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Modeling of Dynamic Spatial Processes

A b s t r a c t. The paper is concerned with econometric modeling of the dynamic spatial processes on the example of the GDP per capita in selected European countries. The considerations of the paper are focused on investigations of the structure of components of the spatio-temporal process. As a result of the analysis some specifications of the dynamic spatial models have been obtained. Next the issues of the estimation and verification of the models are presented. The main conclusion from the analysis is that the econometric models of the spatio-temporal processes ought to be of the dynamic character, e.g. considering the spatial and spatio-temporal trends and spatial, temporal and spatio-temporal autodependence as well.

K e y w o r d s: spatio-temporal trend, autocorrelation, spatial lag model, dynamic spatial model.

1. Introduction

The paper presents the methodology of econometric modeling of the internal structure of dynamic spatial processes. The considerations recapitulate the results of the previous analyses (see: Szulc, 2008, 2009a, 2009b).

An empirical illustration of the considerations is the spatio-temporal distribution of the GDP per capita in selected European countries. They are: Austria, Germany, the Czech Republic, Slovakia and Hungary. The data relate to the established regions according to the European classification system NUTS and they are taken from the data released by EUROSTAT.

In Szulc (2008) the GDP per capita across the separated area in 2004 was analysed. The componential structure of the single “pure” spatial process $Z(\mathbf{s}_i)$, observed on the plane at the spatial locations $\mathbf{s}_i = [x_i, y_i]$, where i = 1, 2, ..., 84, was investigated. In Szulc (2009a) some time aspect was enclosed into the previous analysis, i.e. the changes of the GDP per capita across the separated area in the period: 2000–2006 were considered. The componential structure of the spatial process, in successive years, i.e. $Z_t(\mathbf{s}_i), \mathbf{s}_i = [x_i, y_i]$, i =1, 2, ..., 84, t =1, 2, ..., 7, was investigated. Thus the conditional, in relation to time, approach to the analysis of the spatio-temporal process was undertaken. Then the conclusions concerning the total spatio-temporal structure, leading to the appropriate empirical model, were formulated only in reference to the so-called spatio-temporal trend. Moreover some probable specification of the dynamic spatial model was proposed. In Szulc (2009b) the approaches, mentioned above, were connected with one another by presenting more extended models which described the componential structure of the spatio-temporal process $Z(\mathbf{s}_i, t)$, were $\mathbf{s}_i = [x_i, y_i]$, i = 1, 2, ..., 84, t = 1, 2, ..., 7.

In the investigations the following assumptions were received:

1. The economic spatial processes demonstrate spatial and/or spatio-temporal trends, which are identified as the mean value of the process, changing in space and/or in time.
2. They usually demonstrate autodependence too, which in the structure of the process creates the autoregressive component.
3. The autoregressive component creates the homogeneous/stationary spatial or/and spatio-temporal process.

It means, that for the spatial process $Z(\mathbf{s}_i)$ there is assumed the basic structure of components, which symbolically may be written down in the following general form:

$$Z(\mathbf{s}_i) = P(\mathbf{s}_i) + A(\mathbf{W})Z(\mathbf{s}_i) + \varepsilon(\mathbf{s}_i). \qquad (1)$$

In turn, in the case of the spatio-temporal process $Z(\mathbf{s}_i, t)$, the basic structure of components may be symbolically presented in the form as follows:

$$Z(\mathbf{s}_i, t) = P(\mathbf{s}_i, t) + A(\mathbf{W}, u)Z(\mathbf{s}_i, t) + \varepsilon(\mathbf{s}_i, t). \qquad (2)$$

The symbols in the formulas (1)–(2) signify:

$P(\mathbf{s}_i)$, $P(\mathbf{s}_i, t)$ – respectively, spatial and spatio-temporal trend which is usually expressed in the form of the two-dimensional (three-dimensional) polynomial function of the co-ordinates of the location on plane (and of the time variable); $A(\mathbf{W})$, $A(\mathbf{W}, u)$ – summable spatial and spatio-temporal shift operators, defined in such a way, that **W** (the matrix of spatial connections) causes the variable to be shifted in space, whereas *u* (the backwards shift operator) causes the lag of it in time; $\varepsilon(\mathbf{s}_i)$, $\varepsilon(\mathbf{s}_i, t)$ – spatial and spatio-temporal white-noise processes.

2. Investigating the Trend Structure

In the investigations of spatial trends the hypothesis of two-dimensional polynomial trend was used. The expression of the form:

$$P(\mathbf{s}_i)=\sum_{k=0}^{p}\sum_{m=0}^{p}\theta_{k,m}x_i^k y_i^m \ , \qquad (3)$$

where: $\mathbf{s}_i=[x_i, y_i]$ – the co-ordinates of the location on the plane, i = 1, 2, ..., N – indexes of the investigated spatial units, $k + m \le p$, presents the spatial trend of degree p.

The models with the trend of the 1st , 2nd and 3rd degree were estimated and verified successively. In all cases the models of the 1st degree appeared the best. The results of the estimation and verification of the models are presented in Table 1. Finally it was confirmed that the spatial trends occurred in all years of the investigated period.

Table 1. Estimates of the parameters of the spatial trends of the 1st degree for the GDP per capita in the period: 2000–2006

Parameters	Years						
	2000	20001	2002	2003	2004	2005	2006
$\hat{\theta}_{00}$	15510.4	15943.0	16455.2	17148.2	17925.6	18752.7	19795.4
$S(\hat{\theta}_{00})$	561.122	586.790	597.896	615.903	632.881	680.731	702.129
t_{00}	27.6418	27.1700	27.5218	27.8424	28.3238	27.5479	28.1934
$\hat{\theta}_{10}$	-0.0170	-0.0169	-0.0168	-0.0177	-0.0177	-0.0182	-0.0192
$S(\hat{\theta}_{10})$	0.0017	0.0018	0.0018	0.0019	0.0019	0.0021	0.0021
t_{10}	-10.000	-9.3890	-9.3333	-9.3158	-9.3158	-8.6667	-9.1429
$\hat{\theta}_{01}$	-0.0080	-0.0084	-0.0089	-0.0095	-0.0100	-0.0103	-0.0101
$S(\hat{\theta}_{01})$	0.0024	0.0025	0.0026	0.0027	0.0027	0.0029	0.0030
t_{01}	-3.3333	-3.3600	-3.4231	-3.5185	-3.7037	-3.5517	-3.3667
R^2	0.5676	0.5450	0.5360	0.5470	0.5355	0.5144	0.5216

Figure 1 presents the theoretical surfaces of the trend. Almost parallel location of the surfaces show that the spatial trends of the GDP per capita across the investigated area in substance do not change with regard to the forms in the successive years. The surfaces referring to the consecutive periods are located higher and higher in relation to the axis of the GDP per capita values, which means, that the mean value of the GDP per capita in the regions grows in time.

The results of investigating the spatial trends include the information on the spatio-temporal trend of the GDP. Usually such a trend may be described with the three-dimensional polynomial function of the following general form:

$$P(\mathbf{s}_i, t)=\sum_{k=0}^{p}\sum_{m=0}^{p}\sum_{l=0}^{p}\theta_{k,m,l}x_i^k y_i^m t^l \ , \qquad (4)$$

where: t – time variable, $k + m + 1 \le p$, other significations – like in (3).

In particular, the spatio-temporal trend of the 1st degree takes the form:

$$P(\mathbf{s}_i, t) = \theta_{000} + \theta_{100} x_i + \theta_{010} y_i + \theta_{001} t \; . \qquad (5)$$

The models of the 1st degree trend, obtained for the GDP spatial process in the successive years may be treated as the conditional trends in relation to time. The estimates of the constants in the models show the linear trend of the form: $\hat{\theta}_{00} = \underset{(198.616)}{14512.2} + \underset{(44.412)}{712.314} t$. Other parameters almost do not change.

Conclusion 1

The GDP per capita across the separated area in the investigated period is formed according to the spatio-temporal trend of the 1st degree.

The empirical model of the spatio-temporal trend of the GDP per capita is presented by the equation (6), i.e.:

$$G\hat{D}P_{i,t} = \underset{(509.882)}{14367.2} - \underset{(0.0007)}{0.0177} x_i - \underset{(0.0010)}{0.0093} y_i + \underset{(113.242)}{748.582} t \, , \qquad (6)$$

$R^2 = 0.5504$.

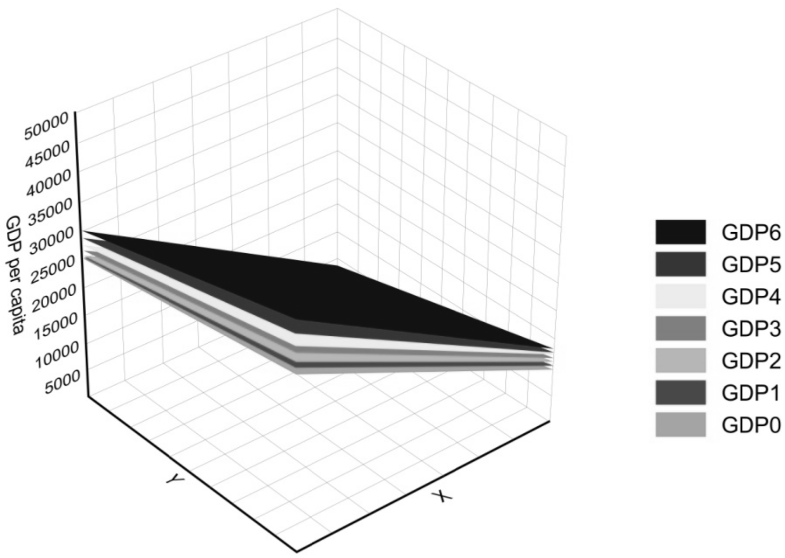

Figure 1. The surfaces of the theoretical values of the GDP per capita according to the spatial trend models in 2000–2006

3. Investigating the Autoregressive Structure

In the investigations of the spatial autocorrelation of the 1st order for the residuals from the previously fitted models of the trend the test Moran's *I*, expressed by the formula (7), was used. In all the years of the investigated period some, not all too high but statistically significant, positive autocorrelation was observed (see, Table 2).

$$I = \frac{N}{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}} \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}\left[Z(\mathbf{s}_i) - \overline{Z}\right]\left[Z(\mathbf{s}_j) - \overline{Z}\right]}{\sum_{i=1}^{N}\left[Z(\mathbf{s}_i) - \overline{Z}\right]^2}, \quad (7)$$

where: $Z(\mathbf{s}_i)$, $Z(\mathbf{s}_j)$ – values of the process of interest at locations i and j, $\overline{Z}$ – the mean value of the process, w_{ij} – the spatial weight of the link between i and j.

Table 2. Testing of spatial autocorrelation

Year	I	E(I)	Var(I)
2000	0.234031	-0.012048	0.004699
	Standardized statistic I 3.6204	p-value = 0.00015	
2001	0.195583	-0.012048	0.004610
	Standardized statistic I 3.0579	p-value = 0.001114	
2002	0.173191	-0.012048	0.004624
	Standardized statistic I 2.724	p-value = 0.003225	
2003	0.171139	-0.012048	0.004638
	Standardized statistic I 2.6899	p-value = 0.003573	
2004	0.164096	-0.012048	0.004647
	Standardized statistic I 2.584	p-value = 0.004884	
2005	0.138682	-0.012048	0.004637
	Standardized statistic I 2.2135	p-value = 0.01343	
2006	0.130453	-0.012048	0.004659
	Standardized statistic I 2.0878	p-value = 0.01841	

Conclusion 2

The values of the GDP per capita in the neighbouring regions are similar to one another.

In the successive years the values of the Moran's statistic were decreasing.

Conclusion 3

The resemblance among the values of the GDP per capita in the neighbouring regions decreases in time.

For investigating the spatial range of the autocorrelation two methods were used. The first one consisted in calculating and verifying significance of the appropriate Moran's statistics, assuming the neighbourhood of different orders, while the second one consisted in using the classic correlation coefficient, calculated for each of the established spatial shift. The significance of the coefficients of the 1st and 5th order or of the 1st , 3rd , 4th and even 5th order was confirmed (according to the used method)[1].

Conclusion 4

The spatial autocorrelation of the GDP per capita across the investigated area may relate not only to the so-called nearest neighbours.

With regard to the diversity of meaning of the results concerning the spatial autocorrelation of the higher orders which were obtained with the help of different methods, the autocorrelation of the 1st order was admitted as the most possible.

4. Modeling of the Trend-Autoregressive Structure of the Spatial Process

The analysis of the trend and autoregressive structure of the GDP per capita across the separated area in the successive years led to the following conclusion:

Conclusion 5

The following form of the spatial econometric model of the GDP per capita should be proposed:

$$GDP_i = \theta_{00} + \theta_{10} x_i + \theta_{01} y_i + \rho \mathbf{W}\left(GDP_i\right) + \varepsilon_i , \tag{8}$$

the same one for each year of the investigated period.

The models of the form (8) are named spatial lag models with regard to the presence of the spatial shifted dependent variable $\mathbf{W}(GDP_i)$. The variable measures the levels of the investigated phenomenon (of the dependent variable) in the neighbouring regions. The results of the estimation and verification of the models with the form (8) for the successive years of the period: 2000–2006 are presented in Table 3.

The obtained empirical models are characterized by significant parameters. The residuals of the models do not show any autocorrelation. Thus, it should be admitted, that the dependence of the 1st order is sufficient to be taken into account in the autoregressive structure.

5. Modeling of the Trend-Autoregressive Structure of the Spatio-Temporal Process

The investigations allow to specify the model referring to the total spatio-temporal structure of the analyzed process. The successive versions of the spatio-temporal models of the GDP process are presented below. The model of the

[1] With regard to the limited volume of the paper (caused by the editorial requirements) the results of estimations of the appropriate coefficients and of verification of their significance are not placed here (for details, see: Szulc, 2009).

form (9) is a direct result of the previous settlements, while the next models came into existence by respecification of this model.

Table 3. The results of the model (8) estimation and verification for the successive years of the period: 2000–2006

Years	Parameters	Estimates of parameters	Standard errors	Statistics z	Pr (> | z |)
2000	θ_{00}	7749	1971.2	3.9316	0.000084
	θ_{10}	-0.008797	0.002455	-3.5832	0.000339
	θ_{01}	-0.003669	0.002272	-1.6150	0.106319
	ρ = 0.51452, test LR = 12.155, p-value = 0.00049				
	Autocorrelation of residuals: test LM = 0.48187, p-value = 0.48758				
2001	θ_{00}	8931	2149.3	4.1552	0.00003
	θ_{10}	-0.009725	0.002611	-3.7250	0.000195
	θ_{01}	-0.004394	0.002462	-1.7843	0.074368
	ρ = 0.45339, test LR = 8.8238, p-value = 0.0029732				
	Autocorrelation of residuals: test LM = 0.049286, p-value = 0.82431				
2002	θ_{00}	9734.9	2275.5	4.2781	0.000019
	θ_{10}	-0.010189	0.002680	-3.8015	0.000144
	θ_{01}	-0.004963	0.002565	-1.9348	0.0530169
	ρ = 0.42117, test LR = 7.2391, p-value = 0.007133				
	Autocorrelation of residuals: test LM = 0.039654, p-value = 0.84216				
2003	θ_{00}	10148	2368.9	4.2840	0.000018
	θ_{10}	-0.010728	0.002798	-3.8349	0.000126
	θ_{01}	-0.005286	0.002655	-1.9907	0.046519
	ρ = 0.42057, test LR = 7.1947, p-value = 0.007312				
	Autocorrelation of residuals: test LM = 0.021646, p-value = 0.88303				
2004	θ_{00}	10898	2505.1	4.3505	0.000014
	θ_{10}	-0.010998	0.002855	-3.8526	0.000117
	θ_{01}	-0.005787	0.002763	-2.0943	0.036232
	ρ = 0.40379, test LR = 6.5691, p-value = 0.01037				
	Autocorrelation of residuals: test LM = 0.0052268, p-value = 0.94237				
2005	θ_{00}	12269	2716.7	4.5162	0.000006
	θ_{10}	-0.01215	0.003076	-3.9493	0.000078
	θ_{01}	-0.006447	0.003011	-2.1412	0.032260
	ρ = 0.35696, test LR = 4.9313, p-value = 0.026374				
	Autocorrelation of residuals: test LM = 0.15626, p-value = 0.69262				
2006	θ_{00}	13268	2891	4.5894	0.000004
	θ_{10}	-0.013137	0.003248	-4.0448	0.000052
	θ_{01}	0.006423	0.003096	-2.0746	0.038030
	ρ = 0.33907, test LR = 4.4004, p-value = 0.035932				
	Autocorrelation of residuals: LM = 0.12389, p-value = 0.72486				

5.1. Model with Spatio-Temporal Trend and Spatial Autocorrelation

The separated spatial analyses for each point in time and the comparison of the obtained results induced to formulate the general conclusion relating to the total spatio-temporal structure of the investigated process in the form of the theoretical model as follows:

$$GDP_{i,t} = \theta_{000} + \theta_{100}x_i + \theta_{010}y_i + \theta_{001}t + \rho\mathbf{W}\left(GDP_{i,t}\right) + \varepsilon_{i,t}\,. \qquad (9)$$

The results of the estimation and verification of the model (9) are presented in Table 4.

Table 4. The results of the model (9) estimation and verification

Parameters	Estimates of parameters	Standard errors	Statistics z	Pr (>|z|)
θ_{000}	8584.5	875.05	9.8103	0.000000
θ_{100}	-0.010805	0.001068	-10.1139	0.000000
θ_{010}	-0.005248	0.001021	-5.1403	0.000000
θ_{001}	447.98	112.72	3.9742	0.000071
ρ = 0.41449 test LR: 49.408, p-value: 0.000000				
Wald statistic: 70.115, p-value: 0.000000 AIC: 11754 (AIC for lm: 11801)				
Autocorrelation of residuals Test LM: 0.003232, p-value: 0.95467				

The model with the spatio-temporal trend and spatial shifts is characterized by significant parameters; the residuals do not show any autocorrelation and it is better than the model which takes into consideration only the trend.

5.2. Model with Spatio-Temporal Trend and with Spatial and also with Time Autoregression

The existence of the very strong time autocorrelation of the GDP per capita (the coefficient of time autocorrelation of the 1st order for the residuals of the model with the spatio-temporal trend of the 1st degree equals 0.9951) justifies including the component $GDP_{i,t-1}$ into the model which describes the structure of the GDP process. Thus, the next specification of the model is following:

$$GDP_{i,t} = \theta_{000} + \theta_{100}x_i + \theta_{010}y_i + \theta_{001}t + \alpha GDP_{i,t-1} + \rho\mathbf{W}\left(GDP_{i,t}\right) + \varepsilon_{i,t}\,. \qquad (10)$$

The results of the estimation and verification of the model (10) are presented in Table 5.

Apart from the improvement in the general degree of the model fitting, it cannot be treated as the final one because the autocorrelation appeared in the residuals.

5.3. Model with the Spatio-Temporal Trend and with Spatial, Time and Spatio-Temporal Autoregression

Just as the coefficients of the spatial and time autocorrelation, the coefficient of the spatio-temporal autocorrelation of the 1st order appeared significant. Its value amounted to 0.1636. Therefore the next model of the GDP spatio-temporal structure additionally takes into consideration the component $\mathbf{W}(GDP_{i,t-1})$. It has the following form:

$$GDP_{i,t} = \theta_{000} + \theta_{100} x_i + \theta_{010} y_i + \theta_{001} t + \alpha GDP_{i,t-1} + \rho \mathbf{W}\left(GDP_{i,t}\right) + \gamma \mathbf{W}\left(GDP_{i,t-1}\right) + \varepsilon_{i,t}. \quad (11)$$

Table 5. The results of the model (10) estimation and verification

Parameters	Estimates of parameters	Standard errors	Statistics z	Pr (>|z|)
θ_{000}	161.68	121.88	1.3265	0.18467
θ_{100}	-0.000219	0.000145	-1.5097	0.13111
θ_{010}	-0.000305	0.000121	-2.5191	0.01176
θ_{001}	125.35	13.627	9.1985	0.00000
α	1.0420	0.004225	246.6556	0.00000
ρ = -0.034921 test LR: 17.139, p-value: 0.000000				
Wald statistic: 17.519, p-value: 0.000000 AIC: 7643.4				
Autocorrelation of residuals Test LM: 49.851, p-value: 0.000000				

The results of the estimation and verification of the model (11) are presented in Table 6.

Table 6. The results of the model (11) estimation and verification

Parameters	Estimates of parameters	Standard errors	Statistics z	Pr (>|z|)
θ_{000}	252.99	115.64	2.1879	0.02868
θ_{100}	-0.000290	0.000137	-2.1088	0.03496
θ_{010}	-0.000275	0.00014	-2.4146	0.01575
θ_{001}	78.610	13.9317	5.6485	0.00000
α	1.0458	0.004019	260.1819	0.00000
γ	-0.42254	0.057249	-7.3808	0.00000
ρ = 0.37102 test LR: 37.425, p-value: 0.000000				
Wald statistic: 44.667, p-value: 0.000000 AIC: 7600.3				
Autocorrelation of residuals Test LM: 0.12572, p-value: 0.72291				

The considered model is characterized by significant parameters. The residuals of the model do not show any autocorrelation. According to its general fitting to the data it is the best among all the models proposed in this paper.

5. Final Remarks

The considerations of the paper confirm that investigating the properties and structures of spatial and spatio-temporal economic processes is important for modeling of them.

The econometric models of the spatio-temporal processes should have the dynamic character. It is expressed in the appropriate specification of the trend-autoregressive structure, characterizing temporal, spatial and spatio-temporal tendencies and the lags and spatial or/and spatio-temporal shifts of the observed dependence.

The GDP per capita across the separated area in the investigated period realizes the spatio-temporal process, which shows the spatio-temporal trend and the spatial and spatio-temporal autodependence. The specification of the dynamic spatial model for the GDP per capita caused that a "good" empirical model was obtained.

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Modelowanie dynamicznych procesów przestrzennych

Z a r y s t r e ś c i. Artykuł przedstawia ekonometryczną analizę procesu przestrzenno-czasowego na przykładzie PKB w wybranych krajach europejskich. Przedmiotem rozważań są przestrzenne i przestrzenno-czasowe trendy oraz autozależności charakteryzujące składnikową strukturę badanego procesu. Składniki te są podstawą do specyfikacji dynamicznych modeli przestrzennych. Zaproponowane w artykule specyfikacje dynamicznych modeli przestrzennych poddaje się empirycznej weryfikacji.

S ł o w a k l u c z o w e: trend przestrzenno-czasowy, autokorelacja, model przesunięć przestrzennych, dynamiczny model przestrzenny.