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TRAINING AIRCRAFT SELECTION FOR DEPARTMENT OF FLIGHT TRAINING IN FUZZY ENVIRONMENT

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Abstract: *The last two decades have seen a growing trend towards the use of aircraft as transportation tools. However, there is a lack of routes because of the insufficient number of pilots. Therefore, the increase in usage of aircraft has been limited. To respond to this increase in Turkey, it indicates a rise in the number of flight academies. Flight academies have emerged as powerful and expensive platforms for flight training. In the new global economy, the aircraft selection problem has become a central issue for Flight Training Departments, which is planned to open in government universities. In this study, an approach based on the fuzzy BWM method is proposed to select more suitable training aircraft in government universities. Criterion weights and alternative aircraft rankings were determined using the fuzzy BWM method. Afterward, a mathematical model was developed to calculate how many aircraft we need to buy under certain constraints. Necmettin Erbakan University, which wants to train new and qualified pilots, needs training aircraft and trainers that can provide pilot training. A case study of training aircraft selection was conducted for the Necmettin Erbakan University Department of Flight Training. As a result, it can be said that 13 aircraft will be sufficient for the Flight Training department to start education.*

Key words: *Training aircraft selection, Flight training, BWM, Fuzzy Sets, Linear programming model.*

1. Introduction

When asked what the term aviation means, the first answer has been usually to travel by aircraft. However, the design, manufacture, and maintenance operations of aircraft required to travel are also included in the aviation term. It is unknown what

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will bring about the combination of advancing time and constantly developing technologies for aeronautics. Today, aircraft have been used for human and cargo transportation, agricultural spraying, and military purposes. For those types of aviation to be used actively, personnel who will design and manufacture aircraft; know-hows; new technologies; pilots to use aircraft; and technicians to undertake maintenance and repair of aircraft are required.

The beginning of aviation history dates back to the 9th century when Abbas Ibn Firnas made the first flying glider (Lienhard, 2019). The Chinese book Poo Phu Tau had been claiming the existence of rotary-wing aircraft in the 4th century. Leonardo da Vinci's glider design, which has survived to the present day, remained only a design in the 15th century but was produced in the 19th century with the materials used in the 15th century. Hezarfen Ahmet Çelebi had traveled 3 km from Galata Tower to the Anatolian side in 1638 with wings he designed inspired by birds. The modern era of aviation history has begun with the hot air balloon designed by the Montgolfier brothers (Kılıç, 2015). Modern aviation history has been continued the development with Alphonse Pénaud's first structurally balanced aircraft model, the first successful flight in history by Felix du Temple, and the first motorized aircraft flight of Orville and Wilbur Wright’s brothers. Airports have begun to be built in many cities during World War II. After World War II, with the pilots' demobilization and the introduction of the aircraft used by the soldiers but surplus to the civilians, there was a great increase in the use of private and commercial aviation, especially in North America. Today, the increase in airlines' use in different transportation types such as passenger transportation, cargo transportation, and dangerous goods transportation continues at an accelerated rate. Even in the coming years, an increase in using space tourism will be observed with the development of services, increased security, and reliability, as in airline tourism (Webber, 2013).

Airline transportation, which is still not widely used in Turkey, is developing rapidly in the world. Transportation with aircraft is a more reliable transportation choice in a shorter time compared to other transportation methods. Although technological developments are of great importance for airline transportation to take superiority over other transportation methods, the number of well-trained pilots is also crucial. Universities working towards the more widespread use of aviation in Turkey have started studies to open flight training departments. Necmettin Erbakan University intends to purchase training aircraft to train pilots in its Department of Flight Training by progressing towards this aim.

However, the increased fuel costs and aircraft costs which have increased due to the economic difficulties experienced in the last few years and the changes in exchange rates, have adversely affected the aviation market. While the aviation market has been affected so much, the number of criteria to be considered for selecting aircraft has increased. For this reason, determining the criteria for the selection of training aircraft is of great importance. Thus, the aircraft selection process has turned into a multi-criteria decision-making (MCDM) problem. There are different options for the solution of MCDM problems.

In this study, an approach based on the fuzzy BWM method is proposed for purchasing training aircraft. A case study of the Necmettin Erbakan University Department of Flight Training has been conducted to show the approach's applicability. The nine crucial criteria affecting aircraft selection problem (Max Cruise Speed, Max Range, Takeoff and Landing Roll, Max Climb Rate, Power Output, Weight, Price, Useful Fuel Capacity, and Time Before Overhaul) has been determined from the surveys conducted with decision-makers who are the academic members of Necmettin Erbakan University Aviation and Space Sciences Faculty. In the next step, according to the experts' evaluations, criterion weights that affect the aircraft selection problem have been determined with the fuzzy BWM method. The most important feature that distinguishes this study from other studies in the literature is developing a linear programming model that determines how many aircraft should be bought under certain constraints (Budget, Minimum flying before overhaul, and Fuel consumption).

An original approach consisting of fuzzy BWM and a mathematical model are proposed. A case study is made on which criteria should be considered in the training aircraft selection process to set an example for developing flight departments in Turkey. A literature study is conducted in the field of aircraft selection, and as a result, a literature matrix is created that relates the studies and the methods used, and it is ensured that the gap in the literature can be seen in future studies. Unlike previous studies, more technical features of training aircraft are discussed.

The rest of this paper is arranged as follows. In Section 2, a literature review is presented. In Section 3, a detailed methodology is presented. Section 4 provides the relevant problem definition and developed mathematical model. In Section 5, a case study of aircraft selection is presented and demonstrated how the proposed approach works. Finally, in Section 6, the conclusion of this paper and suggestions for future work are presented.

2. Literature Review

This section presents a comparative discussion of the former studies on aircraft selection to highlight the proposed study's contributions. This study differs from other studies because the criteria that affect the training aircraft selection problem are carefully determined, determined which type of aircraft should purchase by flight academies, and the number of aircraft required for the flight academy. The literature review is divided into three paragraphs to avoid complexity. In the paragraphs, aircraft selection studies using MCDM methods in crisp, fuzzy, and both crisp and fuzzy environments are given, respectively. Table 1 presents the previous studies and their' criteria and methods used.

See et al. (2004) firstly have demonstrated the strengths and weaknesses of MCDM methods which own theoretical and practical flaws commonly employed, using the speed, max cruise range, and the number of passengers criteria for the airline aircraft selection problem. Then, a method based on hypothetical equivalent has been proposed and expanded to include hypothetical inequivalent. In this study, criteria affecting the problem have not expressed the aircraft selection process sufficiently was observed. Liu & Wu (2010) have proposed an evaluation model based on Information Entropy and Data Envelopment Analysis methods to analyze suitable alternatives for aircraft fleet selection in local transportation airlines. The applicability of the proposed approach has been shown with a numerical example. Six alternative aircraft were evaluated under five main criteria, with ten sub-criteria. The approach has been developed in a fuzzy environment to avoid uncertainties during the decision-making process. Sun et al. (2011) have proposed a new approach for the hypothetical airline aircraft selection problem using ELECTRE, SAW, and TOPSIS methods. Uncertainties may arise in every decision-making problem. The uncertainties have been eliminated using Taguchi loss functions to ensure robustness instead of developing the approach in a fuzzy environment. In this study, robustness has also been added as a criterion. Investigating the effect of robustness as a criterion on the ranking of alternatives has been conducted by sensitivity analysis. Dožić & Kalić

(2015a) have proposed a new approach using two different MCDM methods, AHP and ESM. A hypothetical airline aircraft selection study has demonstrated the applicability of the approach. Sensitivity analysis was performed to show the differences between AHP and ESM results. Dožić & Kalić (2015b) have developed a new fleet planning model for airlines operating on short and medium-haul routes. The fleet planning model consists of three stages: fleet composition, fleet sizing, and aircraft selection. The applicability of the model was demonstrated by a hypothetical airline case study located at the Belgrade airport. Five criteria evaluated seven alternative aircraft. It is inadequate in reaching the appropriate solution in the problem of aircraft selection with selected criteria. Paul et al. (2017) have proposed a TOPSIS method-based approach for fighter aircraft selection. The applicability of the proposed approach is shown with a numerical example. Four alternative fighter aircraft were evaluated with six criteria. The criteria chosen are not sufficient for alternative fighter aircraft selection. Ali et al. (2017) have created a scenario to select new and better aircraft for their existing fleets to develop Pakistan Air Force and Pakistan Air Defense capabilities. They have proposed an approach using the AHP method for the aircraft selection problem. Cost-Benefit Analysis has been carried out for the selected alternative to be compatible with Pakistan's financial budget. Six alternative aircraft have been evaluated with ten criteria. The criteria chosen are not sufficient for alternative fighter aircraft selection. Kiracı & Bakır (2018b) have proposed a TOPSIS method-based approach for choosing the most suitable aircraft for airlines with different flight networks. The applicability of the proposed approach is shown with an example of commercial aircraft selection. Eight decision-makers have evaluated four alternative aircraft types, most demanded by airline companies with five criteria. It is inadequate in reaching the appropriate solution in the problem of aircraft selection with selected criteria. Kiracı & Bakır (2018a) have proposed an approach based on AHP, COPRAS, and MOORA methods, considering cost, performance, and environmental factors for the commercial aircraft most demanded by airline companies. The proposed approach has been shown with a numerical example, and it has been observed that decision-making methods give consistent results. The decision-makers evaluated the four alternative aircraft most demanded by airlines, with seven criteria affecting the selection. No action has been taken to prevent uncertainties that may occur during the decision-making process. Petrovic & Kankaraš (2018) have proposed an approach based on the hybrid DEMATEL and AHP methods to select air traffic protection aircraft. The applicability of the proposed approach is shown with a numerical example. Forty-five decision-makers calculated the weight of 52 sub-criteria under nine main criteria. No precautions have been taken for uncertainties that may arise during the criterion weighting process. Ilgın (2019) has proposed a new approach based on the linear physical programming method to remove the disadvantage of the fact that criteria obtained by different decision-making methods take physically meaningless and subjective values. The applicability of the proposed approach is shown with a numerical example. Five criteria evaluated six alternative aircraft. (Yilmaz et al. (2020) have proposed an approach based on AHP and TOPSIS methods for aircraft selection. Sixteen decision-makers who are teachers in Eskişehir Technical University Flight School evaluated six alternative training aircraft with four main criteria (strategic criteria, financial criteria, operational criteria, and maintenance criteria). The criteria for aircraft selection are not explicitly indicated. For this reason, the study is insufficient on an appropriate decision-making aspect. Hoan & Ha (2021) have proposed a novel decision-making approach with the integration of FUCOM and ARAS methods for aircraft selection. A case study of suitable fighter jets selection for the Vietnam People's Air Force has demonstrated the proposed approach's applicability. Three alternative aircraft (Su-35, Mig-35, and F-16) were examined under 13 criteria. However, the selected criteria do not provide proper outcomes for fighter aircraft selection. For instance, constraints such as the maximum range and the useful fuel capacity have been ignored. Sensitivity analyses have been conducted, and the results are compared with the weighted product method to prove the method's robustness. do Nascimento Maêda et al. (2021) have proposed a hybrid approach based on AHP and TOPSIS methods and the dual normalization procedure for the selection of helicopters to be purchased by the Brazilian Navy, which provides more logistics and combat capacity in naval operations. A real military case study was conducted to improve the performance of the Brazilian armed forces. Among the six helicopters evaluated by considering the attack helicopters used by developed countries during the selection process, the Ah-64E Apache was the most suitable helicopter for the Brazilian armed forces.

Wang & Chang (2007) have proposed an approach based on the TOPSIS method in a triangular fuzzy environment in initial training aircraft selection for Taiwan Air Force. In the case study conducted to demonstrate the approach's applicability, 15 decision-makers have evaluated seven aircraft under 16 criteria. The selection process has been insufficient due to the usage of criteria alike and is not helpful enough to make the right decision for the training aircraft selection. Also, criteria weight has been calculated by taking the average of decision-makers' evaluations, not by any decision-making method. Yeh & Chang (2009) have proposed a new group method for MCDM problems in a fuzzy environment. A case study of Taiwan's domestic airline's empirical aircraft selection has demonstrated the model's applicability. Five alternative aircraft have been evaluated under three main criteria with 11 sub-criteria. The specified criterion has been insufficient to reach an appropriate judgment for the aircraft selection problem. Ozdemir & Basligil (2016) have proposed an approach for the aircraft selection problem using fuzzy ANP and Choquet Integral Method. An aircraft purchase case study has been conducted for a Turkish airline company. Three aircraft were evaluated with ten criteria. The criteria are not conducive to proper aircraft selection. The proposed approach results have been compared with fuzzy AHP, and the same results on F AHP have been obtained in all three methods. (Dožić et al. (2018) have proposed a new methodology to assist in selecting aircraft types that best meet market conditions and airline requirements for estimated travel demand based on known route networks and routes. An AHP-Logarithmic Fuzzy Preference Programming method-based approach has been developed in the fuzzy environment to eliminate human uncertainty. The pairwise comparison matrix was created from interviews with experts from different airlines and universities. The applicability of the methodology has been demonstrated by the case study of regional airline aircraft selection. According to interviews conducted with experts from different airlines and universities, seven alternative aircraft have been evaluated with ten criteria. The selected criteria are not sufficient to reach the most suitable alternative aircraft. Kartika & Hanani (2019) have proposed a new approach based on FGD and AHP methods for aircraft selection. The approach's applicability is proved with a case study of Indonesia's national flag carrier airline company's aircraft selection to be used on new routes. Decision-makers evaluated four alternative aircraft with six criteria. In this study, it is insufficient to decide on the most suitable aircraft with the determined criteria. Ahmed et al. (2020) have proposed a new approach using the AHP and efficiency method in a fuzzy environment to eliminate human uncertainty in the regional aircraft selection problem, considering the environmental design and cost impact. Inspired by Canadian airlines, the framework of the approach was created. Four alternative aircraft were evaluated with 15 sub-criteria under five main criteria.

The consistency of the results of the proposed approach was checked using sensitivity analysis. The study is insufficient in an appropriate decision-making aspect since more emphasis on environmental criteria, and technical criteria essential for aircraft selection remain in the background. Kiracı & Akan (2020) have proposed a new hybrid AHP and TOPSIS approach in the Interval Type 2 fuzzy environment. The applicability of the proposed approach is shown with a numerical example. Four alternative commercial aircraft were evaluated under three main criteria (economic performance, technical performance, and environmental impact) with eight sub-criteria. The technical criteria required for aircraft selection do not fully reflect a real-life choice. Sánchez-Lozano & Rodríguez (2020) have proposed a new hybrid AHP and FRIM approach in a fuzzy environment to the aircraft selection problem. The applicability of the approach has been demonstrated by the Spanish Air Force aircraft selection case study. The necessary evaluations of the proposed approach's application were obtained from the questionnaires conducted with the flight instructors in the 23rd Fighter and Attack Training wing. Four alternative aircraft have been evaluated under 13 criteria. The criteria required for the training aircraft selection have been selected, but some crucial criteria like the time before maintenance or usable fuel capacity seem to have been overlooked. Karamaşa et al. (2021) have proposed an approach based on neutrosophic AHP and MULTIMOORA methods for training aircraft selection for flight training organizations. The aircraft has been evaluated with the help of questionnaires. In order to check the accuracy of the developed approach, a comparative analysis with existing approaches has been made. In line with the comparative analysis, it is observed that the approach produces productive results. Bakır et al. (2021) have proposed an approach based on hybrid PIPRECIA and MARCOS methods in the fuzzy environment for regional aircraft selection. In the case study to demonstrate the approach's feasibility, five decision-makers evaluated six aircraft alternatives under 14 criteria. In addition, a three-stage sensitivity analysis was conducted to demonstrate the accuracy of the approach.

Mello et al. (2012) have proposed a novel approach based on the NAIDE method for the aircraft selection problem. The applicability of the proposed approach has been proved with a numerical illustration of a turboprop aircraft selection. Eight alternative aircraft have been evaluated under 11 criteria which can be stochastic, fuzzy, or crisp measurements. The criteria determined for aircraft selection are not sufficient to lead to the appropriate judgment. The criteria should be elected more specifically. The authors observed a lack of simplicity when approaching diverse types' variables as a weak point of the method. Gomes et al. (2014) have proposed a new approach based on the NAIDE method to aircraft selection. An aircraft selection case study of an airline company investing in regional charter flights in Brazil has demonstrated the applicability of the approach. Eight alternative aircraft have been evaluated under 11 different criteria: crisp, stochastic, and fuzzy. Schwening et al. (2014) have proposed a hybrid AHP and TOPSIS method-based approach for agricultural aircraft selection. The applicability of the model has been demonstrated with a case study. The approach has been developed in a fuzzy environment to avoid uncertainties during the decision-making process. Four alternative agricultural aircraft were evaluated under nine criteria.

Table 1. Summary of previous researches on aircraft selection

Author	MCS	MR	TLGR	MCR	PO	W	P	UFC	TBO	TA	C	F	Method
(See et al., 2004)	✓	✓				✓ (Max Takeoff)					✓		Multi-attribute Method
(Wang & Chang, 2007)	✓		✓	✓				✓		✓		✓	TOPSIS
(Yeh & Chang, 2009)		✓					✓					✓	New Fuzzy group MCDM
(Liu & Wu, 2010)		✓	✓			✓ (Max Takeoff)	✓				✓		Information Entropy and Data Envelopment Analysis
(Sun et al., 2011)	✓					✓ (Max Takeoff)		✓			✓		ELECTRE, SAW, and TOPSIS
(Mello et al., 2012)	✓	✓	✓				✓				✓	✓	NAIADE
(Gomes et al., 2014)	✓	✓	✓				✓				✓	✓	NAIADE
(Schwening et al., 2014)			✓	✓	✓			✓			✓	✓	AHP and TOPSIS
(Dožić & Kalić, 2015a)						✓ (Max Takeoff)	✓				✓		AHP and Even Swaps Method
(Dožić & Kalić, 2015b)						✓ (Max Takeoff)	✓				✓		Even Swaps Method
(Ozdemir & Basligil, 2016)							✓					✓	ANP and Choquet Integral Method
(Ali et al., 2017)	✓					✓ (Max Takeoff)	✓				✓		AHP and Cost Benefit Analysis
(Paul et al., 2017)	✓	✓				✓ (Payload)	✓				✓		TOPSIS
(Dožić et al., 2018)		✓				✓ (Max Takeoff)	✓					✓	AHP and Logarithmic Fuzzy Preference Programming Method
(Kiracı & Bakır, 2018b)	✓	✓					✓	✓			✓		TOPSIS
(Kiracı & Bakır, 2018a)	✓	✓				✓ (Payload)	✓	✓			✓		AHP, COPRAS and MOORA
(Petrovic & Kankaraš, 2018)							✓				✓		DEMATEL and AHP
(Ilgın, 2019)		✓					✓	✓			✓		Linear Physical Programming
(Kartika & Hanani, 2019)						✓ (Max Takeoff)						✓	AHP and TOPSIS
(Yilmaz et al., 2020)						✓ (Max Takeoff)	✓				✓		AHP and TOPSIS
(Ahmed et al., 2020)		✓	✓			✓ (Payload)	✓	✓		✓		✓	AHP and Efficacy Method
(Kiracı & Akan, 2020)	✓	✓	✓			✓ (Max Takeoff)	✓	✓				✓	AHP and TOPSIS
(Sánchez-Lozano & Rodríguez, 2020)		✓	✓			✓ (Max Takeoff)	✓			✓ (Military)		✓	AHP and The Reference Ideal Method
(Hoan & Ha, 2021)	✓			✓	✓	✓ (Max Takeoff)	✓				✓		FUCOM and ARAS
(Do Nascimento Maêda et al., 2021)	✓	✓				✓ (Payload)					✓		AHP, TOPSIS, and 2N
(Karamaşa et al., 2021)			✓				✓					✓	AHP and MULTIMOORA
(Bakır et al.,2021)	✓	✓										✓	PIPRECIA and MARCOS
This Study	✓	✓	✓	✓	✓	✓ (Empty)	✓	✓	✓	✓		✓	BWM and Linear Model

* MCS - Max Cruise Speed; MR - Max Range; TLGR - Takeoff and Landing Ground Roll; MCR - Max Climb Rate; PO - Power Output; W – Weight; P – Price; UFC - Useful Fuel Capacity; TBO - Time Before Overhaul; TA - Training Aircraft; C – Crisp; F - Fuzzy

3. Methodologies

The solution to real-world MCDM problems is too complex to be described with quantitative numbers. This complexity is due to uncertain and conflicting qualitative factors. In MCDM problems, criteria or alternatives are evaluated with qualitative judgments. Human qualitative judgments often contain uncertainty and abstraction. The fuzzy set theory simulates human logic using a mathematical model, and a solution to real-world problems can be provided according to the human thinking style. For this reason, fuzzy sets have been used to provide a more flexible, convenient, and effective solution for the decision-makers and to obtain results more compatible with real situations in the training aircraft selection problem.

In this section, fuzzy set theory, triangular fuzzy numbers, and graded mean integration representation (GMIR) of triangular fuzzy numbers used in our study are briefly mentioned. Moreover, fuzzy BWM suggested by Dong et al. (2021) based upon triangular fuzzy numbers for MCDM is presented in detail.

3.1. Fuzzy set theory

In 1965, L. A. Zadeh noticed that human thinking is primarily fuzzy and interpreted fuzzy sets. Fuzzy set theory has been utilized for modeling decision-making processes based upon vague and uncertain information such as decision-makers' judgments (Kumar et al., 2017; Lima Junior et al., 2014).

3.2. Triangular fuzzy numbers

A fuzzy set is described with a membership function, and all elements of a fuzzy set have membership degrees that range from 0 to 1 (Zadeh, 1965). A triangular fuzzy number is indicated in Figure 1. A triangular fuzzy number is indicated as (l, m, u) with l < m< u (Kargı, 2016; Lima Junior et al., 2014; Alosta et al., 2021).

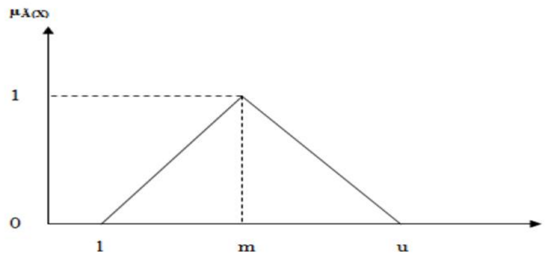

Figure 1. Triangular Membership Function.

A triangular membership function and its elements are represented as follows (Muhammad et al., 2021):

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & for\ x < l, \\ \frac{x-l}{m-l} & for\ l \leq x \leq m, \\ \frac{u-x}{u-m} & for\ m \leq x \leq u, \\ 0 & for\ x > u, \end{cases} \tag{1}$$

3.3. Graded mean integration representation method

GMIR method proposed by Chen & Hsieh (2000). GMIR $R(\tilde{a}_i)$ of a triangular fuzzy number $\tilde{a}_i = (l_i, m_i, u_i)$ can be calculated by

$$R(\tilde{a}_i) = \frac{l_i + 4m_i + u_i}{6} \tag{2}$$

The smaller value of $R(\tilde{a}_i)$, the smaller the triangular fuzzy number $\tilde{a}_i$.

If $\widetilde{w}_j = (w_j^l, w_j^m, w_j^u)$ is a triangular fuzzy number (j=1, 2, …, n). A triangular fuzzy weight vector $\widetilde{w} = [\widetilde{w}_1, \widetilde{w}_2, \dots, \widetilde{w}_n]$ is called a normalized fuzzy weight vector if for every j∈ {1, 2, …, n}, the following holds (Bas et al., 2019; Dong et al., 2021; Guo & Zhao, 2017; Liao et al., 2013):

$$\sum_{j=1}^{n} w_j^m = 1, \quad w_j^u + \sum_{i=1, i\neq j}^{n} w_i^l \leq 1, \; w_j^l + \sum_{i=1, i\neq j}^{n} w_i^u \geq 1 \tag{3}$$

3.4. Fuzzy Best Worst Method

In this section, we present fuzzy BWM suggested by Dong et al. (2021) based upon triangular fuzzy numbers for MCDM. Fuzzy BWM was previously developed by Guo and Zhao (2017) and used in many studies. However, according to Dong et al. (2021), the multiplication and subtraction operations of triangular fuzzy numbers in Fuzzy BWM developed by Guo and Zhao (2017) are not in accordance with the operation laws of triangular fuzzy numbers. Moreover, their final model is a non-linear programming model, whose global optimal solution may not exist. Whereas the Fuzzy BWM model proposed by Dong et al. (2021) is linear in contrast to other papers, it is more reasonable to construct a linear programming model to obtain the optimal fuzzy weights of criteria. We agree with them and therefore used the fuzzy BWM method suggested by Dong et al. (2021) for our study.

3.4.1. Constructing of the mathematical programming model

The optimal weight of all criteria is one where, each pair of w_B/w_j and w_j/w_w have $w_B/w_j = a_{Bj}$ and $w_j/w_W = a_{jW}$ (Rezaei, 2015). However, it is hard to achieve $w_B/w_j = a_{Bj}$ and $w_j/w_W = a_{jW}$ for all j. Because these formulas are equivalent to $w_B = w_j a_{Bj}$ and $w_j = a_{jW} w_W$ respectively, it is anticipated to find the fuzzy weights to ensure $w_B = w_j a_{Bj}$ and $w_j = a_{jW} w_W$ as much as possible. That is,

$$(w_B^l, w_B^m, w_B^u) = (w_j^l, w_j^m, w_j^u)(a_{Bj}^l, a_{Bj}^m, a_{Bj}^u), \tag{4}$$

$$\left(w_j^l, w_j^m, w_j^u\right) = \left(a_{jW}^l, a_{jW}^m, a_{jW}^u\right)(w_W^l, w_W^m, w_W^u), \tag{5}$$

Eq. (4) and Eq. (5) are regarded as fuzzy equations, that is;

$$(w_B^l, w_B^m, w_B^u) \cong (w_j^l a_{Bj}^l, w_j^m a_{Bj}^m, w_j^u a_{Bj}^u), \tag{6}$$

$$\left(w_j^l, w_j^m, w_j^u\right) \cong \left(a_{jW}^l w_W^l, a_{jW}^m w_W^m, a_{jW}^u w_W^u\right), \tag{7}$$

Where ~ denotes the fuzzy number and so, the symbol "≅" is a fuzzy version of "=" for a real number set, and it has the linguistic explanation "fuzzy equal to". Then, Eqs. (6) and (7) are equal fuzzy equations as follows:

$$w_B^l - w_j^l a_{Bj}^l \cong 0,\ w_B^m - w_j^m a_{Bj}^m \cong 0,\ w_B^u - w_j^u a_{Bj}^u \cong 0, \tag{8}$$
$$w_j^l - a_{jW}^l w_W^l \cong 0,\ w_j^m - a_{jW}^m w_W^m \cong 0,\ w_j^u - a_{jW}^u w_W^u \cong 0, \tag{9}$$

For convenience;

$$R(w_j^l) = w_B^l - w_j^l a_{Bj}^l \cong 0,\ R(w_j^m) = w_B^m - w_j^m a_{Bj}^m \cong 0,\ R(w_j^u) = w_B^u - w_j^u a_{Bj}^u \cong 0, \tag{10}$$
$$Q(w_j^l) = w_j^l - a_{jW}^l w_W^l \cong 0,\ Q(w_j^m) = w_j^m - a_{jW}^m w_W^m \cong 0,\ Q(w_j^u) = w_j^u - a_{jW}^u w_W^u \cong 0, \tag{11}$$

The membership functions are constructed below for Eq. (10) and Eq. (11) respectively;

$$\mu(R(w_j^t)) = \begin{cases} 1, & if\ R(w_j^t) = 0 \\ 1 - \frac{R(w_j^t)}{d_j^t}, & if\ 0 \le R(w_j^t) \le d_j^t \\ 1 + \frac{R(w_j^t)}{d_j^t}, & if\ -d_j^t \le R(w_j^t) < 0 \\ 0, & otherwise \end{cases} \tag{12}$$

$$\mu(Q(w_j^t)) = \begin{cases} 1, & if\ Q(w_j^t) = 0 \\ 1 - \frac{Q(w_j^t)}{q_j^t}, & if\ 0 \le Q(w_j^t) \le q_j^t \\ 1 + \frac{Q(w_j^t)}{q_j^t}, & if\ -q_j^t \le Q(w_j^t) < 0 \\ 0, & otherwise \end{cases} \tag{13}$$

Where the tolerance parameters $d_j^t > 0$ and $q_j^t > 0$ ($j = 1, 2, \dots, n; t = l, m, u$). The membership function of the fuzzy equation $R(w_j^t) = w_B^t - w_j^t a_{Bj}^t \cong 0$ is shown in Figure 2.

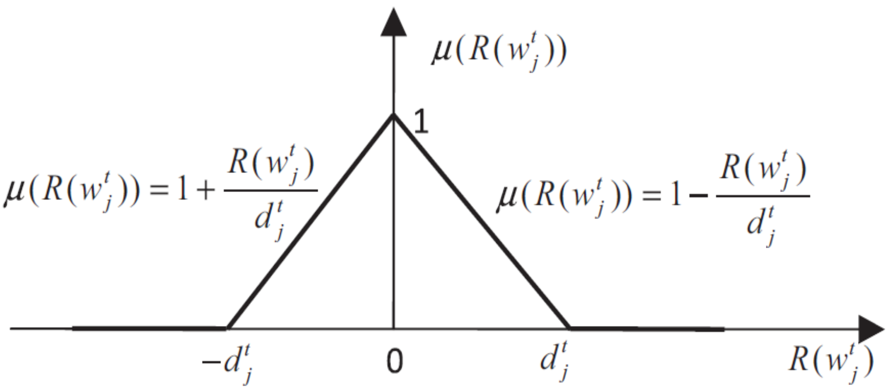

Figure 2. Membership function of the fuzzy equation $R(w_j^t) = w_B^t - w_j^t a_{Bj}^t \cong 0$.

A fuzzy decision S could be considered as a fuzzy set, $S = \{(\tilde{w}, \mu_S(\tilde{w})) | \tilde{w} \in W\}$, where

$$\mu_S(\tilde{w}) = \beta = min\{\mu(R(w_j^t)), \mu(Q(w_j^t)) | j = 1, 2, \dots, n; t = l, m, u\}, \tag{14}$$

Then, Eq. (14) are transformed into:

$$\begin{cases} \mu\big(R(w_j^t)\big) \geq \beta \; (j = 1, 2, \dots, n; t = l, m, u) \\ \mu(Q(w_j^t)) \geq \beta \; (j = 1, 2, \dots, n; t = l, m, u) \\ \qquad 0 \leq \beta \leq 1 \end{cases} \tag{15}$$

Where β indicates the minimum satisfaction degree of the fuzzy constraints. For obtaining the optimal fuzzy weight vector $\widetilde{\mathrm{w}}^* = [\widetilde{w}_1^*, \widetilde{w}_2^*, \dots, \widetilde{w}_n^*]$, the following mathematical programming model, which maximizes the minimum β is proposed (Dong et al., 2021).

$$\begin{aligned} & Max\ \beta \\ s.t. & \begin{cases} \mu\big(R(w_j^t)\big) \geq \beta \; (j = 1, 2, \dots, n; t = l, m, u) \\ \mu(Q(w_j^t)) \geq \beta \; (j = 1, 2, \dots, n; t = l, m, u) \\ \qquad 0 \leq \beta \leq 1 \end{cases} \end{aligned} \tag{16}$$

3.4.2. Solution of the constructed mathematical programming model

Since Eq. (12) and Eq. (13) are piecewise functions, the solution of Eq. (16) depends on the risk attitude of the decision-maker. Therefore, four approaches are proposed to solve Eq. (16).

- C_B is the best criterion, and its weight $\widetilde{w}_B$ should be the maximum. For $R(w_j^t) = w_B^t - w_j^t a_{Bj}^t \cong 0$, *an optimistic decision-maker* might believes $R(w_j^t) = w_B^t - w_j^t a_{Bj}^t > 0$ and for this selects $\mu\big(R(w_j^t)\big) = 1 - \frac{R(w_j^t)}{d_j^t}$ as the membership function, i.e., the right side of Fig. 2 and *a pessimistic decision-maker* might believes $R(w_j^t) = w_B^t - w_j^t a_{Bj}^t < 0$ and for this chooses $\mu\big(R(w_j^t)\big) = 1 + \frac{R(w_j^t)}{d_j^t}$ as the membership function, i.e., the left side of Fig. 2.
- C_W is the worst criterion, and its weight $\widetilde{w}_W$ should be the minimum. For $Q(w_j^t) = w_j^t - a_{jW}^t w_W^t \cong 0$, *an optimistic decision-maker* might believes $Q(w_j^t) = w_j^t - a_{jW}^t w_W^t > 0$ and for this chooses $\mu\big(Q(w_j^t)\big) = 1 - \frac{Q(w_j^t)}{q_j^t}$ as the membership function and *a pessimistic decision-maker* might believes $Q(w_j^t) = w_j^t - a_{jW}^t w_W^t < 0$ and for this selects $\mu\big(Q(w_j^t)\big) = 1 + \frac{Q(w_j^t)}{q_j^t}$ as the membership function.

There are also two cases for *a neutral decision-maker*.

- *Case 1-* The neutral decision-maker chooses $\mu\big(R(w_j^t)\big) = 1 - \frac{R(w_j^t)}{d_j^t}$ and $\mu\big(Q(w_j^t)\big) = 1 + \frac{Q(w_j^t)}{q_j^t}$ as the membership functions for the fuzzy equations $R(w_j^t) = w_B^t - w_j^t a_{Bj}^t \cong 0$ and $Q(w_j^t) = w_j^t - a_{jW}^t w_W^t \cong 0$ respectively.

- *Case 2-* The neutral decision-maker chooses $\mu\left(R(w_j^t)\right) = 1 + \frac{R(w_j^t)}{d_j^t}$ and $\mu\left(Q(w_j^t)\right) = 1 - \frac{Q(w_j^t)}{q_j^t}$ as the membership functions of the fuzzy equations $R(w_j^t) = w_B^t - w_j^t a_{Bj}^t \cong 0$ and $Q(w_j^t) = w_j^t - a_{jW}^t w_W^t \cong 0$ respectively.

For all decision-makers, by plugging $\mu\left(R(w_j^t)\right)$ of Eq. (12) and $\mu\left(Q(w_j^t)\right)$ of Eq. (13), which they have chosen according to their stances above, into Eq. (16), and Eq. (16) is converted into the following linear programming models.

(1) <u>Optimistic approach:</u>

$$\begin{aligned} &Max\ \beta \\ s.t.&\begin{cases} 1 - \frac{w_B^t - w_j^t a_{Bj}^t}{d_j^t} \geq \beta,\ 0 \leq w_B^t - w_j^t a_{Bj}^t \leq d_j^t\ (j = 1, 2, \dots, n; t = l, m, u) \\ 1 - \frac{w_j^t - a_{jW}^t w_W^t}{q_j^t} \geq \beta,\ 0 \leq w_j^t - a_{jW}^t w_W^t \leq q_j^t\ (j = 1, 2, \dots, n; t = l, m, u) \\ 0 \leq \beta \leq 1 \\ \sum_{j=1}^n w_j^m = 1,\ w_j^u + \sum_{i=1, i\neq j}^n w_i^l \ \leq\ 1, w_j^l + \sum_{i=1, i\neq j}^n w_i^u \ \geq\ 1\ (i, j = 1, 2, \dots, n) \end{cases} \end{aligned} \tag{17}$$

(2) <u>Pessimistic approach:</u>

$$\begin{aligned} &Max\ \beta \\ s.t.&\begin{cases} 1 + \frac{w_B^t - w_j^t a_{Bj}^t}{d_j^t} \geq \beta,\ -d_j^t \leq w_B^t - w_j^t a_{Bj}^t \leq 0\ (j = 1, 2, \dots, n; t = l, m, u) \\ 1 + \frac{w_j^t - a_{jW}^t w_W^t}{q_j^t} \geq \beta,\ -q_j^t \leq w_j^t - a_{jW}^t w_W^t \leq 0\ (j = 1, 2, \dots, n; t = l, m, u) \\ 0 \leq \beta \leq 1 \\ \sum_{j=1}^n w_j^m = 1,\ w_j^u + \sum_{i=1, i\neq j}^n w_i^l \ \leq\ 1, w_j^l + \sum_{i=1, i\neq j}^n w_i^u \ \geq\ 1\ (i, j = 1, 2, \dots, n) \end{cases} \end{aligned} \tag{18}$$

(3) <u>Mixed approach I:</u>

$$\begin{aligned} &Max\ \beta \\ s.t.&\begin{cases} 1 - \frac{w_B^t - w_j^t a_{Bj}^t}{d_j^t} \geq \beta,\ 0 \leq w_B^t - w_j^t a_{Bj}^t \leq d_j^t\ (j = 1, 2, \dots, n; t = l, m, u) \\ 1 + \frac{w_j^t - a_{jW}^t w_W^t}{q_j^t} \geq \beta,\ -q_j^t \leq w_j^t - a_{jW}^t w_W^t \leq 0\ (j = 1, 2, \dots, n; t = l, m, u) \\ 0 \leq \beta \leq 1 \\ \sum_{j=1}^n w_j^m = 1,\ w_j^u + \sum_{i=1, i\neq j}^n w_i^l \ \leq\ 1, w_j^l + \sum_{i=1, i\neq j}^n w_i^u \ \geq\ 1\ (i, j = 1, 2, \dots, n) \end{cases} \end{aligned} \tag{19}$$

(4) <u>Mixed approach II:</u>

$$\begin{aligned} &Max\ \beta \\ s.t.&\begin{cases} 1 + \frac{w_B^t - w_j^t a_{Bj}^t}{d_j^t} \geq \beta,\ -d_j^t \leq w_B^t - w_j^t a_{Bj}^t \leq 0\ (j = 1, 2, \dots, n; t = l, m, u) \\ 1 - \frac{w_j^t - a_{jW}^t w_W^t}{q_j^t} \geq \beta,\ 0 \leq w_j^t - a_{jW}^t w_W^t \leq q_j^t\ (j = 1, 2, \dots, n; t = l, m, u) \\ 0 \leq \beta \leq 1 \\ \sum_{j=1}^n w_j^m = 1,\ w_j^u + \sum_{i=1, i\neq j}^n w_i^l \ \leq\ 1, w_j^l + \sum_{i=1, i\neq j}^n w_i^u \ \geq\ 1\ (i, j = 1, 2, \dots, n) \end{cases} \end{aligned} \tag{20}$$

The optimal weight vector $\widetilde{\mathrm{w}}^*$ can be attained by solving Eqs. (17) -(20) separately for all decision-makers. Each of Eqs. (17)–(20) should have a unique optimal solution if the tolerance parameters values d_j^t and q_j^t $(j = 1, 2, \dots, n; t = l, m, u)$ are big enough, and the bigger values of d_j^t and q_j^t, provide the bigger value for the optimal objective value β^*. As per Eq. (14), if the attained optimal objective value β^*= 1, then all of the criteria comparisons are fully consistent, and thus, β^*can be utilized to measure the consistency level of the criteria comparisons (Dong et al., 2021; Pamucar & Savin, 2020; Pamucar & Dimitrijevic, 2021).

3.4.3. Fuzzy consistency index

A comparison is fully consistent when $\tilde{a}_{Bj} \times \tilde{a}_{jW} = \tilde{a}_{BW}$ for all $j = 1, 2, \dots, n$. On the other hand, it is possible for some j which lead to not fully consistent, that is, the following inequality applies (Guo & Zhao, 2017):

$$\tilde{a}_{Bj} \times \tilde{a}_{jW} \neq \tilde{a}_{BW} \tag{21}$$

Let $\tilde{\zeta} = (\zeta^l, \zeta^m, \zeta^u)$ be a triangular fuzzy number that must be subtracted from $\tilde{a}_{Bj} = (a_{Bj}^l, a_{Bj}^m, a_{Bj}^u)$ and $\tilde{a}_{jW} = (a_{jW}^l, a_{jW}^m, a_{jW}^u)$ of Eq. (21) and added to $\tilde{a}_{Bw} = (a_{BW}^l, a_{BW}^m, a_{BW}^u)$ of Eq. (21) to obtain the highest inequality of Eq. (21) (Dong et al., 2021). That is;

$$\left((a_{Bj}^l, a_{Bj}^m, a_{Bj}^u) - (\zeta^l, \zeta^m, \zeta^u)\right) \times \left((a_{jW}^l, a_{jW}^m, a_{jW}^u) - (\zeta^l, \zeta^m, \zeta^u)\right) = (a_{BW}^l, a_{BW}^m, a_{BW}^u) + (\zeta^l, \zeta^m, \zeta^u) \tag{22}$$

As for the minimum consistency $\tilde{a}_{Bj} = \tilde{a}_{jW} = \tilde{a}_{BW}$;

$$\left((a_{BW}^l, a_{BW}^m, a_{BW}^u) - (\zeta^l, \zeta^m, \zeta^u)\right) \times \left((a_{BW}^l, a_{BW}^m, a_{BW}^u) - (\zeta^l, \zeta^m, \zeta^u)\right) = (a_{BW}^l, a_{BW}^m, a_{BW}^u) + (\zeta^l, \zeta^m, \zeta^u) \tag{23}$$

With regards to the operation rules of triangular fuzzy numbers, Eq. (23) could be rewritten as follows (Dong et al., 2021):

$$(a_{BW}^l - \zeta^u, a_{BW}^m - \zeta^m, a_{BW}^u - \zeta^l) \times (a_{BW}^l - \zeta^u, a_{BW}^m - \zeta^m, a_{BW}^u - \zeta^l) = (a_{BW}^l + \zeta^l, a_{BW}^m + \zeta^m, a_{BW}^u + \zeta^u) \tag{24}$$

$$\rightarrow ((a_{BW}^l - \zeta^u)^2, (a_{BW}^m - \zeta^m)^2, (a_{BW}^u - \zeta^l)^2) = (a_{BW}^l + \zeta^l, a_{BW}^m + \zeta^m, a_{BW}^u + \zeta^u) \tag{25}$$

Thus, the following equations can be derived (Dong et al., 2021):

$$\begin{cases} (a_{BW}^l - \zeta^u)^2 = a_{BW}^l + \zeta^l \\ (a_{BW}^m - \zeta^m)^2 = a_{BW}^m + \zeta^m \\ (a_{BW}^u - \zeta^l)^2 = a_{BW}^u + \zeta^u \end{cases} \tag{26}$$

After solving Eq. (26), the FCI $\tilde{\zeta} = (\zeta^l, \zeta^m, \zeta^u)$ is attained for different $\tilde{a}_{BW}$ values, as displayed in Table 2.

Table 2. FCI for fuzzy BWM.

$\tilde{a}_{BW}$	(1, 1, 1)	(2/3, 1, 3/2)	(3/2, 2, 5/2)	(5/2, 3, 7/2)	(7/2, 4, 9/2)
FCI ($\tilde{\zeta}$)	*(0, 0, 0)*	*(0, 0, 1.36)*	*(0.34, 0.44, 2.16)*	*(0.71, 1, 4.29)*	*(1.31, 1.63, 5.69)*
$\tilde{a}_{BW}$	(9/2, 5, 11/2)	(11/2, 6, 13/2)	(13/2, 7, 15/2)	(15/2, 8, 17/2)	(17/2, 9, 19/2)
FCI ($\tilde{\zeta}$)	*(1.96, 2.30, 7.04)*	*(2.65, 3, 8.35)*	*(3.36, 3.73, 9.64)*	*(4.09, 4.47, 10.91)*	*(4.85, 5.23, 12.15)*

3.4.4. Fuzzy consistency ratio

To determine the fuzzy consistency ratio (FCR), we require to minimize the maximum among the deviation between $\widetilde{w}_B^*$ and $\widetilde{w}_j^* \tilde{a}_{Bj}$and the deviation between $\widetilde{w}_j^*$ and $\tilde{a}_{jW}\widetilde{w}_W^*$ for all $j = 1, 2, \ldots, n$. That is, to calculate $minmax\{|\widetilde{w}_B^* - \widetilde{w}_j^* \tilde{a}_{Bj}|,|\widetilde{w}_j^* - \tilde{a}_{jW}\widetilde{w}_W^*|\}$ that is represented by $\tilde{\xi}^* = (\xi^{*l}, \xi^{*m}, \xi^{*u})$. Since subtraction, multiplication, and absolute operations of triangular fuzzy numbers are approximate operations, to attain the exact fuzzy deviation $\tilde{\xi}^* = (\xi^{*l}, \xi^{*m}, \xi^{*u})$ is difficult. Therefore, the approach below is proposed to determine the fuzzy deviation.

$$\xi'^{l} = \frac{1}{2n}\sum_{j=1}^{n}\left(\left|w_B^{*l} - w_j^{*l}a_{Bj}^{l}\right| + \left|w_j^{*l} - a_{jW}^{l}w_W^{*l}\right|\right), \tag{27}$$

$$\xi'^{m} = \frac{1}{2n}\sum_{j=1}^{n}\left(\left|w_B^{*m} - w_j^{*m}a_{Bj}^{m}\right| + \left|w_j^{*m} - a_{jW}^{m}w_W^{*m}\right|\right), \tag{28}$$

$$\xi'^{u} = \frac{1}{2n}\sum_{j=1}^{n}\left(\left|w_B^{*u} - w_j^{*u}a_{Bj}^{u}\right| + \left|w_j^{*u} - a_{jW}^{u}w_W^{*u}\right|\right), \tag{29}$$

Where ξ'^{l}, ξ'^{m} and ξ'^{u} describe the possible lower bound, mode and upper bound of the fuzzy deviation $\tilde{\xi}^* = (\xi^{*l}, \xi^{*m}, \xi^{*u})$, respectively. To ensure $\xi^{*l} \le \xi^{*m} \le \xi^{*u}$, i.e., the attained $\tilde{\xi}^* = (\xi^{*l}, \xi^{*m}, \xi^{*u})$ is a triangular fuzzy number, it is taken;

$$\tilde{\xi}^{*l} = \min\{\xi'^{l}, \xi'^{m}, \xi'^{u}\}, \quad \tilde{\xi}^{*m} = \mathrm{median}\{\xi'^{l}, \xi'^{m}, \mathrm{v}'^{u}\}, \quad \tilde{\zeta}^{*u} = \max\{\xi'^{l}, \xi'^{m}, \xi'^{u}\}, \tag{30}$$

The aim of Eq. (30) is to assure $\xi^{*l} \le \xi^{*m} \le \xi^{*u}$, such that the fuzzy deviation $\tilde{\xi}^* = (\xi^{*l}, \xi^{*m}, \xi^{*u})$ is a triangular fuzzy number.

FCR is identified as

$$FCR = \frac{\tilde{\xi}^*}{\tilde{\zeta}} \tag{31}$$

Where is demonstrated in Table 2 and $\tilde{\xi}^* = (\xi^{*l}, \xi^{*m}, \xi^{*u})$ is attained by Eq. (30). By the operation rules of triangular fuzzy numbers;

$$FCR = \frac{\tilde{\xi}^*}{\tilde{\zeta}} = \frac{(\xi^{*l}, \xi^{*m}, \xi^{*u})}{(\zeta^{l}, \zeta^{m}, \zeta^{u})} = (\frac{\xi^{*l}}{\zeta^{u}}, \frac{\xi^{*m}}{\zeta^{m}}, \frac{\xi^{*u}}{\zeta^{l}}) \tag{32}$$

Then, based on Eq. (2), we could calculate GMIR R(FCR) of the FCR;

$$R(FCR) = \frac{1}{6}(\frac{\xi^{*l}}{\zeta^{u}}, 4\frac{\xi^{*m}}{\zeta^{m}}, \frac{\xi^{*u}}{\zeta^{l}}) \tag{33}$$

- If $R(FCR) \le 0.1$, then the comparisons are acceptable consistent.
- If $R(FCR) = 0$, then all comparisons are fully consistent.
- If $R(FCR) > 0.1$, then the comparisons are unacceptable consistent and some of the comparisons must be identified to be adjusted until $R(FCR) \le 0.1$ (Dong et al., 2021). For this, the identification and adjustment processes are described in detail in Dong et al. (2021).

3.4.5. Steps of fuzzy BWM

Step 1: Determine a set $C = (C_1, C_2, \ldots, C_n)$ of decision criteria.

Step 2: Determine the best (e.g., the most important or the most desirable) criterion C_B and the worst (e.g., the least important or the least desirable) criterion C_W.

Step 3: Determine the fuzzy preference of the best criterion overall the other criteria using the linguistic terms and triangular fuzzy numbers listed in Table 3. The resulting Best-to-Others vector would be $\tilde{A}_B = [\tilde{a}_{B1}, \tilde{a}_{B2}, \dots, \tilde{a}_{Bn}]$ where $\tilde{a}_{Bj}$ demonstrates the fuzzy preference of the best criterion C_B over criterion C_j. $\tilde{a}_{Bj} = \left(a_{Bj}^l, a_{Bj}^m, a_{Bj}^u\right), j = 1, 2, \dots, n$ and $\tilde{a}_{BB} = (1, 1, 1)$.

Step 4: Determine the fuzzy preference of all the other criteria over the worst criterion using the linguistic terms and triangular fuzzy numbers listed in Table 3. The resulting Others-to-Worst vector would be $\tilde{A}_W = [\tilde{a}_{1W}, \tilde{a}_{2W}, \dots, \tilde{a}_{nW}]$ where $\tilde{a}_{jW}$ indicates the fuzzy preference of criterion C_j over the worst criterion C_W. $\tilde{a}_{jW} = \left(a_{jW}^l, a_{jW}^m, a_{jW}^u\right), j = 1, 2, \dots, n$ and $\tilde{a}_{WW} = (1, 1, 1)$ (Dong et al., 2021; Guo & Zhao, 2017; Rezaei, 2015).

Step 5: Determine the suitable tolerance parameters' values d_j^t and q_j^t $(j = 1, 2, \dots, n; t = l, m, u)$ for Eqs. (17)– (20) according to your preference and the decision-making problems' characteristics. In general, d_j^t and q_j^t can take any values from the interval [1, 9].

Step 6: Solve one of Eqs. (17)– (20) according to the risk attitude (i.e., pessimistic, optimistic or neutral) of the decision-maker to get the optimal fuzzy weight vector $\widetilde{w}^* = [\widetilde{w}_1^*, \widetilde{w}_2^*, \dots, \widetilde{w}_n^*]$ and the optimal objective value β^* by using a mathematical software.

Step 7: Compute $\tilde{\xi}^* = (\xi^{*l}, \xi^{*m}, \xi^{*u})$ by Eq. (30).

Step 8: Attain the FCI by Table 2 and calculate FCR by Eq. (32).

Step 9: Calculate GMIR R(FCR) of the attained FCR by Eq. (33).

Step 10: Check the consistency (Dong et al., 2021).

In the step of final ranking of alternatives, with optimal defuzzified weights of the criteria and the normalized scores of the alternatives on the different criteria, X_{ij}; the final aggregate score per alternative, V_i; could be calculated using Eq. (34) (Rezaei et al., 2016);

$$V_i = \sum_j W_j \mathrm{X}_{ij} \tag{34}$$

$$\mathrm{X}_{ij} = \begin{cases} \dfrac{\mathrm{X}_{ij}}{\max\{\mathrm{X}_{ij}\}}, if\ x\ is\ positive(such\ as\ quality), \\ 1 - \dfrac{\mathrm{X}_{ij}}{\max\{\mathrm{X}_{ij}\}}, if\ x\ is\ negative(such\ as\ price). \end{cases} \tag{35}$$

4. Problem Definition

An aircraft selection model was designed to provide training aircraft for a Flight Training department that will start education. For this, in the study, potential training aircraft will be evaluated in terms of important criteria for decision-makers and will decide the number of aircraft to be purchased, taking into account the current constraints and the aircraft weights obtained.

The Flight Training Department is a 4-year undergraduate department that trains professional pilots with the necessary skills, competence, theoretical and practical experience to meet national and international airline companies' needs. In addition to laboratories, students carry out their practical training using training aircraft. Pilots who graduate from this department can find the opportunity to work in many general aviation sectors, especially in domestic and foreign airlines. To deal with this decision problem, that is essential to consider the candidate aircraft and analyze the criteria requirements representing the technical specification that the aircraft should have in-depth.

This study suggests a two-stage solution approach for aircraft evolution. In the first stage, fuzzy BWM is used to get the weights of criteria and then aircraft ranking. Then, the linear programming formulation of aircraft selection is constructed. In the second stage, aircraft' weights (priority scores) are combined into the linear programming model with some resource constraints to determine the optimal order number of aircraft. The aircraft' weights are utilized as coefficients in the objective function to increase purchasing value and how much will be ordered from which plane is determined. All information such as budget, total flying time, and total fuel consumption was assumed fixed and already known.

4.1. Mathematical Model

Index:

i: Set of aircraft $(i = 1, 2, \dots, N)$

Parameters:
P_i: *Unit purchasing cost of aircraft i*
T_i : *Actual flying time before overhaul of aircraft i*
F_i : *Fuel cost per mile of aircraft i*
W_i: *The weight (priority value) of the aircraft i*
B: Total budget allocated to aircraft
M: Total flying time before overhaul
Y: Total fuel consumption cost per mile

Decision Variable:

X_i : *Number of aircraft i*

Objective Function:

$$Max\ Z = \sum_i^N W_i\, X_i \tag{36}$$

Constraints:

$$\sum_i^N P_i X_i \le B \tag{37}$$
$$\sum_i^N T_i\, X_i \ge M \tag{38}$$
$$\sum_i^N F_i\, X_i \le Y \tag{39}$$
$$X_i \ge 0 \text{ and integer }, \quad \forall i \tag{40}$$

The objective function (36) maximizes the total purchasing value. In other words, it allows purchasing aircraft with a higher weight, which means that the best aircraft, according to the criterion evaluation, will be purchased more. Constraint (37) is *the budget constraint* implies that the total purchasing cost of aircraft cannot exceed the allocated budget amount. Constraint (38) is *the minimum flying time before overhaul constraint* means that no aircraft need overhaul until the specified maintenance time, in other words, the flying time of the aircraft should be at least the minimum flying time. Constraint (39) is *the fuel consumption constraint* means that the amount of fuel burned by aircraft per mile should not exceed the budget allocated for it. Constraint (40) is *the non-negativity and integrity constraint*.

5. Case Study: Necmettin Erbakan University, Faculty of Aviation and Space Sciences

Necmettin Erbakan University, Faculty of Aviation and Space Sciences, was established in 2010. Flight Training Undergraduate Program is a department of the Faculty of Aviation and Space Sciences. The faculty also includes aircraft engineering, space and satellite engineering, and aviation management departments. Departments other than the Flight Training department actively give education. The Flight Training department has not started education yet, as it does not have sufficient infrastructure. The university wants to activate its Flight Training department and so should be meet needs such as runway, training aircraft, instructors, maintenance technicians. Therefore, for this reason, the university should first determine examining suitable criteria and alternatives, and expert decision-makers should evaluate training aircraft alternatives, and this is the subject of our study.

The criteria and alternatives for the problem were examined in line with the fuzzy BWM, and the evaluation process was initiated by the instructors' committee consisting of experts. After the preliminary screening, the group of experts (instructors of the Flight Training department) identified nine specification criteria (Max Cruise Speed, Max Range, Take-off and Landing Ground Roll, Max Climb Rate, Power Output, Empty Weight, Price, Useful Fuel Capacity, Time Before Overhand) and eight training aircraft (Cessna Skyhawk SP (172S), Cessna Skylane (182T), Cessna Turbo Stationair HD (206), Cirrus SR22, Cirrus SR20, Diamond DA62, Diamond DA40 NG, Diamond DA42) for the further evaluation process.

5.1. Implementing The Fuzzy BWM

The importance weights for criteria are described with linguistic variables by the decision-makers. The linguistic expressions and their corresponding triangular fuzzy numbers indicating the importance ratings of criteria are given in Table 3.

Table 3. Linguistic variables and fuzzy numbers used in the evaluation of criteria (Dong et al., 2021; Gan et al., 2019; Guo & Zhao, 2017).

Linguistic variables for the importance weight of each criterion	
Linguistic Variables	**Triangular Fuzzy Numbers**
Equally Important (EI)	(1, 1, 1)
Weakly Important (WI)	(2/3, 1, 3/2)
Intermediate-*Weakly to Moderately Important*- (WM)	(3/2, 2, 5/2)
Moderately Important (MI)	(5/2, 3, 7/2)
Intermediate-*Moderately to Strongly Important*- (MS)	(7/2, 4, 9/2)
Strongly Important (SI)	(9/2, 5, 11/2)
Intermediate-*Strongly to Very Important*- (SV)	(11/2, 6, 13/2)
Very Important (VI)	(13/2, 7, 15/2)
Intermediate-*Very to Extremely Important*- (VE)	(15/2, 8, 17/2)
Extremely Important (EEI)	(17/2, 9, 19/2)

The best criterion is the most important one, while the worst criterion is the least important one in aircraft selection based on the opinion of an expert/decision-maker. As a result of interviews with experts, the best criterion was determined as Price, and the worst criterion was determined as Empty Weight for aircraft selection. Next, they performed the criteria comparison by filling out a survey based on the application of the fuzzy BWM, as shown in Table 4.

Table 4. Pairwise comparison vectors for best and worst criteria

BO	Max Cruise Speed	Max Range	Take-off and Landing Ground Roll	Max Climb Rate	Power Output	Empty Weight	Price	Useful Fuel Capacity	Time Before Overhand
Best objective functions: *Price*	VI	MS	WM	VI	MS	EEI	EI	SV	SI

OW	Worst objective functions: *Empty Weight*
Max Cruise Speed	MI
Max Range	SV
Take-off and Landing Ground Roll	VE
Max Climb Rate	MS
Power Output	SV
Empty Weight	EI
Price	EEI
Useful Fuel Capacity	MI
Time Before Overhand	MS

Then, the fuzzy Best-to-Others vector, $\tilde{A}_B = [\tilde{a}_{B1}, \tilde{a}_{B2}, \dots, \tilde{a}_{B9}]$ where $\tilde{a}_{B1}$= (13/2, 7, 15/2), $\tilde{a}_{B2}$= (7/2, 4, 9/2), $\tilde{a}_{B3}$= (3/2, 2, 5/2), $\tilde{a}_{B4}$= (13/2, 7, 15/2), $\tilde{a}_{B5}$= (7/2, 4, 9/2), $\tilde{a}_{B6}$= (17/2, 9, 19/2), $\tilde{a}_{B7}$= (1, 1, 1), $\tilde{a}_{B8}$= (11/2, 6, 13/2) and $\tilde{a}_{B9}$= (9/2, 5, 11/2), and the fuzzy Others-to-Worst vector, $\tilde{A}_W = [\tilde{a}_{1W}, \tilde{a}_{2W}, \dots, \tilde{a}_{9W}]$ where $\tilde{a}_{1W}$= (5/2, 3, 7/2), $\tilde{a}_{2W}$= (11/2, 6, 13/2), $\tilde{a}_{3W}$= (15/2, 8, 17/2), $\tilde{a}_{4W}$= (7/2, 4, 9/2), $\tilde{a}_{5W}$= (11/2, 6, 13/2), $\tilde{a}_{6W}$= (1, 1, 1), $\tilde{a}_{7W}$= (17/2, 9, 19/2), $\tilde{a}_{8W}$= (5/2, 3, 7/2) and $\tilde{a}_{9W}$= (7/2, 4, 9/2) are obtained according to Table 4. The values of all tolerance parameters d_j^t and q_j^t $(j = 1, 2, \dots, n; t = l, m, u)$ are taken as 1.

Four separate linear programming models are constructed for all decision-makers by putting fuzzy preferences $\tilde{a}_{Bj}, \tilde{a}_{jW}$ and tolerance parameters d_j^t , q_j^t given above into Eqs. (17)-(20), and the optimal fuzzy weights $\tilde{w}_j^*$ and minimal satisfaction degrees β are obtained by using the GAMS/CPLEX 24.0 software separately for all approaches as seen in Table 5. Then the values of the fuzzy deviations $\tilde{\xi}^*$, FCR and R(FCR) for all approaches are calculated separately based on Eqs. (30), (32), and (33) respectively and presented in Table 5.

Since all R(FCR) < 0.1, obtained according to the four approaches, it can be seen that the comparisons for all decision makers' approaches are acceptably consistent. In our study, considering that the decision-maker has the optimistic approach with the best consistency, these weights obtained by this approach will be used in the ranking. In this context, the optimistic approach's fuzzy weights are defuzzified using Eq. (2) as follows, and thus they are ready for use in the next step.

$\widetilde{w}_1^*$= 0.052, $\widetilde{w}_2^*$= 0.087, $\widetilde{w}_3^*$= 0.161, $\widetilde{w}_4^*$= 0.052, $\widetilde{w}_5^*$= 0.083, $\widetilde{w}_6^*$= 0.012, $\widetilde{w}_7^*$= 0.396, $\widetilde{w}_8^*$= 0.060 and $\widetilde{w}_9^*$= 0.087.

Table 5. Criteria *weights* ($\tilde{w}_j^*$) *and* minimal satisfaction degrees (*β)* according to the attitude of the decision-maker

Criteria	*The attitude of the decision-maker*			
	Optimistic approach	Pessimistic approach	Mixed approach I	Mixed approach II
Max Cruise Speed	(0.042, 0.054, 0.054)	(0.043, 0.063, 063)	(0.025, 0.033, 0.054)	(0.060, 0.069, 0.077)
Max Range	(0.077, 0.089, 0.089)	(0.081, 0.085, 0.105)	(0.048, 0.091, 0.091)	(0.111, 0.129, 0.129)
Take-off Ground Roll	(0.161, 0. 161, 0. 161)	(0.188, 0.188, 0.188)	(0.120, 0.163, 0.163)	(0.214, 0. 231, 0. 231)
Max Climb Rate	(0.042, 0.054, 0.054)	(0.043, 0.063, 0.063)	(0.025, 0.054, 0.054)	(0.060, 0.077, 0.077)
Power Output	(0.077, 0.083, 0.089)	(0.081, 0.085, 0.105)	(0.048, 0.091, 0.091)	(0.111, 0.129, 0.129)
Empty Weight	(0.012, 0.012, 0.012)	(0.033, 0.036, 0.036)	(0.043, 0.043, 0.043)	(0.017, 0.017, 0.017)
Price	(0.390, 0. 396, 0. 402)	(0.283, 0.322, 0.342)	(0.366, 0. 387, 0. 409)	(0.146, 0. 154, 0. 163)
Useful Fuel Capacity	(0.050, 0.062, 0.062)	(0.051, 0.072, 0.072)	(0.039, 0.063, 0.063)	(0.072, 0.089, 0.089)
Time Before Overhand	(0.077, 0.089, 0.089)	(0.063, 0.086, 0.086)	(0.074, 0.074, 0.074)	(0.105, 0.105, 0.105)
β	0.711	0.871	0.798	0.584
$\tilde{\xi}^*$	(0.084, 0.057, 0.050)	(0.026, 0.063, 0.088)	(0.012, 0.063, 0.078)	(0.108, 0.157, 0.181)
FCR	(0.007, 0.011, 0.010)	(0.002, 0.012, 0.018)	(0.010, 0.012, 0.016)	(0.009, 0.030, 0.037)
R(FCR)	0.010	0.011	0.012	0.028
FCI($\tilde{\zeta}$)	(4.85, 5.23, 12.15) - from Table 2, because $a_{BW}=a_{76}=$ (17/2, 9, 19/2)			

The following steps have been implemented to establish the aircraft selection framework;

First, the aircraft performance on the different criteria is determined by its service centers and decision-makers of the relevant case institution using the technical specification data for aircraft. The scores of aircraft alternatives are shown in Table 6. The data of the training aircraft were obtained in light of the information shared on the web pages of the manufacturing companies (Cessna Aircraft, 2021; Circus Aircraft, 2021; Diamond Aircraft, 2021).

Table 6. Decision Matrix of aircraft performance

	Specification Criteria	1	2	3	4	5	6	7	8	9
	Aircraft Alternatives	Max Cruise Speed (ktas)	Max Range (nm)	Take-off and Landing Ground Roll (ft)	Max Climb Rate (fpm)	Power Output (hp)	Empty Weight (lbs)	Price ($)	Useful Fuel Capacity (gal)	Time Before Overhaul (hours)
1	Cessna Skyhawk SP (172S)	124	640	960	730	180	1690	415000	53	2000
2	Cessna Skylane (182T)	145	915	795	924	230	2000	530000	87	2000
3	Cessna Turbo Stationair HD (206)	161	703	1060	960	310	2365	745000	87	2000
4	Cirrus SR22	183	1169	1082	1270	310	2272	654900	92	2000
5	Cirrus SR20	155	709	1685	781	215	2122	474900	56	2000
6	Diamond DA62	192	1283	1574	1029	180	3505	1290000	86	1800
7	Diamond DA40 NG	154	940	1214	1690	180	1984	535000	48	2000
8	Diamond DA42	197	1215	919	1550	168	3109	869000	76,4	1800

Then, the aircraft scores are normalized using Eq. (35). The normalized scores are summarized in Table 7.

Table 7. Normalized Decision Matrix

Specification Criteria		1	2	3	4	5	6	7	8	9
Aircraft Alternatives		Max Cruise Speed (ktas)↑	Max Range (nm)↑	Take-off and Landing Ground Roll (ft)↓	Max Climb Rate (fpm)↑	Power Output (hp)↑	Empty Weight (lbs)↓	Price ($)↓	Useful Fuel Capacity (gal)↑	Time Before Overhaul (hours)↑
1	Cessna Skyhawk SP (172S)	0.629	0.499	0.430	0.432	0.581	0.518	0.678	0.576	1.000
2	Cessna Skylane (182T)	0.736	0.713	0.528	0.547	0.742	0.429	0.589	0.946	1.000
3	Cessna Turbo Stationair HD (206)	0.817	0.548	0.371	0.568	1.000	0.325	0.422	0.946	1.000
4	Cirrus SR22	0.929	0.911	0.358	0.751	1.000	0.352	0.492	1.000	1.000
5	Cirrus SR20	0.787	0.553	0.000	0.462	0.694	0.395	0.632	0.609	1.000
6	Diamond DA62	0.975	1.000	0.066	0.609	0.581	0.000	0.000	0.935	0.900
7	Diamond DA40 NG	0.782	0.733	0.280	1.000	0.581	0.434	0.585	0.522	1.000
8	Diamond DA42	1.000	0.947	0.455	0.917	0.542	0.113	0.326	0.830	0.900
Weights of Criteria (Optimistic)		0.052	0.087	0.161	0.052	0.083	0.012	0.396	0.060	0.087

Finally, weighted normalized scores (Table 8) and then the overall scores of the alternatives are found using Eq. (34) and, the result is summarized in Table 9.

Table 8. Weighted Normalized Decision Matrix

Specification Criteria		1	2	3	4	5	6	7	8	9
Aircraft Alternatives		Max Cruise Speed (ktas)	Max Range (nm)	Take-off and Landing Ground Roll (ft)	Max Climb Rate (fpm)	Power Output (hp)	Empty Weight (lbs)	Price ($)	Useful Fuel Capacity (gal)	Time Before Overhaul (hours)
1	Cessna Skyhawk SP (172S)	0.033	0.043	0.069	0.022	0.048	0.006	0.269	0.035	0.087
2	Cessna Skylane (182T)	0.038	0.062	0.085	0.028	0.062	0.005	0.233	0.057	0.087
3	Cessna Turbo Stationair HD (206)	0.042	0.048	0.060	0.030	0.083	0.004	0.167	0.057	0.087
4	Cirrus SR22	0.048	0.079	0.058	0.039	0.083	0.004	0.195	0.060	0.087
5	Cirrus SR20	0.041	0.048	0.000	0.024	0.058	0.005	0.250	0.037	0.087
6	Diamond DA62	0.051	0.087	0.011	0.032	0.048	0.000	0.000	0.056	0.078
7	Diamond DA40 NG	0.041	0.064	0.045	0.052	0.048	0.005	0.232	0.031	0.087
8	Diamond DA42	0.052	0.082	0.073	0.048	0.045	0.001	0.129	0.050	0.078

Table 9. Outranking of Alternative Aircraft

	Aircraft	Scores	Normal weights	Ranks
1	Cessna Skyhawk SP (172S)	0.605	0.134	3
2	Cessna Skylane (182T)	0.658	0.144	1
3	Cessna Turbo Stationair HD (206)	0.577	0.126	5
4	Cirrus SR22	0.653	0.143	2
5	Cirrus SR20	0.549	0.120	7
6	Diamond DA62	0.363	0.079	8
7	Diamond DA40 NG	0.605	0.132	4
8	Diamond DA42	0.559	0.122	6

The ranking of the aircraft in the order are *Cessna Skylane (182T), Cirrus SR22, Cessna Skyhawk SP (172S), Diamond DA40 NG, Cessna Turbo Stationair HD (206), Diamond DA42, Cirrus SR20* and, *Diamond DA62*.

The result of the proposed method, *Cessna Skylane (182T)* should be preferred if it is considered to purchase only one aircraft type.

In the next stage, to perform optimal order numbers, the aircraft's weights based on the results of fuzzy BWM will be used as coefficients in the proposed linear programming model.

5.2. Implementing The Proposed Model

Aircraft quantitative information ($\boldsymbol{P_i} = \text{Price(\$)}$, $\boldsymbol{T_i} = \text{Time Before Overhaul(hours)}$, $\boldsymbol{F_i} = \frac{\text{Useful Fuel Capacity}}{\text{Max Range}} * 6{,}44(\$/nm)$) is given in Table 6 (1 gallon fuel fee is calculated as 6,44$), and the weights of aircraft have been obtained as a result of the fuzzy BWM.

Additionally, the maximum acceptable total budget value (***B***) is taken 6000000 $ and the total flying time before overhaul value (***M***) is taken 25000 h, and the maximum acceptable total fuel consumption cost per mile value (***Y***) is taken 1 $/nm in the model. The linear formulation of the case problem is presented as:

$$Z_{max} = 0.134x_1 + 0.144x_2 + 0.126x_3 + 0.143x_4 + 0.120x_5 + 0.079x_6 + 0.132x_7 + 0.122x_8$$

Subject to;

$$415000x_1 + 530000x_2 + 745000x_3 + 654900x_4 + 474900x_5 + 1290000x_6 + 535000x_7 + 869000x_8 \leq 6000000$$

$$2000x_1 + 2000x_2 + 2000x_3 + 2000x_4 + 2000x_5 + 1800x_6 + 2000x_7 + 1800x_8 \geq 25000$$

$$0.535x_1 + 0.612x_2 + 0.799x_3 + 0.509x_4 + 0.509x_5 + 0.431x_6 + 0.328x_7 + 0.406x_8 \leq 1$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0, x_7 \geq 0, x_8 \geq 0$$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$ are integer.

The linear programming model was solved by GAMS/CPLEX 24.0 software package in accordance with these data, and the following results were obtained in Table 10.

Table 9. The optimal solution.

Z_{max}	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1.746	9	1	0	0	0	0	3	0

According to the solution results, it has been decided to purchase 13 aircraft, nine from Cessna Skyhawk SP (172S), one from Cessna Skylane (182T), and three from Diamond DA40 NG aircraft. The proposed model decided to purchase the Cessna Skylane (182T) aircraft with the largest weight, Cessna Skyhawk SP (172S) aircraft with 3rd weight, and Diamond DA40 NG aircraft with 4th weight to maximize the total value of purchasing. On the other hand, the reason for not buying the Cirrus SR22 aircraft, the 2nd in the weight ranking, is that its price value is higher than our budget constraint. Instead of the Cirrus SR22 aircraft, the reason for choosing the Diamond DA40 NG aircraft is that it has a low price and low fuel consumption.

Cessna Turbo Stationair HD (206), mainly because of its high fuel consumption, Diamond DA62 and Diamond DA42 because of high purchase prices, were not preferred.

As a result, it can be said that 13 aircraft will be sufficient for the Flight Training department to start education, and the university is in a position to cover all expenses of the aircraft to be purchased, such as purchasing, maintenance, and fuel with its existing resources.

6. Conclusions

Aircraft selection is a complex process and an important MCDM problem that considers various fundamental issues. In this context, an appropriate solution method should help top management efficiently evaluate various aircraft alternatives based on consistent criteria (Yilmaz et al., 2020). In this paper, we offered a linear programming model for the training aircraft selection problem. We specified the important specification and considered them as aircraft selection criteria. The problem maximizes the number of best aircraft (resulting from the criterion evaluation) to be purchased, while satisfying the budget requirement, fuel consumption limit, and flying time performance constraints. Firstly, we used fuzzy BWM to get the weights for criteria and then used them to evaluate the possible aircraft. Next, we improved a linear programming model to attain the optimum solution for the problem. Finally, we validated the model with a case study by solving it via GAMS/CPLEX 24.0 software. We believe that the proposed model framework is sufficiently valid and strong and could be easily applied in applications for a wide variety of decision-making problems.

In the literature, the BWM method has proven useful in various problems, but it has been applied for the first time in training aircraft selection. This study can be evolved into a commercial aircraft selection study for airline companies by differentiating the criteria in the case study of the proposed approach.

The limitation of the study is that it is limited to nine criteria and eight aircraft alternatives. In future studies, different rankings can be obtained for alternatives by increasing the number of criteria, alternatives, and experts. In particular, all criteria cover the technical features of the aircraft. For example, qualifications such as flight training, the experience of decision-makers, and ergonomics can also be considered as criteria. Moreover, unlike the purchase cost, after the aircraft is purchased, costs such as operation and maintenance occur to the purchasing institution or person. However, this information is not provided by the manufacturers as open source. In addition, there are not enough studies in the literature on the selection of trainer aircraft. For this reason, accessible manufacturer data were considered in the training aircraft purchase process. The limitation of the proposed approach is that the input data expressed in linguistic terms is based on decision-makers' opinions and experiences and therefore includes subjectivity.

Decision-makers practically might not have complete and certain information about objectives and constraints; therefore, for future research, objective function and/or constraints such as budget can be considered fuzzy. Besides, other than the fuzzy BWM method used in this study, other MCDM methods can be used and compared in terms of suitability.

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References

Ahmed, S., Sivakumar, G., Kabir, G., & Ali, S. M. (2020). Regional aircraft selection integrating fuzzy analytic hierarchy process (fahp) and efficacy method. Journal of Production Systems and Manufacturing Science, 1, 63–86.

Ali, Y., Asghar, A., Muhammad, N., & Salman, A. (2017). Selection of a fighter aircraft to improve the effectiveness of air combat in the war on terror: Pakistan air force – A case in point. International Journal of the Analytic Hierarchy Process, 9.

Alosta, A., Elmansuri, O., & Badi, I. (2021). Resolving a location selection problem by means of an integrated AHP-RAFSI approach. *Reports in Mechanical Engineering*, *2*(1), 135-142.

Bakır, M., Akan, Ş., & Özdemir, E. (2021). Regional aircraft selection with fuzzy piprecia and fuzzy marcos: A case study of the Turkish airline industry. Facta Universitatis, Series: Mechanical Engineering, 19, 423–445.

Bas, E., Egrioglu, E., Yolcu, U., & Grosan, C. (2019). Type 1 fuzzy function approach based on ridge regression for forecasting. Granular Computing, 4, 629–637.

Cessna Aircraft. Cessna by textron aviation. (2021). https://cessna.txtav.com / Accessed 15 December 2021.

Chen, S.-H., & Hsieh, C. (2000). Representation, ranking, distance, and similarity of l-r type fuzzy number and application. Aust. J. Intell. Process. Syst., 6, 217–229.

Circus Aircraft. Circus aircraft. (2021). https://cirrusaircraft.com / Accessed 15 December 2021.

Diamond Aircraft. Diamond aircraft. (2021). https://www.diamondaircraft.com/en /Accessed 15 December 2021.

do Nascimento Maêda, S. M., de Araújo Costa, I. P., de Castro Junior, M. A. P., Fávero, L. P., de Araújo Costa, A. P., de Pina Corriça, J. V., … dos Santos, M. (2021). Multi-criteria analysis applied to aircraft selection by Brazilian Navy. Production, 31, 1–13.

Dong, J., Wan, S., & Chen, S.-M. (2021). Fuzzy best-worst method based on triangular fuzzy numbers for multi-criteria decision-making. Information Sciences, 547, 1080–1104.

Dožić, S., & Kalić, M. (2015a). Comparison of two mcdm methodologies in aircraft type selection problem. Transportation Research Procedia, 10, 910–919.

Dožić, S., & Kalić, M. (2015b). Three-stage airline fleet planning model. Journal of Air Transport Management, 46, 30-39.

Dožić, S., Lutovac, T., & Kalić, M. (2018). Fuzzy ahp approach to passenger aircraft type selection. Journal of Air Transport Management, 68, 165–175.

Gan, J., Zhong, S., Liu, S., & Yang, D. (2019). Resilient supplier selection based on fuzzy bwm and gmo-rtopsis under supply chain environment. Discrete Dynamics in Nature and Society, 2019, 1–14.

Gomes, L. F. A. M., de Mattos Fernandes, J. E., & de Mello, J. C. C. B. S. (2014). A fuzzy stochastic approach to the multicriteria selection of an aircraft for regional chartering. Journal of Advanced Transportation, 48, 223–237.

Guo, S., & Zhao, H. (2017). Fuzzy best-worst multi-criteria decision-making method and its applications. Knowledge-Based Systems, 121, 23–31.

Hoan, P., & Ha, Y. (2021). Aras-fucom approach for vpaf fighter aircraft selection. Decision Science Letters, 10, 53–62.

Ilgın, M. A. (2019). Aircraft selection using linear physical programming. Journal of Aeronautics and Space Technologies, 12, 121-128.

Karamaşa, Ç., Karabasevic, D., Stanujkic, D., Kookhdan, A. R., Mishra, A. R., & Ertürk, M. (2021). An extended single-valued neutrosophic ahp and multimoora method to evaluate the optimal training aircraft for flight training organizations. Facta Universitatis, Series: Mechanical Engineering, 19, 555–578.

Kargı, V. (2016). Supplier selection for a textile company using the fuzzy topsis method. Yönetim ve Ekonomi, 23, 789-903.

Kartika, A., & Hanani, A. D. (2019). Analytic hierarchy process (ahp) approach to aircraft type selection on high-frequency domestic routes. International Journal of Engineering and Advanced Technology, 8, 697- 704.

Kılıç, T. (2015). Nasıl Başardılar? AZ Kitap.

Kiracı, K., & Akan, E. (2020). Aircraft selection by applying ahp and topsis in interval type-2 fuzzy sets. Journal of Air Transport Management, 89, 101924.

Kiracı, K., & Bakır, M. (2018a). Application of commercial aircraft selection in aviation industry through multi-criteria decision making methods. Celal Bayar Üniversitesi Sosyal Bilimler Dergisi, 16, 307–332.

Kiracı, K., & Bakır, M. (2018b). Using the multi criteria decision making methods in aircraft selection problems and an application. Journal of Transportation and Logistics, 3, 13–24.

Kumar, D., Rahman, Z., & Chan, F. T. S. (2017). A fuzzy ahp and fuzzy multi-objective linear programming model for order allocation in a sustainable supply chain: A case study. International Journal of Computer Integrated Manufacturing, 30, 535–551.

Liao, M.-S., Liang, G.-S., & Chen, C.-Y. (2013). Fuzzy grey relation method for multiple criteria decision-making problems. Quality & Quantity, 47, 3065–3077.

Lienhard, J. H. No. 1910: Abbas Ibn Fırnas. (2019). https://uh.edu/engines/epi1910.htm

Lima Junior, F. R., Osiro, L., & Carpinetti, L. C. R. (2014). A comparison between fuzzy ahp and fuzzy topsis methods to supplier selection. Applied Soft Computing, 21, 194–209.

Liu, Q., & Wu, T. (2010). A comprehensive evaluation method of aircraft selection for commuter airlines based on iedea model. Integrated Transportation Systems: Green, Intelligent, Reliable, 2010.

Mello, J., Fernandes, J., & Gomes, L. (2012). Multicriteria selection of an aircraft with naiade. Icores 2020.

Muhammad, L. J., Badi, I., Haruna, A. A., & I.A. Mohammed. (2021). Selecting the Best Municipal Solid Waste Management Techniques in Nigeria Using Multi Criteria Decision Making Techniques. *Reports in Mechanical Engineering*, *2*(1), 180-189.

Ozdemir, Y., & Basligil, H. (2016). Aircraft selection using fuzzy anp and the generalized choquet integral method: The Turkish airlines case. Journal of Intelligent & Fuzzy Systems, 31, 589–600.

Pamucar, D. & Savin, L.M. (2020). Multiple-criteria model for optimal off-road vehicle selection for passenger transportation: BWM-COPRAS model. Military Technical Courier, 68(1), 28-64.

Pamucar, D., & Dimitrijevic, S.R. (2021). Multiple-criteria model for optimal anti tank ground missile weapon system procurement. Military Technical Courier, 69(4), 792-827.

Paul, S., Bose, S., Singha, J., Pramanik, D., Midya, R., & Haldar, A. (2017). Fighter aircraft selection using topsis method. National Conference on Recent Trends in Information Technology & Management 2017.

Petrovic, I., & Kankaraš, M. (2018). Dematel-ahp multi-criteria decision making model for the determination and evaluation of criteria for selecting an air traffic protection aircraft. Decision Making: Applications in Management and Engineering, 1, 93-110.

Rezaei, J. (2015). Best-worst multi-criteria decision-making method. Omega (United Kingdom), 53, 49–57.

Rezaei, J., Nispeling, T., Sarkis, J., & Tavasszy, L. (2016). A supplier selection life cycle approach integrating traditional and environmental criteria using the best worst method. Journal of Cleaner Production, 135, 577–588.

Sánchez-Lozano, J. M., & Rodríguez, O. N. (2020). Application of fuzzy reference ideal method (frim) to the military advanced training aircraft selection. Applied Soft Computing, 88, 106061.

Schwening, G. S., Abdalla, Á. M., & EESC-USP. (2014). Selection of agricultural aircraft using ahp and topsis methods in fuzzy environment. Congress of the International Council of Aeronautical Sciences 2014.

See, T.-K., Gurnani, A., & Lewis, K. (2004). Multi-attribute decision making using hypothetical equivalents and inequivalents. Journal of Mechanical Design, 126, 950–958.

Sun, X., Gollnick, V., & Stumpf, E. (2011). Robustness consideration in multi-criteria decision making to an aircraft selection problem. Journal of Multi-Criteria Decision Analysis, 18, 55–64.

Wang, T.-C., & Chang, T.-H. (2007). Application of topsis in evaluating initial training aircraft under a fuzzy environment. Expert Systems with Applications, 33, 870–880.

Webber, D. (2013). Space tourism: Its history, future and importance. Acta Astronautica, 92, 138–143.

Yeh, C.-H., & Chang, Y.-H. (2009). Modeling subjective evaluation for fuzzy group multicriteria decision making. European Journal of Operational Research, 194, 464–473.

Yilmaz, A. K., Malagas, K., Jawad, M., & Nikitakos, N. (2020). Aircraft selection process with technique for order preference by similarity to ideal solution and ahp integration. International Journal of Sustainable Aviation, 6, 220–235.

Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8, 338–353.