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Option Pricing under Sign RCA-GARCH Models

A b s t r a c t. After Black and Scholes's groundbreaking work, the literature concerning pricing options has become a very important area of research. Numerous option valuation methods have been developed. This paper shows how one can compute option prices using Sign RCA-GARCH models for the dynamics of the volatility. Option pricing obtained from Sign RCA-GARCH models, the Black and Scholes's valuation and other selected GARCH option pricing models are compared with the market prices. This approach was illustrated by the valuation of the European call options on the WIG20 index. The empirical results indicated that RCA-GARCH and Sign RCA-GARCH models can be successfully used for pricing options. However none of the models can be indicated as the best one for the option valuations for every period and every time to maturity of the options.

K e y w o r d s: Sign RCA-GARCH models, option pricing, GARCH models.

J E L Classification: G13.

Introduction

Following the seminal work of Black and Scholes (1973) and Merton (1973), the option literature has developed into an important area of research. The Black-Scholes formula (henceforth BS) assumes that stock price varies according to the geometric Brownian motion. The relationship between the geometric Brownian motion and the BS formula presents the following equivalence (Elliott and Kopp, 1999)[1]:

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[1] This equivalence is obtained by applying Itô Lemma with function $f(S_t) = \log S_t$.

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad \Leftrightarrow \quad S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}, \tag{1}$$

where:

S_t – a stock price,

μ – the drift rate, annualized expected value of S_t,

t – time,

W_t – the Wiener process (Brownian motion),

$\sigma > 0$ – the annualized volatility of S_t.

The BS formula assumes that the returns of the underlying asset (stock price) follow a normal distribution with constant volatility. Empirical evidence has shown, however, that the model is in conflict with facts, especially for short-run returns[2]. The financial markets research indicated that financial series, such as stock returns, foreign exchange rates and others, exhibit leptokurtosis and volatility varying in time. Hence the assumption of constant volatility is often strongly violated. Therefore, several option valuation models have been developed to incorporate stochastic volatility. One approach is to use continuous-time stochastic volatility models. Another approach is to use discrete-time generalized autoregressive conditionally heteroskedastic (GARCH) models (amongst others Engle,1982; Bollerslev,1986).

The choice of discrete-time GARCH models for this study was motivated by two facts, that:

– the inclusion of linear autoregressive dynamics, AR(1), affects option prices (Hafner and Herwartz, 2001),
– the random coefficient autoregressive models with the sign function (Sign RCA) are straightforward generalization of the constant coefficient autoregressive models (Thavaneswaran et al., 2006a).

The random coefficient and the sign function have influenced the unconditional kurtosis. The value of unconditional kurtosis in the RCA-GARCH and the Sign RCA-GARCH models is bigger in comparison with ordinary AR-GARCH. In addition, the sign function allows the modelling of asymmetry, such as response of returns on various information from the market.

The purpose of this work is to apply the Sign RCA-GARCH models to pricing European call options, and compare these results and results obtained from the Black-Scholes model and from other selected GARCH options pricing models with the market prices. Such use of Sign RCA-GARCH

[2] More frequent than monthly.

models as far as we know has not been applied in option pricing except the work by Górka (2012).

1. Theoretical Framework

1.1. Option Pricing

As a consequence of the equation (1), the price of a European call option is given by equation:

$$c_t = S_t N(d_1) - Ke^{-r\tau} N(d_2), \tag{2}$$

where:

$$d_1 = \frac{ln\frac{S_t}{K} + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau},$$

S_t – the stock price at time t,
K – the exercise price,
r – the risk-free interest rate,
τ – the time to maturity of the option,
$N(\cdot)$ – the cumulative normal density function,
σ – the volatility of rate of the return on the stock.

The valuation of derivative is about moving to the world free of risk, in which risky assets have the same return as the risk-free. The general idea of the valuation of derivatives is based on the following theorem.

Theorem (Elliott and Kopp, 1999).
If the process S satisfies the equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{3}$$

then also satisfies the equation

$$dS_t = rS_t dt + \sigma S_t d\widetilde{W}_t, \tag{4}$$

where is r the risk-free interest rate, $\widetilde{W}_t = W_t - \frac{r-\mu}{\sigma}t$ is a Wiener process.

Duan (1995) introduced the GARCH option pricing model by generalizing the traditional risk neutral valuation methodology to the case of conditional heteroskedasticity.

Letting the conditional mean μ_t and conditional variance σ_t^2 be measurable functions with respect to the information set (F), the general model under the data generating probability measure P is given by (Hafner and Herwartz, 2001):

$$\begin{aligned} & y_t = \mu_t + \sigma_t \varepsilon_t, \\ & \varepsilon_t \sim i.i.d.\, N(0,1), \\ & \sigma_t^2 = f\left(\sigma_s^2, \varepsilon_s; -\infty < s < t; \theta\right), \end{aligned} \tag{5}$$

where f is a parametric function with parameter vector θ.

In GARCH models, it is not possible to find a risk-neutralization procedure that leaves unchanged the marginal variance of the process or the conditional variance beyond one period. Therefore Duan (1995) introduced the local risk-neutral valuation relationship (LRNVR; equivalent martingale measure Q). The local risk-neutral valuation relationship is an essential feature of the equivalence of the conditional variances under the data generating probability measure P (historical measure) and the equivalent martingale Q.

Under the measure Q, the model is as follows (Hafner and Herwartz, 2001):

$$\begin{aligned} & y_t = \mu_t + \sigma_t(\varsigma_t - \lambda_t), \\ & \varsigma_t \sim i.i.d.\, N(0,1), \\ & \sigma_t^2 = f\left(\sigma_s^2, \varepsilon_s; -\infty < s < t; \theta\right), \\ & \varepsilon_t = \varsigma_t - \lambda_t, \\ & \lambda_t = \frac{\mu_t - r}{\sigma_t}, \end{aligned} \tag{6}$$

where:

r is the risk-free interes rate,

$\mathrm{E}^{\mathrm{Q}}[y_t \mid \mathrm{F}_{t-1}] = r$,

$\mathrm{var}^{\mathrm{P}}[y_t \mid \mathrm{F}_{t-1}] = \mathrm{var}^{\mathrm{Q}}[y_t \mid \mathrm{F}_{t-1}]$.

This procedure leaves unchanged the one period ahead conditional variance and the conditional expected future return is equal to the risk-free interest rate at each time t. Discounted asset price under the measure Q is a martingale.

1.2. Option Pricing under Sign RCA-GARCH Models

The RCA-GARCH models were proposed by Thavaneswaran et al., (2006a). The RCA(1)-GARCH(1,1) model has the following form:

$$y_t = (\phi + \delta_t) y_{t-1} + \xi_t, \tag{7}$$

$$\xi_t = \sigma_t \varepsilon_t, \tag{8}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \xi_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{9}$$

where ϕ, α_0, α_1, β_1 are parameters of model, $\varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right)$, $\delta_t \sim i.i.d.\left(0, \sigma_\delta^2\right)$, $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$.

Theoretical properties of this model can be found among others: Górka (2012), Thavaneswaran et al., (2006a, 2006b, 2008, 2009). It is worth noting, that the value of unconditional variance and kurtosis increases in comparison with ordinary AR(1)-GARCH(1,1).

We can define the RCA(1)-GARCH(1,1) option pricing model under the historical measure P:

$$\begin{aligned} y_t &= (\phi + \delta_t) y_{t-1} + \sigma_t \varepsilon_t, \\ \varepsilon_t &\sim N(0,1), \\ \sigma_t^2 &= \alpha_0 + \left(\alpha_1 \varepsilon_{t-1}^2 + \beta_1\right) \sigma_{t-1}^2, \end{aligned} \tag{10}$$

and under the measure Q:

$$\begin{aligned} y_t &= r_1 + \sigma_t \varsigma_t, \\ \varsigma_t &\sim N(0,1), \\ \sigma_t^2 &= \alpha_0 + \left[\alpha_1 (\varsigma_{t-1} - \lambda_{t-1})^2 + \beta_1\right] \sigma_{t-1}^2, \\ \lambda_{t-1} &= \frac{(\phi + \delta_{t-1}) y_{t-2} - r_1}{\sigma_{t-1}}, \end{aligned} \tag{11}$$

where r_1 is the one-day risk-free interest rate.

For the RCA(1)-GARCH(1,1) model, like for the AR(1)-GARCH(1,1) (Hafner and Herwartz, 2001), we can obtain the unconditional variance under the measure Q.

Proposition 1 (Górka 2012). Under the measure Q, the unconditional variance of y_t under stationarity is finite if

$$\alpha_1\left(1 + \phi^2 + \sigma_\delta^2\right) + \beta_1 < 1,$$

and

$$\operatorname{var}^Q(y_t) = \frac{\alpha_0 + \alpha_1 r_1^2 \left[(1-\phi)^2 + \sigma_\delta^2\right]}{1 - \alpha_1\left(1 + \phi^2 + \sigma_\delta^2\right) - \beta_1}.$$

The Sign RCA(1)-GARCH(1,1) models proposed by Thavaneswaran, et al., (2006a) have the following form:

$$y_t = (\phi + \delta_t + \Phi s_{t-1}) y_{t-1} + \xi_t, \tag{12}$$

$$\xi_t = \sigma_t \varepsilon_t, \tag{13}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \xi_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{14}$$

where ϕ, Φ, α_0, α_1, β_1 are parameters of model, $\varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right)$, $\delta_t \sim i.i.d.\left(0, \sigma_\delta^2\right)$, $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$,

$$s_t = \begin{cases} 1 & \text{for } y_t > 0 \\ 0 & \text{for } y_t = 0 \\ -1 & \text{for } y_t < 0 \end{cases}.$$

The sign function (s_t) has the interpretation: if $\phi + \delta_t > |\Phi|$, the negative value of Φ means that the negative (positive) observation values at time $t-1$ correspond to a decrease (increase) of observation values at time t. In the case of stock returns it would suggest (for returns) that after a decrease of stock returns, the higher decrease of stock returns occurs than expected, and in the case of the increase of stock returns the lower increase in stock returns occurs than expected.

Theoretical properties of this model can be found among others: Górka (2012), Thavaneswaran et al., (2006a, 2006b, 2008, 2009). It is worth noting, that the adding the sign function has influence the increase of unconditional variance and kurtosis in comparison with RCA(1)-GARCH(1,1), and therefore also with ordinary AR(1)-GARCH(1,1).

Under the historical measure P the Sign RCA(1)-GARCH(1,1) option pricing model can be defined:

$$\begin{aligned} & y_t = (\phi + \delta_t + \Phi s_{t-1}) y_{t-1} + \sigma_t \varepsilon_t, \\ & \varepsilon_t \sim N(0,1), \\ & \sigma_t^2 = \alpha_0 + \left(\alpha_1 \varepsilon_{t-1}^2 + \beta_1\right) \sigma_{t-1}^2, \end{aligned} \tag{15}$$

Under the martingale measure Q the Sign RCA(1)-GARCH(1,1) option pricing model takes the form:

$$\begin{aligned} & y_t = r_1 + \sigma_t \varsigma_t, \\ & \varsigma_t \sim N(0,1), \\ & \sigma_t^2 = \alpha_0 + \left[\alpha_1 (\varsigma_{t-1} - \lambda_{t-1})^2 + \beta_1\right] \sigma_{t-1}^2, \\ & \lambda_{t-1} = \frac{(\phi + \delta_{t-1} + \Phi s_{t-2}) y_{t-2} - r_1}{\sigma_{t-1}}, \end{aligned} \tag{16}$$

For the Sign RCA(1)-GARCH(1,1) model, like for the RCA(1)-GARCH(1,1), we can obtain unconditional variance under the measure Q.

Proposition 2 (Górka 2012). Under the measure Q, the unconditional variance of y_t under stationarity is finite if

$$\alpha_1\left(1+\phi^2+\sigma_\delta^2+\Phi^2\right)+\beta_1<1,$$

and

$$\operatorname{var}^Q\left(y_t\right)=\frac{\alpha_0+\alpha_1 r_1^2\left[(1-\phi)^2+\sigma_\delta^2+\Phi^2\right]}{1-\alpha_1\left(1+\phi^2+\sigma_\delta^2+\Phi^2\right)-\beta_1}.$$

1.3. Monte Carlo Simulations

GARCH models are very popular and effective for modeling the volatility dynamics in many asset markets. Unfortunately, existing GARCH models do not have closed-form solutions for option prices. These models are typically solved by simulation. The Monte Carlo simulation procedure for option pricing can be described in following steps (Duan, 1995; Hafner and Herwartz, 2001; Lehar et al., 2002; Piontek, 2002, 2004):

1. Parameter estimation under the empirical measure P.
2. Simulation of sample paths for the underlying asset price under the equivalent martingale measure Q (50000 paths), i.e.

$$S_{i,n}=S_{t_0}e^{nr_1-0.5\sum_{s=1}^{n}\sigma_{i,t0+s}^2+\sum_{s=1}^{n}\sigma_{i,t0+s}\varsigma_{i,t0+s}},$$

where: i – i-th path, $\varsigma_{i,t_{0+s}}$ – the current values of the innovation, $\sigma_{i,t_{0+s}}^2$ – the current values of the variance.

3. Correction to the standard Monte Carlo simulation procedure (empirical martingale simulation, Duan and Simonato, 1998), i.e.

$$S_{i,n}^*=S_{t_0}e^{nr_1}\frac{S_{i,n}}{\frac{1}{m}\sum_{i=1}^{m}S_{i,n}},$$

where: m – number of paths.

4. Discounting the expected payoffs to yield of the Monte Carlo price of option, i.e.

$$c_{t_0}=e^{nr_1}\mathrm{E}^{\mathrm{Q}}\left[\max\left(S_{i,n}^*-K,0\right)|\,\mathsf{F}_{t_0}\right]$$

where: c_{t_0} – the corresponding call price obtained by Monte Carlo simulation, E^{Q} – the risk-neutral conditional expectation operator.

The risk-free interest rate (r_1) was approximated on the basis of the interest rate of the WIBID[3] and the WIBOR[4].

To quantify the deviation of theoretical option prices from the prices observed at the market the statistical error measures were applied (Lehar et al., 2002):

– the relative pricing error

$$\mathrm{RPE} = \frac{\hat{c}_t - c_t}{c_t},$$

– the absolute relative pricing error

$$\mathrm{ARPE} = \frac{|\hat{c}_t - c_t|}{c_t},$$

where c_t and $\hat{c}_t$ denote the observed price and the model price, respectively.

The RPE is a measure of the bias of the pricing model. A non-zero RPE may therefore indicate the existence of systematic errors. The ARPE measures both the bias and the efficiency of pricing (Lehar et al., 2002).

2. An Empirical Analysis

The data used in the empirical study were the WIG20 index and prices of the European call options on WIG20 index on the Warsaw Stock Exchange (WSE). The sample period for the WIG20 runs from 19-th of November 2003 to 21-th of February 2011. Evolution of the WIG20 index was displayed in Figure 1. The WIG20 index went up during the first 4 years, after that it rapidly went down and since 2009 started to increase again.

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[3] Warsaw Interbank Offered Rate.

[4] Warsaw Interbank Bid Rate.

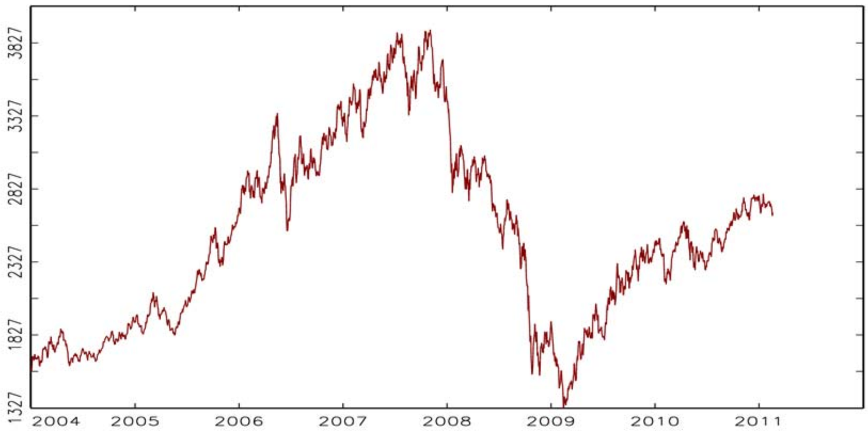

Figure 1. The WIG20 index (November 19, 2003 – February 21, 2011)

Two periods were chosen to calculate option prices. The first one was at the turn of 2007–2008 (it is about 3 months long) and the second one was at the turn of 2010–2011. First day of the option pricing at the turn of year 2007–2008 was made on 22-th of November 2007 (valuation on the November 23, 2007) and 16-th of November 2010 at the turn of 2010–2011 (valuation on the November 17, 2010).

Option pricing were made using:

- the standard Black-Scholes (BS),
- Monte Carlo simulation (MCS; Duan's method).

For comparison, the valuation of options were also made using the AR(1)-GARCH(1,1) model and the AR(1)-GJR-GARCH(1,1) model. For all model specifications, parameter values were obtained from the MLE using WIG20 index daily logarithmic returns. The sample sizes on which models were estimated are as follows:

- 252 observations (~ year),
- 504 observations (~ 2 years),
- 1008 observations (~ 4 years).

All computation were made using authors codes written in GAUSS 6.0.

Example results of the valuation of European call options for WIG20 stock index on a particular day (November 17, 2010), on three month to maturity and different sample sizes, and market prices of these options (the closing price) are shown in Table 1.

Table 1. European call option prices for WIG20 stock index on three months to maturity and market prices of these options

Strike (K)	BS	RCA-GARCH	Sign RCA-GARCH	AR-GARCH	AR-GJR-GARCH	Market price 17.12.2010
252 observations						
2300	**476.72**	491.21	493.56	524.49	494.79	483.50
2400	**377.64**	399.56	403.62	433.41	405.20	388.50
2500	**278.57**	314.31	320.34	348.30	322.02	294.90
2600	179.50	**238.20**	246.04	271.50	247.50	221.05
2700	80.43	**173.31**	182.39	204.68	183.29	133.00
2800	1.31	**120.84**	130.42	149.05	130.64	78.00
2900	0.00	**80.65**	89.98	104.83	89.51	45.10
3000	0.00	51.66	59.92	71.13	58.89	23.00
3100	0.00	31.79	38.66	46.57	37.26	9.10
504 observations						
2300	476.72	487.08	489.88	**486.94**	487.81	483.50
2400	377.64	390.36	392.63	**389.99**	391.45	388.50
2500	278.57	297.02	298.39	**296.19**	298.16	294.90
2600	179.50	211.25	211.15	209.70	**211.28**	221.05
2700	80.43	138.46	136.63	136.25	**135.72**	133.00
2800	1.31	82.89	79.92	80.40	**76.51**	78.00
2900	0.00	**45.48**	42.27	43.25	36.75	45.10
3000	0.00	**23.27**	20.53	21.61	14.78	23.00
3100	0.00	11.24	**9.37**	10.18	4.87	9.10
1008 observations						
2300	**476.72**	495.32	495.34	495.31	519.97	483.50
2400	**377.64**	405.43	405.45	405.42	440.70	388.50
2500	**278.57**	322.00	322.00	321.97	368.18	294.90
2600	179.50	247.43	**247.40**	**247.40**	303.11	221.05
2700	80.43	183.66	**183.60**	183.63	245.94	133.00
2800	1.31	131.79	**131.69**	131.76	196.75	78.00
2900	0.00	91.65	91.51	91.62	155.33	45.10
3000	0.00	62.22	62.06	62.19	120.91	23.00
3100	0.00	41.44	41.29	41.41	93.08	9.10

Note: The bold number denotes the theoretical option prices the closest to the market price.

Option prices calculated by the BS formula were underestimated, while option prices calculated by other models were overestimated for the sample size of 252 observations and 1008 observations. For the sample size of 504 observations the option prices were overestimated for some strikes, but for other – underestimated. It depends of the type of option and of the model on which theoretical option prices were calculated. For the out-of-the-money options differences between market prices and theoretical option prices calculated by models on the simple size of 504 observations were small, while for others simple sizes these differences were greater. In this study, for the sample size of 504 observations the theoretical option prices were closest to the market prices. It holds for all model specifications.

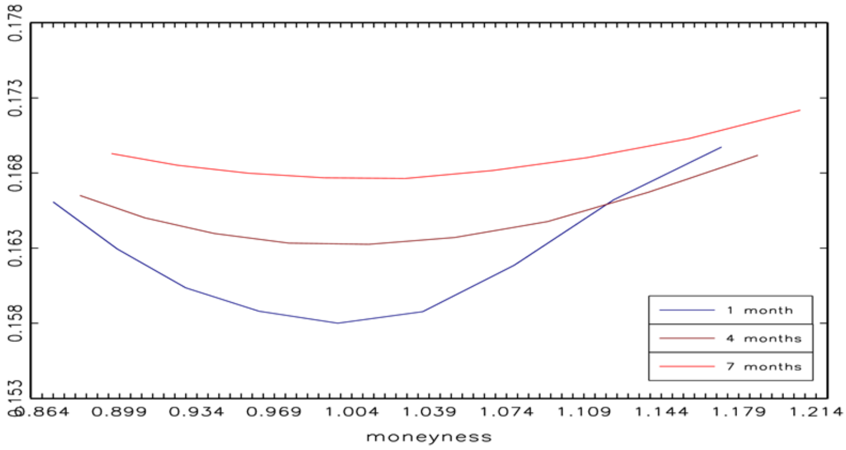

Figure 2. The annualized implied volatility of the RCA-GARCH option pricing model (504 observations; day of the option pricing – November 16, 2010)

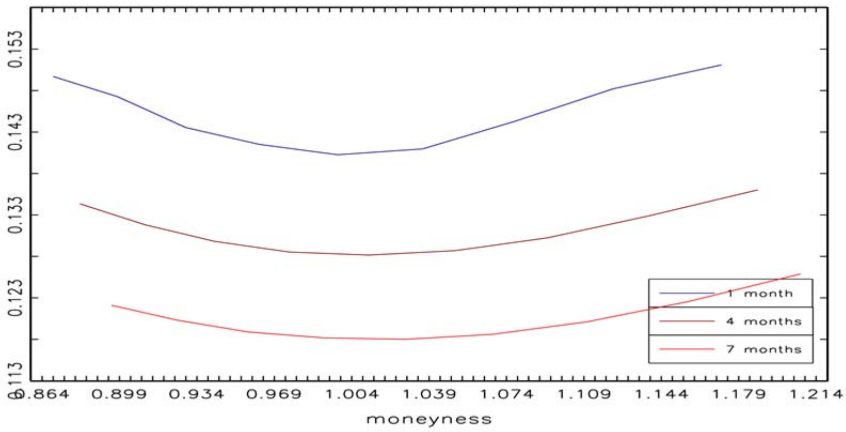

Figure 3. The annualized implied volatility of the Sign RCA-GARCH option pricing model (504 observations; day of the option pricing – November 16, 2010)

Figures 2 and 3 show the relationship between implied volatility, the exercise price and the time to maturity of the option. This shape resembles a smile and is called the volatility smile. It is often observed in financial markets (Lehar et al., 2002; Piontek, 2002). When the time to maturity increases, the smile tends to become flatter. With the increase of the time to maturity of the option the increase (Figure 3) or decrease (Figure 2) of volatility for options with the same moneyness is often observed. This result is similar to the result obtained from other GARCH models (Piontek, 2002, 2004; Hafner and Herwartz, 2001; Duan, 1995).

On the basis of the valuation of options for a day, it is difficult to draw more general conclusions on the usefulness of RCA-GARCH and Sign RCA-GARCH models. Therefore, the valuation of options in the two periods were made using different models as a result of progressive estimation models. In the first period (November 23, 2007 – January 25, 2008) the values of OW20C8[5] and OW20F8 options with different exercise prices for the next 63 days were determined. In the second period (November 17, 2010 – February 21, 2011) the valuation of OW20C1 and OW20F1 options with different exercise prices for the next 67 days was made.

Table 2. The mean option pricing errors $(\tau > 180)$

Error	Options	RCA-GARCH	Sign RCA-GARCH	AR-GARCH	AR-GJR-GARCH
		252 observations			
RPE	ITM	0.8079	0.1860	0.1988	0.2022
	ATM	1.8958	0.4736	0.5088	0.4358
	OTM	5.8016	1.1948	1.2816	1.0673
ARPE	ITM	0.8079	0.1873	0.1988	0.2067
	ATM	1.8958	0.4736	0.5088	0.4408
	OTM	5.8016	1.1948	1.2816	1.2015
		504 observations			
RPE	ITM	0.0764	0.0911	0.2228	0.2022
	ATM	0.1517	0.1830	0.5339	0.4358
	OTM	0.3341	0.4280	1.4065	1.0673
ARPE	ITM	0.0812	0.0996	0.2228	0.2067
	ATM	0.1596	0.1979	0.5339	0.4408
	OTM	0.3589	0.4911	1.4198	1.2015
		1008 observations			
RPE	ITM	0.3268	0.3114	0.3268	0.3628
	ATM	0.8260	0.7756	0.8260	0.8607
	OTM	2.2219	2.0916	2.2217	2.1698
ARPE	ITM	0.3268	0.3176	0.3268	0.3628
	ATM	0.8260	0.7884	0.8260	0.8607
	OTM	2.2219	2.1179	2.2217	2.1698

Note: ATM – at-the-money, ITM – in-the-money, OTM – out-of-the-money. The bold number indicate minimum of absolute error.

The obtained values of option prices were subsequently split according to the time to maturity of the options (in days), i.e.

[5] Type of call options.

– short maturity ($\tau \leq 60$),
– medium maturity ($60 < \tau \leq 180$),
– long maturity ($\tau > 180$).

Then, in the first place the option at-the-money (ATM) was determined, and then statistical measures of errors for the four options in-the-money (ITM) and four options out-of-the-money (OTM) were calculated[6]. The results for the second period were shown in Tables 2–4.

Table 3. The mean option pricing errors $(60 < \tau \leq 180)$

Error	Options	RCA-GARCH	Sign RCA-GARCH	AR-GARCH	AR-GJR-GARCH
		252 observations			
RPE	ITM	0.2549	0.1153	0.1117	0.1435
	ATM	0.6812	0.3362	0.3339	0.4123
	OTM	4.5922	1.6938	1.6740	1.8805
ARPE	ITM	0.2585	0.1209	0.1147	0.1441
	ATM	0.6966	0.3676	0.3392	0.4123
	OTM	4.6231	1.7395	1.6768	1.8805
		504 observations			
RPE	ITM	–0.0002	0.0426	0.1000	0.1532
	ATM	–0.1188	0.0651	0.2646	0.4006
	OTM	0.1759	0.5314	1.5774	2.2177
ARPE	ITM	0.0913	0.0685	0.1166	0.1676
	ATM	0.3081	0.1690	0.3375	0.4734
	OTM	0.7708	0.6223	1.6448	2.4192
		1008 observations			
RPE	ITM	0.1702	0.1901	0.1702	0.2003
	ATM	0.5001	0.5692	0.5001	0.5282
	OTM	2.5682	3.2605	2.5682	2.2497
ARPE	ITM	0.1715	0.2031	0.1715	0.2035
	ATM	0.5010	0.6349	0.5010	0.5448
	OTM	2.5682	3.3311	2.5682	2.2857

Note: ATM – at-the-money, ITM – in-the-money, OTM – out-of-the-money. The bold number indicate minimum of absolute error.

Obtained results depend mainly on the time to maturity and size of sample. However, the smallest absolute error values were received for the sample of 504 observations regardless of the choice of the model for the theoretical option prices (see Table 2–4). For each time to maturity and size of

[6] Firstly, 4 options in-the-money and out-of-the-money were right next to the option at-the-money. Second, for each day of option pricing the error measures had had 3 values, one of each type of option. For the whole of the period (for example, a option with medium maturity), the result was the average of the results for the option of this period (for example, for the option of $60 < \tau \leq 180$).

sample different conclusions may be drawn. For example, for long time to maturity (Table 2) and the sample of 504 observations the smallest values of the mean pricing error (absolute and relative) were obtained for RCA-GARCH models, while for the short time to maturity (Table 4) – for Sign RCA-GARCH models. This holds for each type of options. In this study, regardless of the sample size, the out-of-the-money options for the short time to maturity were substantially overestimated. The similar results for the long time to maturity for the first period (November 23, 2007 – January 25, 2008) were found.

Table 4. The mean option pricing errors $(\tau \leq 60)$

Error	Options	RCA-GARCH	Sign RCAGARCH	AR-GARCH	AR-GJR-GARCH
		252 observations			
RPE	ITM	0.0266	0.0287	0.0248	0.0517
	ATM	0.1834	0.1548	0.1891	0.2992
	OTM	1.7632	1.7607	1.8024	1.8375
ARPE	ITM	0.0442	0.0452	0.0474	0.0622
	ATM	0.2039	0.2035	0.2277	0.3092
	OTM	1.7765	1.9044	1.8163	1.8760
		504 observations			
RPE	ITM	0.0250	0.0211	0.0479	0.0520
	ATM	0.1704	0.0942	0.3498	0.2910
	OTM	2.0401	1.1652	4.3350	3.5110
ARPE	ITM	0.0503	0.0471	0.0725	0.0751
	ATM	0.2180	0.1922	0.4135	0.3458
	OTM	2.1327	1.3513	4.4305	3.6669
		1008 observations			
RPE	ITM	0.0413	0.0434	0.0413	0.0542
	ATM	0.2634	0.2738	0.2636	0.2903
	OTM	2.8569	2.9869	2.8585	1.6641
ARPE	ITM	0.0544	0.0565	0.0544	0.0651
	ATM	0.2747	0.2851	0.2748	0.3048
	OTM	2.8630	2.9928	2.8645	1.7021

Note: ATM – at-the-money, ITM – in-the-money, OTM – out-of-the-money. The bold number indicate minimum of absolute error.

Comparing the RPEs for the four different models (Table 2–4), one can see systematic overpricing across all models (except the RCA-GARCH model for the ITM and ATM options for $60 < \tau \leq 180$ and the sample of 504 observations – Table 3). In other words, the volatility of the underlying asset price was systematically overestimated.

In some cases the differences between the mean option pricing errors were small (e.g. for the sample of 1008 observations in Table 2, for the sample of 252 observations in Table 4), while in other cases these differences were substantial (e.g. for the sample of 504 observations in Table 2 or 4). It is worth noting, that Sign RCA-GARCH models outperform the selected GARCH models, because the absolute error were substantially lower (e.g. for the sample of 504 or 1008 observations in Table 2, for the sample of 504 or 1008 observations in Table 4). However, the better performance of Sign RCA-GARCH models was not well established and depends on the time to maturity, size of sample or the period of the data.

Conclusions

This paper has applied Sign RCA-GARCH models to compute theoretical option prices. This approach was illustrated by the valuation of the European call options on the WIG20 index, together with a comparison of their values obtained on the selected GARCH models. It is difficult to make general remarks, nevertheless the empirical results showed that:

- the Black-Scholes model cannot explain the prices of out-of-the-money options,
- RCA-GARCH and Sign RCA-GARCH models can be successfully applied in pricing options,
- none of the models can be indicate as the best one for the option valuations for every period and every time to maturity of the options,
- the choice of a sample size for estimating the option pricing model has a significant impact on the option pricing,
- the choice of the volatility model is important for achieving a satisfying pricing performance.

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Modele Sign RCA-GARCH w wycenie opcji

Z a r y s t r e ś c i. Po ukazaniu się przełomowej pracy Blacka i Scholesa literatura dotycząca wyceny opcji stała się bardzo ważnym obszarem w badaniach. Zostały opracowane liczne metody wyceny opcji. W artykule tym pokazano, jak można obliczyć ceny opcji wykorzystując model Sign RCA-GARCH do opisu dynamiki zmienności. Wyceny opcji uzyskane przedstawioną metodą oraz wyceny opcji uzyskanych z wykorzystaniem modelu Blacka-Scholesa i wybranych modeli GARCH zostały porównane z ceną rynkową. Podejście to zostało zilustrowane wyceną europejskich opcji kupna na indeks WIG20. Empiryczne wyniki wskazują, że modele RCA GARCH i Sign RCA GARCH mogą być z powodzeniem stosowane do wyceny opcji. Jednak żadnego z przedstawionych modeli nie można wskazać jako najlepszego do wyceny opcji dla dowolnej wielkości próby czy dowolnego czasu pozostającego do wygaśnięcia opcji.

S ł o w a k l u c z o w e: model Sign RCA-GARCH, wycena opcji, modele GARCH.