A SENSITIVITY ANALYSIS IN MCDM PROBLEMS: A STATISTICAL APPROACH

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Abstract: *This study provides a model for result consistency evaluation of multi-criteria decision-making (MDM) methods and selection of the optimal one. The study presents the results of an analysis of the sensitivity of decision-making based on the rank methods: SAW, MOORA, VIKOR, COPRAS, CODAS, TOPSIS, D'IDEAL, MABAC, PROMETHEE-I,II, ORESTE-II with variations in the elements in the decision matrix within a given error (imprecision). It is suggested to use multiple simulation of the elements estimations of the decision matrix within a given error for calculating the ranks of alternatives, which allows obtaining statistical estimates of ranks. Based on the statistics of simulations, decision-making can be carried out not only on the alternatives statistics having rank I but also on the statistics of alternatives having the largest total I and II rank or I, II and III ranks. This is especially true when the difference in rank values is not large and is distributed evenly among the first three ranks. The calculations results for the task of selecting the adequate location of 8 objects by 11 criteria are presented here. The main result shows that the alternatives having I, II and III ranks for some ranking methods are not distinguishable within the selected error value of the elements in the decision matrix. A quantitative analysis can only narrow the number of effective alternatives for a final decision. A statistical analysis makes the number of options estimation possible in which an alternative has a priority. Additional criteria that take into account both aggregate priorities and the availability of possible priorities for other alternatives with small variations in the decision matrix provide additional important information for the decision-maker.*

Key Words: *Multi-criteria Decision-making, SAW, MOORA, VIKOR, COPRAS, CODAS, TOPSIS, D'IDEAL, MABAC, PROMETHEE-I,II, ORESTE-II, Sensitivity Analysis*

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1 Introduction

Decision-making processes are present in all activities of daily life. The decision attempts aim at solving problems in a particular case in the best way but it is worth remembering that this process is complex and takes place in an environment of uncertainty.

The multi-criteria decision-making methods (MCDMM) are the tool for reducing subjectivity in decision-making by creating a series of filters selection and helping to make choice among the complex alternatives. They are characterized by a particular mathematical apparatus which makes the application of different methods to the same problem often result in different solutions. Consequentially, the alternative choice does not depend solely on the criteria that one uses to evaluate those alternatives but on the MCDMM that one uses as well (Pamučar et al, 2017).

There is no consensus on how to determine the sensitivity analysis, i.e. the "quality" of a decision method and the reliability of the results. The sensitivity analysis can be defined as stability or behavior of the solution to small changes in preferences which occur during the resolution process or to small changes in the values taken for parameters; it is what some authors consider as efficiency multicriteria decision method (Pamučar & Ćirović, 2015).

Barron and Schmidt (1988) recommended two procedures to accomplish a sensitivity analysis in multi-attribute value models (entropy based procedure and a least squares procedure). These procedures calculate, for a given pair of alternatives, the best alternative, the *closest* set of weights that equates their ranking. Watson and Buede (1987) illustrate a sensitivity analysis in a decision modeling strategy. Von Winterfeldt and Edwards (1986) cover the sensitivity analysis in the traditional way for those problems which can be approached by using a multi-attribute utility theory (MAUT) or a Bayesian model. They define the *Flat Maxima Principle* for MAUT problems, which states that the existence of dominance makes the sensitivity analysis almost unnecessary.

Evans (1984) investigates a linear programming-like sensitivity analysis in the decision theory. His approach is based on the geometric characteristics of optimal decision regions in the probability space. Also, in Triantaphyllou (1992) the sensitivity analysis approach is described for a class of inventory models. The methodology for the sensitivity analysis in multi-objective decision-making is described in Ríos (1990). That treatment introduced a general framework for the sensitivity analysis which expanded results of the traditional Bayesian approach to decision-making. Likewise, that work contains an analysis of why the flat maxima principle is not valid. Samson (1988) presents a whole new approach to the sensitivity analysis. He proposed that it should be part of the decision analysis process *thinking in real time*.

Triantaphyllou and Mann (1989) emphasize two criteria for MDM methods analysis. The first criterion refers to fulfillment of result consistency conditions in the case when the method is applied to a multi-dimensional problem while the second criterion refers to the stability conditions of the best ranked alternative. In their study, Triantaphyllou and Mann (1989) compare four methods (WSM-weighted sum model, WPM-weighted product model, AHP-analytic hierarchy process and Revised AHP-revised hierarchy process). Those two authors conclude that none of the considered methods is completely effective in terms of both evaluative criteria. In 1996, Triantaphyllou and Lin examined five fuzzy multi-attribute decision-making methods (fuzzified WSM, WPM, AHP, revised AHP and TOPSIS) in terms of the same two evaluative criteria, adapted to fuzzy environment. Just like the previous study,

when four crisp methods were compared, they came to same conclusions: that none of the examined fuzzy methods is perfectly effective in terms of both evaluative criteria and that precision methods decrease with increasing complexity of the decision-making problem.

In the last couple of years, there have been frequent comparative analyses by the authors who conduct comparison of the results gained through use of several different MDMM (Rodrigues et al., 2014; Anojkumar et al., 2014; Liu et al, 2013; Wang & Tzeng, 2012; Peng et al., 2011; Yang et al., 2008). However, the fact that there are multiple methods that recommend the same choice is not a satisfactory warranty of rationality and quality of the calculated solution (Pavličić, 1997).

 Examples of analysis of ranking results accordance obtained through different methods can be seen in Rodrigues et al. (2014), Liu et al. (2013), Peng et al. (2011), Yang et al. (2008). It should be noted that the results of this kind of research depend on the observed method choice and characteristics of problems that those methods are being applied to. In accordance with that, there are different conclusions made by different authors. In the works in which robustness and stability analysis of obtained solution is conducted in MDM, besides comparison with the solutions gained thorough other methods and techniques, the analysis is often based on an appropriate sensitivity analysis of the results to changes of certain parameters in the decision-making model (Yu et al. (2012); Stevens-Navarro et al. (2012); Li et al. (2013a); Li et al. (2013b); Corrente et al. (2014); Kannan et al. (2014)).

As specified in the shown research studies, the selection of an optimal MCDMM is a very complex problem which without any prior sensitivity analysis of the solution can have a wrong selection. Therefore, it is necessary to define the model for the sensitivity analysis of MCDMM. This article presents a study of estimating the variation of alternatives according to the criteria for the results of ranking alternatives, and in connection with this, the approach to improving the reliability of decision-making (reduce the risk of making an unsound decision) is discussed in detail. The model was tested on the example of logistical center location selection and the results of are presented in section 4. It is necessary to emphasize that the results presented in section 4 refer only to the observed example of the logistical center location selection and cannot be generalized.

The remainder of this paper is structured as follows: Section 2 gives a brief idea of the research methodology. Section 3 proposes preliminary methods for multi-attribute decision-making and techniques. Sections 4 and 5 present an illustrative example and discussion of the sensitivity model results. Finally, Section 6 presents the conclusions, highlighting directions for further research.

2 Research Methodology

The MCDM problem is usually solved in a two phase process: (1) The rating, that is, the aggregation of the values of criteria for each alternative, and (2) The ranking or ordering between the alternatives, with respect to the global consensual degree of satisfaction. The step-by-step sequence of the problem of multi-criteria decision-making is defined as follows (Triantaphyllou, 2000; Tzeng & Huang, 2011):

(1) Choice of alternatives (A_i; $i = 1, 2, ..., m$);

(2) Choice of evaluating criteria (C_j; $j = 1, 2, ..., n$);

(3) Acceptance of scales of an estimation of alternatives on each criterion;

(4) Determination of priorities (weights) of criteria (w_j; $j = 1, 2, .., n$);

(5) Determination evaluation matrix, i.e. decision matrix $X = \left[a_{ij} \right]_{m \times n}$;

(6) Choosing a method for ranking alternatives.

Careful consideration of each step is the key to the success of the final choice. The first three and the last of the steps relate exclusively to a specific subject area and imply involvement of qualified specialists in the field under consideration. The remaining steps are formalized (partially or completely) and require involvement of specialists in applied mathematics. Accordingly, there are 5 main factors affecting the outcome for ranked decision-making methods for MCDMM for which variations in the form of a formalized procedure or method are possible. These are: (1) the choice of scales of criteria; (2) evaluation of the weights of the criteria; (3) evaluation of alternatives according to the criteria; (4) the method of normalizing the decision matrix and (5) method of ranking. Earlier, in Pamučar et al (2017), the sensitivity of the choice of criteria scales and the evaluation of the weights of the criteria on the results of the ranking of alternatives, convincingly confirming the above thesis, was investigated.

It seems obvious that for real decision-making tasks none of the alternatives can be accurately measured for each of the criteria. The reason for this is the fundamental uncertainty of nature. The correct wording shows how accurately the alternative is evaluated by the criterion. Therefore, $a_{ij} = a_{ij} \cdot (1 \pm \delta_{ij})$, where $\delta_{ij} \in (0,1)$ is the relative error of the estimate. Taking this into account, if we use the linear algebraic transformations of the elements of the decision-making matrix (preliminary normalization of the elements a_{ij} is necessary) to obtain the final ranks of the alternatives, or the class of methods based on the quasi-arithmetic transformations of the decision matrix elements, it is obvious that the degree of reliability of the result depends on the degree of reliability of the elements of matrix D. In the absence of errors of other values, the error of the final ranking will not be less than $\max_{ij}(a_{ij} \cdot \delta_{ij})$. In the simplest case of the OWA (Ordered Weighted Averaging) criteria aggregation method, the reliability of the result is estimated by the order value $\max(\delta_{ij})$. Thus, the final ranks r_i ($i = 1, 2, ..., m$) are calculated with an error and are stochastic values. Then the question of the priority of one alternative over another should be solved in a statistical way.

Let alternatives A_k and A_s have r_k and r_s ranks, respectively, and $r_k \approx r_s$. The question is whether they are significant. The answer can be obtained if we use the t-Test about the equality of two average normal populations. The lack of reliable information about δ_{ij} will not allow such a test to be performed. We consider the following method of partially solving the problem of estimating the error in calculating the ranks of alternatives.

Step 1 An approximate estimate of the maximum total error in the choice problem, for example, $\delta_{ij} = 0.1$ (or 10%), which is similar to specifying the risk.

Step 2 Multiple simulation of r_i ranks (for example, 1000 simulations) for the variation of the elements of the decision matrix X: $a_{ij} = a_{ij} \cdot (1 \pm \delta_{ij} \cdot rnd())$ using the random number generator Rnd [0, 1].

Step 3 Calculate the mean and variance for r_i and test the performance of the paired t-Test for alternatives having 1, 2, and 3 ranks.

Calculations show that for different variants of calculations, the ranks of alternatives change. For example, suppose that in the 1000 decision matrix simulations the 1 rank of alternative A_k took 780 points, alternative A_s was 200, and alternative A_p was 20 points. The ratios are 3.9 and 39 times more in favor of alternative A_k. But this is only with a superficial (trivial) approach. After all, the first 20 ranks of alternative A_p are obtained for specific 20 implementations of the decision matrix. It is possible that the true values of the estimates of alternatives are according to the criteria from the same set. Then there is the possibility of not making the best decision although this chance (risk) is about 2%. Therefore, the value of the approach assuming statistical variations of estimates of the alternatives by the given criteria consists in additional information for the decision-maker regarding the magnitude of the risks.

Having a statistical picture of the assessment of ranks, the decision-making can be carried out not only on the statistics of alternatives having rank 1, but it can also use statistics of the alternatives having the largest total 1 and 2 rank, or 1, 2 and 3 ranks. This is especially true when the difference in rank values is not large and is distributed on an average evenly between the first three ranks. For example, suppose that for A_k the number of first places is 40%, the second 10%, and the third 5%; for A_s the number of first places is 36%, the second 25%, and the third 7%; for A_p the number of first places is 25%, the second 20%, and the third 20%. Then:

(1) A_k is better than A_s and A_p in the number of 1 ranks (40> 36> 25);

(2) A_k is worse than A_s and better than A_p by the amount of the sum of 1 and 2 ranks (50<51, 50>45);

(3) A_k is worse than A_s and worse than A_p in the amount of the sums of 1, 2 and 3 ranks (55<62<65).

The above example shows complexity (and subtlety) of the procedure for selecting alternatives for the decision-maker in this scenario.

3 Preliminary methods for used multi-attribute decision-making methods

Before any further explanation of the recommended model, we are going to explain the basic setup of methods used in this work. Five methods were used: SAW, MOORA, VIKOR, COPRAS, CODAS, TOPSIS, D'IDEAL, MABAC, PROMETHEE-I,II, ORESTE-II. Before a statistical analysis of the above presented multi-criteria methods we define some preliminary benchmarks important for this research:

(1) In this research alternatives A_i are unformalized linguistic variables and criteria (C_j) are non-formalized linguistic variables. For each criterion it is necessary to determine the direction of growth, i.e. *max* (beneficial) = (+1) or *min* (cost)= (– 1) as $sg_j = sign C_j = \{\pm 1\}_1^n$; $j = 1, ..., n$.

(2) The methods for normalizing the decision matrix:

(1) Max	(2) Sum	(3) Max-Min	(4) Vector
For benefit attributes			
$(1)\ x_{ij} = \dfrac{a_{ij}}{a_j^{\max}};$	$(2)\ x_{ij} = \dfrac{a_{ij}}{\sum\limits_{i=1}^{m} a_{ij}};$	$(3)\ x_{ij} = \dfrac{a_{ij} - a_j^{\min}}{a_j^{\max} - a_j^{\min}};$	$(4)\ x_{ij} = \dfrac{a_{ij}}{\sqrt{\sum\limits_{j=1}^{n} a_{ij}^2}}$
For cost attributes			
$(1')\ x_{ij} = 1 - \dfrac{a_{ij}}{a_j^{\max}};$	$(2')\ x_{ij} = 1 - \dfrac{a_{ij}}{\sum\limits_{i=1}^{m} a_{ij}};$	$(3')\ x_{ij} = \dfrac{a_j^{\max} - a_{ij}}{a_j^{\max} - a_j^{\min}};$	$(4')\ x_{ij} = 1 - \dfrac{a_{ij}}{\sqrt{\sum\limits_{j=1}^{n} a_{ij}^2}}$
Variations			
$(1'')\ x_{ij} = \dfrac{a_j^{\min}}{a_{ij}};$	$(2'')\ x_{ij} = \dfrac{1/a_{ij}}{(1/a_{ij})/\sum\limits_{i=1}^{m}(1/a_{ij})};$		
$(1''')\ x_{ij} = -\dfrac{a_{ij}}{a_j^{\max}};$			

(3) Selecting a metric to measure the remoteness of two m-dimensional objects C and D

$$L_p(C, D) = \left[\sum_{i=1}^{m} (c_i - d_i)^p \right]^{1/p}, 1 \le p \le \infty; L_\infty(C, D) = \max_i |c_i - d_i| \qquad (1)$$

3.1 SAW (Simple Additive Weighting) method

Simple Additive Weighting (SAW) method is probably the best known and most widely used MADM method (Anupama et al, 2015). The SAW method also known as a scoring method is one of the best and simplest types of multiple attribute decision-making method. The basic logic of the SAW method is to obtain a weighted sum of performance ratings of each alternative over all attributes. An evaluation score is calculated for each alternative by multiplying the scaled value given to the alternative of that attribute with the weights of relative importance directly assigned by the decision maker followed by summing up of the products for all criteria. The advantage of this method is that it is proportional linear transformation of the raw data which means that the relative order of magnitude of the standardized scores remains equal. The step wise procedure is given below (Kaklauskas et al, 2006):

Step 1 Construct a decision matrix $X = \left[a_{ij} \right]_{m \times n}$ that includes m personnel and n criteria. Calculate the normalized decision matrix for benefit/cost criteria:
$$a_{ij} : norm(1 - 1' < or\,1'' or\,1''' >, 2 - 2', 3 - 3', or\ 4 - 4') \qquad (2)$$

Step 2 Evaluate each alternative, A_i by the following formula:

$$A_i = \sum_{j=1}^{n} w_j a_{ij}; \quad \sum_{j=1}^{n} w_j = 1 \tag{3}$$

where a_{ij} is the normalized value of the *i*-th alternative with respect to the *j*-th criteria, w_j is the weighted criteria (Kaklauskas et al, 2006).

3.2 MOORA (MultiObjective Optimization on the basis of Ratio Analysis) method

The method starts with a matrix of responses of different alternatives on different objectives x_{ij}; where x_{ij} represents the response of alternative *i* on objective *j*.

MOORA goes for a ratio system in which each response of an alternative on an objective is compared to a denominator, which is representative for all alternatives concerning that objective. The step wise procedure is given below (Brauers & Zavadskas, 2006; Kalibatas & Turskis, 2008; Brauers, 2008; Brauers et al., 2008):

Step 1 Construct a decision matrix $X = \left[a_{ij} \right]_{m \times n}$ that includes m personnel and *n* criteria. Calculate the normalized decision matrix for benefit/cost criteria:

$$a_{ij} : norm(1 \ or \ 2 \ or \ 3 \ or \ 4) \tag{4}$$

Step 2 Evaluate each alternative, A_i by the following formula:

$$Q_i = \sum_{j=1}^{n} sg_j \cdot w_j \cdot a_{ij}; \quad \sum_{j=1}^{n} w_j = 1 \tag{5}$$

where a_{ij} is the normalized value of the *i*-th alternative with respect to the *j*-th criteria, w_j is the weighted criteria. These normalized responses of the alternatives on the objectives belong to the interval [0,1].

Step 3 For optimization these responses are added in case of maximization and subtracted in case of minimization (Brauers, 2008):

$$Q_i = \max_j (v_{ij}); \quad a_{ij} : norm(4); \quad r_j = \left| \max_i \left(sg_j \cdot a_{ij} \right) \right|, \ v_{ij} = \omega_j \cdot \left| r_j - a_{ij} \right|; \quad \max_i Q_i \tag{6}$$

where a_{ij} is the normalized value of the *i*-th alternative with respect to the *j*-th criteria, w_j is the weighted criteria.

3.3 VIKOR (VIsekriterijumsko KOmpromisno Rangiranje) method

VIKOR method represents an often used method for multicriteria ranking and suitable for solving different decision-making problems. It is especially suitable for those situations where the criteria of quantitative nature are prevalent. The VIKOR method was developed based on the elements of compromise programming. The method starts from the "border" forms of L_p metrics (Opricović & Tzeng, 2004). It seeks the solution that is the closest to the ideal. In order to find the distance from the ideal point it uses the following function:

$$L_p \left(F^*, F \right) = \left\{ \sum_{j=1}^{n} \left[f_j^* - f_j \left(x \right) \right]^p \right\}^{1/p}, 1 \le p \le \infty \tag{7}$$

This function represents the distance between ideal point F^* and point $F(x)$ in space of criteria functions.

The essence of the VIKOR method is that for every action it finds the value of Q_i, and then it chooses the action which has the lowest listed value (the smallest distance from the "ideal" point). The step wise procedure is given below:

Step 1 Determine "ideal" and "anti-ideal" object

$$a_j^+ = \{\max_i a_{ij} \left| if \ j \in C_j (\max); \ \min_i a_{ij} \right| if \ j \in C_j (\min)\};$$

$$a_j^- = \{\min_i a_{ij} \left| if \ j \in C_j (\max); \ \max_i a_{ij} \right| if \ j \in C_j (\min)\}. \tag{8}$$

where a_j^+ and a_j^-, respectively, present ideal and anti-ideal object.

Step 2 Weighted Normalization: *norm(3)*

$$w_j \cdot x_{ij} = \begin{cases} w_j \cdot \dfrac{a_{ij} - a_j^+}{a_j^+ - a_j^-} & if \ x_{ij} \in B; \\ \\ w_j \cdot \dfrac{a_j^+ - a_{ij}}{a_j^+ - a_j^-} & if \ x_{ij} \in C. \end{cases} \tag{9}$$

Where *B* and *C,* respectively, present beneficial and cost group of criteria.

Step 3 The strategies of maximal *R* and group utility *S*

$$S_i = \sum_{j=1}^{n} x_{ij}; \ S^* = \min_i S_i; \ S^- = \max_i S_i \tag{10}$$

$$R_i = \max_j x_{ij}; \ R^* = \min_i R_i; \ R^- = \max_i R_i$$

Step 4 Calculate the values of Q_i

$$Q_i = v \cdot \frac{S_i - S^*}{S^- - S^*} + (1-v)\frac{R_i - R^*}{R^- - R^*} \tag{11}$$

where *v* plays the role of the balancing factor between the overall benefit (*S*) and the maximum individual deviation (*R*). Smaller values of *v* emphasize group gain, while larger values increase the weight determined by individual deviations. "Voting by majority rule" (*v*> 0.5); or "by consensus" (for *v* = 0.5); or "with a veto" (for *v* <0.5).

Step 5 The result of the procedure comprises three rating lists: *S*, *R* and *Q*. The alternatives are evaluated by sorting values of *S*, *R* and *Q* by the criterion of the minimum value. The best alternatives:

$$\min_i \{Q_i, S_i, R_i\} \tag{12}$$

Step 6 As a compromise solution, an alternative A_1 is proposed which is best estimated by Q (minimum) if the following two conditions are met:

<u>Condition C1</u>: "Allowable advantage": $Q(A_2) – Q(A_1) >= 1/(m – 1)$, where A_2 is an alternative to the second position in the *Q* ranking list.

<u>Condition C2:</u> "Acceptable stability in decision-making": Alternative A_1 should also be best estimated by *S* or / and *R*.

Step 7 If one of the conditions - 1 or 2 - is not satisfied, then a set of compromise solutions is proposed, which consists of:

- alternatives A_1 and A_2, if condition C2 is not met, or,

- alternatives A_1, A_2, ..., A_k if condition C1 is not satisfied; A_k is determined by relation $Q(Ak-1) - Q(A1) < 1/(m-1) \ \& \ Q(Ak) - Q(A1) \geq 1/(m-1)$.

3.4 COPRAS (COmplex PRoportional ASsessment) method

Ranking alternatives by the COPRAS method assumes direct and proportional dependence of significance and priority of the investigated alternatives on a system of criteria (Ustinovichius et al, 2007). The selection of significance and priorities of alternatives, by using the COPRAS method, can be expressed concisely using four stages (Viteikiene & Zavadskas, 2007). For normalization in the COPRAS method we use $x_{ij} : norm(1 \, or \, 2 \, or \, 3 \, or \, 4)$.

In the COPRAS method, each alternative is described with the sum of maximizing attributes S_{+i}. In order to simplify calculation of S_{+i} and S_{-i} in the decision-making matrix columns, the maximizing criteria are placed first, followed by the minimizing criteria. In such cases, S_{+i} and S_{-i} are calculated as follows (Viteikiene & Zavadskas, 2007):

$$S_{+i} = \sum_{j=1}^{n} x_{ij} \left| for \, j \in C_j(\max); \right.$$

$$S_{-i} = \sum_{j=1}^{n} x_{ij} \left| for \, j \in C_j(\min). \right.$$

(13)

Relative weight Q_i of the i-th alternative is calculated as follows:

$$Q_i = S_{+i} + \frac{\sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{1}{S_{-i}}}$$

(14)

The priority order of the compared alternatives is determined on the basis of their relative weight (higher relative weight higher priority/rank).

3.5 TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method

The basic principle of the TOPSIS method is that the best alternative should have the shortest distance from the ideal solution and the farthest distance from the anti-ideal solution. A relative distance of each alternative from ideal and anti-ideal solutions is obtained as (Chang et al, 2010)

$$Q_i = \frac{S_i^-}{S_i^+ + S_i^-}, i = 1,...,n$$

(15)

where S_i^+ and S_i^- are separation measures of alternative i from the ideal and anti-ideal solution, respectively; Q_i is the relative distance of alternative i to the ideal solution, and $Q_i \in [0,1]$.

The largest value of criterion Q_i correlates with the best alternative. The best ranked, or the most preferable, alternative A_{TPS}^* can be determined as $A_{TPS}^* \left\{ A_i = \max_i Q_i \right\}$.

For normalization in the TOPSIS method we use r_{ij} : $norm(1\ or\ 2\ or\ 3\ or\ 4)$. The separation measures of each alternative, from the ideal and anti-ideal solutions, are computed using the following formulae (Chang et al, 2010):

$$S^+ = \left\{ \sum_{j=1}^{n} \left[w_j \left(r_{ij} - r_i^+ \right) \right]^2 \right\}^{1/2} \tag{16}$$

$$S^- = \left\{ \sum_{j=1}^{n} \left[w_j \left(r_{ij} - r_i^- \right) \right]^2 \right\}^{1/2} \tag{17}$$

where element r_{ij} represents the performance of alternative A_i in relation to criterion C_j. For m criteria ($C_1, C_2, ..., C_m$) and n alternatives ($A_1, A_2, ..., A_n$) matrix R has shape $R = \left[r_{ij} \right]_{nxm}$. Values ($w_1, w_2, ..., w_m$) represent weight values of the criteria that satisfy condition $\sum_{i=1}^{n} w_i$.

Ideal A^+ and anti-ideal A^- solution in the TOPSIS method can be determined using formulas (8) and (9), respectively.

$$A^+ = \left\{ (\max v_{ij} \mid j \in G), (\min v_{ij}, j \in G'), i = 1, .., n \right\} = \left\{ v_1^+, v_2^+, ..., v_m^+ \right\} \tag{18}$$

$$A^- = \left\{ (\min v_{ij} \mid j \in G), (\max v_{ij}, j \in G'), i = 1, .., n \right\} = \left\{ v_1^-, v_2^-, ..., v_m^- \right\} \tag{19}$$

It can be seen from equations (16) and (17) that the ordinary TOPSIS method is based on the Euclidean distance (Chang et al., 2010; Shanian & Savadogo, 2006).

3.6 D'IDEAL (Displaced Ideal Method)

An "ideal" object is formed from the most preferable values of the criteria and so are "anti-ideals" from the least preferred values. The distances of the objects from the original set to the "anti-ideal" are determined, on the basis of which the "worst" objects are allocated. After excluding the "worst" objects, we return to the stage of formation of the "ideal", and it changes, approaching the real objects. The procedure ends when there remain a small number of objects, which are considered to be the most preferable. The step wise procedure is given below:

Step 1 Determine an "ideal" object and an "anti-ideal" one

$$a_j^+ = \{ \max_i a_{ij} \left| if\ j \in C_j(\max); \ \min_i a_{ij} \right| if\ j \in C_j(\min) \}; \ j = 1, ..., n \tag{20}$$

$$a_j^- = \{ \min_i a_{ij} \left| if\ j \in C_j(\max); \ \max_i a_{ij} \right| if\ j \in C_j(\min) \}; \ j = 1, ..., n \tag{21}$$

$$w_j \cdot x_{ij} = \begin{cases} w_j \cdot \dfrac{a_{ij} - a_j^+}{a_j^+ - a_j^-} \ if\ x_{ij} \in B; \\ \\ w_j \cdot \dfrac{a_j^+ - a_{ij}}{a_j^+ - a_j^-} \ if\ x_{ij} \in C. \end{cases} \tag{22}$$

Step 2 Calculate the distance of the objects to the "anti-ideal" using metrics for different values of *p*, for example, $p = \{1, 2, \infty\}$

$$L_i^p = \left\{ \sum_{j=1}^{n} x_{ij}^{\ p} \right\}^{1/p}, \ L_i^\infty = \max_j \left| x_{ij} \right| \tag{23}$$

Step 3 Exclude "hopeless" options. For this, for each p, all objects are ordered in proximity to the "ideal" in magnitude L_i^p. The more L_i^p, the further A_i is from the anti-ideal, and the higher the rank of the alternative A_i (rank 1 is higher).

$$Q_i = \sum_p (L_i^p / L_{max}^p); L_{max}^p = \max_i L_i^p \ ;$$

$$R_i = \sum_p r_i^p; \ r_i^p = rank(L_i^p \mid \{L_1^p, L_2^p, ..., L_m^p\})$$

(24)

Exclude one (two or three, depending on the number of alternatives) of "unpromising" variants that have the greatest total rank R_i. These are objects that, at different metrics (different p), are at the end of the ordered series. The procedure ends when there remain a small number of objects, which are considered to be the most preferable. The best alternative is $\max_i Q_i$.

3.7 MABAC (Multi-Attributive Border Approximation area Comparison)

The MABAC method is developed by Pamucar & Cirovic (2015). The basic setting of the MABAC method consists in defining the distance of the criteria function of every observed alternative from the border approximate area. The step wise procedure is given below:

Step 1 Normalization of the initial matrix elements.

$$x_{ij} = \begin{cases} \dfrac{a_{ij} - a_j^+}{a_j^+ - a_j^-} \ if \ x_{ij} \in B; \\ \dfrac{a_j^+ - a_{ij}}{a_j^+ - a_j^-} \ if \ x_{ij} \in C. \end{cases}$$

(25)

where, a_j^+ and a_j^- represent the elements of the initial decision matrix.

Step 2 Calculation of the weighted matrix elements. The elements of the weighted matrix are calculated on the basis of the expression (26)

$$v_{ij} = (x_{ij} + 1) \cdot w_j$$

(26)

where v_{ij} represents the elements of the normalized matrix, w_j represents the weighted coefficients of the criterion.

Step 3 Determination of the approximate border area matrix. The border approximate area for every criterion is determined by expression (27):

$$g_j = \left(\prod_{i=1}^m v_{ij} \right)^{1/m}, \ i = 1, m; \ j = 1, n$$

(27)

where v_{ij} represents the elements of the weighted matrix, m represents total number of alternatives.

After calculating the value of g_j by criteria, a matrix of border approximate areas G is developed in the form n x 1.

Step 4 Ranking of alternatives. The calculation of the values of the criteria functions by alternatives is obtained as the sum of the distance of alternatives from the border approximate areas. The final values of the criteria function of alternatives are obtained as follows

$$Q_i = \sum_{j=1}^{n} (v_{ij} - g_j) \tag{28}$$

where n represents the number of criteria.

Step 5 The best alternative is $\max_i Q_i$.

3.8 ORESTE (Organisazion, RangEment ot SynTEze de donnecs relationnelles) method

The ORESTE method was developed by Roubens (1978). The aim of this method is to find a global preference structure of a set of alternatives by evaluating them by each criterion and the preference among the criteria. This method generally defines criteria and alternatives, constructs a global complete and partial preorder of the alternatives by performing indifference and conflict analyses. In this research normalization matrix is performed by using $r_{ij} : norm(3)$. The step wise procedure is given below (Roubens (1978)):

Step 1 Transition from matrix DM to matrix of ranks (the columns of the matrix are replaced by their ranks)

$$r_{ij} = rank(a_{ij} \mid \{a_{1j}, a_{2j}, ..., a_{mj}\}), \; \forall i, j \quad (i = 1, ..., m; j = 1, ..., n) \tag{29}$$

Step 2 Determine ranks of criteria

$$rc_j = rank(C_j \mid \{C_1, C_2, ..., C_n\}), \; \forall j = 1, ..., n \; ; or$$
$$rc_j = rank(w_j \mid \{w_1, w_2, ..., w_n\}) \tag{30}$$

Step 3 Compute the projections of ranks

$$d_{ij} = \left[(1 - \alpha) \cdot r_{ij}^{\,p} - \alpha \cdot rc_j^{\,p} \right]^{1/p}, \; \alpha \in (0;1)$$

$$p = 1, \; Average \; (Mean);$$
$$p = -1, \; Medium \; Harmonic;$$
$$p = 2, \; Mean \; Square \tag{31}$$
$$p = \inf, \; \max(R, w);$$
$$p = -\inf, \; \min(R, w)$$

Step 4 Calculating ranks d_{ij}

$$Rd_{ij} = rank(d_{ij} \mid \{d_{ij}\}_{i=1:m; \, j=1:n}), \; R_i = \sum_{j=1}^{n} Rd_{ij} \tag{32}$$

Step 5 Calculate ranks R_i

$$OutR_i = rank(R_i \mid \{R_1, R_2, ..., R_m\}) \tag{33}$$

Step 6 Calculate preference factors C_{ik}

$$C_{ik} = \frac{1}{2 \cdot n^2 \cdot (m-1)} \cdot \sum_{j=1}^{n} (Rd_{ij} - Rd_{kj} + \mid Rd_{ij} - Rd_{kj} \mid) \tag{34}$$

$$r_{ij} = \underset{j}{rank}(a_{ij}); \; R_{ij} = \underset{j}{sort}(a_{ij}, if \; sg_j = +1, 'descend', \; if \; sg_j = -1, 'ascend',);$$

Step 7 The best alternative is $\min_i Q_i$.

3.9 PROMETHEE (Preference Ranking Organisation METHod for Enrichment Evaluations)
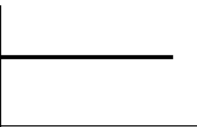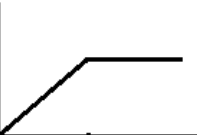
The PROMETHEE method was developed at the beginning of the 1980s and has been extensively studied and refined since then (Figueira et al., 2005). It has particular application in decision-making, and is used around the world in a wide variety of decision scenarios, in the fields such as business, governmental institutions, transportation, healthcare and education.
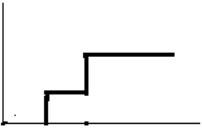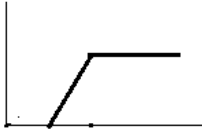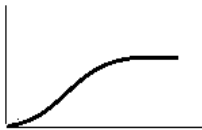
The PROMETHEE method helps the decision makers find the alternative that best suits their goal and their understanding of the problem. It provides a comprehensive and rational framework for structuring a decision problem, identifying and quantifying its conflicts and synergies, clusters of actions, and highlights the main alternatives and the structured reasoning behind them. The step wise procedure is given below:

Step 1 Set the preference function for two objects for each criterion $H_j = H(d_{is}, p, q)$. As a rule, they have two parameters: p - indifference threshold, it reflects the fact that if difference of d_{is} values of two alternatives i and s is unimportant, then objects by criterion j are equivalent. If the difference in threshold value p is exceeded, a preference relation is established between the objects. If the difference in threshold q is exceeded, the preference function corresponds to the "strong preference" of variant i with respect to variant s with respect to criterion j. With the difference of d_{is} in the interval from p to q, the preference function is less than 1, which corresponds to a "weak preference".

The choice of the preference function is determined by the decision-makers. Some types of functions are preferred $H(d)$ are presented below (Table 1): 1) regular- 0 if $d< =0$, 1 if $d>0$; 2) U-Shape ([p 0] $p>0$); 3) V-Shape ([p 0] $p>0$); (p is indifference threshold); 4) Level criterion([p g]); p, $q>0$ (q is the preference threshold); 5) Linear criterion([p g]); $p, q>0$; 6) Gaus criterion([p p]) p=sigma (Table 1).

Table 1 Preference functions of PROMETHEE

Function	Shape	Threshold	Formula
Usual		No threshold	$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases}$;
U-shape		q threshold	$f(x) = \begin{cases} 1, & x > q \\ 0, & x \leq q \end{cases}$;
V-shape		p threshold	$f(x) = \begin{cases} x/p, & x \leq p \\ 1, & x > p \end{cases}$;

Function	Shape	Threshold	Formula
Level	1	p and q threshold	$f(x) = \begin{cases} 0, \ x \le p \\ 0.5, \ p < x < q; \\ 1, \ x \ge q \end{cases}$
Linear	1	p and q threshold	$f(x) = \begin{cases} 0, \ x \le p \\ (x-p)/(q-p), \ p < x < q; \\ 1, \ x \ge q \end{cases}$
Gaussian	1	s threshold	$f(x) = 1 - \exp\left(-\dfrac{x^2}{2s^2}\right);$

Step 2 Calculate the difference in the values of the criteria for the two objects and calculate preference indexes V

$$d_{is} = a_{ij} - a_{sj}; \ H_j = H_j(d_{is}, p, q,); \ V_{is} = \sum_{j=1}^{n} w_j \cdot H_j \quad -[m \times m] - Matrix \tag{35}$$

Step 3 Determine the preference factors

$$\Phi_i^+ = \sum_{s=1, s \ne i}^{m} V_{is}; \ \Phi_i^- = \sum_{s=1, s \ne i}^{m} V_{si}; \ Q_i = \Phi_i^+ - \Phi_i^- . \tag{36}$$

Step 4 The best alternative is $\max_i Q_i$.

3.10 CODAS (COmbinative Distance-based ASsessment) method

The CODAS method is an efficient and updated decision-making methodology introduced by Keshavarz Ghorabaee et al. (2016). The desirability of alternatives in the CODAS is determined based on l^1-norm and l^2-norm indifference spaces for criteria. According to these spaces, in the procedure of this method, a combinative form of the Euclidean and Taxicab distances is utilized for calculation of the assessment score of alternatives. The step wise procedure is given below:

Step 1 Construct the Weighted Normalized Decision Matrix

$$x_{ij} = \begin{cases} w_j \cdot \dfrac{a_{ij}}{a_j^{\max}} \ if \ x_{ij} \in B; \\ w_j \cdot \dfrac{a_j^{\min}}{a_{ij}} \ if \ x_{ij} \in C. \end{cases} \tag{37}$$

Step 2 Determine the negative-ideal solution as given in equation. Construct *min* vector for criteria

$$r_j = \min_i x_{ij}; \ j = 1,...,n; \ i = 1,...,m \tag{38}$$

Step 3 Calculate the Euclidean and Taxicab distances of alternatives from the negative-ideal solution

$$E_i = \left[\sum_{j=1}^{n} (x_{ij} - r_j)^2 \right]^{1/2} \tag{39}$$

$$T_i = \sum_{j=1}^{n} | x_{ij} - r_j | \tag{40}$$

Step 5 Construct the relative assessment matrix

$$H_{ik} = (E_i - E_k) + \psi(E_i - E_k) \cdot (T_i - T_k), \quad i,k = 1,...,m \tag{41}$$

where ψ denotes a threshold function

$$\psi(x) = \begin{cases} 1, & if \ |x| \geq \tau \\ 0, & if \ |x| \prec \tau \end{cases} \tag{42}$$

τ is the threshold parameter that can be set by the decision maker. It is suggested to set this parameter as a value between 0.01 and 0.05. If the difference between the Euclidean distances of two alternatives is less that τ, the two alternatives are also compared by the Taxicab distance.

Step 6 Calculate the assessment score of each alternative

$$H_i = \sum_{k=1}^{m} H_{ik} \tag{43}$$

Step 7 Rank the alternatives according to the decreasing values assessment score *H*. The alternative with the highest *H* is the best choice among the alternatives.

4 An illustrative example: the location selection of tri-modal LC and logistical flows

The sensitivity analysis model is tested on an example of the logistical center (LC) location selection (Pamučar et al, 2017). The goal is to find a location which generates lowest expenses, offers highest efficiency and at the same time fulfills operational and strategic needs.

3.1 Alternatives and criteria weighting

In our example the authors used 11 criteria which were identified in Pamučar et al (2017) based on which the location selection of tri-modal LC is going to be conducted (Table 2).

Table 2 Criteria for LC selection (Pamučar et al, 2017)

Criterion	Criterion name	w_i	Unit of Measurement
C_1	Connectivity to Multimodal Transport	0.109	Linguistic Variable
C_2	Infrastructure Development Evaluation	0.105	Infrastructure Development (%)
C_3	Environment effect	0.101	Linguistic Variable
C_4	Conformity with Spatial Plans and Strategy Of Economic Development	0.097	Linguistic Variable
C_5	Gravitating Intermodal Transport Unit - ITU	0.094	Number of Gravitating ITUs (ITU/year)

Criterion	Criterion name	w_i	Unit of Measurement
C_6	Reload capacity of an LC	0.094	Number of Reloaded ITUs (ITU/h)
C_7	Available Area For Future Development and Capacity Expansion Of Lcs	0.093	LC Development Area (m^2)
C_8	User's Distance from an LC	0.088	Linguistic Variable
C_9	Traffic Safety	0.084	Linguistic Variable
C_{10}	Length of Railroad Reload Front	0.071	Reload Front Length (m)
C_{11}	Evaluation of Quality of Traffic Approaches for Interpellant Means of Transportation	0.063	Linguistic Variable

A total of eight locations were considered. Table 3 shows characteristics of eight locations (alternatives) for the tri-modal LC development on the Danube River.

Table 3 A mixed data matrix corresponding to example (Pamučar et al, 2017)

Alternative	Criteria										
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
LC 1	4	71	4	3	45000	150	1056	2	4	478	4
LC 2	4	85	4	4	58000	145	2680	2	5	564	4
LC 3	4	76	4	4	56000	135	1230	2	4	620	3
LC 4	3	74	3	4	42000	160	1480	4	3	448	5
LC 5	5	82	3	5	62000	183	1350	2	4	615	4
LC 6	4	81	3	5	60000	178	2065	2	3	580	4
LC 7	4	80	3	5	59000	160	1650	3	5	610	4
LC 8	3	82	4	4	54000	120	2135	3	4	462	5

The weight coefficients of the criteria are obtained based on the Sun (2012), Zare at al (2013) and Rahmaniani et al (2013):

$$w_j = (0.109; 0.105; 0.101; 0.097; 0.094; 0.094; 0.093; 0.088; 0.084; 0.071; 0.063)$$

with criteria sign $C_s = (1; 1; -1; 1; 1; 1; 1; -1; 1; 1; 1)$, where "1" marks criteria of the "benefit" type (bigger criterion value is preferable), whereas "-1" marks criteria of the "cost" type (lower criterion value is preferable).

Variations in the values of the alternatives of the presented example are carried out for the criteria C3, C5-C7, C10. For software implementation, it is sufficient to specify a vector-switch, according to the number of criteria. For the realized example, this is the vector [0 0 1 0 1 1 1 0 0 1 0], where 1- on, 0-off.

3.2 Statistical experiment

Statistics of effectiveness indicators of the alternatives for each criterion is made by the following calculation formula:

$$D_k = D_0 + (-1 + 2 \cdot rnd()) \cdot \delta \cdot D_0 \tag{44}$$

where D_0 is the initial evaluation of the decision matrix; the function $rnd()$ returns a uniformly distributed random number from [0,1]; δ is relative error of estimating alternatives for each criterion; $k = 1, ..., N$ is the number of variations in decision matrix D_k.

For each variation of matrix D_k, a general evaluation of the alternatives' effectiveness for all the criteria was made by using one of the above described aggregation methods: SAW, MOORA, VIKOR, COPRAS, CODAS, TOPSIS, D'IDEAL, MABAC, PROMETHEE-I,II, ORESTE-II. The calculations are performed in the MATLAB system. The software protocols (m-files) and the user's manual are publicly available in the file exchange of website of the company MathWorks (Mukhametzyanov, 2018a, 2018b, 2018c, 2018d). The volume of the statistical experiment is N=1024. Relative error values δ varied from 5, to 25% {0.05, 0.10, 0.15, 0.20, 0.25}.

Thus, in statistical experiments, N values of the overall evaluation of the effectiveness of alternatives $\left\{C_i^*\right\}_k$ for all the criteria and the ranks (priorities) $\left\{r_i^*\right\}_k$ of alternatives A_i ($i = 1, m$) corresponding to these values for each of the considered MCDM methods are obtained. For each sample of N values C_i^*, mean $\overline{C}_i^*$ and standard deviation $std(C_i^*)$ are calculated.

5 Results of sensitivity analysis of MDM methods

5.1 Distribution of the overall evaluation of the effectiveness of alternatives

In accordance with the central limit theorem (Lyapunov CLT) and considering that C_i^* aggregation is carried out additively for all alternatives, the distribution of random variable C^* obeys the normal distribution law. Figs. 1 and 2 show typical histograms of values C_i^* obtained for different values of the relative error of the computational experiment.
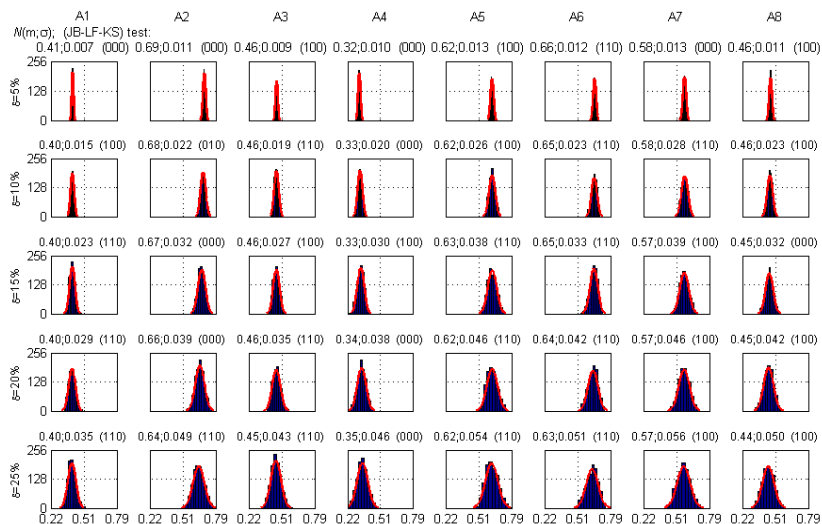
Fig. 1 Histogram of the relative closeness to the ideal solution depending on the relative error in the data (δ,%). (1000 Simulation of DM Matrix; m, σ- parameters of normal distribution)

Fig. 1 shows point estimates of unknown mean-variance parameters and also the logical values of three tests of normal distribution. This is Jarque-Bera test, Lilliefors test and Kolmogorov-Smirnov test. The null hypothesis that the sample is in vector Q comes from normal distribution with unknown mean and variance, with the alternative that it does not come from normal distribution. Three (JB-LF-KS): the figure is represented by a set of 0 and 1 for each of the tests. The test returns the logical value h = 1 if it rejects the null hypothesis at the 5% significance level, and h = 0 if it cannot.
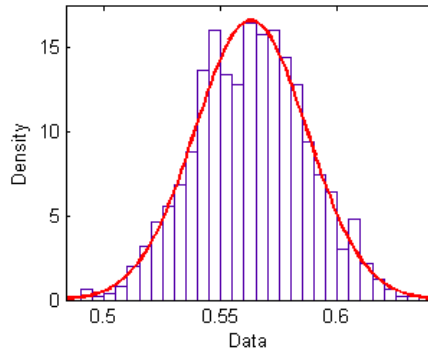
Fig. 2 Fit distributions to C_5^* (LC5, COPRAS, δ=20%, 1000 Simulation of DM Matrix)

For all the methods, a slight decrease $\overline{C}_i^*$ and increase $std(C_i^*)$ is observed with increasing values δ . The dynamics $\overline{C}_i^*$ and $std(C_i^*)$ is shown in Fig. 3.
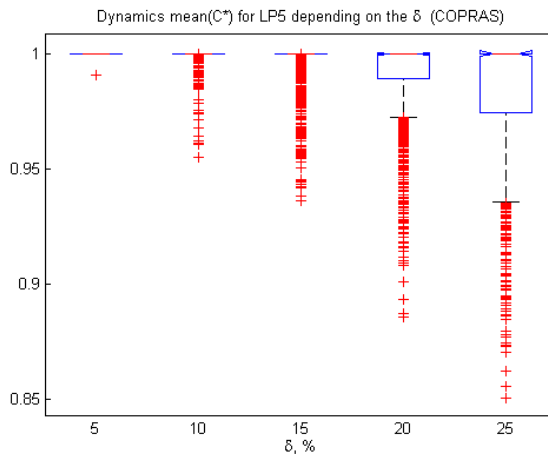
Fig. 3 Dynamics mean (C_5^*) depending on δ. (A5, COPRAS, 1000 Simulation of DM Matrix).

To determine the distribution law, use the statistical function Statistics Toolbox MATLAB - dfittool - Interactive distribution fitting - opens a graphical user interface for displaying fit distributions to data.

Distribution parameters referring to Fig. 2 is: Distribution: Normal; Log likelihood: 2309.1; Domain: -Inf <y <Inf; Mean: 0.563; Variance: 0.0006; Parameter Estimate: mu 0.563, Std. Err. Mu 0.00076, Sigma 0.024, Std. Err. Sigma: 0.0005; 95% confidence intervals for mu: (0.5614; 0.5644). Similar statistical results hold for all A_i, all Methods, and all δ.

The distributions of statistics for the SAW, MOORA, ORESTE-2, TOPSIS, MABAC, PROMETHEE methods are described by the normal distribution law. The CODAS and VIKOR methods are not very stable to the variation of the initial data - multimodality, distribution asymmetry, or incomprehensible distribution laws are observed. For the COPRAS method, "leaders" alternatives A2, A5, A6 have deviations from normality due to strong asymmetry. For the D'Ideal method, distributions for alternatives A5 and A8 are not stable.

4.2 Ranking of alternatives

Changing the initial decision matrix at random in the calculations for a given value of the relative error (not more than δ) in a number of experiments, the priorities of the alternatives change. For example, Tables 4 and 5 show the results of the COPRAS calculations for various initial data. At δ = 10%, the first priority has alternatives LP5 in 91.8% simulations, LP2 in 5.2% simulations and LP6 in 3.0% simulations.

Table 4 COPRAS ranking C* for various initial data of the decision matrix for δ = 0.1; 1024 simulations (Fragment)

K	LC1	LC2	LC3	LC4	LC5	LC6	LC7	LC8
1	0.112	0.135	0.119	0.110	0.138	0.135	0.131	0.118
2	0.112	0.135	0.119	0.109	0.138	0.137	0.130	0.118
3	0.112	0.135	0.118	0.111	0.138	0.135	0.132	0.118
4	0.115	0.133	0.118	0.110	0.140	0.135	0.130	0.118
5	0.113	0.134	0.117	0.110	0.138	0.131	0.134	0.121
6	0.112	0.135	0.119	0.110	0.136	0.136	0.131	0.121
7	0.112	0.137	0.118	0.109	0.138	0.134	0.131	0.120
8	0.113	0.135	0.118	0.110	0.139	0.133	0.132	0.119
9	0.112	0.135	0.119	0.108	0.139	0.139	0.129	0.119
10	0.113	0.137	0.119	0.110	0.136	0.134	0.132	0.118
				...				
51	0.112	0.134	0.120	0.110	0.138	0.136	0.130	0.119
				...				
74	0.112	0.134	0.119	0.110	0.139	0.135	0.131	0.118
				...				
102	0.112	0.136	0.120	0.109	0.137	0.134	0.131	0.119
				...				
1023	0.113	0.137	0.120	0.110	0.138	0.134	0.131	0.118
1024	0.114	0.134	0.120	0.110	0.137	0.136	0.130	0.118
First rank (%)	8.6	-	-	79.3	12.1	-	-	
Second rank (%)	-	-	-	-	44.2	-	-	
Third rank (%)	48.4	-	-	-	-	-	-	

Table 5 Summarized results of the COPRAS calculations for various initial data ($\delta = 0.1$; N=1024)

Rank	Alternatives							
	LP1	LP2	LP3	LP4	LP5	LP6	LP7	LP8
I	0	88	0	0	812	124	0	0
II	0	391	0	0	172	453	8	0
III	0	496	0	0	40	410	78	0
IV	0	49	0	0	0	37	938	0
V	0	0	615	0	0	0	0	409
VI	1	0	409	0	0	0	0	614
VII	954	0	0	69	0	0	0	1
VIII	69	0	0	955	0	0	0	0
Total	1024	1024	1024	1024	1024	1024	1024	1024

In some cases, it may turn out that average efficiencies $\overline{C}_i^*$ for various alternatives are not statistically indistinguishable; hence, for the ranking it is necessary to consider alternatives having the largest number of the second and third ranks. We denote them as II(1) and III(1). Priorities of alternatives C_i^* are stochastic values. Therefore, when using the ranking procedure, the criterion for meaningful distinguishability of values C_i^* should be used. For example, the question is how much the value C_i^* is for the seventh experiment (refer to: Table 2). To correctly answer this question, it is necessary to construct interval estimates for C_i^* and to make a t-Test of the Student of significant difference between the two averages. Thus, the problem of measuring the error of the result is current, provided that the error (error) in the initial data is known (estimated). Otherwise, we cannot guarantee the priority of any alternative, no matter what method we use.

Having a statistical picture of the ranks assessment, the decision-making can be carried out not only on the statistics of alternatives having rank I, but it can also use the statistics of alternatives having, for example, the largest total rank. The sums of the first three alternatives are relevant. The total rank of such alternatives is denoted by I+II(1) and I+II+III(1).

For various variants of calculations for a fixed δ, the ranks of alternatives change. Suppose that in the N simulation experiments DM, I the rank of alternative A_k took n_k points (times), alternative A_s took n_s, and alternative A_p took n_p ($n_k > n_s > n_p$). It seems that the choice is in favor of alternative A_k. But this holds for only when the approach is superficial (trivial). After all, n_p of the first ranks of alternative A_p are obtained for concrete n_p implementations of the decision matrix. It is possible that the true values of the estimates of alternatives are according to the criteria from the same set. Therefore, it is necessary to take into account such options. The following variations of the ranks I(2), I(3) - alternatives having rank I and, respectively, 2 and 3, the number (points) of realizations in N experiments are relevant. Alternatives I+II(2), I+II(3) and I+II+III(2) - alternatives having I, II and III rank are also relevant, and having respectively 2 and 3 the number of total realizations in N experiments.

Figs. 4 and 5 show the distribution of the points (number) of realizations of effective alternatives (%) to the total number of N experiments having ranks I, II, III, and the sums I+II, I+II+III ranks.
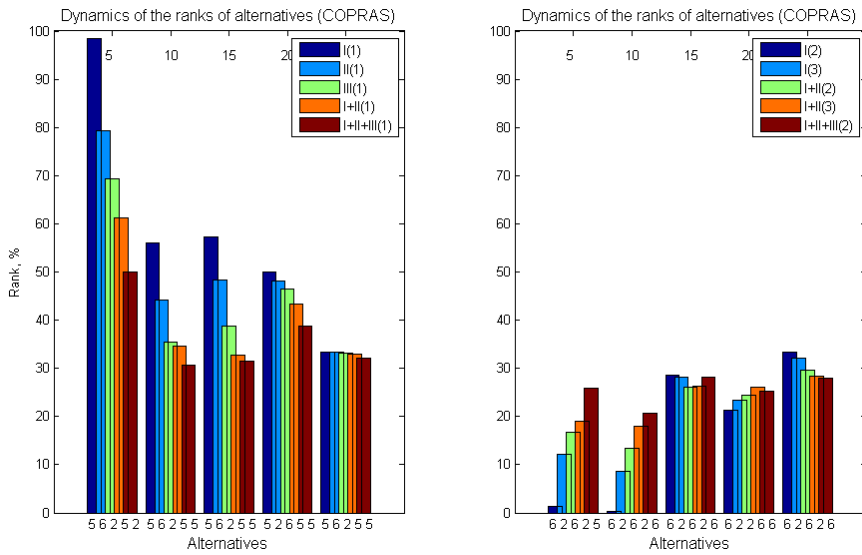
Fig. 4 Distribution of the point of realizations of effective alternatives in%
to the total number of *N* experiments having ranks I, II, III, and the sums
I+II, I+II+III ranks (LC5, COPRAS, 1024 Simulation of DM Matrix, δ = 5-
25%)

The results show that in more options, the alternative is LC5. However, for a significant fraction of the total number of experiments = *N*, the alternatives are LC3 and LC6.
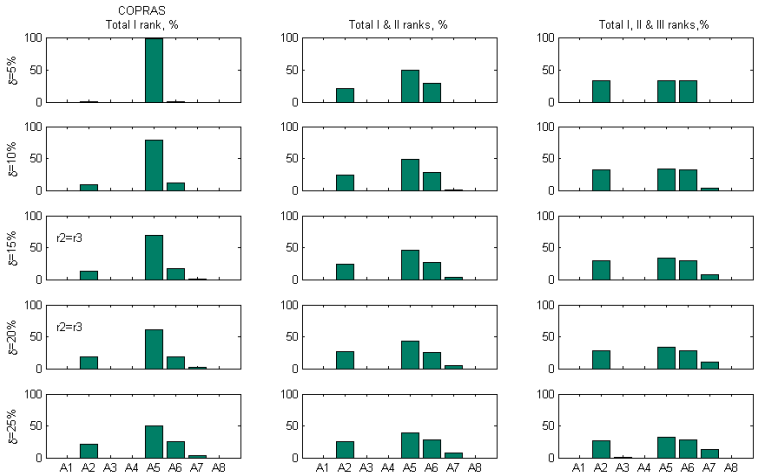
Fig. 5 Distribution of the point of realizations of effective alternatives in%
to the total number of N experiments having ranks I(1) and the sums I +
II(1), I + II + III(1) ranks. (COPRAS, 1024 Simulation of DM Matrix, δ = 5-
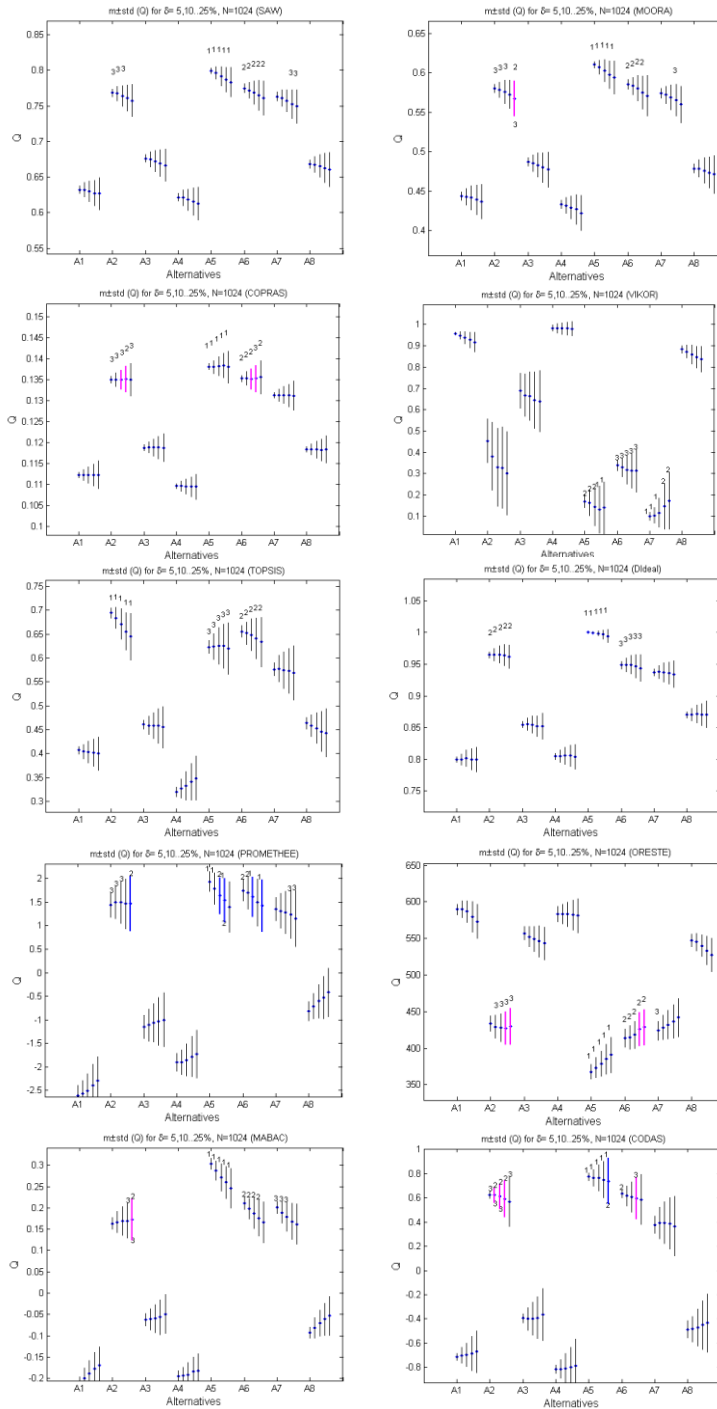25%)

Fig. 6 Point and interval estimates of the integral index of alternatives for various rank methods and for different values of the random deviation of the elements of the decision matrix (δ = 5, 10, 15, 20 and 25%)

Figs. 1, 2 and 3 are marked, respectively, 1, 2 and 3 ranks. In blue, statistically indistinguishable values of the integral index of the 1st and 2nd rank are distinguished; in red, there are statistically indistinguishable values of the integral index of the 2nd and 3rd ranks.

4.3 Distribution of the overall effectiveness evaluation of alternatives for various MCDM methods

Table 6 shows the distribution of the overall effectiveness evaluation of alternatives for various MCDM methods for δ = 10% (similarly for the other δ). The results show a strong sensitivity of the selection procedure from the selected MCDM method and from the selection criterion.

Table 6 Distribution of the overall efficiency evaluation of alternatives for various MCDM methods for δ = 10% (N=1024)

					Rank(point)						
	δ,%		I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
SAW	5	A:	5	6	2	5	5	6	1	6	2	6
		%	99.8	78.4	60.5	50.0	33.3	0.2	0.0	39.3	8.9	32.2
	10	A:	5	6	2	5	5	6	2	6	2	6
		%	95.0	53.5	39.3	49.8	33.3	3.2	1.7	28.4	15.8	29.1
	15	A:	5	6	2	5	5	6	2	6	2	6
		%	79.3	39.9	35.6	47.3	32.9	11.3	5.9	25.6	17.9	26.7
	20	A:	5	6	7	5	5	6	2	6	2	6
		%	66.7	31.3	32.4	45.0	32.6	17.0	11.4	24.1	20.6	25.7
	25	A:	5	6	7	5	5	6	2	6	2	6
		%	59.5	28.9	33.0	42.7	31.6	19.0	14.5	24.0	21.1	25.3
			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
MOORA	5	A:	5	6	2	5	5	1	1	6	2	6
		%	100.0	75.6	58.0	50.0	33.3	0.0	0.0	37.8	10.5	32.5
	10	A:	5	6	2	5	5	6	2	6	2	6
		%	93.9	51.7	42.8	49.7	33.3	3.9	1.6	27.8	16.5	28.3
	15	A:	5	6	2	5	5	6	2	6	2	6
		%	77.4	35.4	33.8	47.4	33.1	13.4	6.6	24.4	19.4	26.9
	20	A:	5	6	7	5	5	6	2	6	2	6
		%	68.0	30.4	34.0	44.8	32.6	15.7	11.3	23.0	20.8	24.7
	25	A:	5	2	2	5	5	6	2	6	2	6
		%	60.7	28.6	31.2	42.3	31.9	19.3	12.5	23.7	20.6	25.1
			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
COPRAS	5	A:	5	6	2	5	2	6	2	6	2	5
		%	98.4	56.1	57.3	50.0	33.3	1.3	0.3	28.7	21.3	33.3
	10	A:	5	6	2	5	5	6	2	6	2	6
		%	79.3	44.2	48.4	48.0	33.3	12.1	8.6	28.2	23.4	32.1
	15	A:	5	6	2	5	5	6	2	6	2	6
		%	69.2	35.5	38.8	46.5	33.2	16.8	13.5	26.2	24.4	29.6
	20	A:	5	2	6	5	5	6	2	2	6	6
		%	61.2	34.7	32.8	43.3	32.9	19.0	18.0	26.3	26.0	28.3
	25	A:	5	6	2	5	5	6	2	6	2	6
		%	49.9	30.7	31.4	38.8	32.1	25.8	20.7	28.2	25.3	27.9
			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
	5	A:	7	5	6	5	5	5	6	7	6	7
		%	100.0	1.5	1.4	50.0	33.3	98.5	84.6	50.0	42.3	33.3
	10	A:	7	5	6	7	7	5	6	5	6	5
		%	99.5	9.9	8.3	50.0	33.3	89.0	72.4	49.4	36.4	33.3
	15	A:	7	5	6	7	5	5	6	5	6	7
		%	90.4	11.4	13.1	48.6	33.0	84.3	59.7	47.9	30.9	33.0
	20	A:	5	7	6	5	5	7	6	7	6	7
		%	79.9	13.9	21.1	46.0	32.3	74.9	49.6	44.4	27.7	31.5
	25	A:	5	7	6	5	5	7	2	7	6	7

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			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
		%	76.3	12.5	18.4	43.8	31.7	67.7	45.1	40.1	26.9	29.8
VIKOR	5	A:	2	6	5	2	2	6	1	6	5	5
		%	98.9	98.5	99.6	50.0	33.3	1.1	0.0	49.8	0.2	33.3
	10	A:	2	6	5	2	2	6	5	6	5	6
		%	80.8	71.7	80.7	47.6	33.3	16.1	3.1	43.9	8.5	33.3
	15	A:	2	6	5	2	6	6	5	6	5	2
		%	61.0	53.9	54.4	40.8	32.6	25.0	13.8	39.5	18.5	32.1
	20	A:	2	6	5	6	6	6	5	2	5	2
		%	50.8	44.6	41.8	36.5	31.2	28.4	19.1	36.0	23.5	30.6
	25	A:	2	6	5	6	6	6	5	2	5	2
		%	44.5	39.3	35.8	34.4	30.0	29.5	22.5	33.4	25.0	29.2
			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
TOPSIS	5	A:	5	2	6	5	2	1	1	2	6	5
		%	100.0	99.9	97.0	50.0	33.3	0.0	0.0	50.0	0.0	33.3
	10	A:	5	2	6	5	5	1	1	2	6	2
		%	100.0	92.6	75.9	50.0	33.3	0.0	0.0	46.3	3.5	33.2
	15	A:	5	2	6	5	5	2	6	2	6	2
		%	97.9	80.1	60.8	50.0	33.3	2.1	0.1	41.1	8.3	32.9
	20	A:	5	2	6	5	5	2	6	2	6	2
		%	94.1	70.7	49.8	49.7	33.3	4.7	0.9	37.7	10.1	31.8
	25	A:	5	2	6	5	5	2	6	2	6	2
		%	89.6	61.1	44.4	49.2	33.3	7.9	2.1	34.5	11.3	31.2
			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
D'Ideal	5	A:	5	6	7	5	5	1	1	6	7	6
		%	100.0	66.9	63.1	50.0	33.3	0.0	0.0	33.4	16.3	33.0
	10	A:	5	6	7	5	5	6	7	6	7	6
		%	98.2	51.2	41.0	50.0	33.3	1.3	0.4	26.2	18.2	28.5
	15	A:	5	6	7	5	5	6	7	6	7	6
		%	89.1	38.9	36.4	49.0	33.3	6.6	2.8	22.8	16.2	25.3
	20	A:	5	6	2	5	5	6	7	6	2	6
		%	82.8	32.5	33.5	47.9	33.2	7.0	5.7	19.8	16.4	23.6
	25	A:	5	2	2	5	5	2	6	2	6	2
		%	71.1	28.8	31.9	44.9	32.6	11.3	10.3	20.1	18.2	24.0
			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
MABAC	5	A:	5	6	7	5	5	6	7	6	7	6
		%	99.3	63.1	44.5	50.0	33.3	0.5	0.1	31.8	14.3	29.9
	10	A:	5	6	2	5	5	6	7	6	7	6
		%	93.2	52.8	36.5	49.7	33.3	4.8	1.2	28.8	11.4	27.9
	15	A:	5	6	2	5	5	6	2	6	2	6
		%	87.4	43.7	36.6	48.6	33.1	6.8	3.1	25.2	14.2	27.0
	20	A:	5	6	2	5	5	6	2	6	2	6
		%	79.5	35.3	33.8	47.0	32.9	8.6	6.5	21.9	18.8	24.9
	25	A:	5	6	2	5	5	6	2	6	2	2
		%	72.4	31.4	34.3	45.8	32.5	11.2	10.1	21.3	19.6	24.5
			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
ORESTE	5	A:	5	6	2	5	5	6	2	6	2	6
		%	67.7	47.9	36.0	46.4	32.9	24.2	6.2	36.0	12.2	31.9
	10	A:	5	6	2	5	5	6	2	6	2	6
		%	47.2	33.3	32.0	36.9	30.5	30.6	14.8	31.9	19.3	29.2
	15	A:	6	5	2	5	5	5	2	6	2	6
		%	33.6	31.3	27.0	31.1	28.6	30.8	23.0	30.1	23.7	28.0
	20	A:	5	5	7	5	5	6	2	6	2	6
		%	31.9	28.6	26.7	30.3	27.7	29.8	25.5	26.6	26.0	26.6
	25	A:	6	2	7	2	2	2	5	6	5	6
		%	28.6	27.0	26.3	27.7	26.9	28.4	27.8	27.3	27.2	26.4

			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
PROMETEE-II	5	A:	5	6	2	5	5	6	1	6	2	2
		%	99.8	56.2	56.3	50.0	33.3	0.2	0.0	28.2	21.8	33.3
	10	A:	5	2	2	5	5	6	2	6	2	2
		%	90.9	45.6	47.9	49.4	33.3	6.4	2.6	25.5	24.1	32.1
	15	A:	5	2	2	5	5	6	2	2	6	2
		%	74.7	39.7	40.2	47.6	33.1	14.0	10.4	25.1	24.1	30.1
	20	A:	5	2	6	5	5	6	2	2	6	6
		%	64.5	36.1	34.7	43.6	32.7	19.3	13.4	24.8	24.6	27.9
	25	A:	5	5	2	5	5	6	2	6	2	6
		%	54.7	29.5	34.5	42.1	31.9	22.4	17.8	25.7	23.3	27.2
			I(1)	II(1)	III(1)	I+II(1)	I+II+III(1)	I(2)	I(3)	I+II(2)	I+II(3)	I+II+III(2)
Total	5	A:	5	6	6	5	5	2	6	6	2	6
		%	86.4	54.3	30.7	44.7	33.3	12.2	11.2	32.7	18.7	32.0
	10	A:	5	6	6	5	5	6	2	6	2	6
		%	79.0	41.0	32.2	44.1	33.0	15.1	14.7	28.1	22.7	29.4
	15	A:	5	6	6	5	5	6	2	6	2	6
		%	70.4	32.7	31.3	43.4	32.4	18.7	17.2	25.7	24.2	27.6
	20	A:	5	2	6	5	5	6	2	2	6	6
		%	64.8	31.3	30.4	42.1	32.0	19.5	18.8	25.0	24.0	26.2
	25	A:	5	2	6	5	5	2	6	2	6	2
		%	58.4	29.0	28.2	40.2	31.2	21.3	21.2	25.2	24.1	25.7

5 Conclusion

Despite a significant number of developed and new methods, the problem of multicriteria choice is still not trivial. Following the obtained results, the evaluation of alternatives according to the criteria and the choice of the criterion for ranking alternatives using different ranks have a profound effect on the final choice.

Multiple simulation of the estimations of the decision matrix elements within a given error for calculating the ranks of alternatives allows one to obtain statistical estimates of ranks. Based on the simulations statistics, the decision-making can be carried out not only on the statistics of alternatives having rank 1, but also by using alternatives statistics having the largest total I and II rank or I, II and III ranks. This is especially true when the difference in rank values is not large and is distributed evenly among the first three ranks.

Apparently, a quantitative analysis can be used only to narrow the set of effective alternatives for the final decision-making. A statistical analysis makes an estimation of the number options possible in which an alternative has a priority. Additional criteria that take into account both aggregate priorities and the availability of possible priorities for other alternatives with small DM variations provide additional important information for the decision-maker.

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