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Bayesian Analysis of the Box-Cox Transformation in Stochastic Volatility Models†

A b s t r a c t. In the paper, we consider the Box-Cox transformation of financial time series in Stochastic Volatility models. Bayesian approach is applied to make inference about the Box-Cox transformation parameter (λ). Using daily data (quotations of stock indices), we show that in the Stochastic Volatility models with fat tails and correlated errors (FCSV), the posterior distribution of parameter λ strongly depends on the prior assumption about this parameter. In the majority of cases the values of λ close to 0 are more probable a posteriori than the ones close to 1.

K e y w o r d s: Box-Cox transformation, SV model, Bayesian inference.

1. Introduction

The continuously compounded rates of return (or logarithmic returns) as well as the simple rates of return are commonly used in econometric analyses of financial data. These two types of data transformation are applied arbitrarily. In the derivatives pricing literature there is the tradition of using logarithmic returns, but when the logarithmic return is modelled as a conditionally Student-t distributed random variable, the conditional expected simple rate of return is infinite. It violates the finite second moment condition for the asset payoff in call option pricing (see Duan, 1999). Duan (1999) uses the generalized error distribution (GED) for the logarithmic returns that also exhibits fat tails and includes the normal distribution as a special case. Other researchers build model with sample returns instead of log-returns and with the Student-t distribution (see e.g. Hafner, Harwartz, 1999; Härdle, Hafner, 2000; Bauwens, Lubrano, 2002). However, both the logarithmic return and simple one are variants of the well-known Box-Cox transformation of the x_t/x_{t-1} ratio (where x_t denotes the asset price at time t) with parameter 0 and 1, respectively. In the paper, we con-

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sider the Box-Cox transformation of financial time series in Stochastic Volatility (SV) models. Bayesian approach is applied to make inference about the Box-Cox transformation parameter (λ). As parameter λ is estimated along with other unknown parameters, information in the data is used to determine which transformation is appropriate for the data.

The structure of the article is as follows: section 2 consists of a short presentation of the Bayesian SV model with fat-tails correlated errors for the transformed data, section 3 focuses on the empirical results, and finally, section 4 incorporates the conclusions.

2. Bayesian AR(1)-FCSV Model for the Transformed Data

Let x_t denote the price of an asset at time t, $t = 0, 1, ..., T$. The Box-Cox transformation of the x_t/x_{t-1} ratio is defined as:

$$B(x_t / x_{t-1}, \lambda) = \begin{cases} \frac{(x_t / x_{t-1})^\lambda - 1}{\lambda} & \lambda > 0 \\ \ln(x_t / x_{t-1}) & \lambda = 0 \end{cases}, \text{ t} = 1, ..., T.$$

For $B(x_t / x_{t-1}, \lambda)$ we use an autoregressive structure[1]:

$$B(x_t / x_{t-1}, \lambda) - \delta_1 = \rho_1 [B(x_{t-1} / x_{t-2}, \lambda) - \delta_1] + \varepsilon_t, \quad t = 1, ..., T, \qquad (1)$$

where $\{\varepsilon_t\}$ is the stochastic volatility process with fat-tails and correlated errors (FCSV), introduced by Jacquier et. al., (2004). The discrete-time FCSV process can be written as:

$$\varepsilon_t = u_t \sqrt{h_t / \omega_t}, \qquad (2)$$

$$\ln h_t = \gamma + \phi \ln h_{t-1} + \sigma_h \eta_t, \qquad (3)$$

$$\omega_t \sim \chi^2(\nu)/\nu, \quad \omega_t \perp (u_l, \eta_l), \quad t, l \in \{1, \ldots, T\},$$

$$(u_t, \eta_t)' \sim IN\left(0, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right), \quad t = 1, \ldots, T.$$

where the abbreviation "*IN*" denotes that the random vectors concerned are independent and normally distributed, ⊥ denotes stochastic independence.

In the FCSV process, when ρ is equal to zero, h_t is the inverse precision in the conditional distribution, $p(\varepsilon_t|h_t)$, that is, $(\nu/\nu\text{-}2)h_t$ (for $\nu > 2$) is the conditional variance. Thus, the FCSV model specifies a log-normal autoregressive process for the conditional variance factor (h_t) with correlated innovations in the conditional mean and conditional variance equations, i.e. in (2) and (3), respectively.

[1] We use the autoregressive structure, because financial time series such as stock market indices often present positive autocorrelation of order one of the returns (see Campbell et al., 1997).

One interpretation for the latent variable h_t is that it represents the random, uneven and autocorrelated flow of new information into financial markets (see Clark, 1973). The parameter ϕ is related to the volatility persistence, and σ_h is the volatility of the log-volatility. The above model captures the leverage effect when the correlation ρ is negative. In fact, if ρ is negative, then a negative innovation u_t is associated with higher contemporaneous and subsequent volatilities. On the other hand, a positive innovation u_t is connected with a decrease in volatility (see Jacquier et al., 2004).

The Bayesian model is characterized by the joint probability density function of the untransformed x_t/x_{t-1} ratios (i.e. $\mathbf{y} = (y_1, ..., y_T)'$, where $y_t = x_t/x_{t-1}$), the latent variables (i.e. $\mathbf{h} = (h_1, ..., h_T)'$, $\boldsymbol{\omega} = (\omega_1, ..., \omega_T)'$), and of the parameter vector $\boldsymbol{\theta}$:

$$p(\mathbf{y},\mathbf{h},\boldsymbol{\omega},\boldsymbol{\theta} \mid \mathbf{y}_{(0)}) = p(\mathbf{y},\mathbf{h},\boldsymbol{\omega} \mid \boldsymbol{\theta},\mathbf{y}_{(0)})\, p(\boldsymbol{\theta}), \tag{4}$$

where

$$p(\mathbf{y},\mathbf{h},\boldsymbol{\omega} \mid \boldsymbol{\theta},\mathbf{y}_{(0)}) = p(\boldsymbol{\omega} \mid \nu)\exp\left\{-\frac{1}{2}\, tr\left(\boldsymbol{\Sigma}^{*-1}\sum_{t=1}^{T}\mathbf{r}_t\mathbf{r}_t'\right)\right\} | J(\lambda,\mathbf{y}) | \times$$

$$\times | \boldsymbol{\Sigma}^* |^{-0.5T}\ (2\pi)^{-T}\prod_{t=1}^{T}\omega_t^{\,0.5} h_t^{\,-1.5},$$

$$p(\boldsymbol{\omega} \mid \nu) = \prod_{t=1}^{T}\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)^{-1}\ \omega_t^{\frac{\nu}{2}-1}\ \exp\left(-\frac{\nu}{2}\omega_t\right) I_{(0,+\infty)}(\omega_t),$$

$$\boldsymbol{\Sigma}^* = \begin{pmatrix} 1 & \rho\sigma_h \\ \rho\sigma_h & \sigma_h^2 \end{pmatrix},\ \mathbf{r}_t = (u_t, \sigma_h\eta_t)',\ \ \boldsymbol{\theta} = (\delta_1,\rho_1,\gamma,\phi,\sigma_h^2,\rho,\nu,\lambda)',$$

$\mathbf{y}_{(0)}$ denotes initial values. The Jacobian $J(\lambda, \mathbf{y})$ is $J(\lambda,\mathbf{y}) = \prod_{t=1}^{T} y_t^{\lambda-1}$.

Our model specification gets completed by assuming the following prior structure:

$$p(\delta_1,\rho_1,\gamma,\phi,\sigma_h^2,\rho,\nu,\lambda) = p(\delta_1)\,p(\rho_1)\,p(\gamma)\,p(\phi)\,p(\sigma_h^2,\rho)\,p(\nu)\,p(\lambda),$$

where we use proper prior densities of the following distributions:

$\delta_1 \sim N(0,\ 1)$, $\rho_1 \sim$ U(-1,1) $\gamma \sim N(0,\ 100)$, $\phi \sim N(0,\ 100)\ I_{(-1,1)}(\phi)$, $\nu \sim$ Exp(0.1), $\tau \sim IG(1,\ 0.005)$, $\psi|\tau \sim N(0,\ \tau/2)$, $\psi = \sigma_h\rho,\ \ \tau = \sigma_h^2(1-\rho^2)$.

The prior distribution for δ_1 is standardized normal, U(-1,1) denotes the uniform distribution over (-1,1). The prior distribution for ϕ is normal, truncated by the restriction that the absolute value of ϕ is less than one ($I_{(-1,\,1)}(.)$ denotes the indicator function of the interval (-1, 1), which is the region of stationarity of lnh_t). The symbol $IG(v_0,\ s_0)$ denotes the inverse Gamma distribution with mean

$s_0/(v_0\text{-}1)$ and variance $s_0^2/[(v_0-1)^2(v_0-2)]$ (thus, when $\rho = 0$, the prior mean for σ_h^2 does not exist, but the precision, σ_h^{-2}, has a Gamma prior with mean 200 and standard deviation 200). The symbol Exp(a) denotes the exponential distribution with mean $1/a$ (thus the prior mean for v is equal to 10 with the standard deviation equals 10). The prior distribution for (ψ, τ) induces a prior distribution for (ρ, σ_h^2), which has the following form:

$$p(\sigma^2, \rho) = s_0^{v_0}\Gamma(v_0)^{-1} p_0^{0.5}(2\pi)^{-0.5}(\sigma_h^{-2})^{v_0+1} e^{-\frac{s_0}{(1-\rho^2)\sigma_h^2}} (1-\rho^2)^{-v_0-1.5} e^{-\frac{(\rho\sigma-\psi_0)^2 p_0}{2(1-\rho^2)\sigma_h^2}},$$

$v_0 = 1$, $s_0 = 0.005$, $\psi_0 = 0$, $p_0 = 2$ (similar to Jacquier et al., 2004).

As far as the prior distribution for λ, we assume that our prior information regarding this parameter can be represented by the following:

a) a non-standard distribution on the interval [0, 1]: $p(\lambda) \propto e^{-\beta x(1-x)}$, where $\beta = 30$. This prior distribution is symmetrical and U-shaped, as shown in Figure 1.
b) the beta distribution with parameters 0.5 and 0.5;
c) the uniform distribution on the interval [0, 1];
d) the exponential distribution with mean 1;

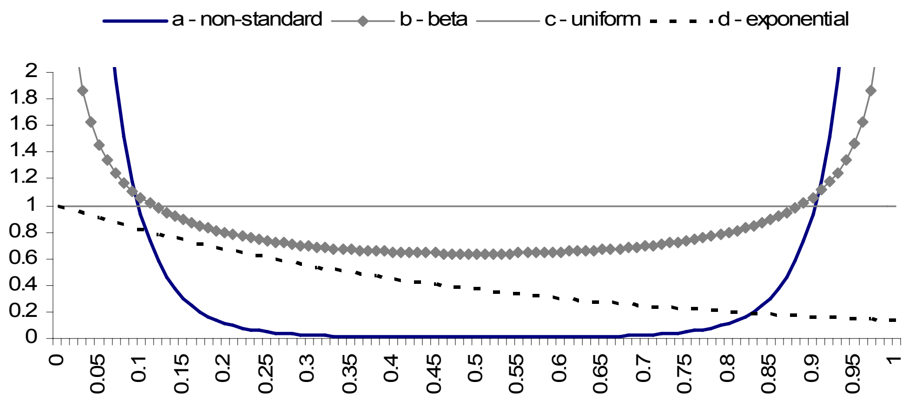

Figure 1. Prior distributions for the Box-Cox transformation parameter (λ)

As regards the initial condition for h_t, i.e. h_0, we assume that it is equal to 1. The joint posterior distribution is then

$$p(\mathbf{h}, \boldsymbol{\omega}, \boldsymbol{\theta} \mid \mathbf{y}, \mathbf{y}_{(0)}) \propto p(\boldsymbol{\theta})p(\boldsymbol{\omega} \mid v) \times$$

$$\times \exp\left\{-\frac{1}{2} tr\left(\boldsymbol{\Sigma}^{*-1}\sum_{t=1}^{T}\mathbf{r}_t\mathbf{r}_t'\right)\right\} | J(\lambda, \mathbf{y}) || \boldsymbol{\Sigma}^* |^{-0.5T} \prod_{t=1}^{T}\omega_t^{0.5}h_t^{-1.5}.$$

The posterior probability density function is used to make inference about the parameters and latent variables.

3. Empirical Results

We consider ten international stock market indices, namely the S&P 500, NASDAQ 100, DJIA (for the US), NIKKEI (for Japan), the CAC 40 (for France), the DAX (for Germany), the FTSE 100 (for the UK), WIG 20 (for Poland), HANG SENG (for China), SPTSE 60 (for Canada).

The data set consists of the daily closing quotations of the stock market indices from January 2001 (or 2002) until February (or March) 2009 (see Table 1). Basic descriptive characteristics of the daily price ratios are presented in Table 1. All series of x_t/x_{t-1} ratio exhibit strong kurtosis, and they have highly non-normal (truncated by zero) empirical distributions.

Table 1. Sample characteristics for the data sets used

time series (x_t/x_{t-1} ratio of:)	average	std. dev.	kurtosis	*period from: - to:*	*# obs. T*
WIG 20	1.0000	0.0162	4.9800	02.01.2001 – 13.02.2009	2035
S&P 500	0.9998	0.0139	13.3286	03.01.2002 – 06.03.2009	1805
NIKKEI 225	0.9999	0.0163	11.3553	07.01.2002 – 06.03.2009	1760
FTSE 100	0.9999	0.0137	10.9021	03.01.2002 – 06.03.2009	1813
DAX	1.0000	0.0169	8.6642	03.01.2002 – 06.03.2009	1825
NASDAQ 100	1.0000	0.0178	7.8639	03.01.2002 – 06.03.2009	1808
CAC 40	0.9998	0.0159	9.7031	03.01.2002 – 06.03.2009	1838
SPTSE 60	1.0001	0.0132	14.2830	03.01.2002 – 06.03.2009	1798
HANG SENG	1.0002	0.0164	15.0382	03.01.2002 – 06.03.2009	1789
DJIA	0.9999	0.0130	12.5726	02.01.2001 – 13.02.2009	2039

Note: The data were downloaded from the website http://finance.yahoo.com.

In Table 2 we present the posterior means and standard deviations (in parenthesis) of the parameters, in the case of the AR(1)-FCSV model with the uniform prior for λ on [0, 1]. Our posterior results are obtained in Gauss 9.0 using MCMC methods: Metropolis-Hastings within the Gibbs sampler (see, e.g. Pajor 2003 and Jacquier et al., 2004 for detail).[2] First, for more series the autoregressive parameters seem to be insignificantly different from zero. The posterior distributions of δ and ρ_1 are located close to zero. Second, all indices have persistent volatility as shown by ϕ - the lowest posterior mean is 0.927 (for the WIG20 index), the highest one is 0.97 (for NASDAQ). It means that the half-life of shock to volatility, $HL = \ln(0.5)/\ln(\phi)$, is equal to about 9 days for the WIG20 index and 20 days for the NASDAQ index. We observed that the NASDAQ index exhibits a lower variability of volatility as shown by the precision, σ_h^{-2}. As regards the leverage effect parameter, ρ, the posterior means of ρ are negative, from -0.15 for the WIG20 index to -0.62 for the CAC40 index.

[2] The results are obtained using 100 000 burnt-in and 1000 000 final Gibbs passes.

The parameter ρ is estimated precisely with a standard deviation around 0.068. Almost all the posterior mass of ρ is in the negative region. Thus, the leverage effect is strong for all indices excluding the WIG20 index, for which it is significantly lower. The posterior means of the degrees of freedom are between 16 (for the HANG SENG index) and 39 (for the FTSE 100 index). The HANG SENG index has the lowest posterior mean of degrees of freedom of the Student-t distribution. For the remaining indices the posterior mean of ν is above 23, indicating that the normal conditional distribution would not be strongly rejected by the data.

Table 2. Posterior means and standard deviations (in parenthesis) of the parameters of the AR(1)-FCSV model, in the case of $\lambda \sim U[0, 1]$

parameter	WIG 20	S&P 500	NIKKEI 225	FTSE 100	DAX	NASDAQ 100	CAC 40	SPTSE 60	HANG SENG	DJIA
$\delta_1 * 10^4$	4.74 (3.15)	4.74 (1.65)	6.52 (2.56)	5.43 (1.63)	9.70 (2.15)	5.77 (2.63)	6.59 (1.99)	8.68 (1.78)	7.20 (2.37)	4.14 (1.60)
ρ_1	0.027 (0.023)	-0.095 (0.023)	-0.034 (0.024)	-0.092 (0.024)	-0.059 (0.023)	-0.070 (0.024)	-0.080 (0.023)	-0.060 (0.024)	0.005 (0.023)	-0.074 (0.022)
γ	-0.622 (0.119)	-0.332 (0.048)	-0.429 (0.068)	-0.357 (0.052)	-0.328 (0.049)	-0.297 (0.048)	-0.335 (0.047)	-0.493 (0.074)	-0.408 (0.069)	-0.348 (0.051)
ϕ	0.928 (0.014)	0.965 (0.005)	0.951 (0.008)	0.962 (0.006)	0.963 (0.006)	0.966 (0.006)	0.963 (0.005)	0.948 (0.008)	0.955 (0.008)	0.963 (0.005)
σ_h^{-2}	22.233 (5.675)	16.972 (2.809)	17.711 (3.395)	13.728 (2.123)	15.277 (2.592)	25.790 (5.279)	15.973 (2.491)	13.672 (2.447)	15.958 (3.156)	17.871 (2.996)
ρ	-0.153 (0.081)	-0.607 (0.063)	-0.55 (0.065)	-0.578 (0.061)	-0.612 (0.059)	-0.492 (0.081)	-0.62 (0.063)	-0.484 (0.067)	-0.372 (0.075)	-0.55 (0.064)
ν	23.04 (9.96)	27.20 (11.45)	38.47 (14.52)	39.75 (14.61)	31.77 (12.76)	30.34 (12.06)	31.33 (12.55)	37.85 (14.55)	16.04 (7.22)	26.99 (11.17)
λ	0.397 (0.255)	0.472 (0.265)	0.504 (0.266)	0.448 (0.263)	0.405 (0.256)	0.387 (0.253)	0.399 (0.255)	0.497 (0.266)	0.401 (0.256)	0.433 (0.262)

Table 3. Posterior means and standard deviations (in parenthesis) of λ, in the case of the exponential prior distribution for λ (d)

parameter	WIG 20	S&P 500	NIKKEI 225	FTSE 100	DAX	NAS-DAQ 100	CAC 40	SPTSE 60	HANG SENG	DJIA
λ	0.431 (0.371)	0.666 (0.556)	0.801 (0.624)	0.579 (0.497)	0.45 (0.382)	0.412 (0.35)	0.44 (0.377)	0.798 (0.652)	0.437 (0.376)	0.531 (0.453)

Finally, we consider the posterior evidence regarding the Box-Cox transformation parameter. Figure 2 shows the prior and posterior distributions for λ in the case of the WIG20 index. We see from the graphs that the prior distribution for λ strongly affects the posterior distribution for this parameter, e.g., a U-shaped prior distribution implies the U-shaped posterior distribution. In the case of the uniform prior for λ on the interval [0; 1], for most stock indices (considered here) the posterior mean is smaller than the prior mean, but the dispersion of posterior distribution is close to that of the prior distribution (in the case of c, the prior mean is equal to 0.5, the prior standard deviation is equal to 0.288). Even though the prior distribution is symmetrical, in the majority of cases the posterior distributions are asymmetrical. The values of λ from the interval [0, 0.5] are more probable a posterior than those from [0.5, 1] (see the quantiles of the posterior distributions of the Box-Cox transformation parameter in Table 4).

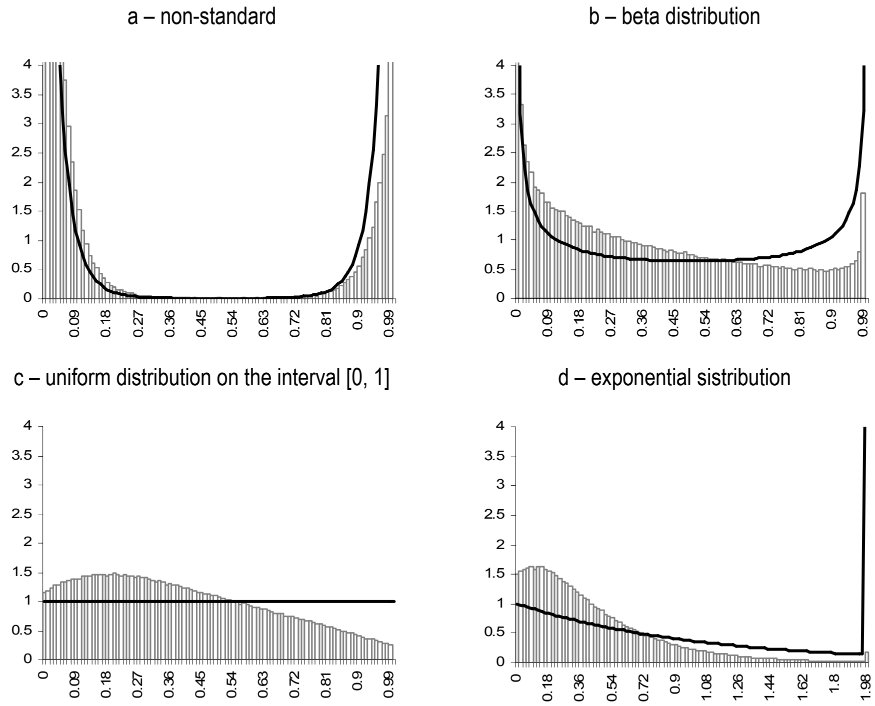

Figure 2. Prior (solid line) and posterior (bars) distributions for λ (the WIG20 index)

In the case of the non-standard prior distribution for λ considered in (a), except for the NIKKEI and SPTSE 60 indices, the posterior medians are below 0.1, but the probability that λ is less than 0.9 is not zero. In Table 5 we present the posterior probabilities that λ is in the interval [0, 0.01] and in the interval [0.99, 1]. Except for the NIKKEI index, the values of λ from the interval [0, 0.01] are more probable a posterior than those from the interval [0.99, 1]. Thus the data transformations which are close to the log-return are more probable a posterior than those which lead to the simple return.

Table 4. Posterior quantiles for λ

	quantile of order	WIG 20	S&P 500	NIKKEI 225	FTSE 100	DAX	NASDAQ 100	CAC 40	SPTSE 60	HANG SENG	DJIA
a	0.05	0.003	0.004	0.005	0.004	0.003	0.003	0.003	0.005	0.003	0.004
	0.25	0.016	0.023	0.033	0.021	0.016	0.015	0.016	0.031	0.016	0.019
	0.5	0.042	0.082	0.824	0.062	0.044	0.039	0.042	0.665	0.042	0.055
	0.75	0.135	0.957	0.972	0.937	0.161	0.111	0.144	0.97	0.133	0.914
	0.95	0.988	0.995	0.996	0.993	0.989	0.985	0.988	0.996	0.988	0.992
b	0.05	0.005	0.011	0.016	0.008	0.005	0.005	0.004	0.013	0.004	0.007
	0.25	0.088	0.163	0.217	0.131	0.095	0.081	0.088	0.194	0.087	0.116
	0.5	0.277	0.433	0.516	0.375	0.294	0.26	0.28	0.489	0.278	0.343
	0.75	0.573	0.746	0.806	0.693	0.597	0.545	0.575	0.787	0.576	0.658
	0.95	0.939	0.981	0.987	0.973	0.945	0.926	0.939	0.985	0.938	0.963
c	0.05	0.042	0.063	0.076	0.055	0.044	0.04	0.042	0.072	0.043	0.051
	0.25	0.185	0.252	0.286	0.229	0.192	0.178	0.186	0.278	0.188	0.215
	0.5	0.362	0.462	0.506	0.429	0.374	0.35	0.366	0.496	0.369	0.409
	0.75	0.585	0.686	0.723	0.656	0.597	0.571	0.589	0.716	0.591	0.636
	0.95	0.864	0.914	0.929	0.901	0.871	0.855	0.865	0.926	0.867	0.891
d	0.05	0.034	0.058	0.077	0.047	0.036	0.032	0.035	0.071	0.034	0.044
	0.25	0.158	0.252	0.322	0.213	0.167	0.153	0.161	0.307	0.16	0.196
	0.5	0.332	0.52	0.652	0.445	0.351	0.32	0.341	0.63	0.337	0.411
	0.75	0.598	0.932	1.13	0.805	0.629	0.574	0.615	1.122	0.61	0.74
	0.95	1.172	1.771	2.037	1.577	1.208	1.11	1.186	2.095	1.186	1.435

Note: Prior distributions for λ: a – non-standard, U-shaped on the interval [0, 1], b – beta distribution, c – uniform distribution on [0, 1], d – exponential distribution.

Table 5. Posterior results for λ

case	index:	WIG 20	S&P 500	NIKKEI 225	FTSE 100	DAX	NASDAQ 100	CAC 40	SPTSE 60	HANG SENG	DJIA
a	u=Pr(λ<0.01|y)	0.173	0.127	0.102	0.142	0.172	0.178	0.172	0.105	0.174	0.149
	v=Pr(λ>0.99|y)	0.041	0.083	0.109	0.068	0.044	0.038	0.042	0.100	0.040	0.060
	u/v	4.238	1.531	0.934	2.088	3.942	4.693	4.111	1.052	4.370	2.492
b	u=Pr(λ<0.01|y)	0.077	0.050	0.040	0.060	0.073	0.081	0.079	0.045	0.081	0.064
	v=Pr(λ>0.99|y)	0.018	0.035	0.044	0.030	0.019	0.016	0.018	0.039	0.017	0.024
	u/v	4.294	1.418	0.913	1.999	3.829	4.924	4.402	1.137	4.769	2.656
c	u=Pr(λ<0.01|y)	0.011	0.007	0.006	0.008	0.011	0.012	0.011	0.006	0.011	0.009
	v=Pr(λ>0.99|y)	0.003	0.005	0.006	0.004	0.003	0.002	0.003	0.006	0.003	0.004
	u/v	4.485	1.546	0.933	2.084	3.733	5.293	4.222	1.027	4.184	2.604

Note: Prior distributions for λ: a – non-standard, U-shaped on the interval [0, 1], b – beta distribution, c – uniform distribution on [0, 1].

It is important to stress that even though the prior distribution of λ has a strong effect on the posterior distribution of λ, it does not affect the posterior distribution of the remaining parameters. Thus in Table 3 we present the posterior characteristics only of λ, obtained in the AR(1)-FCSV model with the exponential distribution for the Box-Cox transformation parameter. Although the prior mean is equal to 1, for all series the posterior mean is less than 1.

Finally, in Table 6 we present the results of the formal Bayesian model comparison. We consider three AR(1)-FCSV models: with, respectively, $\lambda = 0$ (M_1), $\lambda = 1$ (M_2), and $\lambda \sim U(0, 1)$ (M_3). If $\lambda = 1$, the relation (1) is linear in the simple returns. If $\lambda = 0$, it is linear in the logarithmic returns. To obtain the marginal data densities we use the Newton and Raftery method (see Newton and Raftery 1994). The Newton and Raftery estimator is quite stable for all our models. The drawback of this method in the FCSV models is that the models differ from one another by quite a few orders of magnitude.

For all series, assuming equal prior model probabilities, the AR(1)-FCSV model with $\lambda = 0$ (log-returns) is more probable a posterior than with $\lambda = 1$ (simple returns). Only for the DJIA index, the AR(1)-FCSV model with the uniform prior distribution of λ is quite a few orders of magnitude better than that with $\lambda = 0$.

Table 6. Posterior probabilities (under equal prior model probabilities) and marginal data densities of the observation vector **y** in M_i model (based on the Newton – Raftery method)

Index	M_1: $\lambda = 0$	M_2: $\lambda = 1$	M_3: $0 < \lambda < 1$*	$p(y\|M_1)$	$p(y\|M_2)$	$p(y\|M_3)$*
WIG 20	0.9754	0.0000	0.0246	$2.4\cdot10^{-170}$	$1.8\cdot10^{-176}$	$6.0\cdot10^{-172}$
S&P 500	0.9995	0.0000	0.0005	$2.8\cdot10^{-176}$	$2.0\cdot10^{-186}$	$1.4\cdot10^{-179}$
NIKKEI 225	0.9997	0.0000	0.0003	$1.5\cdot10^{-188}$	$7.5\cdot10^{-196}$	$4.2\cdot10^{-192}$
FTST 100	0.9931	0.0069	0.0000	$4.1\cdot10^{-161}$	$2.8\cdot10^{-163}$	$4.3\cdot10^{-177}$
DAX	1.0000	0.0000	0.0000	$3.4\cdot10^{-122}$	$2.4\cdot10^{-141}$	$1.3\cdot10^{-129}$
NASDAQ 100	1.0000	0.0000	0.0000	$9.3\cdot10^{-103}$	$1.6\cdot10^{-108}$	$1.7\cdot10^{-113}$
CAC 40	1.0000	0.0000	0.0000	$5.5\cdot10^{-192}$	$2.4\cdot10^{-202}$	$3.3\cdot10^{-197}$
SPTSE 60	1.0000	0.0000	0.0000	$3.3\cdot10^{-45}$	$2.5\cdot10^{-53}$	$1.1\cdot10^{-55}$
HANG SENG	1.0000	0.0000	0.0000	$3.3\cdot10^{-50}$	$5.2\cdot10^{-55}$	$2.9\cdot10^{-55}$
DJIA	0.0000	0.0000	1.0000	$2.3\cdot10^{-202}$	$7.8\cdot10^{-204}$	$2.6\cdot10^{-195}$

Note: *The results are obtained in the AR(1)-FCSV model with the uniform prior for λ on the interval (0, 1).

4. Conclusions

The paper presents the stochastic volatility models with the Box-Cox transformation of financial time series. The widely used logarithmic and simple returns are nested into the Box-Cox transformation by setting $\lambda = 0$ and $\lambda = 1$, respectively. Using daily data, we show that in the stochastic volatility model with fat tails and correlated errors, the posterior distribution of the Box-Cox transformation parameter strongly depends on the prior assumption about this parameter. Our empirical results show that in the majority of cases the values of λ close to 0 are more probable a posteriori than the ones close to 1. The formal Bayesian model comparison indicates that the Box-Cox transformation with $\lambda = 0$ (log-return) is preferred by the data in the FCSV model. However, the posterior distributions of λ show that the simple returns are not completely inappropriate.

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Bayesowska analiza transformacji Boxa i Coxa dla w modelach o zmienności stochastycznej

Z a r y s t r e ś c i. Celem artykułu jest statystyczna analiza transformacji Boxa i Coxa ilorazu cen instrumentów finansowych w modelach FCSV. Stosowane jest podejście bayesowskie, które pozwala zbadać, w jakim stopniu dane modyfikują wstępne przekonanie o parametrze transformacji. Wyniki empiryczne pokazują, że założenia o rozkładzie a priori parametru transformacji ma istotny wpływ na kształt brzegowego rozkładu a posteriori tego parametru. Jednak w większości rozważanych przypadków rozkłady te, w porównaniu z rozkładami a priori, są przesunięte w kierunku zera. Zatem transformacje ilorazu cen dające wartości bliskie logarytmicznej stopie zwrotu są bardziej prawdopodobne a posteriori niż transformacje prowadzące do prostej stopy zwrotu.

S ł o w a k l u c z o w e: transformacja Boxa i Coxa, model SV, wnioskowanie bayesowskie.