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METHODS FOR ASSIGNING WEIGHTS TO DECISION MAKERS IN GROUP AHP DECISION-MAKING

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Abstract: *The method known as the analytical hierarchy process (AHP), a theoretical and methodological concept of multi-criteria analysis, is increasingly used in solving various decision-making problems. AHP is an excellent support to both the individual and group decision-making process, however, the involvement of a greater number of decision makers complicates the process and requires a different approach to when an individual decides alone. The synthesis of individual decisions within a group can be done in various ways, but the problem is how to deal with different levels of consistency when there are a number of decision makers. Thus, this paper presents some of the methods for defining the individual weights of decision makers in group AHP decision making.*

Key words*: weights; decision makers; analytical hierarchy process; group decision making.*

1. Introduction

Decision making is as old as humanity itself. People have always made decisions (without even being aware of it), since decision making is, in fact, an integral part of everyday life. However, as life has become increasingly complex over time, it has also become necessary to master new knowledge in order to make the right decisions.

In order to support the work of individual or group decision makers with complex sets of diverse information that cross over at the psychological, technical and other levels during the decision-making process, various mathematical and computer tools have been developed to support the decision-making process. One of these tools is AHP.

Considering that the AHP is based on individual (subjective) opinion of a decision maker (DM) about decision-making issue, it is always better to make a decision in

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group context, as this reduces the risk of wrong assessment, as the problem is approached from different perspectives based on different knowledge and experience of decision makers, and finally, the decision made has greater legitimacy to be realized.

The objective of the research presented in this paper is to indicate the difference in significance of individual decision makers in group AHP synthesis. Basic assumption is founded on the attitude that individuality brings participants' subjectivity (education, knowledge, concentration, desire, etc.) into decision-making process, so a quality methodological procedure is necessary that would objectivise final (group) decision. Knowing the possibilities to define weights of decision makers directly contributes to the transparency of group decision.

The paper with introduction and conclusion consists of four parts. In the second part of the paper titled Analytical hierarchical process, mathematical basis of the AHP is presented. In the third section of the paper, a case study is used to present some of the methods for assigning individual weights to decision makers in group AHP. In the Conclusion - the fourth section of this paper, are pointed out key contributions of the conducted research and the directions for future research.

2. Analytical Hierarchy Process (AHP)

The Analytical Hierarchical Process (Saaty, 1980) is a method of multi-criteria analysis that is widely used in the world to support individual and group decision making (Eskobar et al., 2004; Vaidya & Kumar, 2006; Altuzarra et al., 2007; Ho, 2008; Arnette et al., 2010; Subramanian & Ramanathan, 2012; Bernasconi et al., 2014). The method is both "analytic" and "hierarchical" because a decision maker decomposes complex problem of decision-making into several decision-making elements between which he establishes hierarchy relation. The word "process" in the name of the method suggests that after the formation of the initial hierarchy of a decision making issue are allowed its iterative modifications (Saaty, 1999). The hierarchy of the decision making issue has several levels, with the goal at the top of the hierarchy; the following level contains the criteria, while the alternatives are at the bottom. Such hierarchical setting refers to standard decision-making problem, but there are also cases where the hierarchy has four and more levels, respectively, when there are sub-criteria between criteria and alternatives. Also, there are decision making issues in which the hierarchy has two levels, and then only alternatives are below the goal.

After setting the hierarchy, the decision maker compares pairs of elements at a given level of hierarchy with respect to all the elements at the higher level (superiors), in order to determine their mutual importance. In standard AHP, the elements are compared by providing linguistic (semantic) evaluations of mutual importance in relation to the elements at the higher level of the hierarchy using basic scale in the Table 1 (Saaty, 1980).

In addition to Saaty's scale, other scales can also be used, such as Lootsma's (Lootsma, 1988; Lootsma, 1990; Lootsma et al., 1990), Ma & Zheng's (Ma & Zheng, 1991), balanced, etc., but the Saaty 's scale is used mostly. Linear part of the Saaty's scale consists of integers [1,9], and non-linear part of its reciprocal values [1,1/9].

When a DM at the given level of hierarchy evaluates n elements of the decision-making process as compared to the superior element according to the scale shown in the Table 1, its semantic ratings according to the definitions in the left column are expressed as numerical values from the right column and recorded in a square matrix A.

Table 1. Saaty 's relative importance scale

Definition	Numerical value
Absolute dominance of the element i over the element j	9
Very strong dominance of the element i over the element j	7
Strong dominance of the element i over the element j	5
Weak dominance of the element i over the element j	3
The same importance of the elements i and j	1
Weak dominance of the element j over the element i	1/3
Strong dominance of the element j over the element i	1/5
Very strong dominance of the element j over the element i	1/7
Absolute dominance of the element j over the element i	1/9
(Intervalues)	(2,4,6,8)

The matrix is positive and reciprocal (symmetrical in relation to the main diagonal). In other words, the elements from the top of triangle of the matrix are reciprocal to the elements from the bottom of triangle, and the elements on the main diagonal are equal to 1 ($a_{ij} = 1/a_{ij}$, for every i and j; $a_{ii} = 1$ for every i), as shown in the relation 1.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \quad (1)$$

If using standard Saaty 's scale, then every a_{ij} can have one of 17 values from a discrete interval [1/9,9]. Determining weights of the compared elements based on numerical values of the matrix A is called prioritization. Prioritization shows a process of determining of priority vectors $w = (w_1, \dots, w_n)^T$ from the matrix A, where every $w_i > 0$ and it's true $\sum_{i=1}^{n} w_i = 1$. There are several matrix and optimization methods of prioritization (Table 2), but the most commonly used methods are eigenvalue method, logarithmic least square method and the method of additive normalization (Blagojević, 2015).

Table 2. Prioritization methods and their authors

Prioritization methods	Authors of the method
Eigenvector method – EV	Saaty (1980)
Additive normalization method – AN	Saaty (1980)
Weighted least squares method – WLS	Chu et al. (1979)
Logarithmic least squares method – LLS	Crawford & Williams (1985)
Logarithmic goal programming method – LGP	Bryson (1995)
Fuzzy preference programming method – FPP	Mikhailov (2000)

Due to its simplicity and frequent use, as the example shown in this paper is used the method of additive normalization (AN). To obtain a priority vector w it is enough to divide each element from the given column of the matrix A with the sum of the elements of this column (normalization), then to sum up the elements in each row and finally to divide each resulting sum with the rank of the matrix A. This procedure is described by the relations 2 and 3:

$$a_{ij}' = \sum_{i=1}^{n} a_{ij}, ij = 1, 2, \ldots, n \tag{2}$$

$$w_i = \frac{\sum_{j=1}^{n} a_{ij}'}{n}, i = 1, 2, \ldots, n \tag{3}$$

Based on the evaluation, by selected prioritization method are determined local weights of decision-making elements, and by synthesis, that is, additive synthesis, at the end are determined weights of alternatives at the lowest level in relation to the element at the highest level (goal), thus completing individual deciding using the AHP. The additive synthesis is presented with the relation 4:

$$u_i = \sum_j w_j d_{ij} \tag{4}$$

where in:

- u_i – final (global) priority of the alternative i;
- w_j – weight of the criterion j;
- d_{ij} – local weight of the alternative i in relation to the criterion j;

In addition to the prioritization method, one of essential characteristics of the AHP is that at all levels of the hierarchy consistency of the decision makers' evaluation is checked. For testing consistency, Saaty (1977) proposed consistency ratio (CR) used in the AN prioritization method. Calculating the consistency ratio consists of two steps. In the first step, the consistency index (CI) is calculated using the relation 5:

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{5}$$

where in:

- n – the rank of the matrix;
- λ_{max} – the maximum eigenvalue of the comparison matrix;

In the second step, the consistency ratio (CR) is calculated as the relationship of the consistency index (CI) and the random index (RI):

$$CR = \frac{CI}{RI} \tag{6}$$

The random index (RI) depends on the rank of the matrix and its values are obtained in random generation of 500 matrices (Table 3).

Table 3. Random index values depending on the matrix rank

n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

If the consistency ratio (CR) is lower or equal to 0, 10 the result indicates that the decision maker was consistent and there is no need for the re-evaluation (Jandrić Srđević, 2000). If the consistency ratio (CR) is higher than 0.10, the decision maker should repeat (or modify) his evaluation in order to improve consistency.

Important feature of the AHP is sensitivity analysis of the final solution. The sensitivity analysis is carried out in order to see the extent to which the changes in the input data reflect the changes in the obtained results (Nikolić & Borović, 1996). In order to conclude whether the ranking list of the alternatives is sufficiently stable in relation to acceptable changes in input data, it is recommended to check the priority of alternatives for different combinations of input data. This analysis is very easily performed using software packages (softwares) to support decision making. One of the most commonly used is Expert Choice, which offers five sensitivity analysis options: Dynamic, Performance, Gradient, Head to head and 2D . The analysis can be done based on the goal or any other element in the hierarchy. Sensitivity analysis based on the goal node shows the sensitivity of alternatives to all elements in the hierarchical tree structure.

The stability of the results is performed using dynamic sensitivity analysis (option Dynamic). If the rank of the alternatives remains unchanged when alternating the importance of the main criteria by 5% in all combinations, the result is considered to be stable (Hot, 2014) . The AHP algorithm implementation is shown in the Figure 1.

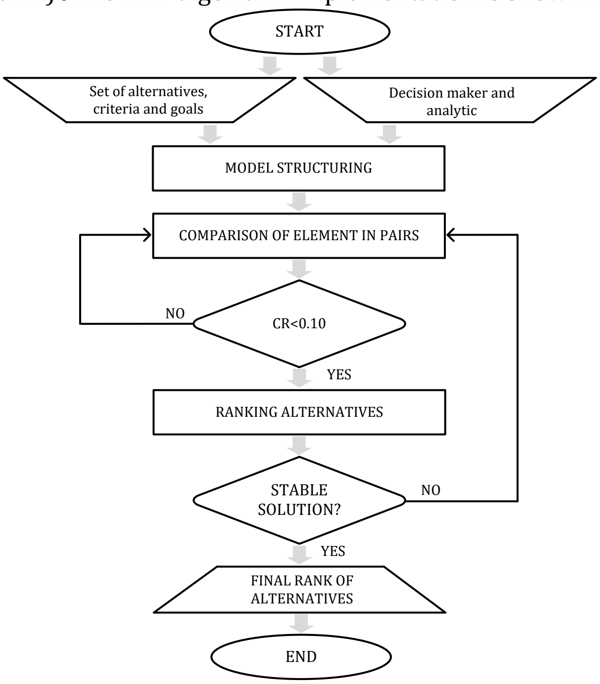

Figure 1. The AHP algorithm (Hot, 2014)

In the AHP there are several ways to consolidate individual decisions in group equivalents (Blagojević, 2015):

- Aggregation of Individual Priorities - AIP;
- Aggregation of Individual Judgments – AIJ;
- Consensus Model Convergence – CCM;
- Geometric Cardinal Consensus model – GCCM;

However, the synthesis of individual results of the AHP application and making group decision requires prior determination of individual weights of decision makers. This is a specific problem, which is especially difficult if there is no institutional framework defining this issue. Therefore, in this paper several possibilities of determining the criteria for defining weights of individual members of the group are presented.

3. Possibilities of defining individual weights of group members

According to the described methodology for the implementation of the AHP, it is discussed the hierarchy of decision making problems taken from (Lukovac, 2016), Figure 2, which consists of three levels.

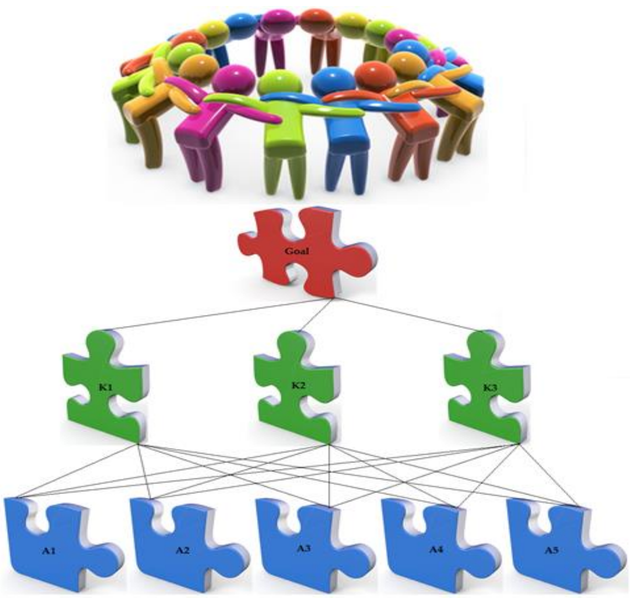

Figure 2. Hierarchy of decision making problems (Lukovac, 2016)

The goal is to "rank" the persons who can be included as assessors in the process of assessing the performance of drivers, and it is at the top of the hierarchy.

The ranking criteria are at the following – intermediate level, which include:

- Knowledge of the work to be evaluated (K1);
- The best possible insight into the work to be evaluated (K2);
- Objectivity, impartiality, in the evaluation process (K3);

Alternatives (participants in evaluating assessors) represent the subjects of ranking and they are at the lowest level of the hierarchy, which are:

- Superior (A1);

– Dispatcher (A2);
– Colleague (A3);
– Client (A4);
– Self-assessment (A5);

The expert group consisted of twenty decision makers (DMs) to which the assessment of the performances of direct executors - the drivers, was one of the obligations arising from the functional duty they performed. The decision makers compared the elements presented in a hierarchy in pairs using Expert Choice 2000 software, which automatically calculates the reciprocal values, so consequently only the elements in the so-called upper triangles of the comparison matrices are evaluated. The comparison matrices of the decision makers are presented in Tables 4 to 10.

Table 4. The comparison matrices of DM 1to DM 3

DM 1						DM 2						DM 3					
Goal						Goal						Goal					
	K1	K2	K3				K1	K2	K3				K1	K2	K3		
K1	1	1	1			K1	1	1	1			K1	1	1/2	1/2		
K2		1	1			K2		1	1			K2		1	1		
K3			1			K3			1			K3			1		
K1						K1						K1					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	2	5	3	A1	1	1	1	6	3	A1	1	2	1	6	2
A2		1	2	5	3	A2		1	1	6	3	A2		1	2	6	3
A3			1	3	2	A3			1	5	1	A3			1	4	1
A4				1	1	A4				1	1/4	A4				1	1/3
A5					1	A5					1	A5					1
K2						K2						K2					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	4	6	4	A1	1	1	4	6	4	A1	1	3	5	6	3
A2		1	4	6	4	A2		1	4	6	4	A2		1	2	5	2
A3			1	5	3	A3			1	5	3	A3			1	5	3
A4				1	1	A4				1	1	A4				1	1/2
A5					1	A5					1	A5					1
K3						K3						K3					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	3	5	7	A1	1	1	4	6	9	A1	1	2	5	7	9
A2		1	3	5	7	A2		1	4	6	9	A2		1	2	6	8
A3			1	3	5	A3			1	3	6	A3			1	4	7
A4				1	3	A4				1	6	A4				1	5
A5					1	A5					1	A5					1

Table 5. The comparison matrices of DM 4 to DM 6

DM 4				DM 4				DM 6			
Goal				Goal				Goal			
	K1	K2	K3		K1	K2	K3		K1	K2	K3
K1	1	1	1	K1	1	1	1/2	K1	1	1	1
K2		1	1	K2		1	1/2	K2		1	1
K3			1	K3			1	K3			1

K1						K1						K1					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	2	3	5	3	A1	1	2	2	9	2	A1	1	1	3	7	3
A2		1	2	6	2	A2		1	1	9	1	A2		1	2	8	2
A3			1	4	1	A3			1	9	2	A3			1	6	1
A4				1	½	A4				1	1/7	A4				1	1/2
A5					1	A5					1	A5					1

K2						K2						K2					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	2	4	6	3	A1	1	3	3	5	3	A1	1	1	2	5	1
A2		1	2	4	2	A2		1	3	5	3	A2		1	2	5	3
A3			1	4	3	A3			1	3	2	A3			1	6	3
A4				1	1	A4				1	1/2	A4				1	1/2
A5					1	A5					1	A5					1

K3						K3						K3					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	3	4	7	A1	1	2	4	6	9	A1	1	2	3	5	9
A2		1	3	4	7	A2		1	2	5	7	A2		1	3	5	8
A3			1	4	7	A3			1	3	6	A3			1	5	7
A4				1	4	A4				1	1/2	A4				1	4
A5					1	A5					1	A5					1

Table 6. The comparison matrices of DM 7 to DM 9

DM 7				DM 8				DM 9			
Goal				Goal				Goal			
	K1	K2	K3		K1	K2	K3		K1	K2	K3
K1	1	1/2	½	K1	1	1	1	K1	1	1/2	1/2
K2		1	1	K2		1	1	K2		1	1
K3			1	K3			1	K3			1

K1						K1						K1					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	3	7	3	A1	1	1	2	5	2	A1	1	1	1	5	1
A2		1	3	7	3	A2		1	2	5	2	A2		1	1	5	1
A3			1	7	1	A3			1	5	1	A3			1	5	1
A4				1	1/7	A4				1	1/3	A4				1	1/5
A5					1	A5					1	A5					1

K2						K2						K2					
	A1	A 2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1/2	2	7	2	A1	1	1	2	6	3	A1	1	1	2	5	2
A2		1	3	7	3	A2		1	2	6	3	A2		1	2	5	2
A3			1	3	2	A3			1	5	2	A3			1	4	1
A4				1	1/3	A4				1	1	A4				1	1
A5					1	A5					1	A5					1
K3						K3						K3					
	A1	A 2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	4	6	7	A1	1	1	1	5	7	A1	1	1	2	5	9
A2		1	4	6	7	A2		1	1	5	7	A2		1	2	5	9
A3			1	3	7	A3			1	5	7	A3			1	4	7
A4				1	3	A4				1	4	A4				1	4
A5					1	A5					1	A5					1

Table 7. The comparison matrices of DM 10 toDM 12

DM 10						DM 11						DM 12					
Goal						Goal						Goal					
	K1	K2	K3				K1	K2	K3				K1	K2	K3		
K1	1	1	1			K1	1	1	1			K1	1	1	1		
K2		1	1			K2		1	1			K2		1	1		
K3			1			K3			1			K3			1		
K1						K1						K1					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	1	4	1	A1	1	1	2	5	3	A1	1	2	2	7	4
A2		1	1	4	1	A2		1	2	5	3	A2		1	2	7	3
A3			1	4	1	A3			1	5	1	A3			1	5	1
A4				1	½	A4				1	1/4	A4				1	1/3
A5					1	A5					1	A5					1
K2						K2						K2					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	2	6	3	A1	1	1	2	5	3	A1	1	1	2	3	3
A2		1	2	6	3	A2		1	2	5	3	A2		1	2	3	3
A3			1	5	1	A3			1	3	1	A3			1	3	1
A4				1	1	A4				1	1/2	A4				1	1/3
A5					1	A5					1	A5					1
K3						K3						K3					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	2	3	4	8	A1	1	1	3	3	7	A1	1	1	2	3	7
A2		1	**3**	4	8	A2		1	3	3	7	A2		1	2	3	7
A3			1	4	8	A3			1	3	7	A3			1	3	7
A4				1	2	A4				1	3	A4				1	3
A5					1	A5					1	A5					1

Table 8. The comparison matrices of DM 13 to DM 15

DM 13					
Goal					
	K1	K2	K3		
K1	1	1	1		
K2		1	1		
K3			1		
K1					
	A1	A2	A3	A4	A5
A1	1	1	1	4	1
A2		1	1	4	1
A3			1	4	1
A4				1	½
A5					1
K2					
	A1	A2	A3	A4	A5
A1	1	1	2	5	3
A2		1	2	5	3
A3			1	5	3
A4				1	½
A5					1
K3					
	A1	A2	A3	A4	A5
A1	1	1	2	3	5
A2		1	2	3	5
A3			1	3	5
A4				1	3
A5					1

DM 14					
Goal					
	K1	K2	K3		
K1	1	1	1		
K2		1	1		
K3			1		
K1					
	A1	A2	A3	A4	A5
A1	1	1	1	3	2
A2		1	1	3	2
A3			1	3	1
A4				1	1/3
A5					1
K2					
	A1	A2	A3	A4	A5
A1	1	1	2	4	3
A2		1	2	4	3
A3			1	3	1
A4				1	1/3
A5					1
K3					
	A1	A2	A3	A4	A5
A1	1	1	2	4	6
A2		1	2	4	6
A3			1	3	5
A4				1	3
A5					1

DM 15					
Goal					
	K1	K2	K3		
K1	1	1	1		
K2		1	1		
K3			1		
K1					
	A1	A2	A3	A4	A5
A1	1	1	2	6	3
A2		1	2	6	3
A3			1	6	4
A4				1	1/3
A5					1
K2					
	A1	A2	A3	A4	A5
A1	1	1	2	3	3
A2		1	2	3	3
A3			1	5	2
A4				1	1/3
A5					1
K3					
	A1	A2	A3	A4	A5
A1	1	1	2	4	6
A2		1	2	4	6
A3			1	3	5
A4				1	3
A5					1

Table 9. The comparison matrices of DM 16 to DM 18

DM 16					
Goal					
	K1	K2	K3		
K1	1	1	1		
K2		1	1		
K3			1		
K1					
	A1	A2	A3	A4	A5
A1	1	1	1	6	2
A2		1	1	6	2
A3			1	6	2
A4				1	¼
A5					1

DM 17					
Goal					
	K1	K2	K3		
K1	1	1/2	1/2		
K2		1	1		
K3			1		
K1					
	A1	A2	A3	A4	A5
A1	1	1	1	5	2
A2		1	1	5	2
A3			1	5	2
A4				1	1/3
A5					1

DM 18					
Goal					
	K1	K2	K3		
K1	1	1	1		
K2		1	1		
K3			1		
K1					
	A1	A2	A3	A4	A5
A1	1	2	3	7	4
A2		1	2	6	3
A3			1	5	2
A4				1	1/3
A5					1

К2						К2						К2					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	2	4	3	A1	1	1	2	5	4	A1	1	1	2	3	4
A2		1	2	4	3	A2		1	2	5	4	A2		1	2	3	4
A3			1	4	2	A3			1	4	2	A3			1	3	2
A4				1	1/3	A4				1	1/3	A4				1	1/3
A5					1	A5					1	A5					1
К3						К3						К3					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	2	4	6	8	A1	1	1	2	4	8	A1	1	1	3	4	6
A2		1	3	5	7	A2		1	2	4	8	A2		1	3	4	6
A3			1	4	8	A3			1	3	7	A3			1	2	5
A4				1	3	A4				1	4	A4				1	3
A5					1	A5					1	A5					1

Table 10. The comparison matrices of DM 19 to DM 20

DM 19						DM 20					
Goal						Goal					
	К1	К2	К3				К1	К2	К3		
К1	1	1	1			К1	1	1	1		
К2		1	1			К2		1	1		
К3			1			К3			1		
К1						К1					
	A1	A2	A3	A4	A5		A1	A2	A3	A4	A5
A1	1	1	2	6	3	A1	1	2	2	6	3
A2		1	2	5	3	A2		1	1	4	3
A3			1	4	2	A3			1	4	2
A4				1	1/3	A4				1	1/3
A5					1	A5					1
К2						К2					
	A 1	A 2	A 3	A 4	A5		A 1	A 2	A 3	A 4	A5
A1	1	2	3	7	4	A1	1	1	3	8	5
A2		1	2	4	3	A2		1	3	8	5
A3			1	3	1	A3			1	7	2
A4				1	1/3	A4				1	1/3
A5					1	A5					1
К3						К3					
	A 1	A 2	A 3	A 4	A5		A 1	A 2	A 3	A 4	A5
A1	1	3	4	6	7	A1	1	2	3	5	7
A2		1	3	4	6	A2		1	3	4	6
A3			1	3	6	A3			1	3	5
A4				1	3	A4				1	3
A5					1	A5					1

Table 11 presents the vectors of the priority alternatives for each decision maker, obtained by means of relations (1)–(6) based on data from the comparison matrices (Tables 4 to 10).

Table 11. Vectors of alternatives priorities by decision makers

Decision maker	Alternatives					*CR*
	Superior	Dispatcher	Colleague	Client	Self-assessment	
DM 1	0.351	0.351	0.165	0.066	0.068	0.02
DM 2	0.341	0.341	0.177	0.056	0.085	0.04
DM 3	0.433	0.257	0.171	0.055	0.083	0.05
DM 4	0.390	0.287	0.172	0.068	0.082	0.03
DM 5	0.424	0.253	0.181	0.045	0.098	0.03
DM 6	0.341	0.313	0.193	0.054	0.099	0.04
DM 7	0.327	0.381	0.150	0.051	0.090	0.03
DM 8	0.309	0.309	0.225	0.063	0.094	0.01
DM 9	0.303	0.303	0.203	0.067	0.127	0.02
DM 10	0.309	0.284	0.206	0.067	0.134	0.03
DM 11	0.330	0.330	0.170	0.068	0.101	0.02
DM 12	0.340	0.307	0.181	0.073	0.099	0.02
DM 13	0.286	0.286	0.224	0.076	0.128	0.01
DM 14	0.301	0.301	0.198	0.076	0.124	0.01
DM 15	0.317	0.317	0.213	0.066	0.088	0.02
DM 16	0.329	0.292	0.218	0.054	0.107	0.02
DM 17	0.319	0.319	0.211	0.067	0.084	0.01
DM 18	0.360	0.316	0.168	0.071	0.085	0.02
DM 19	0.405	0.284	0.16	0.057	0.094	0.02
DM 20	0.388	0.301	0.177	0.054	0.079	0.02

As during the implementation of dynamic alteration in sensitivity analysis of all important criteria by 5% in all combinations (optional Dynamic in Expert Choice software), there was no change in ranking of alternatives, the final results of the conducted individual AHP can be considered stable.

Since all DMs performed all the evaluations, the information base is complete, and thus is fulfilled one of the conditions for starting group syntheses of individual priority vectors from the Table 11. However, the synthesis of the individual results of the AHP application into a group decision requires prior defining of individual weights of DMs. By means of the case study, five methods for assigning individual weights to decision makers are considered (Lukovac, 2016).

"The first method" is to assign equal weights to all DMs and then synthesize a group decision (Srđević et al. 2004). This approach, however, does not treat individual DM consistency and is subject to manipulation and other irregularities. For example, if a DM had personal motive (relative-friendly relations, possibility of corruption, etc.), his ratings could be adjusted and/or inconsistent (to better rank the desired candidates) and would not have suffered any consequences in relation to his inconsistency (weights would remain the same as at other DMs).

According to this approach, in the specific case the weight (α_k) of all DM would be 0. 05 ($\alpha_k = 1/20$).

"The second method" is to assign to DMs the weights based on the values of Spearman's correlation coefficient which shows the compatibility of the individual

DM with the reference group decision where also his decision was taken into account (Srđević et al., 2009). Spearman's correlation coefficient (S) is calculated according to the relation 7.

$$S = 1 - \frac{6\sum_{i=1}^{n} D_a^2}{n(n^2-1)} \tag{7}$$

D_a is the difference between U_a and V_a, where U_a and V_a are the ranks for the alternative a by reference list and by list compared to the reference, and n is the number of alternatives. In a group context, the relation 7 is applied to each of the combinations (group list, the list for kth member of the group, that is, the Spearman's coefficient is calculated according to the number of the members of the group). Spearman's coefficient value may vary between theoretical values of – 1 and 1. When the value approaches to 1, the indication is that the ranks are the same or similar, and when the value approaches to zero and – 1, the ranks are reverse, or negatively correlated.

In this case, the highest weight obtains the DM whose decision was the closest to the group decision (the DM having the highest value of Spearman's coefficient), while the smallest weight obtains the DM whose decision was the furthest from the group decision. All DMs are scaled according to the value of Spearman's coefficient.

For the purpose of calculating the weights of DMs under this possibility, in the considered case, the first thing to be done is making group decision. Two basic and most commonly used ways for obtaining group decision in the AHP are the AIP and the AIJ (Ramanathan & Ganesh, 1994; Forman & Peniwati, 1998).

For consolidating individual decisions into the group one, in this case, the AIP method is used, which is characteristic for two aggregations:

(a) Weight Arithmetic Mean Weight Method – WAMM. It is provided the alternative A_i and its weight value (priority) $w_i^{(k)}$ for the k-th decision-maker. If all members of the group (g) are assigned appropriate weights α_k, the weight arithmetic mean is:

$$w_i^{(g)} = \sum_{k=1}^{m} w_i^{(k)} \alpha_k \tag{8}$$

where in:

– $w_i^{(g)}$ final (composite) priority of the alternative A_i.

– m number of decision makers (group members);

Assuming, individual weights α_k of the members of the group were previously additionally normalized, i.e., $\sum_{k=1}^{m} \alpha_k = 1$.

(b) Geometric Mean Method – GMM. In this method, the aggregation consists in applying the following expression:

$$w_i^{(g)} = \prod_{k=1}^{m} \left(w_i^{(k)} \right)^{\alpha_k} \tag{9}$$

The weights of group members (α_k) are also previously additionally normalized.

In the Table 12 are shown the results of the AIP synthesis of the individual DM priority vectors from the Table 11 in the case where DMs are assigned equal weights ($\alpha_k = 0.05$).

Table 12. The AIP synthesis for $\alpha_k = 0.05$

AIP	Superior	Dispatcher	Colleague	Client	Self-assessment
WAMM	0.345	0.307	0.188	0.063	0.097
GMM	0.345	0.307	0.188	0.063	0.097

From the Table 12 it can be seen that the identical values of group vector are obtained of alternatives priorities for both aggregations of the AIP synthesis (WAMM and GMM). With the synthesis carried out it is fulfilled the condition for calculating Spearman's correlation coefficient for every DM, respectively, for comparing individual DM decisions with a reference, group decision. In the Table 13 are shown the weights (α_k) assigned to the DM based on the obtained Spearman's coefficient of DM based on the correlation 7.

Table 13. DM weights based on S value

Decision maker	**S**	$\boldsymbol{\alpha_k}$
DM 1	0.975	0.050
DM 2	0.975	0.050
DM 3	1	0.051
DM 4	1	0.051
DM 5	1	0.051
DM 6	1	0.051
DM 7	0.9	0.046
DM 8	0.975	0.050
DM 9	0.975	0.050
DM 10	1	0.051
DM 11	0.975	0.050
DM 12	1	0.051
DM 13	0.975	0.050
DM 14	0.975	0.050
DM 15	0.975	0.050
DM 16	1	0.051
DM 17	0.975	0.050
DM 18	1	0.051
DM 19	1	0.051
DM 20	1	0.051

"The third method" is to determine the weights of the DMs based on their competency for solving given decision making problem (Lukovac, 2016). According to this approach, the competency coefficient for each DM is calculated. The obtained competence coefficients are later additionally normalized and assigned as weights of DMs. In the Table 14 are shown the weights (α_k) assigned to the DMs by the value calculated according to their competence ration according to the approach developed in (Lukovac, 2016).

Table 14. DM weights based on K value

Decision maker	**K**	α_k
DM 1	0.6706	0.054
DM 2	0.5624	0.046
DM 3	0.561	0.046
DM 4	0.5833	0.048
DM 5	0.5621	0.046
DM 6	0.5619	0.046
DM 7	0.5598	0.046
DM 8	0.6918	0.056
DM 9	0.7195	0.058
DM 10	0.5686	0.046
DM 11	0.6141	0.050
DM 12	0.6319	0.051
DM 13	0.6946	0.056
DM 14	0.6738	0.055
DM 15	0.6341	0.051
DM 16	0.6018	0.049
DM 17	0.6754	0.054
DM 18	0.5888	0.048
DM 19	0.6242	0.051
DM 20	0.5673	0.046

"The fourth method" is to assign to DMs weights obtained by normalizing reciprocal values of their consistency ratios (CR) (Srđević, 2008), Table 15.

Table 15. DM weights based on CR value

Decision maker	CR	1/CR	α_k
DM 1	0.02	50	0.047
DM 2	0.04	25	0.024
DM 3	0.05	20	0.019
DM 4	0.03	33.3333	0.032
DM 5	0.03	33.3333	0.032
DM 6	0.04	25	0.024
DM 7	0.03	33.3333	0.032
DM 8	0.01	100	0.095
DM 9	0.02	50	0.047
DM 10	0.03	33.3333	0.032
DM 11	0.02	50	0.047
DM 12	0.02	50	0.047
DM 13	0.01	100	0.095
DM 14	0.01	100	0.095
DM 15	0.02	50	0.047
DM 16	0.02	50	0.047
DM 17	0.01	100	0.095
DM 18	0.02	50	0.047
DM 19	0.02	50	0.047
DM 20	0.02	50	0.047

"The fifth method" is that the weights of the DMs are determined by the consistency ratio (CR) and total Euclidean distance (ED), explained in (Srđević et al., 2009). This possibility was developed in (Blagojević et al. , 2010) as a method consisting of the following steps:

1. For every DM, CR and ED are calculated from all comparison matrices;
2. All CR values for every DM are summed separately, and then the same procedure is repeated for ED values;
3. The reciprocal values of the CR and ED values are calculated for every DM;
4. Additive normalization is performed (the reciprocal value of a sum for one DM is divided by a sum of reciprocal values of the sums of all DMs), especially for CR and ED ;
5. For every DM, the mean value of the normalized values of CR and ED is calculated and it is adopted as its weight in the group AHP decision, respectively, $\alpha_k = (\text{NormCr} + \text{NormED}) / 2$.

Based on the data from the comparison matrices shown in (Lukovac, 2016), for the considered AHP example, in the Tables 16 – 19 the calculation of the weights of DMs based on CR and ED is described.

Table 16. Consistency and total Euclidean distance DM 1-DM 5

	DM 1		**DM 2**		**DM 3**		**DM 4**		**DM 5**	
	CR	ED	CR	ED	CR	ED	CR	ED	CR	ED
Goal	0	0	0	0	0	0	0	0	0	0
K1	0.008	1.109	0.032	2.026	0.044	1.867	0.023	2.896	0.022	4.364
K2	0.056	3.775	0.099	6.483	0.074	4.897	0.054	3.048	0.050	3.226
K3	0.031	3.532	0.079	6.979	0.086	8.032	0.065	4.885	0.063	4.955
Σ	0.096	8.416	0.210	15.488	0.204	14.796	0.142	10.829	0.135	12.546
1/Σ	10.43	0.12	4.75	0.06	4.90	0.07	7.02	0.09	7.41	0.08
Norm	0.046	0.052	0.021	0.028	0.022	0.030	0.031	0.041	0.033	0.035
α_k	0.049		0.025		0.026		0.036		0.034	

Table 17. Consistency and total Euclidean distance DM 6-DM 10

	DM 6		**DM 7**		**DM 8**		**DM 9**		**DM 10**	
	CR	ED	CR	ED	CR	ED	CR	ED	CR	ED
Goal	0	0	0	0	0	0	0	0	0	0
K1	0.029	3.098	0.046	5.800	0.012	2.025	0.000	0.000	0.013	1.514
K2	0.054	2.910	0.021	2.565	0.023	2.206	0.039	2.339	0.042	2.634
K3	0.076	5.836	0.064	5.757	0.032	3.441	0.026	3.638	0.059	4.867
Σ	0.158	11.844	0.130	14.122	0.067	7.672	0.065	5.977	0.114	9.015
1/Σ	6.32	0.08	7.71	0.07	14.93	0.13	15.42	0.17	8.78	0.11
Norm	0.028	0.037	0.034	0.031	0.066	0.057	0.068	0.074	0.039	0.049
α_k	0.032		0.033		0.062		0.071		0.044	

Table 18. Consistency and total Euclidean distance DM 11-DM 15

	DM 11		**DM 12**		**DM 13**		**DM 14**		**DM 15**	
	CR	ED	CR	ED	CR	ED	CR	ED	CR	ED
Goal	0	0	0	0	0	0	0	0	0	0
K1	0.026	3.057	0.024	3.049	0.013	1.514	0.018	1.284	0.034	2.836
K2	0.006	0.910	0.034	2.403	0.018	1.985	0.018	1.964	0.052	3.248
K3	0.047	3.900	0.019	2.601	0.027	2.469	0.020	2.482	0.020	2.482
Σ	0.079	7.866	0.078	8.053	0.059	5.968	0.055	5.730	0.106	8.567
1/Σ	12.63	0.13	12.88	0.12	16.95	0.17	18.02	0.17	9.43	0.12
Norm	0.056	0.056	0.057	0.055	0.075	0.074	0.079	0.077	0.041	0.051
α_k	0.056		0.056		0.074		0.078		0.046	

Table 19. Consistency and total Euclidean distance DM 16-DM 20

	DM 16		**DM 17**		**DM 18**		**DM 19**		**DM 20**	
	CR	ED	CR	ED	CR	ED	CR	ED	CR	ED
Goal	0	0	0	0	0	0	0	0	0	0
K1	0.002	0.874	0.001	0.456	0.019	3.260	0.011	1.724	0.017	1.906
K2	0.024	2.291	0.020	2.450	0.051	2.961	0.015	2.050	0.026	4.422
K3	0.069	6.330	0.020	3.294	0.021	3.172	0.070	5.971	0.045	4.003
Σ	0.096	9.495	0.041	6.200	0.092	9.393	0.096	9.745	0.087	10.331
1/Σ	10.45	0.11	24.33	0.16	10.91	0.11	10.41	0.10	11.43	0.10
Norm	0.046	0.046	0.107	0.071	0.048	0.047	0.046	0.045	0.050	0.043
α_k	0.046		0.089		0.047		0.046		0.046	

4. Conclusions

Decision making, especially at the strategic level, requires more participants in the decision-making process (experts), who have different preferences depending on institutional placements, interests, skills, education and the like. In order to maximally objectify group context, in the procedure of synthesis of individual decisions , in this paper, using specific case, several possibilities for grading individual preferences of decision makers in group AHP synthesis are presented.

It is important to emphasize also the difference between the terms "joint" and "group" decision. In the first case it is implied the consensus, and in the second not necessarily. Group context treated in this paper fits to another case, no harmonization is performed, no consultation among participants, and the results of individual evaluations are consolidated later.

Further research should be directed towards analyzing the AHP synthesis results for the shown possibilities of assigning weights to decision makers. The subject of the research should also be directed towards the consensus of decision makers and the so-called joint decision.

References

Altuzarra A., Moreno–Jimenez J.M., & Salvador M.A. (2007). Bayesian priorization procedure for AHP-group decision making. European Journal of Operational Research, 182(1), 367–382.

Arnette, A., Zobel C., Bosch D., Pease J., & Metcalfe T. (2010), Stakeholder ranking of watershed goals with the vector analytic hierarchy process: Effects of participant grouping scenarios. Environmental Modelling and Software, 25, 1459–1469.

Bernasconi, M., Choirat, C., & Seri, R. (2014). Empirical properties of group preference aggregation methods employed in AHP: Theory and evidence. European Journal of Operational Research, 232(3), 584–592.

Blagojević, B. (2015). Minimizacija odstupanja grupne od individualne odluke primenom inteligentnih stohastičkih algoritama u problemima vodoprivrede i poljoprivrede, Doktorska disertacija, Univerzitet u Novom Sadu, Poljoprivredni fakultet, 15. (In Serbian).

Blagojević, B., Srđević, Z., Srđević, B., & Suvočarov, K. (2010). Rangiranje korisnika kredita za opremu za navodnjavanje pomoću Analitičkog hijerarhijskog procesa, Vodoprivreda, 42, 213-222. (In Serbian).

Escobar, M.T., Aguarón, J., & Moreno–Jiménez J.M. (2004). A note on AHP group consistency for the row geometric mean priorisation procedure. European Journal of Operational Research, 153, 318–322.

Forman E., & Peniwati K. (1998). Aggregating individual judgments and priorities with the analytic hierarchy process. European Journal of Operational Research, 108, 165–169.

Ho, W. (2008). Integrated analytic hierarchy process and its applications–A literature review. European Journal of Operational Research, 186(1), 211–228.

Hot, I. (2014). Upravljanje izradom generalnih projekata u oblasti infrastrukture primenom višekriterijumske analize, Doktorska disertacija, Fakultet tehničkih nauka Univerziteta u Novom Sadu. (In Serbian).

Jandrić, Z., & Srević, B. (2000). Analitički hijerarhijski proces kao podrška odlučivanju u vodoprivredi. Vodoprivreda, 186-188, 327–335. (In Serbian).

Lootsma, F.A. (1988). Numerical scaling of human judgment in pairwise–comparison methods for fuzzy multi–criteria decision analysis, Mathematical Models for Decision Support. NATO ASI Series F, Computer and System Sciences, Springer–Verlag, Berlin, Germany, 57–88.

Lootsma, F.A. (1990). T French and the American school in multi–criteria decision analysis. Operations Research, 24(3), 263–285.

Lootsma, F.A., Mensch, T.C.A., & Vos, F.A. (1990). Multi–criteria analysis and budget reallocation in long–term research planning. European Journal of Operational Research, 47, 293–305.

Lukovac, V. (2016). Model za otklanjanje grešaka u sistemu procene performansi vojnih vozača, Doktorska disertacija, Univerzitet odbrane, Vojna akademija, Beograd. (In Serbian).

Ma, D., & Zheng, X. (1991). 9/9–9/1 scale method of the AHP, Proceedings of the 2nd International Symposium on the AHP, Pittsburgh, 197–202.

Nikolić, I., & Borović, S. (1996). Višekriterijumska optimizacija–metode, primena u logistici, softver, Vojnoizdavački zavod, Beograd. (In Serbian).

Ramanathan, R., & Ganesh, L.S. (1994). Group preference aggregation methods employed in AHP: an evaluation and an intrinsic process for deriving members' weightages. European Journal of Operational Research, 79, 249–265.

Saaty, T. L. (1980). The Analytic Hierarchy Process, McGraw-Hill, New York.

Saaty, T.L. (1977). A scaling method for priorities in hierarchical structures. Journal of Mathematical Psychology, 15, 234–281.

Saaty, T.L. (1999). Decision making for leaders: the analytic hierarchy process for decisions in a complex world, 2, RWS publications.

Srđević, B, Potkonjak, S., Srđević, Z., Škorić, M., & Zoranović, T. (2004). Simulacija grupnog odlučivanja u izboru tehnologije navodnjavanja, Poljoprivreda između suša i poplava, 190-197, Poljoprivredni fakultet, Novi Sad. (In Serbian).

Srđević, B, Suvočarov, K., & Srđević, Z. (2009). Analitički hijerarhijski process: Individualna i grupna konzistentnost donosilaca odluka, Vodoprivreda, 41(1-3), 13-21. (In Serbian).

Srđević, B., Suvočarev K., & Srđević, Z. (2008). AHP grupno odlučivanje bez konsenzusa: Primer planiranja segmentacije mokrog polja, Vodoprivreda, 40, 51-58. (In Serbian).

Subramanian, N., & Ramanathan, R. (2012). A review of applications of Analytic Hierarchy Process in operations management. International Journal of Production Economics, 138(2), 215–241.

Vaidya, O.S., & Kumar, S. (2006). Analytic hierarchy process: An overview of applications. European Journal of Operational Research, 169(1), 1–29.