives in the problem is higher, the approach also maintains its performance in ranking fewer alternatives. Meanwhile, we cannot have a conclusion on which the other compared methods have an edge over others.

As mentioned earlier in this study, the AMTRD approach should consider the individual MCDM rankings' that constitute it without any difference between them and the similarity rate in the rankings should be as high as possible in all of them. In this context, it is desirable to minimize the Max TRD between individual MCDMs and this would be a minimax strategy to perform. So, AMTRD outperformed CC, CRITIC, and EM in the context of minimax strategy in all cases and models. So, these results indicate that removing any MCDM did not crucially affect the performance of the AMTRD and also reveal the robustness of the proposed method.

## 5. Results and Discussions

This paper aimed to develop an aggregating method that has its original point of view and appropriately aggregates the MCDMs in line with its assumptions. Some aggregation methods have been proposed in the past. However, they mostly relied on the thought that MCDMs which have similar rankings should be more important than the others in the aggregated rankings. Contrary to this idea, we proposed a new approach that does not underestimate any MCDM method, assuming that there is no superiority between MCDM methods, which is one of the important assumptions in decision theory, in the evaluation process. To prevent positive bias towards the methods with similar rankings, the basis of the proposed AMTRD method is structured to minimize the Max TRD of final rankings from any rankings of MCDM methods. Owing to this approach, overall, the importance of the methods remains relatively the same. Because the methods with similar rankings have interaction and so their compound significance highly affects the final rankings. However, the methods with different rankings offset this high effect with their higher individual weights that were acquired by AMTRD. In the AMTRD approach, the decision-maker can select a group of MCDM methods that he/she considers reliable and uses their scores to obtain aggregated rankings in line with the minimax strategy.

The two cases were used in the study to demonstrate the effectiveness of the proposed approach in both MCDM problems with a big and small number of alternatives. Different models were handled with AMTRD and as indicated by the overall results the AMTRD aggregated groups of MCDM methods reliably in line with the minimax strategy, whether the number of methods in the group is small or big, or which methods compromise the group. While the proposed approach had significantly better results from any weighting scheme in Case 1 with 38 alternatives, the AMTRD maintained its assignments even the number of alternatives decreased to 8 as in Case 2. Furthermore, even when the number of MCDMs used in AMTRD was reduced from five to four in Case 1, it can produce similar ranking results with the model that has five MCDM ratings and these outcomes display the robustness of the AMTRD. In addition, the AMTRD outperformed the other well-known approaches in minimizing the Max TRD of aggregated rankings between any individual MCDM methods. Besides, a balance, rather than a net correlation, was established between the correlation of AMTRD and MCDM ranking results and the weight of MCDMs in the AMTRD. So there is no such guarantee that if the weight of the MCDM is high or low then its rank correlation with AMTRD also would be high or low. These overall results display that the desired reconciliation with AMTRD is achieved in the aggregation of MCDM methods. In future studies, there are sufficient opportunities for reconciling different MCDMs with different weighting approaches in the decision-making process.

**Author Contributions:** Research problem, O.P.; Methodology, O.P.; Formal Analysis, O.P.; Resources, O.P.; Writing – Original Draft Preparation, O.P.; Writing – Review & Editing, O.P.

**Funding:** This research received no external funding.

**Conflicts of Interest:** The authors declare that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix 1. MATLAB Script of AMTRD Optimization Model.

```

clc; clear;
%%%% AMTRD Method %%%
%R decision matrix(nxm) (n=number of alternatives, m=number of MCDMs)
%Example CASE 2 with seven MCDMs
R=[0.50271 97.03645 0.46362 1.80552 0.46416 0.54666 0.38165
0.42847 84.69563 0.42089 1.74793 0.41581 0.46510 0.36653
0.47157 84.63413 0.31088 2.67523 0.37096 0.45064 0.29127
0.41977 86.87727 0.57101 2.45927 0.41881 0.44734 0.39028
0.41959 72.21532 0.29427 1.99619 0.39069 0.43345 0.34793
0.51769 100.00000 0.60762 2.28269 0.45815 0.55642 0.35988
0.31515 69.23272 0.17624 1.80032 0.29286 0.31062 0.27509
0.44897 84.07422 0.39773 2.79296 0.41257 0.45042 0.37472];
%R(:,[3,4])=[]; You can extract any MCDMs.
%But also do not forget make necessary changes in constraint functions
n=size(R,1); %number of alternatives
m=size(R,2); %number of criteria
% Min-Max Normalization [0,1]
for i=1:n
for j=1:m
R1(i,j)= (R(i,j)-min(R(:,j)))/(max(R(:,j))-min(R(:,j)));
end
end
RN=R1;
% Standard Deviation

```



```

for j =1:m
    b=0;
    for i=1:n
        b=b+(RN(i,j)-mean(RN(:,j)))^2;
    end
sigma(j,1)=sqrt((1/(n))*b);
end
% Correlation Coefficients
for j=1:m
for k=1:m
    aw=0;
    bw=0;
    cw=0;
    for i=1:n
        aw=aw+(RN(i,j)-mean(RN(:,j)))*(RN(i,k)-mean(RN(:,k)));
        bw=bw+(RN(i,j)-mean(RN(:,j)))^2;
        cw=cw+(RN(i,k)-mean(RN(:,k)))^2;
    end
RNC(j,k) = aw/(sqrt(bw*cw));
end
eksiRNC=(1-RNC).^(1/1);
Cj(j)=sum(eksiRNC(j,:));
CRj(j)=sigma(j)*sum(eksiRNC(j,:));
end
for j=1:m
    cc(j)=Cj(j)/sum(Cj); %CC
    CR(j)=CRj(j)/sum(CRj); %CRITIC
end
%Sorting rows for each MCDMs scores
RN1=sortrows(RN,1,'descend');
RN2=sortrows(RN,2,'descend');
RN3=sortrows(RN,3,'descend');
RN4=sortrows(RN,4,'descend');
RN5=sortrows(RN,5,'descend');
RN6=sortrows(RN,6,'descend');
RN7=sortrows(RN,7,'descend');
H=1000; % A sufficient large number
N=0.0001; % A sufficient small number
% Variable Definitions
wprob = optimproblem;
% weights
wler = optimvar('wler',m,'LowerBound',0,'UpperBound',1);
% Variables Y
y1 = optimvar('y1',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
y2 = optimvar('y2',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
y3 = optimvar('y3',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
y4 = optimvar('y4',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
y5 = optimvar('y5',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
y6 = optimvar('y6',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
y7 = optimvar('y7',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
% Variables YY
yy1 = optimvar('yy1',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
yy2 = optimvar('yy2',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
yy3 = optimvar('yy3',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
yy4 = optimvar('yy4',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
yy5 = optimvar('yy5',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
yy6 = optimvar('yy6',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
yy7 = optimvar('yy7',(n-1)*n/2,'Type','integer','LowerBound',0,'UpperBound',1);
% Objective Variable
zler= optimvar('zler','LowerBound',0);
% Constraint Predefinitions
A1 = optimconstr((n-1)*n/2);
A2 = optimconstr((n-1)*n/2);
A3 = optimconstr((n-1)*n/2);
A4 = optimconstr((n-1)*n/2);
A5 = optimconstr((n-1)*n/2);
A6 = optimconstr((n-1)*n/2);
A7 = optimconstr((n-1)*n/2);
C1 = optimconstr((n-1)*n/2);
C2 = optimconstr((n-1)*n/2);
C3 = optimconstr((n-1)*n/2);
C4 = optimconstr((n-1)*n/2);
C5 = optimconstr((n-1)*n/2);

```

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```

C6 = optimconstr((n-1)*n/2);
C7 = optimconstr((n-1)*n/2);
B1 = optimconstr(1);
B2 = optimconstr(1);
B3 = optimconstr(1);
B4 = optimconstr(1);
B5 = optimconstr(1);
B6 = optimconstr(1);
B7 = optimconstr(1);
A11 = optimconstr(1);
% Creating Constraint Functions
iter=0;
itera=0;
for i=1:n
    for j=1:n
        if i<j
            iter=iter+1;
            A1(iter)=(RN1(i,:)*wler)>=(RN1(j,:)*wler)-H*y1(iter)+N;
            A2(iter)=(RN2(i,:)*wler)>=(RN2(j,:)*wler)-H*y2(iter)+N;
            A3(iter)=(RN3(i,:)*wler)>=(RN3(j,:)*wler)-H*y3(iter)+N;
            A4(iter)=(RN4(i,:)*wler)>=(RN4(j,:)*wler)-H*y4(iter)+N;
            A5(iter)=(RN5(i,:)*wler)>=(RN5(j,:)*wler)-H*y5(iter)+N;
            A6(iter)=(RN6(i,:)*wler)>=(RN6(j,:)*wler)-H*y6(iter)+N;
            A7(iter)=(RN7(i,:)*wler)>=(RN7(j,:)*wler)-H*y7(iter)+N;
        elseif i>j
            itera=itera+1;
            C1(itera)=(RN1(i,:)*wler)<=(RN1(j,:)*wler)+H*yy1(itera)-N;
            C2(itera)=(RN2(i,:)*wler)<=(RN2(j,:)*wler)+H*yy2(itera)-N;
            C3(itera)=(RN3(i,:)*wler)<=(RN3(j,:)*wler)+H*yy3(itera)-N;
            C4(itera)=(RN4(i,:)*wler)<=(RN4(j,:)*wler)+H*yy4(itera)-N;
            C5(itera)=(RN5(i,:)*wler)<=(RN5(j,:)*wler)+H*yy5(itera)-N;
            C6(itera)=(RN6(i,:)*wler)<=(RN6(j,:)*wler)+H*yy6(itera)-N;
            C7(itera)=(RN7(i,:)*wler)<=(RN7(j,:)*wler)+H*yy7(itera)-N;
        end
    end
end
B1(1)=zler(1)>=sum(y1)+sum(yy1);
B2(1)=zler(1)>=sum(y2)+sum(yy2);
B3(1)=zler(1)>=sum(y3)+sum(yy3);
B4(1)=zler(1)>=sum(y4)+sum(yy4);
B5(1)=zler(1)>=sum(y5)+sum(yy5);
B6(1)=zler(1)>=sum(y6)+sum(yy6);
B7(1)=zler(1)>=sum(y7)+sum(yy7);
A11(1)=sum(wler)==1;
% Constraint Definitions
wprob.Constraints.A1 = A1;
wprob.Constraints.A2 = A2;
wprob.Constraints.A3 = A3;
wprob.Constraints.A4 = A4;
wprob.Constraints.A5 = A5;
wprob.Constraints.A6 = A6;
wprob.Constraints.A7 = A7;
wprob.Constraints.C1 = C1;
wprob.Constraints.C2 = C2;
wprob.Constraints.C3 = C3;
wprob.Constraints.C4 = C4;
wprob.Constraints.C5 = C5;
wprob.Constraints.C6 = C6;
wprob.Constraints.C7 = C7;
wprob.Constraints.B1 = B1;
wprob.Constraints.B2 = B2;
wprob.Constraints.B3 = B3;
wprob.Constraints.B4 = B4;
wprob.Constraints.B5 = B5;
wprob.Constraints.B6 = B6;
wprob.Constraints.B7 = B7;
wprob.Constraints.A11 = A11;
% Objective Function Definition
wprob.Objective=zler;
% Solving mixed integer linear programming model by branch and bound algorithm
opts = optimoptions('intlinprog','MaxNodes',100000);
[so1,fval]=solve(wprob,'options',opts);

```

**Appendix 2.** Comparison between different weighting schemes for Case 1

MCDM Weights	CC		CRITIC		EM	
	Weights	Rank	Weights	Rank	Weights	Rank
ARAS	0.14867		0.15838		0.20000	
COPRAS	0.18256		0.15295		0.20000	
EDAS	0.35370		0.32373		0.20000	
MOOSRA	0.13523		0.15328		0.20000	
WASPAS	0.17985		0.21166		0.20000	
	Final Scores	Rank	Final Scores	Rank	Final Scores	Rank
Australia	0.68758	5	0.69803	6	0.68720	8
Austria	0.37239	19	0.39501	19	0.44873	18
Belgium	0.46863	15	0.48468	16	0.50779	16
Canada	0.69335	3	0.70389	5	0.69424	7
Chile	0.11067	36	0.11846	36	0.13562	36
Czech Republic	0.29736	26	0.31079	26	0.33688	24
Denmark	0.54402	12	0.56606	11	0.60717	12
Estonia	0.30404	25	0.31258	25	0.32400	26
Finland	0.47071	14	0.48767	15	0.51259	15
France	0.32083	22	0.34197	22	0.39101	20
Germany	0.46444	16	0.48860	14	0.53885	14
Greece	0.14928	33	0.15744	33	0.17622	33
Hungary	0.14504	34	0.15211	34	0.16973	34
Iceland	0.62404	8	0.65690	8	0.72598	5
Ireland	0.43761	17	0.45302	17	0.47534	17
Israel	0.31301	23	0.32216	24	0.33529	25
Italy	0.23413	28	0.24724	28	0.27894	28
Japan	0.40509	18	0.41780	18	0.44392	19
Korea	0.32805	21	0.34607	21	0.38925	22
Latvia	0.18713	31	0.19328	31	0.20683	31
Luxembourg	0.54733	11	0.57200	10	0.61321	11
Mexico	0.23208	30	0.23190	30	0.22354	30
Netherlands	0.69301	4	0.71999	2	0.79547	2
New Zealand	0.52850	13	0.54319	13	0.55662	13
Norway	0.63783	7	0.66856	7	0.72578	6
Poland	0.23318	29	0.24233	29	0.25887	29
Portugal	0.15882	32	0.16754	32	0.18939	32
Slovak Republic	0.26684	27	0.27419	27	0.28305	27
Slovenia	0.35542	20	0.36851	20	0.39006	21
Spain	0.30652	24	0.32373	23	0.36007	23
Sweden	0.68554	6	0.70795	4	0.74223	3
Switzerland	0.69697	2	0.71760	3	0.73651	4
Turkey	0.10874	37	0.11488	37	0.12834	37
United Kingdom	0.55583	9	0.58110	9	0.63934	10
United States	0.83409	1	0.83637	1	0.79608	1
Brazil	0.11379	35	0.12024	35	0.13868	35
Russia	0.54998	10	0.55046	12	0.65332	9
South Africa	0.00000	38	0.00000	38	0.00000	38



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