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The Sign RCA Models: Comparing Predictive Accuracy of VaR Measures†

A b s t r a c t. Evaluating Value at Risk (VaR) methods of predictive accuracy in an objective and effective framework is important for both efficient capital allocation and loss prediction. From this reasons, finding an adequate method of estimating and backtesting is crucial for both the regulators and the risk managers'. The Sign RCA models may be useful to obtain the accurate forecasts of VaR. In this research one briefly describes the Sign RCA models, the Value at Risk and backtesting. We compare the predictive accuracy of alternative VaR forecasts obtained from different models. Empirical example is mainly related to the PBG Capital Group shares on the Warsaw Stock Exchange.

K e y w o r d s: Family of Sign RCA Models, Value at Risk, backtesting, loss function.

1. Introduction

Nowadays, accurate modelling of risk is very important in risk management. This is a result of the globalisation of financial market, the evolution of the derivative markets and the technological development. Value at Risk (VaR) has become the standard measure to quantify market risk[1]. This measure can be used by the financial institutions to assess their risks or by a regulatory committee to set margin requirements.

In the literature, many parametric VaR models and many forecasting accuracy assessments for VaR methods exist. The important representation of the parametric VaR models are the generalized autoregressive conditional heteroskedasticity models (GARCH) (Bollerslev, 1986; Engle, 1982). These models describe non-linear dynamics of financial time series. A different, alternative approach to the description of financial time series represent the

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[1] It was introduced by JP Morgan in 1996.

random coefficient autoregressive models (RCA) (which were proposed by Nicholls, Quinn, 1982). Thavaneswaran et al. proposed a number of expansions of the random coefficient autoregressive model order one. The new models, such as Sign RCA(1), RCAMA(1,1), Sign RCAMA(1,1), RCA(1)-GARCH(1,1) and Sign RCA(1)-GARCH(1,1) can be used to obtain Value-at-Risk measure.

The aim of this paper is to use the family of Sign RCA models to obtain the VaR forecasts and compare the results obtained from Sign RCA models with other selected VaR models.

2. The Family of Sign RCA Models

Random coefficient autoregressive models (RCA) are straightforward generalization of the constant coefficient autoregressive models. A full description of this class of models including their properties, estimation methods and some applications can be found in Nicholls and Quinn (1982).

Thavaneswaran, Appadoo and Bector (2006) proposed a first order random coefficient autoregressive model with a first order moving average component, i.e. RCAMA(1,1). In another paper Thavaneswaran and Appadoo (2006) proposed to add the sign function to RCA(1) and RCAMA(1,1) models. The last modification is based on assumption that residuals from the RCA model or the Sign RCA model can be described by a GARCH model. In this way, the RCA(1)-GARCH(1,1) model and Sign RCA(1)-GARCH(1,1) model were created. All these modifications influence the increase of variance and kurtosis of processes[2].

In Table 1 equations of individual models from the family of Sign RCA models and their names are presented.

To ensure the existence of the I-VI models (Table 1) the following assumptions must be satisfied:

$$\begin{pmatrix} \delta_t \\ \varepsilon_t \end{pmatrix} \sim^{iid} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\delta^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{pmatrix} \right), \tag{1}$$

$$\phi^2 + \sigma_\delta^2 < 1 . \tag{2}$$

The sign function, described by the following formula

$$s_t = \begin{cases} 1 & \text{for } y_t > 0, \\ 0 & \text{for } y_t = 0, \\ -1 & \text{for } y_t < 0, \end{cases} \tag{3}$$

[2] Theoretical properties of the family of Sign RCA models can be found in articles, i.e.: Appadoo, Thavaneswaran, Singh (2006), Aue (2004), Górka, (2008), Thavaneswaran, Appadoo, Bector (2006), Thavaneswaran, Appadoo (2006), Thavaneswaran, Appadoo, Ghahramani, (2009), Thavaneswaran, Peiris, Appadoo (2008).

has the interpretation: if $\phi + \delta_t > |\Phi|$, the negative value of Φ means that the negative (positive) observation values at time $t-1$ correspond to a decrease (increase) of observation values at time t. In the case of stock returns it would suggest (for returns) that after a decrease of stock returns, the higher decrease of stock returns occurs than expected, and in the case of the increase of stock returns the lower increase in stock returns occurs than expected.

Table 1. The family of Sign RCA models (without conditions)

Model	Model equations	No.
RCA(1)	$y_t = (\phi + \delta_t)y_{t-1} + \varepsilon_t$	I
Sign RCA(1)	$y_t = (\phi + \delta_t + \Phi s_{t-1})y_{t-1} + \varepsilon_t$	II
RCAMA(1,1)	$y_t = (\phi + \delta_t)y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$	III
Sign RCAMA(1,1)	$y_t = (\phi + \delta_t + \Phi s_{t-1})y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$	IV
RCA(1)-GARCH(1,1)	$y_t = (\phi + \delta_t)y_{t-1} + \varepsilon_t$, $\varepsilon_t = \sqrt{h_t} z_t$ $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}$	V
Sign RCA(1)-GARCH(1,1)	$y_t = (\phi + \delta_t + \Phi s_{t-1})y_{t-1} + \varepsilon_t$, $\varepsilon_t = \sqrt{h_t} z_t$ $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}$	VI

Note: s_t – sign function is described by equation (3); ϕ, θ, Φ, α_i, β_1 – model parameters.

Condition (2) is necessary and sufficient for the second-order stationarity of process described by equation I, however conditions (1)-(2) ensure strict stationarity of this process. If conditions (1)-(2) are satisfied, then processes described by equations II-IV are stationary in mean.

If residuals from the RCA model are described by a GARCH model, then the RCA(1)-GARCH(p,q) model described by equation V, where $z_t \sim N(0, \sigma_z^2)$, $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$, is obtained. If the sign function is added to the RCA-GARCH model, then the process described by equation VI is obtained. The conditions ensuring the positive value of conditional variance of this process are the following: $z_t \sim N(0, \sigma_z^2)$, $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $|\Phi| \leq \alpha_0$.

Predictors of the conditional mean and conditional variance of Sign RCA models are presented in Table 2 and 3 respectively.

Table 2. Conditional mean predictors

Models	Conditional mean
RCA(1), RCA(1)-GARCH(1,1)	$y^P_{t+1\|t} = E\left(y_{t+1}\|F_t\right) = \varphi y_t$
Sign RCA(1), Sign RCA(1)-GARCH(1,1)	$y^P_{t+1\|t} = E\left(y_{t+1}\|F_t\right) = \left(\varphi + \Phi s_t\right) y_t$
RCAMA(1,1)	$y^P_{t+1\|t} = E\left(y_{t+1}\|F_t\right) = \varphi y_t + \theta\varepsilon_t$
Sign RCAMA(1,1)	$y^P_{t+1\|t} = E\left(y_{t+1}\|F_t\right) = \left(\varphi + \Phi s_t\right) y_t + \theta\varepsilon$

Table 3. Conditional variance predictors

Models	Conditional variance
RCA(1), Sign RCA(1), RCAMA(1,1), Sign RCAMA(1,1)	$\sigma^2_{t+1\|t} = E\left(u^2_{t+1}\|F_t\right) = \sigma^2_\varepsilon + \sigma^2_\delta y^2_t$
RCA(1)-GARCH(1,1), Sign RCA(1)-GARCH(1,1)	$\sigma^2_{t+1\|t} = E\left(u^2_{t+1}\|F_t\right) = \sigma^2_z E(h_t) + \sigma^2_\delta y^2_t$

3. Value-at-Risk

Value-at-Risk (VaR) is used as a tool for measuring market risk. It is defined as „the maximum potential loss that a portfolio can suffer within a fixed confidence level during a holding period".

Formal definition of VaR is following (Artzner, Delbaen, Eber, Heath, 1999):

$$\mathrm{VaR}_\alpha\left(X\right) = \inf\left\{x : F_X\left(x\right) \geq \alpha\right\} = \inf\left\{x : P\left(X > x\right) \leq 1-\alpha\right\}, \tag{4}$$

where $\alpha \in (0,1)$ is a particular confidence level, F_X – the cumulative density function.

Consider a time series of daily *ex post* returns ($r_t = 100\left(\ln P_t - \ln P_{t-1}\right)$ where P_t is the share price at time *t*) and corresponding time series of *ex ante* VaR forecasts (VaR_α), the formula (4) takes the form:

$$P\left(r_{t+1} \leq -\mathrm{VaR}_\alpha\right) = \alpha\ . \tag{5}$$

The negative sign arises from the convention of reporting VaR as a positive number.

One-step-ahead conditional forecasts of Value-at-Risk are calculated by the formula:

$$\mathrm{VaR}^l_{t+1}\left(\alpha\right) = \mu_{t+1|t} + \sigma_{t+1|t} z_\alpha, \tag{6}$$

where $\mu_{t+1|t}$, $\sigma_{t+1|t}$ are one-step-ahead conditional forecasts of mean and volatility respectively.

3.1. Estimation Methods for *VaR*

This section briefly describes the alternative models that we use for estimating VaR forecasts in this paper.

The following models are used in the research to obtain VaR forecasts:

- The historical simulation (HS)[3]. The VaR is estimated as the α-th quantile of the empirical distribution of returns. HS is based on the assumption that returns are *iid* time series of an unknown distribution.
- The equally weighted moving average (EWMA) model, i.e.

$$\sigma_{t+1|t}^2 = \frac{1}{k}\sum_{i=t-k+1}^{t} r_i^2 \,, \tag{7}$$

  where k – size of window, r_i^2 – returns. The returns are assumed to be normally distributed.
- The RiskMetrics (RM) model, i.e.

$$\sigma_{t+1|t}^2 = (1-\lambda)\sum_{i=t-k+1}^{t} \lambda^{t-i} r_i^2 = \lambda\sigma_t^2 + (1-\lambda) r_t^2 \,, \tag{8}$$

  where $\lambda \in (0,1)$ is known as the decay factor, $\lambda\sigma_t^2$ is the previous volatility forecast weighted by the decay factor, and $(1-\lambda)r_t^2$ is the latest squared returns weighted by $(1-\lambda)$. The VaR is estimated under the assumption that returns are normally distributed (as in the case of EWMA).
- The AR(1)-GARCH(1,1) model, i.e.

$$r_t = \phi r_{t-1} + \varepsilon_t \,, \tag{9}$$

  where $\varepsilon_t = z_t \sigma_t$, $z_t \sim N(0,1)$,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \,. \tag{10}$$

  In this case, returns series is assumed to be conditionally normally distributed.
- Models from the family of Sign RCA models[4].

[3] HS is the oldest and still very popular estimator of the VaR.

[4] They were presented in previous section.

3.2. Backtesting VaR Estimates

Backtesting is based on testing whether the VaR estimates are statistically accurate.

The „failure process" is defined as:

$$I_t = 1\left(r_t < -\text{VaR}_t^l\right), \qquad t = T+1, ..., T+N , \tag{11}$$

where $1(*)$ denotes the indicator function returning a unit if the argument is true, and zero otherwise; T is the size of the sample used to estimate parameters of the model; N is the number of one-step-ahead VaR forecasts computed. The VaR forecasts are accurate if the $\{I_t\}$ series is *iid* with mean α, i.e. $E\left[I_{t|t-1}\right] = \alpha$. To test the statistical accuracy we used the standard likelihood ratio tests:

1. The proportion of failures test – LR_{pof} (Kupiec, 1995)[5]:

$$H_0: \ E[I_t] = \alpha \qquad vs. \qquad H_1: \ E[I_t] \neq \alpha ,$$

$$LR_{pof} = -2\ln\left[\left(\frac{1-\alpha}{1-\hat{\alpha}}\right)^{N-n}\left(\frac{\alpha}{\hat{\alpha}}\right)^{n}\right] \quad \sim \chi_1^2 , \tag{12}$$

where n is the number of failures VaR, $\hat{\alpha}$ is the MLE of α, i. e. $\frac{n}{N}$.

2. The Christoffersen independence test – LR_{ind} (Christoffersen, 1998):

$$H_0: \ \alpha_{01} = \alpha_{11} ,$$

$$LR_{ind} = -2\ln\frac{\left(1-\overline{\alpha}\right)^{T_{00}+T_{10}}\ \overline{\alpha}^{T_{01}+T_{11}}}{\left(1-\widehat{\alpha}_{01}\right)^{T_{00}}\ \widehat{\alpha}_{01}^{T_{01}}\left(1-\widehat{\alpha}_{11}\right)^{T_{10}}\ \widehat{\alpha}_{11}^{T_{11}}} \quad \sim \chi_1^2 , \tag{13}$$

where:

$$\hat{\alpha}_{ij} = \frac{T_{ij}}{T_{i0}+T_{i1}}, \quad \overline{\alpha} = \frac{T_{01}+T_{11}}{T_{00}+T_{10}+T_{01}+T_{11}} ,$$

T_{ij} – number of *i* values followed by a *j* value in the I_t series $(i, j = 0,1)$.

3. The time between failures test – LR_{tbf} (Haas, 2001)[6]:

$$LR_{tbf} = -2\sum_{i=1}^{N}\ln\left[\left(\frac{1-\alpha}{1-\widehat{\alpha}_i}\right)^{v_i-1}\frac{\alpha}{\widehat{\alpha}_i}\right] \quad \sim \chi_N^2 , \tag{14}$$

[5] Similar, the LR test of unconditional coverage by Christoffersen (1998) was proposed. Other symbol of this test is the LR_{uc}.

[6] Haas extended the Kupiec's time until first failure test (TUFF test) by adding test for every exception (second and next).

where $\hat{\alpha}_i = \frac{1}{v_i}$, v_1 – time until first failure, v_i – time between exception $(i-1)$ and exception i for $i = 2, ..., N$.

If, in above tests the null hypothesis is not rejected, then a particular model gives accurate forecasts of VaR. However, if more than one model is deemed adequate, we cannot conclude which of VaR model should be selected.

Lopez (1998) suggested measuring the accuracy of VaR forecasts on the basis of distance between observed returns and forecasted VaR values. This approach does not give any formal statistical selection of model adequacy but it allows to rank the models.

Let $f = \sum_{t=1}^{N} f_t$ means a total loss function. A model which minimizes the total loss function is preferred over the other models. In the literature, different loss functions were proposed (see Lopez, 1998, 1999; Blanco and Ihle, 1998; Sarma, Thomas and Shah, 2003, Caporin, 2003; Angelidis, Benos and Degiannakis, 2004). In this paper, the loss functions used to compare the accurate VaR forecasts are as follows:

– The regulatory loss function – RL (Lopez, 1999)[7]:

$$f_{t+1} = \begin{cases} 0 & r_{t+1} > -\mathrm{VaR}_{r,t}, \\ 1 + \left(r_{t+1} + \mathrm{VaR}_{r,t}\right)^2 & r_{t+1} \le -\mathrm{VaR}_{r,t}. \end{cases} \tag{15}$$

– The firm's loss function – FL (Sarma, Thomas, Shah, 2003):

$$f_{t+1} = \begin{cases} c\mathrm{VaR}_{r,t} & r_{t+1} > -\mathrm{VaR}_{r,t}, \\ 1 + \left(r_{t+1} + \mathrm{VaR}_{r,t}\right)^2 & r_{t+1} \le -\mathrm{VaR}_{r,t}. \end{cases} \tag{16}$$

where c is a measure of cost of capital opportunity.

Sarma, Thomas and Shah (2003) proposed testing for superiority of a model *vis-á-vis* another in terms of the loss function. They suggested a two-stage VaR model selection procedure. The first stage consists in testing the statistical accuracy for the competing VaR models. In the second stage of the VaR model selection procedure, the firm's loss function is used to evaluate statistically VaR models[8].

[7] This name comes from Sarma, Thomas and Shah (2003) who explain that (16) is able to express the regulatory concerns in model evaluation. However, no score is attached in case if exception does not occur.

[8] Only that VaR model for which the average number of failures was equal to the expected and these failures are independently distributed is included in the second stage.

Consider two VaR models, *i* and *j*. The hypotheses are:

$$H_0: \quad \theta = 0 \qquad vs. \qquad H_1: \quad \theta < 0,$$

where θ is the median of the distribution of $z_t = f_{i,t} - f_{j,t}$, where $f_{i,t}$ and $f_{j,t}$ are the values of loss function generated by model *i* and model *j* respectively. Negative values of z_t indicate a superiority of model *i* over *j*.

The testing procedure is as follows:

1. Define an indicator variable $\psi_t = 1(z_t \geq 0)$ and the number of non-negative z_t's, as $S_{ij} = \sum_{t=T+1}^{T+N} \psi_t$.
2. Calculate the statistics as:

$$STS_{ij} = \frac{S_{ij} - 0.5N}{\sqrt{0.25N}} \qquad \sim N(0,1) \text{ asymptotically}, \tag{17}$$

STS_{ij} is based on assuming that the z_t is *iid*[9].

Alternatively, we can compare competing VaR models using the predictive quantile loss function (see Giacomini and Komunjer, 2005; Bao et al., 2006). The expected loss function is given by:

$$Q_\alpha = \frac{1}{N}\sum_{i=1}^{N}\left[\alpha - 1\left(r_i < -\mathrm{VaR}_i\right)\right]\left(r_i + \mathrm{VaR}_i\right). \tag{18}$$

The selected model is the VaR model which has the minimum of Q_α.

4. Empirical application

The data used in the empirical application are daily prices of twenty Polish firms' shares from the WIG20 portfolio on the Warsaw Stock Exchange (WSE). The data were obtained from bossa.pl for the period from 23-rd September 2005 to 18-th February 2009, which yields 852 observations. However, one of shares was excluded because it was not quoted on 23 September 2005. To analyze daily percentage log returns of each share were used.

This empirical study was composed of two parts. The first part (Analysis I) was carried out with regard to all of twenty shares from WSE. The research procedure was the following:

1. For the first 500 observations of each returns series the descriptive statistics and some tests were calculated. Next, returns series with

[9] For details on the sign test see Diebold and Mariano (1995).

autocorrelation and kurtosis bigger than for normal distribution were chosen[10].

2. Parameters of six models from the family of Sign RCA were estimated for the first 500 observations of time series selected in step one. Next, only models with statistically significant parameters were used.
3. The estimation of parameters for models selected in step 2 was performed for rolling window of 100, 150, 200, 250, 300, 400, 500 observations. In the same way the estimation of AR(1)-GARCH(1,1) models was obtained.
4. For all models from step 3 and for the historical simulation (HS), the equally weighted moving average (EWMA) model, the RiskMetrics (RM) models (with $\lambda = 0.95$ and $\lambda = 0.99$) VaR measures were calculated[11]. One-step-ahead forecasts of *VaR* (that is 751, 701, 651, 601, 551, 451, 351 forecasts, respectively) were calculated on the basis of these models.
5. The traditional VaR tests and loss functions for each model and window were calculated.
6. The obtained results in above step were compared.

In the second part (Analysis II) only the PBG shares (PBG Capital Group) was chosen. All presented models of VaR for the last 250 observations were calculated[12]. For obtained VaR forecasts the two-stage VaR model selection procedure was applied.

All model parameters (Analysis I and II) were estimated using maximum likelihood (MLE) with the BFGS algorithm**.** Calculations were carried out in the Gauss program.

4.1. Results of the Analysis I

Selected results of the descriptive statistics and some tests are given in Table 4. All series have a mean between -0.052 and 0.561, kurtosis bigger than for normal distribution. The standard deviations are different, ranging from 1.955 for PGNIG to 5.354 for BIOTON. The skewness and kurtosis differ among all series. Only 8 of 19 returns series have autocorrelation. The LBI test rejects the null hypothesis of non random coefficient to four stock returns.

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[10] This method of the elimination of initially selected companies can impact on the results. It would be worth to check out which results might be obtained for the whole set of companies. However, such analysis was omitted in this paper.

[11] The returns series were assumed either to be normally distributed or conditionally normally distributed, respectively.

[12] The set of 250 observations corresponds to roughly one year of trading days and according to the Basel II Accord requirement the minimum of 250 VaR forecasts should be used to the backtesting approach. Therefore, one-step-ahead forecasts of VaR at the same period (250 observations) were calculated. Parameters were estimated for rolling windows of 125, 250, 375 observations each. The returns series were assumed either to be conditionally normally distributed or normally distributed respectively.

Next, the 7 different models were estimated for 8 returns series. Further, only models with statistically significant parameters were chosen. In this way models like RCA and Sign RCA were chosen.

To present backtesting results for VaR forecasts of the PBG shares was chosen because for that share the autoregressive parameter in the RCA models for all returns series has been the biggest. It is very important because we can expect the Sign RCA model to be better than other models.

The traditional VaR tests and loss functions for the PBG for all models are presented in Table 5 and the 5% at significance level. One can see that the accuracy test rejects the null hypothesis for windows size of 500, 400 observations for HS, EWMA model, AR(1)-GARCH(1,1) model, RCA model and Sign RCA model. For example, for window size 250 the regulatory loss function is the smallest for RM ($\lambda = 0.95$). Next position in this ranking have AR(1)-GARCH(1,1) model, EWMA model, RCA model, Sign RCA model and the last position has RM ($\lambda = 0.99$). The HS method is not taken into consideration because the accuracy test rejects the null hypothesis for windows size of 250 observations. On the other hand, the firm's loss function is the smallest for RM ($\lambda = 0.99$) and the next positions in ranking have Sign RCA model, RCA model, EWMA model, AR(1)-GARCH(1,1) model and RM ($\lambda = 0.95$).

The differences between values of the firm's loss function are small for estimated models. To compare these results, the tests for superiority of a model *vis-á-vis* another were used only for models included into the second stage at Sarma, Thomas and Shah procedure. The results are presented in Table 6. For the window size 300 we can see that the Sign RCA model is significantly better than other models, i.e. the null hypothesis is rejected in the test of superiority between the Sign RCA model and the other models presented in subsection 3.1. However, as the size of windows decreases the RM model ($\lambda = 0.99$) outperforms the Sign RCA model. RCA and Sign RCA models are statistically the same for the window size 100. In cases when results with HS are compared one can see that HS is almost everywhere significantly better than others.

The Table 7 includes the results of the VaR tests and the loss function at the 2,5% significance level which are similar to the results obtained at the 5% significance level. Only for HS with the window size 250 and for RCA model with the window size of 300 observations, some differences can be noticed, i. e. In the case of HS the accuracy at the 2.5% is better than at the 5% significance level (except RCA model). For the loss function conclusions are the same with one exception, i. e. the HS has the last rank for regulatory loss function and the first rank for firm's loss function.

At the 1% significance level we obtained more differences (see Table 8). Firstly, Risk Metrics models are accurate only for windows size 500 and 500, 400 for $\lambda = 0.95$, $\lambda = 0.99$, respectively. The RCA, Sign RCA and EWMA models are accurate for small windows (size 200, 150, 100). The regulatory loss function is the smallest for HS. The firm's loss function has the lowest values for Sign RCA models for the window size 200. Very strange results were obtained for HS and therefore we are not able to find any rules for accuracy and value of the regulatory loss function.

4.2. Results of the Analysis II

Firstly, we calculated the 250 one-step-ahead forecasts of VaR of the PBG share using all models of VaR (presented in 3.1)[13]. The VaR forecasts were received from different models estimated for the different window sizes, i.e, T =125, 250 and 375.

Secondly, the competing VaR models were testing for statistical accuracy. For the established period of forecasting, only Sign RCAMA(1,1) models (for T = 375 and all significance level, for T = 250 and α = 2.5%, 1%), Sign RCA(1)-GARCH(1,1) models (for α = 1% and rolling window size T = 375, 125) and Risk Metrics models (for λ= 0,99 and α = 1% and T = 125) did not fulfill the conditions used at first stage of Sarma, Thomas and Shah procedure (the null hypothesis was rejected at least for one test, see (12)-(14)). For other models, the firm's loss function (see the Table 9), the STS test and the predictive quantile loss function (see the Table 10) were calculated. Lower values of the firm's loss function for VaR forecasts were received from RCAMA(1,1), RCA(1) and Sign RCAMA(1,1) (if it was included at second stage) models (with the exception of the HS for α = 5% and T = 375, 250 and with the exception of the RM(λ= 0,99) for T = 125 and α = 5%, 2.5%). The test for superiority of a model *vis-á-vis* another indicates that:

1. At the 5% significance level, for different rolling window sizes, each of models having first rank is superior over other models.
2. At the 2.5% significance level, for rolling windows size of 125 observations, the RM (λ= 0,99) is superior over other models. The RCAMA(1,1) model is better than almost all other models (with the exception of HS method and RCA(1) model for T = 375 and with the exception of the RCA(1)-GARCH(1,1) model for T = 250, for which the predictive ability is equal).
3. For the α = 1%, for different rolling window sizes, each of models having first rank is superior over other models (with the exception of RCAMA(1,1) and RCA(1) models for T = 375 that have equal predictive ability).

Other conclusions are formulated based on the predictive quantile loss function (Table 10), which yields different position in the ranking. For VaR forecasts of the PBG share, for established forecasting period, the choice of the

[13] One-step ahead forecasts on the period 19.02.2008-18.02.2009 were computed.

best model from the competing models depends on the significance level and rolling window sizes. For the Sign RCA models the rolling window size of 125 observations seemed too small. This conclusion is similar to one from Analysis I.

5. Conclusions

Evaluating forecasts based solely on one criterion yield the limited information regarding the accuracy method. Thus, in the literature is commonly accepted that results of each evaluation criterion are presented separately and then best performing method is selected. However, it can be noticed that the different evaluation criteria give the different choice of the best estimation method of VaR. Therefore, it is difficult to make general remarks, nevertheless the empirical results showed that:

1. None of the presented methods gave a satisfactory VaR estimates.
2. The results showed no domination of either forecasting methods of VaR.
3. Bigger sample did not lead to the better results.
4. It seems that the family of Sign RCA models should be used for the sample size of 150 to 300 observations.
5. In terms of the firm's loss function the Sign RCA model was significantly better than the AR-GARCH model, RM ($\lambda = 0.95$) model and EWMA model. The Sign RCA model was not worse than the standard RCA model.
6. One should treat every share individually and use different methods and models for obtaining a good forecast of VaR.
7. The historical simulation gave better results (in terms of accuracy) at the 1% significance level than for other significance levels. It seems that the minimum window size should be 250 observations but smaller than 500 observations.
8. The RCAMA(1,1) model can be competitive to other VaR measures from the firm's loss function point of view.
9. The Sign RCA models with GARCH errors did not give better forecasts of VaR for the PBG share.

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Modele Sign RCA: Porównanie trafności prognoz VaR

Z a r y s t r e ś c i. Obiektywna i skuteczna ocena trafności prognozowania wartości narażonej na ryzyko (Value at Risk – *VaR*) jest bardzo ważna zarówno dla efektywnego zarządzania kapitałem jak i do prognozowania strat. Z tego powodu znalezienie odpowiednich metod estymacji i weryfikacji *VaR* jest kluczowe zarówno dla instytucji nadzorujących jak i dla menadżerów. Modele Sign RCA mogą być użyteczne do otrzymywania trafnych prognoz *VAR*. W artykule, pokrótce przedstawione są modele Sign RCA, wartość narażona na ryzyko i weryfikacja prognoz *VaR*. Porównana jest trafność prognoz *VaR* otrzymanym z różnych alternatywnych modeli. Przykład empiryczny skoncentrowany jest głównie na cenach akcji spółki PBG notowanej na Giełdzie Papierów Wartościowych w Warszawie.

S ł o w a k l u c z o w e: Modele klasy Sign RCA, Value at Risk, testowanie wsteczne, funkcja strat.

Table 4. Results of the descriptive statistics, Box-Ljung tests and locally best invariant test

Company	Mean	Std. Dev.	Skewness	Kurtosis	B-L (1)	B-L (2)	LBI
AGORA	-0,052	2,451	-0,204	4,853	8,925***	9,029***	1,672
ASSECOPOL	0,169	2,493	-0,582	13,015	6,953***	8,357***	2,848**
BIOTON	-0,036	5,354	-8,286	138,621	1,673	2,111	-0,028
BRE	0,256	1,972	0,263	4,055	3,915**	4,025	2,378**
BZWBK	0,175	2,442	-0,135	3,472	1,478	2,738	1,034
CERSANIT	0,247	2,361	0,567	6,312	0,156	1,887	1,639
GETIN	0,231	2,646	0,523	11,370	0,008	0,837	0,954
GTC	0,255	2,737	0,461	5,383	1,510	8,046***	0,461
KGHM	0,212	3,011	-0,591	5,303	0,001	4,766	1,156
LOTOS	0,036	2,174	-0,329	4,835	1,596	2,249	-0,078
PBG	0,363	2,094	0,095	5,344	3,466*	3,468	1,909
PEKAO	0,071	2,160	0,219	3,616	0,005	0,044	0,273
PGNIG	0,068	1,955	0,192	4,413	0,284	4,870*	2,929**
PKNORLEN	-0,021	2,170	-0,069	3,853	0,017	3,680	0,508
PKOBP	0,117	2,055	0,324	3,912	3,625*	3,647	0,002
POLIMEXMS	0,366	2,420	-0,172	6,835	2,402	3,945	1,449
POLNORD	0,561	5,290	-1,387	28,269	2,085	2,489	-0,047
TPSA	-0,022	1,978	-0,161	3,775	0,310	1,757	1,109
TVN	0,145	2,242	-0,083	3,716	3,004*	3,250	3,218**

Note: *, **, *** indicate rejection of H_0 at the 10% ,5% and 1% significant level, respectively. B-L (1) – estimates of the Box-Ljung test statistics of order 1. B-L (2) – estimates of the Box-Ljung test statistics of order 2. LBI – estimates of the locally best invariant test statistics.

Table 5. Results of the VaR tests (95% VaR for PBG) and the loss function

Model	$\hat{\alpha}$	LR_{pof}	LR_{ind}	LR_{tbf}	RL	FL
SH						
500	10,54%	17,451***	1,370	41,329	205,55	1366,59
400	9,09%	12,929***	1,139	33,525	240,15	1722,11
300	7,62%	6,923***	0,603	34,955	279,97	2106,98
250	7,15%	5,213**	0,494	34,272	281,66	2246,19
200	5,84%	0,914	0,026	32,805	261,26	2462,33
150	5,56%	0,453	0,016	40,548	263,62	2668,63
100	4,79%	0,068	0,394	35,815	224,13	2913,60
EWMA						
500	9,12%	10,179***	1,967	32,075	186,89	1427,20
400	7,98%	7,207***	0,350	28,098	208,58	1818,34
300	6,35%	1,961	0,027	26,143	236,04	2235,86
250	5,82%	0,817	0,723	28,900	229,42	2428,40
200	5,07%	0,007	0,348	31,243	223,04	2628,37
150	4,42%	0,512	0,121	32,300	222,40	2856,85
100	4,26%	0,907	0,117	35,007	215,48	3079,17

Table 5. Continued

Model	$\hat{\alpha}$	LR_{pof}	LR_{ind}	LR_{tbf}	RL	FL
RM (λ= 0,95)						
500	7,12%	2,959*	3,850*	25,133	117,94	1629,97
400	6,43%	1,788	0,544	25,727	148,73	2036,69
300	5,99%	1,070	0,657	27,728	206,95	2394,60
250	5,49%	0,296	0,481	29,504	206,95	2546,11
200	5,07%	0,007	0,348	33,443	206,96	2686,53
150	4,85%	0,033	0,326	36,837	210,83	2856,07
100	4,66%	0,186	0,310	37,665	213,93	3052,66
RM (λ= 0,99)						
500	6,55%	1,630	3,238*	24,787	137,97	1576,26
400	5,99%	0,872	0,306	25,381	172,62	1948,88
300	6,17%	1,484	0,796	28,060	230,84	2272,02
250	6,16%	1,581	0,041	28,334	240,02	2397,50
200	6,14%	1,678	0,104	35,573	255,05	2502,31
150	6,56%	3,293	0,000	46,305	287,75	2571,93
100	7,19%	6,720***	0,253	55,251	346,13	2559,78
AR(1)-GARCH(1,1)						
500	8,26%	6,628**	5,247**	25,450	168,05	1497,28
400	7,54%	5,331**	1,411	29,023	185,14	1884,31
300	6,72%	3,095*	0,117	31,296	233,15	2267,60
250	5,66%	0,525	0,596	27,342	219,27	2460,78
200	5,38%	0,190	0,550	31,366	214,92	2629,52
150	5,14%	0,027	0,514	36,663	215,14	2868,68
100	4,26%	0,907	0,117	36,932	221,00	3138,36
RCA						
500	8,83%	8,924***	1,693	26,278	187,36	1442,89
400	7,76%	6,238**	1,630	28,136	204,92	1828,93
300	6,53%	2,498	0,065	27,313	237,79	2223,04
250	5,82%	0,817	0,723	26,199	230,09	2410,01
200	5,07%	0,007	0,348	25,277	221,17	2594,08
150	4,99%	0,000	0,415	37,995	227,54	2812,48
100	4,39%	0,604	0,171	34,631	220,41	3025,38
Sign RCA						
500	8,83%	8,924***	1,693	26,278	186,61	1438,09
400	7,54%	5,331**	5,564**	27,559	204,61	1838,01
300	6,53%	2,498	0,065	27,312	239,14	2219,78
250	5,82%	0,817	0,723	26,199	230,74	2404,58
200	5,07%	0,007	0,348	25,277	221,40	2586,80
150	4,99%	0,000	0,038	39,013	227,35	2801,30
100	4,79%	0,068	0,864	37,365	228,79	3003,25

Note: *, **, *** indicate rejection of H_0 at the 10% ,5% and 1% significant level, respectively, LR_{pof} – the values of the proportion of failures test statistics, LR_{ind} – the values of the independence test statistics, LR_{tbf} – the values of the time between failures test statistics, RL – regulatory loss function, FL – firm's loss function.

Table 6. The test for superiority of a model *vis-á-vis* another

Sample:	300						
↓ better →	Sign RCA	RCA	AR-GARCH	RM(0.99)	RM(0.95)	EWMA	HS
Sign RCA	x	-7,455*	-8,052*	-10,863*	-9,330*	-12,397*	
RCA	7,455	x	-3,962*	-10,182*	-9,159*	-3,451*	
AR-GARCH	8,052	3,962	x	-2,343*	-9,245*	0,724	
RM(0.99)	10,863	10,182	2,343	x	-8,989*	8,904	
RM(0.95)	9,330	9,159	9,245	8,989	x	8,563	
EWMA	12,397	3,451	-0,724	-8,904*	-8,563*	x	
HS							x
Sample:	250						
↓ better →	Sign RCA	RCA	AR-GARCH	RM(0.99)	RM(0.95)	EWMA	HS
Sign RCA	x	-5,262*	-4,691*	1,020	-8,199*	-6,323	
RCA	5,262	x	-4,691*	1,999	-7,954*	-5,099	
AR-GARCH	4,691	4,691	x	4,854	-7,954*	-0,612	
RM(0.99)	-1,020	-1,999	-4,854*	x	-9,994*	-6,159	
RM(0.95)	8,199	7,954	7,954	9,994	x	6,078	
EWMA	6,323	5,099	0,612	6,159	-6,078*	x	
HS							x
Sample:	200						
↓ better →	Sign RCA	RCA	AR-GARCH	RM(0.99)	RM(0.95)	EWMA	HS
Sign RCA	x	-5,369*	-5,056*	11,640	-4,194*	-8,662*	12,895
RCA	5,369	x	-3,253*	12,581	-3,880*	-5,683*	13,992
AR-GARCH	5,056	3,253	x	14,619	-1,842	-2,234*	13,522
RM(0.99)	-11,640*	-12,581*	-14,619*	x	-10,308*	-16,971*	4,586
RM(0.95)	4,194	3,880	1,842	10,308	x	1,999	11,092
EWMA	8,662	5,683	2,234	16,971	-1,999	x	14,619
HS	-12,895*	-13,992*	-13,522*	-4,586*	-11,092*	-14,619*	x
Sample:	150						
↓ better →	Sign RCA	RCA	AR-GARCH	RM(0.99)	RM(0.95)	EWMA	HS
Sign RCA	x	-0,567	-2,984*	19,905	-2,002*	-8,120*	10,462
RCA	0,567	x	-3,059*	21,113	-1,775	-7,063*	12,502
AR-GARCH	2,984	3,059	x	20,887	2,379	-2,757*	13,257
RM(0.99)	-19,905*	-21,113*	-20,887*	x	-14,315*	-24,135*	-6,761*
RM(0.95)	2,002	1,775	-2,379*	14,315	x	-0,944	10,613
EWMA	8,120	7,063	2,757	24,135	0,944	x	15,448
HS	-10,462*	-12,502*	-13,257*	6,761	-10,613*	-15,448*	x
Sample:	100						
↓ better →	Sign RCA	RCA	AR-GARCH	RM(0.99)	RM(0.95)	EWMA	HS
Sign RCA	x	-1,715	-5,218*		-2,153*	-9,159*	4,634
RCA	1,861	x	-5,729*		-1,861	-6,386*	6,313
AR-GARCH	5,218	5,729	x		4,415	2,007	9,086
RM(0.99)				x			
RM(0.95)	2,153	1,861	-4,415*		x	0,401	6,240
EWMA	9,159	6,386	-2,007*		-0,401	x	9,597
HS	-4,634*	-6,313*	-9,086*		-6,240*	-9,597*	x

Note: * indicate rejection of H_0 at the 10% and 5% significant level.

Table 7. Results of the VaR tests (97.5% VaR for PBG) and the loss functions

Model	$\hat{\alpha}$	LR_{pof}	LR_{ind}	LR_{tbf}	RL	FL
SH						
500	5,98%	12,642***	2,683	35,191**	123,33	1628,86
400	5,10%	9,659***	0,031	30,168	135,69	2074,56
300	4,54%	7,587***	0,019	27,865	171,95	2467,40
250	3,66%	2,912*	0,047	17,641	155,85	2764,23
200	3,69%	3,289*	0,015	24,976	154,39	2950,27
150	3,14%	1,086	0,130	20,965	158,94	3208,25
100	3,06%	0,911	0,117	23,765	157,49	3400,58
EWMA						
500	4,84%	6,234**	1,736	28,034**	114,91	1661,45
400	4,88%	8,226***	0,006	29,047	130,49	2111,82
300	3,81%	3,358*	0,049	25,024	151,85	2597,32
250	3,33%	1,533	0,156	20,585	145,23	2833,12
200	3,07%	0,816	0,217	21,926	144,02	3070,24
150	2,85%	0,343	0,281	23,676	144,58	3332,11
100	2,53%	0,003	0,455	25,225	138,45	3610,65
RM (λ = 0,95)						
500	3,13%	0,536	0,714	12,178	60,26	1932,02
400	3,10%	0,628	0,582	10,837	80,65	2404,45
300	3,27%	1,214	0,256	13,397	129,14	2810,04
250	3,00%	0,569	0,337	12,575	129,14	2990,57
200	2,76%	0,181	0,419	13,916	129,15	3157,89
150	2,85%	0,343	0,281	21,614	132,36	3352,30
100	2,80%	0,261	0,255	21,282	134,38	3584,68
RM (λ = 0,99)						
500	3,99%	2,710	1,167	23,185*	80,16	1836,63
400	3,77%	2,586	0,186	21,844	104,95	2268,16
300	3,63%	2,537	0,099	21,407	148,64	2644,88
250	3,33%	1,533	0,156	20,585	153,45	2799,71
200	3,23%	1,291	0,143	25,544	164,68	2925,99
150	3,71%	3,668*	0,001	31,614	189,66	3000,01
100	4,93%	14,208***	0,018	54,397**	240,05	2947,31
AR(1)-GARCH(1,1)						
500	5,41%	9,215***	2,182	26,878	103,79	1732,20
400	4,21%	4,517***	1,676	22,917	109,42	2199,89
300	3,63%	2,537	0,099	21,407	148,91	2650,81
250	3,33%	1,533	0,156	20,585	139,01	2877,29
200	3,07%	0,816	0,217	21,926	136,56	3081,41
150	3,14%	1,086	0,130	27,158	134,82	3359,94
100	3,06%	0,911	0,117	22,612	142,15	3662,46

Table 7. Continued

Model	$\hat{\alpha}$	LR_{pof}	LR_{ind}	LR_{tbf}	RL	FL
RCA						
500	4,84%	6,234**	1,736	24,499*	115,61	1674,19
400	4,66%	6,888***	2,057	29,051	127,91	2125,85
300	3,99%	4,277**	0,017	25,205	154,07	2583,65
250	3,33%	1,533	0,156	20,585	146,76	2814,50
200	3,07%	0,816	0,217	21,926	142,87	3032,33
150	2,85%	0,343	0,281	23,676	144,83	3292,28
100	2,80%	0,261	0,255	24,688	141,09	3540,01
Sign RCA						
500	5,13%	7,666***	1,953	26,390	115,69	1665,44
400	3,99%	3,494*	1,500	23,079	126,41	2142,75
300	3,81%	3,358*	0,049	21,490	154,02	2582,48
250	3,33%	1,533	0,156	20,585	147,33	2807,82
200	3,23%	1,291	0,143	24,923	143,88	3020,79
150	3,00%	0,665	1,299	27,904	145,29	3276,84
100	2,93%	0,539	0,179	27,440	145,11	3517,04

Note: *, **, *** indicate rejection of H_0 at the 10% ,5% and 1% significant level, respectively, LR_{pof} – the values of the proportion of failures test statistics, LR_{ind} – the values of the independence test statistics, LR_{tbf} – the values of the time between failures test statistics, RL – regulatory loss function, FL – firm's loss function.

Table 8. Results of the VaR tests (99% VaR for PBG) and the loss functions

Model	$\hat{\alpha}$	LR_{pof}	LR_{ind}	LR_{tbf}	RL	FL
SH						
500	2,56%	6,056**	0,475	17,577**	42,00	2163,96
400	1,77%	2,218	0,290	13,426*	56,23	2775,29
300	1,27%	0,375	0,180	4,422	77,63	3452,54
250	2,16%	6,162**	0,576	17,518	91,70	3464,14
200	1,08%	0,036	0,152	1,888	67,93	4456,03
150	2,00%	5,459**	0,571	24,181**	105,19	4229,94
100	0,93%	0,036	0,132	3,400	60,19	5611,84
EWMA						
500	3,13%	10,313***	0,714	23,320**	66,52	1933,39
400	2,66%	8,633***	0,658	25,503**	74,00	2476,85
300	2,00%	4,285**	0,449	13,755	90,20	3046,29
250	2,00%	4,676**	1,436	11,420	86,18	3320,07
200	1,54%	1,624	0,313	7,374	86,15	3617,66
150	1,14%	0,135	0,185	3,069	84,75	3936,79
100	1,07%	0,032	0,173	3,535	81,86	4268,12

Table 8. Continued

Model	$\hat{\alpha}$	LR_{pof}	LR_{ind}	LR_{tbf}	RL	FL
RM (λ = 0,95)						
500	1,99%	2,719*	0,286	9,927	26,51	2271,53
400	2,22%	5,014**	1,581	12,666	39,05	2820,60
300	2,36%	7,441***	1,053	14,764	77,73	3286,66
250	2,16%	6,162**	1,181	12,415	77,73	3500,93
200	2,00%	5,067**	1,303	12,230	77,73	3699,53
150	2,00%	5,459**	1,184	14,802	78,92	3935,00
100	1,86%	4,516**	1,288	12,587	79,58	4213,58
RM (λ = 0,99)						
500	1,71%	1,472	0,209	5,627	38,83	2171,52
400	1,55%	1,188	0,221	7,810	54,51	2679,38
300	2,00%	4,285**	0,449	14,208	89,53	3103,32
250	2,00%	4,676**	0,490	12,402	93,77	3280,42
200	2,15%	6,547**	1,074	15,053	104,02	3421,39
150	2,85%	16,200***	0,281	39,564	124,04	3485,92
100	3,06%	20,831***	0,117	48,907	157,58	3431,58
AR(1)-GARCH(1,1)						
500	2,28%	4,259**	0,374	11,647	55,22	2048,00
400	2,22%	5,014**	0,455	15,475	58,34	2590,51
300	1,81%	2,977*	0,370	11,299	89,91	3118,01
250	1,83%	3,360*	0,411	9,867	81,74	3386,59
200	1,69%	2,592	0,379	7,917	81,97	3635,11
150	1,71%	2,958	0,419	14,533	79,46	3965,85
100	1,33%	0,755	0,270	9,869	79,37	4329,54
RCA						
500	3,13%	10,313***	0,714	23,320**	67,06	1947,01
400	2,88%	10,707***	0,774	25,957**	73,93	2487,60
300	2,18%	5,778**	0,535	18,055	92,98	3031,22
250	2,00%	4,676**	0,490	13,800	88,61	3302,35
200	1,69%	2,592	0,379	11,307	86,68	3571,25
150	1,43%	1,138	0,290	9,825	86,71	3881,87
100	1,20%	0,281	0,219	6,357	82,19	4187,14
Sign RCA						
500	3,13%	10,313***	0,714	23,320**	65,98	1940,98
400	2,88%	10,707***	0,774	25,957**	75,11	2494,86
300	2,00%	4,285**	0,449	13,755	92,42	3029,72
250	2,00%	4,676**	0,490	13,800	88,94	3293,79
200	1,69%	2,592	0,379	11,307	86,81	3560,67
150	1,43%	1,138	0,290	11,433	85,97	3867,81
100	1,33%	0,755	0,270	11,177	84,15	4156,81

Note: *, **, *** indicate rejection of H_0 at the 10% ,5% and 1% significant level, respectively, LR_{pof} – the values of the proportion of failures test statistics, LR_{ind} – the values of the independence test statistics, LR_{tbf} – the values of the time between failures test statistics, RL – regulatory loss function, FL – firm's loss function.

Table 9. Results of the firm's loss function

Model	T = 375		T = 250		T = 125	
	FL	rank	FL	rank	FL	rank
			α = 5%			
Sym, Hist,	1075,956	1	1141,764	2	1263,7779	10
EWMA	1109,013	5	1192,913	9	1257,8114	8
RM (λ= 0,95)	1229,387	9	1251,9575	10	1263,0397	9
RM (λ= 0,99)	1189,228	8	1191,2823	8	1101,7305	1
AR(1)-GARCH(1,1)	1154,158	7	1169,4068	6	1207,2458	4
RCA	1101,740	3	1153,1165	4	1212,1169	6
Sign RCA	1103,836	4	1172,3837	7	1239,823	7
RCAMA	1098,295	2	1149,6521	3	1204,0552	3
Sign RCAMA	-	-	1102,4462	1	1180,7179	2
RCA GARCH	1132,959	6	1158,4801	5	1209,4749	5
Sign RCA GARCH	1296,929	10	1339,8815	11	1404,0839	11
			α = 2,5%			
Sym, Hist,	1284,699	5	1364,1283	5	1521,6798	10
EWMA	1282,948	4	1377,3036	7	1454,5161	8
RM (λ= 0,95)	1447,782	9	1473,5854	9	1481,442	9
RM (λ= 0,99)	1379,883	8	1378,9041	8	1283,0513	1
AR(1)-GARCH(1,1)	1334,391	7	1366,5462	6	1426,3841	6
RCA	1272,254	2	1341,8047	2	1406,9397	5
Sign RCA	1274,395	3	1358,9357	4	1433,1749	7
RCAMA	1270,903	1	1337,7536	1	1397,165	3
Sign RCAMA	-	-	-	-	1361,558	2
RCA GARCH	1305,188	6	1349,0442	3	1405,9143	4
Sign RCA GARCH	1549,821	10	1577,6011	10	1630,4145	11
			α = 1%			
Sym, Hist,	1754,0863	9	1754,7449	9	2234,0469	9
EWMA	1506,0564	4	1623,2342	7	1723,7757	7
RM (λ= 0,95)	1697,2481	8	1719,5178	8	1727,7362	8
RM (λ= 0,99)	1625,3806	7	1619,3369	6	-	-
AR(1)-GARCH(1,1)	1569,7932	6	1611,8139	5	1688,9632	5
RCA	1494,4265	2	1573,8864	2	1663,8987	3
Sign RCA	1496,858	3	1593,0414	4	1698,2509	6
RCAMA	1492,2845	1	1568,9729	1	1650,9211	2
Sign RCAMA	-	-	-	-	1600,3206	1
RCA GARCH	1535,3552	5	1589,0235	3	1664,72	4
Sign RCA GARCH	-	-	1849,3472	10	-	-

Note: T denotes the rolling window size, *FL*– the firm's loss function.

Table 10. Results of the the predictive quantile loss function

Model	*T* = 375		*T* = 250		*T* = 125	
	Q_α	rank	Q_α	rank	Q_α	rank
			α = 5%			
Sym. Hist	0,3183	7	0,3159	7	0,3202	7
EWMA	0,3169	5	0,3141	5	0,3181	5
RM (λ= 0,95)	0,3191	8	0,3182	9	0,3189	6
RM (λ= 0,99)	0,3163	4	0,3153	6	0,3213	9
AR(1)-GARCH(1,1)	0,3215	9	0,3167	8	0,3122	1
RCA	0,3156	1	0,3131	3	0,3173	3
Sign RCA	0,3161	3	0,3122	1	0,3169	2
RCAMA	0,3174	6	0,3130	2	0,3206	8
Sign RCAMA	-	-	0,3363	11	0,3253	10
RCA GARCH	0,3158	2	0,3137	4	0,3179	4
Sign RCA GARCH	0,3421	10	0,3283	10	0,3657	11
			α = 2,5%			
Sym. Hist	0,1945	5	0,1908	2	0,1938	6
EWMA	0,1943	4	0,1910	3	0,1914	2
RM (λ= 0,95)	0,1916	1	0,1914	5	0,1915	3
RM (λ= 0,99)	0,1923	2	0,1931	9	0,1995	9
AR(1)-GARCH(1,1)	0,1985	9	0,1913	4	0,1845	1
RCA	0,1947	6	0,1925	6	0,1956	7
Sign RCA	0,1943	3	0,1896	1	0,1927	4
RCAMA	0,1957	8	0,1931	8	0,1982	8
Sign RCAMA	-	-	-	-	0,2016	10
RCA GARCH	0,1953	7	0,1926	7	0,1928	5
Sign RCA GARCH	0,2108	10	0,2048	10	0,2358	11
			α = 1%			
Sym. Hist	0,1015	9	0,0992	9	0,0957	6
EWMA	0,0978	3	0,0947	1	0,0941	4
RM (λ= 0,95)	0,0973	2	0,0962	5	0,0958	7
RM (λ= 0,99)	0,0957	1	0,0960	3	-	-
AR(1)-GARCH(1,1)	0,1000	7	0,0961	4	0,0909	1
RCA	0,0997	6	0,0963	6	0,0950	5
Sign RCA	0,0988	5	0,0950	2	0,0934	2
RCAMA	0,1004	8	0,0967	7	0,0969	8
Sign RCAMA	-	-	-	-	0,1022	9
RCA GARCH	0,0980	4	0,0967	8	0,0938	3
Sign RCA GARCH	-	-	0,1143	10	-	-

Note: *T* denotes the rolling window size, Q_α – the predictive quantile loss function.