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APPLICATION OF HYBRID MODEL FUZZY AHP - VIKOR IN SELECTION OF THE MOST EFFICIENT PROCEDURE FOR RECTIFICATION OF THE OPTICAL SIGHT OF THE LONG-RANGE RIFLE

Marko Radovanović[1*], Aca Ranđelović[1] and Željko Jokić[1]

[1] University of Defence, Military academy, Belgrade, Serbia

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Abstract: *The paper presents a decision support model when choosing the most efficient rectification procedure of the optical sight of the long - range rifle. The model is based on the fuzzy AHP method and the VIKOR method. Using the fuzzy AHP method, coefficient values of the criteria were defined. Fuzzification of the AHP method was performed by combining data obtained from experts - comparison of criteria in pairs and the degree of confidence in the comparison. Using the VIKOR method, the best alternative was selected. Through the paper, the criteria that condition this choice are elaborated and the application of the method in a specific situation is presented. Also, the paper presents the sensitivity analysis of the developed model.*

Key words: *Fuzzy AHP, VIKOR, multi-criteria decision-making, rectification, long-range rifle.*

1. Introduction

The Serbian Army is a complex organizational system, where the decision-making process is a very important element. Therefore, the application of multi-criteria decision-making methods is an indispensable segment in this process. This paper presents a model for selecting the most efficient rectification method of a 12.7 mm M93 long - range rifle optical sight.

A long-range rifle is a weapon to support infantry platoons in attack and defense. It is a type of small arms that is specially designed for fire action on people, non-combat and lightly armored combat vehicles, at distances up to 1800 m (Randjelovic et al. 2019a). It is a weapon of high accuracy and precision and achieves its firepower on targets by direct shooting.

Successful rectification of sights achieves the accuracy and precision of a long-range rifle. Based on accuracy and precision, the probability of hitting the target is

* Corresponding author.
E-mail addresses: markoradovanovicgdb@yahoo.com (M. Radovanović), aca.r.0860.ar@gmail.com (A. Ranđelović), antras1209@gmail.com (Ž. Jokić)

determined, which affects the efficiency of long-range rifle 12.7 mm M93 solving fire tasks in operations. Having in mind the importance of rectification of the optical sight of a long - range rifle for performing combat actions, the most efficient rectification procedure was selected by applying the method of multi - criteria decision - making.

2. Problem description

Through this paper, a model is presented which determines the most efficient and most economical procedure of rectification of the optical sight of a long - range rifle. Procedures for rectification of the optical sight of the 12.7 mm M93 long-range rifle are defined on the basis of the provisions of the technical and temporary instructions for the optical sight of the long-range rifle and the instructions for use for the optical sight of the long-range rifle (Long-range rifle 12.7 mm M93 (description, handling and maintenance), 2010; Purpose, description and handling of the 12.7 mm long-range rifle, 1999; The long-rifle Optical sight ON M93 for the long-range rifle "Zastava" 12.7 mm M93, 1998).

In addition to the above, as one alternative, a modeled rectification procedure was taken, which was reached on the basis of the results of previous research in this area, presented in detail in Radovanović (2016), Radovanović et al. (2016) and Randjelovic et al. (2019a). The aim of this paper is to select the most efficient rectification procedure using the method of multi-criteria decision-making in order to indirectly increase the efficiency of realization of fire tasks with a long-range rifle. The results used for the analysis were obtained on the basis of realized shootings at the training field "Pasuljanske livade".

Most units of the Serbian Army for the process of rectification of the optical sight of the long-rifle 12.7 mm M93, use the model shown in the temporary instructions for long-range rifle (Purpose, description and handling of long-range rifle 12.7 mm, 1999). To a lesser extent, other methods of rectification are used in the units. According to the above, it can be concluded that there is no universality regarding the rectification of the optical sight of a long-range rifle. Comparisons regarding quality, but also other parameters of rectification have not been performed so far. In other words, there are several satisfactory ways of rectification, but so far no detailed analysis has been made as to which way (model) would be the most acceptable from several aspects (quality, price, required resources, etc.). Accordingly, it is clear that the presented problem is an ideal field for the application of multi-criteria decision-making methods.

In the literature available to the authors, it was found that there is not a large number of papers dealing with this issue. Radovanović (2016) models a new rectification procedure and the software program Correction of sights. In the paper Radovanović et al. (2016) performed a numerical analysis of different ways of rectification in relation to certain criteria such as ammunition consumption, time and price of rectification. Randjelovic et al. (2019a) show the dependence of the rectification procedure on the execution of fire tasks in a counter-terrorist operation. The available literature describes only a part of the criteria on the basis of which the most efficient rectification procedure is selected.

3. Description of applied methods

The hybrid model, applied when solving the problem of choosing the most efficient rectification method of the long - range rifle optical sight, was defined by a combination of the fuzzy AHP and VIKOR methods. This part of the paper describes the methods used in the paper. The fuzzy AHP method was used to define the coefficient values, while the VIKOR method was used to select the best alternative. Figure 1 shows the phases through which this model was realized.

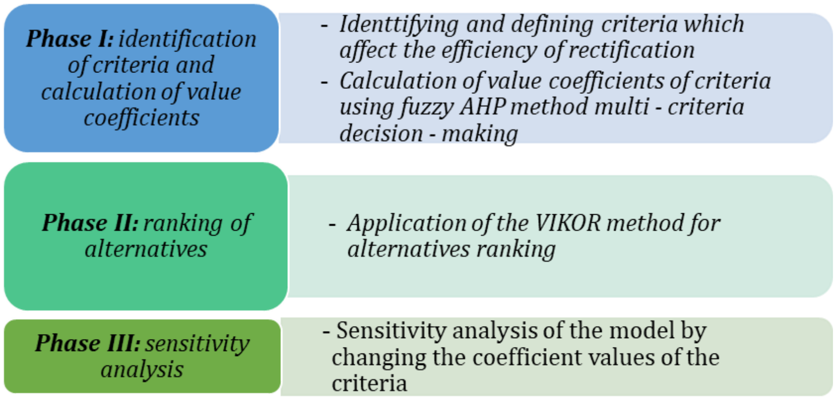


Figure 1. Appearance of the model for rectification of the optical sight of a long-range rifle

3.1. Fuzzy AHP method

The AHP method was developed by Thomas Saaty (1980). To date, this method has undergone a large number of modifications (Božanić et al., 2013; Stević et al., 2017; Petrović et al., 2018; Chatterjee et al., 2019; Afriliansyah et al., 2019; Osintsev et al., 2020; Zhu et al., 2020;), but in some cases it is still used in its original form (Radovanović et al., 2019; Radovanović and Stevanović, 2020; Ranđelović et al., 2019b) both in the individual (Badi and Abdulshahed, 2019) and in group decision making (Srđević and Zoranović, 2003).

Analytical hierarchical process is a method based on the decomposition of a complex problem into a hierarchy, with the goal at the top, criteria, sub-criteria and alternatives at the levels and sublevels of the hierarchy (Saaty, 1980). For comparisons in pairs, which is the basis of the AHP method, the Saaty's scale is usually used, Table 1.

Table 1. Saaty's pair-wise comparison scale

Standard values	Definition	Inverse values
1	Same meaning	1
3	Weak dominance	1/3
5	Strong dominance	1/5
7	Very strong dominance	1/7
9	Absolute dominance	1/9
2, 4, 6, 8	Intermediate values	1/2, 1/4, 1/6, 1/8

The comparison in pairs leads to the initial decision matrices. The Saaty's scale is most commonly used to determine the coefficient values of the criteria, but can also be used to rank alternatives.

Very often when taking values from the Saaty's scale in the pair-wise comparison process, decision makers hesitate between the values they will assign to a particular comparison. In other words, it happens that they are not sure of the comparison they are making. Due to the above, various modifications of the Saaty's scale are often made. One of them is the application of fuzzy numbers.

There are different approaches in the fuzzification of the Saaty scale, and in principle they can be divided into two groups: sharp (hard) and soft fuzzification (Božanić et al., 2015b). Fasification can be done with different types of fuzzy numbers, and is most often done using a triangular fuzzy number Figure 2.

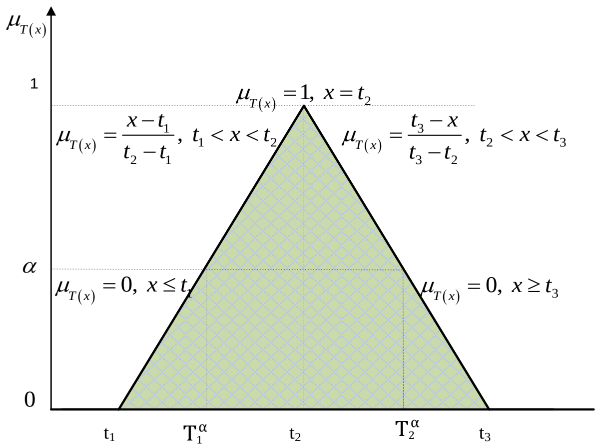


Figure 2. Triangular phase number T (Pamučar et al., 2016b)

By "sharp" fuzzification is meant that a fuzzy number $T=(t_1,t_2,t_3)$ is a predetermined confidence interval, that is, it is predetermined that the value of the fuzzy number will not be greater than t_3 or less than t_1 (Božanić et al., 2015b). Based on the predefined fuzzy Saaty's scale, a comparison is made in pairs. In soft fuzzification, the confidence interval of the values in the Saaty's scale is not predetermined, but is defined during the decision-making process, based on additional parameters.

The definition of the coefficient values of the criteria in this paper was performed by applying the phased Saaty's scale presented in the works of Božanić et al. (2016), Pamučar et al. (2016a), Božanić (2017), Božanić et al. (2018), Bojanic et al. (2018) and Bobar et al. (2020). The starting elements of this fuzzification are (Bobar et al., 2020):

1) introducing the fuzzy numbers instead of classic numbers of the Saaty scale,
2) introducing the degree of confidence of decision makers/analysts/experts (DM/A/E) in the statements they make when comparing in pairs - γ.

The degree of confidence (γ) is defined at the level of each comparison in pairs. The value of the degree of confidence belongs to the interval $\gamma\in[0,1]$, where $\gamma=1$ describes the absolute confidence of DM/A/E in the defined comparison. The decrease in the confidence of DM/A/E in the performed comparison is accompanied by a decrease in the degree of confidence γ_{ji}. Forms for calculating fuzzy numbers are given in Table 2.

Table 2. Fuzzification of the Saaty's scale using the degree of confidence (Bobar et al., 2020)

Definition	Standard values	Fuzzy number	Inverse values of fuzzy number
Same meaning	1	(1, 1, 1)	(1, 1, 1)
Weak dominance	3	$\left(3\gamma_{ji},3,\left(2-\gamma_{ji}\right)3\right)$	$\left(1/\left(2-\gamma_{ji}\right)3,1/3,1/3\gamma_{ji}\right)$
Strong dominance	5	$\left(5\gamma_{ji},5,\left(2-\gamma_{ji}\right)5\right)$	$\left(1/\left(2-\gamma_{ji}\right)5,1/5,1/5\gamma_{ji}\right)$
Very strong dominance	7	$\left(7\gamma_{ji},7,\left(2-\gamma_{ji}\right)7\right)$	$\left(1/\left(2-\gamma_{ji}\right)7,1/7,1/7\gamma_{ji}\right)$
Absolute dominance	9	$\left(9\gamma_{ji},9,\left(2-\gamma_{ji}\right)9\right)$	$\left(1/(2-\gamma_{ji})9,1/9,1/9\gamma_{ji}\right)$
Intermediate values	2, 4, 6, 8	$\left(x\gamma_{ji},x,\left(2-\gamma_{ji}\right)x\right)$, $x=2,\ 4,\ 6,\ 8$	$\left(1/\left(2-\gamma_{ji}\right)x,1/x,1/x\gamma_{ji}\right)$ $x=2,\ 4,\ 6,\ 8$

An example of the appearance of a fuzzy number with different degrees of confidence is given in Figure 3. For example, the value of low dominance from the Saaty's scale and degrees of confidence γ=1, γ=0.7 and γ=0.3 are taken.

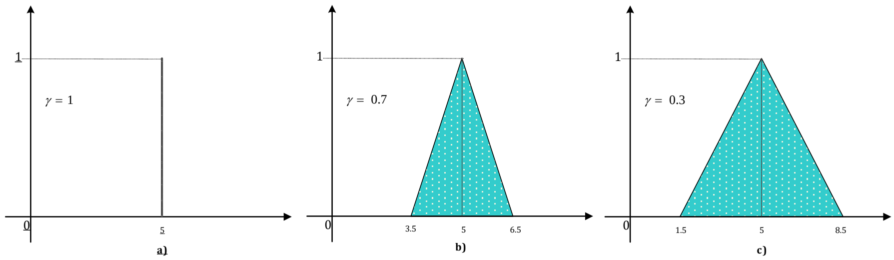


Figure 3. Dependence of fuzzy number on degree of confidence

By introducing different values of the degree of confidence, the left and right distributions of fuzzy comparisons change according to the expression (Bobar et al., 2020):

$$T=\left(t_1,t_2,t_3\right)=\begin{cases} t_1=\gamma t_2, & t_1\le t_2, \quad t_1,t_2\in\left[1/9,9\right] \\ t_2=t_2, & t_2\in\left[1/9,9\right] \\ t_3=\left(2-\gamma\right)t_2, & t_3\le t_2, \quad t_2,t_3\in\left[1/9,9\right] \end{cases} \tag{1}$$

where the value of t_2 represents the value of the linguistic expression from the classical Saaty's scale, which in the fuzzy number has the maximum affiliation t_2=1.

Fuzzy number $T=\left(t_1,t_2,t_3\right)=\left(x\gamma,x,\left(2-\gamma\right)x\right)$, $x\in\left[1,9\right]$ is defined by expressions (Božanić, 2017):

$$t_1=x\gamma=\begin{cases} x\gamma, & \forall\ 1\le x\gamma\le x \\ 1, & \forall\ x\gamma<1 \end{cases} \tag{2}$$

$$t_2=x,\ \forall\ x\in\left[1,9\right] \tag{3}$$

$$t_3 = \left(2-\gamma_{ji}\right)x, \forall x \in \left[1,9\right] \tag{4}$$

Inverse fuzzy number $T^{-1} = \left(1/t_1, 1/t_2, 1/t_3\right) = \left(1 \big/ \left(2-\gamma_{ji}\right)x, 1/x, 1 \big/ \gamma_{ji}x\right)$, $x \in \left[1,9\right]$ is defined as (Božanić, 2017):

$$1/t_3 = 1 \big/ \left(2-\gamma_{ji}\right)x = \begin{cases} 1 \big/ \left(2-\gamma_{ji}\right)x, \ \forall \ 1 \big/ \left(2-\gamma_{ji}\right)x < 1 \\ 1, \ \forall \ 1 \big/ \left(2-\gamma_{ji}\right)x \geq 1 \end{cases}, x \in \left[1,9\right] \tag{5}$$

$$1/t_2 = 1/x, \ \forall \ 1/x \in \left[1,9\right] \tag{6}$$

$$1/t_3 = 1 \big/ \gamma_{ji}x, \forall 1/x \in \left[1,9\right] \tag{7}$$

Accordingly, the initial decision matrix has the following form (Božanić et al., 2015a):

$$A = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} a_{11};\gamma_{11} & a_{12};\gamma_{12} & \cdots & a_{1n};\gamma_{1n} \\ a_{21};\gamma_{21} & a_{22};\gamma_{22} & \cdots & a_{2n};\gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1};\gamma_{n1} & a_{n2};\gamma_{n2} & \cdots & a_{nn};\gamma_{nn} \end{bmatrix} \end{array} \tag{8}$$

where $\gamma_{ji}=\gamma_{ij}$. Reaching the final results implies further application of the standard steps of the AHP method. At the end of the application, the fuzzy number is converted to a real number. Numerous methods are used for this procedure (Herrera and Martinez, 2000). Some of the known terms for defuzzification are (Liou and Wang, 1992; Seiford, 1996):

$$A = ((t_3 - t_1) + (t_2 - t_1))/3 + t_1 \tag{9}$$

$$A = \left[\lambda t_3 + t_2 + \left(1-\lambda\right)t_1\right]/2 \tag{10}$$

where λ represents the optimism index, which can be described as the belief/ratio DM/A/E in decision-making risk. Most often, the optimism index is 0, 0.5 or 1, which corresponds to the pessimistic, average or optimistic view of the decision maker (Milićević, 2014).

3.2. VIKOR method

VIKOR (**VI**šekriterijumsko **KO**mpromisno **R**angiranje) is a method of multi-criteria decision-making whose use is very common. It was developed by Serafim Opricović (1986). It is suitable for solving various decision-making problems. It is especially emphasized for situations where criteria of a quantitative nature prevail.

The VIKOR method starts from the "boundary" forms of Lp - metrics, where the choice of the solution that is closest to the ideal is made. The presented metric represents the distance between the ideal point F* and the point F (x) in the space of criterion functions (Opricović, 1986). Minimizing this metric determines a compromise solution. As a measure of the distance from the ideal point, the following is used:

$$L_p(F^*, F) = \left\{ \sum_{j=1}^{n} \left[f_j^* - f_j(x) \right]^p \right\}^{1/p}, 1 \le p \le \infty \quad (11)$$

The VIKOR method has been applied in a large number of papers in its original form (Nisel, 2014; Kuo and Liang, 2011; Opricović and Tzeng, 2004; Jokić et al., 2019, Radovanović et al. 2020), but also in fuzzy (Chatterjeea and Chakrabortyb, 2016; Ince, 2007; Shemshadi et al., 2011;) and a rough (Li and Song, 2016; Wang et al. 2018) environment.

When applying the VIKOR method, the following terms are used:

- n – number of criteria
- m – number of alternatives for multicriteria ranking
- f_{ij} – the values of the i criterion function for the j alternative,
- w_j – the value of the j criterion function,
- v – the weight of the strategy, meeting most of the criteria,
- i – ordinal number of the alternative, i = 1, ..., m.,
- j – ordinal number of the criteria, j = 1, ..., n,
- Q_i – measure for multi-criteria ranking of the j alternative.

For each alternative, there are Q_i values, after which the alternative with the lowest Q_i value is selected. The measure for multi-criteria ranking of the i action (Q_i) is calculated according to the expression (Opricović, 1998):

$$Q_i = v * QS + (1 + v) * QR_i \quad (12)$$

where:

$$QS_i = \frac{S_i - S^*}{S^- - S^*} \quad (13)$$

$$QR_i = \frac{R_i - R^*}{R^- - R^*} \quad (14)$$

By calculating the QS_i, QR_i, and Q_i values for each alternative, it is possible to form three independent rankings. The QS_i value, is a measure of deviation that displays the requirement for maximum group benefit (first ranking list). QRi value is a measure of deviation that shows the requirement to minimize the maximum distance of an alternative from the "ideal" alternative (second ranking list). Q_i value represents the establishment of a compromise ranking list that combines QS_i and QR_i values (third ranking list). By choosing a smaller or larger value for v (the weight of strategies to meet most criteria), the decision maker can favor the influence of QS_i value or QR_i value in the compromise ranking list. For example, higher values for v (v > 0.5) indicate that the decision maker gives greater relative importance to the strategy of satisfying most of the criteria (Nikolić et al., 2010). Modeling the preferential dependence of criteria usually includes the weights of individual criteria. If the given values are weights $w_1, w_2, \ldots, w_n$, the multi-criteria ranking by the VIKOR method is realized by using the measure S_i and R_i. In the previous terms, the labels used have the following meanings:

$$S_i = \sum_{j=1}^{n} w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right) = \sum_{j=1}^{n} w_j d_{ij} \quad (15)$$

$$R_i = \max_j \left[w_i \left(f_i^* - f_{ij} \right) / \left(f_i^* - f_i^- \right) \right] = \max_j w_j d_{ij} \quad (16)$$

i = 1,2, ..., m, j=1,2,...,n, and where:

$S^* = \min_i S_i$

$S^- = \max_i S_i$

$R^* = \min_i R_i$

$R^- = \max_i R_i$

$f^* = \max_i f_{ij}$

$f^- = \min_i f_{ij}$

Alternative a_i is better than alternative a_k according to *j* criterion, if:

– $f_{ij} \succ f_{kj}$ (for max f_j, that is when the criterion has a maximum requirement),
– $f_{ij} \prec f_{kj}$ (for min f_j, that is when the criterion has a minimum requirement).

In multi-criteria ranking by the VIKOR method, alternative ai is better than alternative a_k (in total, according to all criteria), if: $Q_i < Q_k$. A compromise ranking list for the value v = 0.5 is taken as an acceptable ranking list according to the VIKOR method.

If an alternative is in the first position on such a compromising ranking list, it still does not mean that this alternative is considered the best. In order for an alternative to be adopted as the best, it must be first on the compromise ranking list and meet two conditions: condition C1 and condition C2.

Condition C1:

The first alternative on the compromise ranking list for the value v = 0.5, must have a "sufficient advantage" over the action from the next position. "Advantage" is calculated as the difference of measures Q_i for the value v = 0.5. Alternative a' has a "sufficient advantage" over the following a" from the ranking list, if fulfilled:

$$Q(a`) - Q(a``) \geq DQ, \tag{17}$$

$$DQ = \min(0.25, \frac{1}{m-1}) \tag{18}$$

where:

– *DQ* – "sufficient advantage" threshold value
– *m* – number of alternatives,
– 0,25 – a "sufficient advantage" threshold value that limits the threshold value for cases with a small number of alternatives.

Condition C2:

The first alternative on the compromise ranking list for the value v = 0.5, must have a "sufficiently stable" first position with a change in value v. The first alternative on the compromise ranking list has a "sufficiently stable" position, if it meets at least one of the following conditions:

– has the first position on the ranking list according to *QS*,
– has the first position on the ranking list according to *QR*,
– has the first position on the ranking list according to *Q* for v = 0.25 and v = 0.75.

If the first action from the compromise ranking list does not meet one or both conditions (C1 and C2), it is considered that it is not "sufficiently" better than the action from the second position and possibly some more actions. In such cases, a set of compromise solutions is formed, which consists of the first, second and possibly some other actions (third, fourth ...). If the first action does not meet only the condition C2, then only the first and second actions are included in the set of compromise solutions. However, if the first action does not meet condition C1 (or both conditions, both C1 and C2), then the set of compromise solutions contains actions from the compromise ranking list to the action that meets condition C1, that is to the one over which the first action has a "sufficient advantage" via *DQ*.

The results of the VIKOR method are:

- Ranking lists according to QS_i, QR_i and Q_i measures,
- A set of compromise solutions (in case the conditions C1 and/or C2 are not met).

These results represent the basis for deciding and adopting the final solution.

4. Description of the criteria and calculation of the coefficient values of the criteria

Through the first phase of the model application, the criteria that influence the selection of the optimal alternative, that is the rectification procedure, were defined. When defining the criteria for the selection of rectification methods, it is necessary to include all relevant facts of the optimized system, which is further important for determining the weight coefficients of the criteria. The criteria are defined on the basis of a study of the available literature and the views of experts. Six criteria are defined and presented in this part of the paper.

The rectification time (C_1) represents the total time from the moment of placing the long-range rifle in the firing position, settings for shooting, shooting, and setting the optical sight to the end of rectification, and is expressed in units of time or minutes (Radovanović et al., 2016). The stated criterion is of numerical character and "cost" type (smaller values are more desirable).

Ammunition consumption (C_2) represents the number of bullets needed to perform shooting in order to realize the rectification of the optical sight of a long-range rifle (Radovanović, 2016). The specified criterion is of the "cost" type. Ammunition consumption directly affects the economic characteristics of long-range rifle rectification, such as the cost of the rectification procedure. The criterion is numerical and is expressed by the number of bullets required for the realization of the rectification of the optical sight.

Shooting accuracy (C_3) represents the measured size of the image of scattering hits limited by four probable deflections (Vs) in each side of the middle hit (Kokelj and Randjelovic, 2018). The smaller the scatter, the smaller the image of the beam trajectory, which makes the weapon more accurate. Shooting accuracy is prescribed in accordance with the size of the image of the hits, and is expressed in millimeters (mm). The criterion is of the "cost" type.

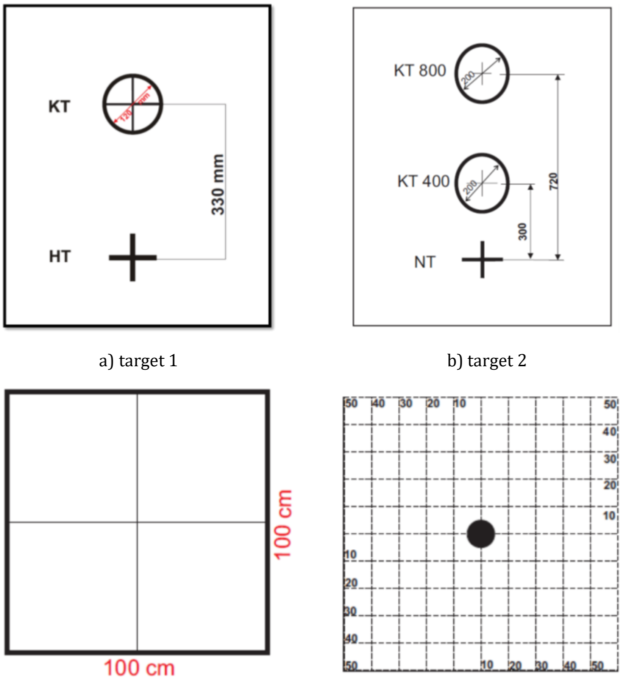
The number of engaged persons (C_4) is a criterion that affects the economic characteristics of rectification, and is expressed in the minimum number of people needed to realize the rectification of a long-range rifle. The criterion is of the "cost" type.

The type of target (C_5) is related to the characteristics of the target and represents a criterion that is directly related to the rectification model and directly affects the efficiency of the rectification performed based on the target model (Radovanović, 2016). Depending on the appearance and size of the target point on the target, the shooting results may also differ. The criterion is of a linguistic character, where higher values are more desirable. Qualitative scores are quantified using the scale shown in Table 3.

Table 3. Scale of converting quantitative criteria into qualitative ones

Quantitative	Type of target	Note
0.9	target 1	modeled target (Figure 4a)
0.7	target 2	target defined by technical instruction (Figure 4b)
0.5	target 3	target defined by the instructions for use of the M93 optical sight (Figure 4c)
0.3	target 4	target defined by the temporary instruction for the 12.7 mm M93 long-range rifle (Figure 4d)

The appearance of these targets is given in Figure 4.



a) target 1 b) target 2

c) target 3 d) target 4

Figure 4. Rectification target models

Shooting accuracy (C6) is the measured value between the scattering of the beam trajectory and the target being shot. The criterion is numerical and is expressed in millimeters. It is defined by the distance of the image of scattering hits and the image of the target at a certain distance. The conclusion about accuracy is made on the basis of the magnitude of the deviation of the middle hit (Mh) from the center of the target (Ct). The shooting is more accurate because the deviation of the middle goal from the center of the target is smaller, and vice versa. The criterion is of the "cost" type. The accuracy of shooting depends on the work of the shooter, the meteorological conditions in which the shooting takes place, the completeness and correctness of accessories and instruments and ammunition (Randjelovic et al., 2019a).

Using the fuzzy AHP method shown in the previous section, the coefficient values of the criteria were defined. The coefficient values were calculated for each expert separately, and the obtained values were aggregated into one. The obtained coefficient values of the criteria are given in Table 4.

Table 4. Coefficient values of the criteria

Criteria	Coefficient values
C_1	0.143
C_2	0.110
C_3	0.376
C_4	0.026
C_5	0.049
C_6	0.295

5. Choosing the best alternative

The analysis of the literature defines seven alternatives, that is seven models of rectification of the optical sight of the 12.7 mm M93 long-range rifle, which is in use in the units of the Serbian Army:

– A1 - Model defined in Optical sight ON PD 12.7 M93 for long-range rifle "Zastava" 12.7 mm M93, temporary instruction, (1998).
– A2 - Model defined in Optical sight ON PD 12.7 M93 for long-range rifle "Zastava" 12.7 mm M93, temporary instruction, (1998) with the use of rectifiers (R) defined in Radovanović, (2016).
– A3 - Model defined in Long-range rifle 12.7 mm M93, technical manual, (2010).
– A4 - Model defined in Long-range rifle 12.7 mm M93, technical manual, (2010), with the use of rectifiers (R) defined in Radovanović, (2016).
– A5 - Model defined in Purpose, description and handling of the 12.7 mm M93 long-range rifle, temporary instruction, (1999).
– A6 - Model defined in Purpose, description and handling of a 12.7 mm M93 long-range rifle, temporary instruction, (1999), with the use of rectifiers (R) defined in Radovanović, (2016).
– A7 - Model defined in Radovanović, (2016).

Assessments of alternatives according to the criteria are given in the initial decision matrix (Table 5).

Table 5. Initial decision matrix

	C_1 (min)	C_2 (min)	C_3 (min)	C_4 (min)	C_5 (max)	C_6 (min)
	w=0.143	w=0.110	w=0.376	w=0.026	w=0.049	w=0.295
A_1	90	20	47	1	target 3	22
A_2	70	16	38	2	target 3	18
A_3	52	18	40	1	target 2	29
A_4	31	12	33	2	target 2	19
A_5	90	20	52	1	target 4	45
A_6	70	16	41	2	target 4	40
A_7	23	8	36	2	target 1	15

Table 6 shows a quantified initial decision matrix, where for the criterion of target type (C_5) the conversion from qualitative to quantitative assessments was performed based on the scale shown in Table 3.

Table 6. Quantified initial decision matrix

	C_1 (min)	C_2 (min)	C_3 (min)	C_4 (min)	C_5 (max)	C_6 (min)
	w=0.143	w=0.110	w=0.376	w=0.026	w=0.049	w=0.295
A_1	90	20	47	1	0.5	22
A_2	70	16	38	2	0.5	18
A_3	52	18	40	1	0.7	29
A_4	31	12	33	2	0.7	19
A_5	90	20	52	1	0.3	45
A_6	70	16	41	2	0.3	40
A_7	23	8	36	2	0.9	15

Using the expression 11-16, the final values of the alternatives were obtained, Table 7.

Table 7. Calculated values for QS_i, QR_i, Q_i (v=0,5), Q_i (v=0,75), (v=0,25)

	QS_i	QR_i	$Q_i(v=0,5)$	$Q_i(v=0,75)$	$Q_i(v=0,25)$
A_1	0.615	0.688	0.651	0.633	0.669
A_2	0.310	0.129	0.220	0.265	0.175
A_3	0.406	0.250	0.328	0.367	0.289
A_4	0.090	0.030	0.060	0.075	0.045
A_5	1.000	1.000	1.000	1.000	1.000
A_6	0.639	0.589	0.614	0.627	0.601
A_7	0.000	0.000	0.000	0.000	0.000

Based on the obtained values from Table 7, the ranking of alternatives was performed, Table 8. As can be seen from Table 8, the best alternative is A7. In order to choose a certain alternative as the best, it is necessary that it meets the conditions C1 and C2. Testing of condition C1 was performed, which was not fulfilled because:

$Q(A_4) - Q(A_7)$= 0,060 -0,00= 0,060 < DQ= 1/(7-1)=0,167

Alternative A4 enters a set of compromise solutions, because the first alternative from the ranking list A7 does not have a "sufficient advantage" over the second-ranked alternative A4. Other alternatives are not included in the set of compromise solutions, because alternative A7 has a "sufficient advantage" over the third-ranked alternative A2, other alternatives do not need to be tested according to the stated condition.

Table 8. Ranking lists of alternatives based on QS_i, QR_i, Q_i values

	QS_i	QR_i	$Q_i(v=0,5)$	$Q_i(v=0,75)$	$Q_i(v=0,25)$
A_1	5	6	6	6	6
A_2	3	3	3	3	3
A_3	4	4	4	4	4
A_4	2	2	2	2	2
A_5	7	7	7	7	7
A_6	6	5	5	5	5
A_7	1	1	1	1	1

Condition C2 is met if alternative A7 has a "sufficiently stable" first place according to two criteria:

– alternative A_7 has the first position on the ranking list according to QR and

– alternative A_7 takes the first position on the ranking list for Q (v = 0.25) and Q (v = 0.75)

Based on the obtained results, the final solution is defined by a set of compromise solutions in which there are alternatives A7 and A4. In this case, the decision maker can choose the alternative A7 - Rectification Model described in Radovanović (2016), and as the first back-up rectification procedure, the alternative A4 - Model defined in 12.7 mm M93 long-range rifle, technical instruction, (2010), is proposed, with the use of rectifiers (R) defined in Radovanović, (2016).

6. Sensitivity analysis

During the last phase, the sensitivity of the applied mathematical model was examined, in order for the decision maker to receive confirmation of the rationality and quality of the obtained solution, that is to determine how changes in the significance of criteria lead to changes in the ranks of alternatives (Tešić and Božanić, 2018). Checking the stability of the MCDM methods used is an indispensable step in the process of developing a model to support decision-making (Pamučar et al., 2017). Table 9 shows six scenarios (from S1 to S6) of changing the coefficient values of the criteria, on the basis of which the alternatives were ranked using the VIKOR method.

Table 9. Criteria for changing the significance of the criteria

	C_1	C_2	C_3	C_4	C_5	C_6
S_1	0,30	0,14	0,14	0,14	0,14	0,14
S_2	0,14	0,30	0,14	0,14	0,14	0,14
S_3	0,14	0,14	0,30	0,14	0,14	0,14
S_4	0,14	0,14	0,14	0,30	0,14	0,14
S_5	0,14	0,14	0,14	0,14	0,30	0,14
S_6	0,14	0,14	0,14	0,14	0,14	0,30

Table 10 shows the ranks of alternatives obtained by applying different scenarios.

Table 10. Ranks of alternatives obtained by applying different scenarios

	S_1	S_2	S_3	S_4	S_5	S_6
A_1	6	6	6	2	5	5
A_2	4	3	4	6	4	4
A_3	3	4	3	1	2	3
A_4	2	2	2	5	3	2
A_5	7	7	7	3	7	7
A_6	5	5	5	7	6	6
A_7	1	1	1	4	1	1

In order to establish the correlation of the ranks obtained by different types of scenarios, the Spiraman's coefficient was used:

$$S = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2 - 1)} \tag{19}$$

where D_i represents the difference of rank according to the given scenario and rank in the corresponding scenario, and n the number of ranked elements. The Spiraman's coefficient belongs to the value interval [-1,1]. When the ranks of the alternatives completely match the Spirman's coefficient is 1 ("ideal positive correlation"), when the ranks are completely opposite the Spiraman's coefficient is -1 ("ideal negative correlation"), is when S = 0 the ranks are uncorrelated. The values of the Spirman's coefficient in this case are shown in Table 11.

Table 11. Spirman's coefficient values

	S_0	S_1	S_2	S_3	S_4	S_5	S_6
S_0	1	0.964	1	0.964	-0.286	0.857	0.929
S_1		1	0.964	1	-0.107	0.929	0.964
S_2			1	0.964	-0.286	0.857	0.929
S_3				1	-0.107	0.929	0.964
S_4					1	0.214	0.071
S_5						1	0.964
S_6							1

Based on the results in Table 12, it is concluded that the values of the Spiraman's coefficient are extremely high for most cases, that is there is an ideal positive correlation of ranks in most cases. Deviation from the ideal positive correlation is observed in scenario S_4, compared to other scenarios. Negative correlation in scenario S_4 occurs as a consequence of two causes: 1) Criterion C_4 has the lowest coefficient value in scenario S_0, 2) criterion C_4 can have values of 1 or 2, which directly affects the negative correlation of scenario S_4. During the change of the coefficient values of the criteria of the first-ranked alternative A_7, not counting the scenario S_4 did not change its rank by changing the significance of the criteria. Based on all the above, it is possible to conclude that the model has sufficient sensitivity.

7. Conclusion

The paper successfully applied the fuzzy AHP-VIKOR hybrid model to the selection of the most efficient rectification method of the 12.7 mm M93 long-range rifle optical sight. In this way, a more detailed review of the presented problem was performed. The paper presents the phases of development and application of multi-criteria decision-making models. The definition of criteria of importance for the selection of the rectification model and the calculation of coefficient values using the fuzzy AHP method was performed. The selection of the most efficient model of rectification of the optical sight of a long - range rifle was performed using the VIKOR method. The final results indicate a set of compromise solutions (alternatives A_7 and A_4).

The paper analyzes the sensitivity of the presented model, changes in the significance of coefficients of the criteria (through several scenarios by favoring one criterion). The results of the analysis indicate sufficient sensitivity of the model.

The contribution of this work is reflected in the selection of the most efficient model of rectification of the optical sight of a long-range rifle, whose application would increase the efficiency of long-range rifle squad, reduce rectification time, reduce rectification cost and achieve universality of long-range rifle rectification. The presented model can be further improved by a more detailed analysis of the criteria and the application of other methods of multicriteria analysis in the problem of choosing the most efficient rectification procedure and defining the values of the criteria.

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