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Empirical Verification of World's Regions Profitability in Dynamic International Investment Strategy

A b s t r a c t. The main goal of the work is to present the empirical verification of the investment attractiveness in a given world financial region. The attractiveness of a region is represented by the share of assets from this region in the optimal portfolio. The multivariate GARCH model has been used to describe international dependencies. Optimal portfolios based on Value at Risk and Expected Shortfall minimization have been compared to the Markowitz portfolio. Indications, which should be taken into account by investors willing to invest in different world regions, have been presented as the result.

K e y w o r d s: optimal portfolio, Value at Risk, Expected Shortfall, international dependency.

J E L Classification: C52, G11, G15, G32.

Introduction

The trading digitization of the last two decades has allowed investors to invest their money into financial markets of a given region of the world with no impediments. The easy access to the knowledge about the economic situation of a given region helps to decide if it is worthy to invest in it. Moreover, the capital flow between markets from different parts of the world leads to evaluation of the attractiveness of an investing on a global scale. There are questions to be answered: how to define the attractiveness of a region and

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how to measure it. The attractiveness of a region could be understood as an amount of invested assets of this region in profitable investments. However, accepting this definition of the attractiveness of a region makes its empirical verification difficult because of numerous possibilities of asset investing in this region. Thus, we suggest building a global index characterizing financial conditions of a region and then the attractiveness of this region could be understood as a share of the corresponding global index in the optimal portfolio of all global indices. The optimal portfolio is assumed to be the least risky one in the class of portfolios with the assumed return.

The first attempt to build the optimal portfolio was undertaken by Markowitz (1959). This method is based on a historical data and it assumes the multivariate normal distribution of variables. The risk of a financial position is described by the variance. Despite of the serious simplification, the model is commonly in use. Some approaches to a construction of optimal portfolios discussed in the literature are based on other measures of risk. The optimal portfolios based on Value-at-risk (*VaR*) or Expected Shortfall (*ES*) values were discussed by Rockafellar and Uryasev (2000, 2002). The *VaR* is the most popular measure of risk and it achieved the high status of being written into industry regulations (see, for instance, Jorion (1996)). However, *VaR* is unstable and it is difficult to determine even numerically when variables have non-normal distributions. Moreover, *VaR* fails to be coherent in the sense of Artzner et al. (1999). The Expected Shortfall (*ES*) is another risk measure, with an economic interpretation similar to *VaR*, which avoids most of the *VaR*'s drawbacks. It seems that the construction of optimal portfolios based on *VaR* or *ES* values is more reasonable than Markowitz portfolio as it takes into consideration time series distributions which usually differ from normal distributions. That is why in the case of constructing optimal portfolio based on *VaR* or *ES* values it is necessary to know the distribution of time series being a multivariate financial data.

Researchers have proposed numerous models in order to describe a multivariate financial data. The empirical research suggests using a dynamic multidimensional model to describe the relationship among financial time series. The multivariate GARCH model was proposed by Bollerslev (1990), where the conditional correlation was assumed to be constant (CCC model). In the literature there are proposed models where the conditional correlation is dynamic, such as the DCC model of Engle (2002) and Tse and Tsui (2002) where the correlation matrix changes at every point of the time. Pelletier (2006) proposed the Regime Switching Dynamic Correlation (RSDC) model where the covariance was decomposed into correlations and standard deviations and both the correlations and the standard deviations were dynamic.

Capiello et al. (2006) described the model with asymmetric dynamics of dependences among considered financial time series. The multivariate GARCH model has been used in the construction of optimal portfolios by e.g. Billio et al. (2006) and Palomba (2008).

The main goal of this article is to carry out empirical verification of the attractiveness of regions understood as a share of the corresponding global index in the optimal portfolio of all global indices. The following five regions were taken into consideration: Northern America, Pacific Asia, Japan, Western and Eastern Europe. The index characterizing the financial condition of each region has been constructed as the capitalization weighted rate of regional stock market indices. To build the optimal portfolio three approaches have been implemented: the minimization of *VaR*, the minimization of *ES* and the standard Markowitz procedure, all of them under the fixed expected return assumption. In the empirical study, the short selling has been permitted. The multivariate GARCH (DCC) model with a vector autoregressive mean has been chosen to model the multidimensional time series.

As an additional result of this empirical work the comparison of the properties of the three optimal portfolios has been obtained. According to our knowledge, this is the first analysis of this type.

The paper is organized as follows: In Section 1, the model is presented (Section 1.1), the risk measure theory is briefly depicted (Section 1.2) and the portfolio optimization methods used in further analysis are described in Section 1.3. The empirical study is presented in Section 2, the data set is described in Section 2.1, the estimated parameters of the model are shown in Section 2.2 and the outcome of the optimal investment strategy is discussed in Section 2.3. The obtained results are summarized and interpreted in conclusions.

1. Methodology

1.1. Multivariate Model

Consider the N dimensional stochastic vector process $\{\mathbf{r}_t\}$ satisfying the following Vector Autoregressive formula:

$$\mathbf{r}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{r}_{t-1} + \boldsymbol{\varepsilon}_t, \quad (1)$$

where $\boldsymbol{\mu}$ and \mathbf{A} denote the constant vector and the autoregression matrix, respectively, and $\boldsymbol{\varepsilon}_t$ denotes the error term. Let Ω_{t-1} denotes the information set generated by the observed series $\{\mathbf{r}_t\}$ up to the time $(t - 1)$. We assume that the process $\boldsymbol{\varepsilon}_t$ is conditionally heteroscedastic, represented by:

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2} \mathbf{z}_t, \quad (2)$$

where \mathbf{H}_t is the dynamic covariance matrix at the time t , and \mathbf{z}_t is a sequence of N dimensional i.i.d. random vector, such that $E(\mathbf{z}_t) = \mathbf{0}$ and $E(\mathbf{z}_t \mathbf{z}_t^T) = \mathbf{I}_N$. Therefore $E(\boldsymbol{\varepsilon}_t | \Omega_{t-1}) = \mathbf{0}$ and $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^T | \Omega_{t-1}) = \mathbf{H}_t$.

Furthermore, let us assume that $\mathbf{z}_t \sim G(0, \mathbf{I}_N)$, where G is a continuous density function.

There are various parametric formulations to specify the covariance matrix \mathbf{H}_t introduced. In this paper, the Dynamic Conditional Correlation (DCC) specification, introduced by Engle (2002) and Tse, Tsui (2002), is considered. Hence, the covariance matrix \mathbf{H}_t is decomposed as follows:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \quad (3)$$

where \mathbf{R}_t is the time varying conditional correlation matrix of the vector $\boldsymbol{\varepsilon}_t$ and $\mathbf{D}_t = \{\text{diag}(\mathbf{H}_t)\}^{1/2}$ is the diagonal matrix whose i -th diagonal entry is given by the conditional standard deviation $\sqrt{h_{ii,t}}$ of $\varepsilon_{i,t}$. Conditional variances $h_{ii,t}$ can be estimated separately and written in the following vector form based on GARCH(1,1) model:

$$h_{ii,t} = \omega + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1}, \quad i = 1, 2, \dots, N. \quad (4)$$

In order to model the joint distribution, the most popular of DCC models has been used, due to Engle (2002), where the correlation matrix \mathbf{R}_t is presented as follows:

$$\mathbf{R}_t = \text{diag}(\mathbf{Q}_t)^{-1/2} \mathbf{Q}_t \text{diag}(\mathbf{Q}_t)^{-1/2}, \quad (5)$$

where the proxy process \mathbf{Q}_t is defined by:

$$\mathbf{Q}_t = (1 - \alpha - \beta) \bar{\mathbf{Q}} + \alpha \mathbf{z}_{t-1} \mathbf{z}_{t-1}^T + \beta \mathbf{Q}_{t-1}, \quad \beta \geq 0, \alpha + \beta < 1, \quad (6)$$

where $\mathbf{z}_t = \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t$, and $\bar{\mathbf{Q}}$ denotes the unconditional matrix of the standardized errors \mathbf{z}_t .

In this paper, the density function G is assumed to be the multidimensional Student's-t distribution. The model parameters are estimated by Maximum Likelihood Method according to the procedure described by Ghalanos (2013) and implemented in the R environment. The estimation of the realized correlation is conducted by the recursive procedure.

1.2. Risk Measures

In this paper two measures of risk are discussed: Value at Risk (*Var*) and Expected Shortfall (*ES*). We follow the notation presented by Föllmer

and Schied (2011). For the financial position X and $\lambda \in (0,1)$, Value at Risk at level λ is defined as:

$$VaR_\lambda(X) := \inf\{m | P[X + m < 0] \leq \lambda\}. \quad (7)$$

Value at Risk is the smallest amount of capital which, if added to X and invested in the risk-free asset, keeps the probability of a negative outcome below some fixed level. Generally, Value at Risk is not a convex risk measure excluding the case when the set of all attainable positions consists of normally distributed financial positions. The absence of the convexity in the case of non-normal distributions is a substantial objection. It appears that, so called Expected Shortfall (ES), is a convex risk measure. This measure has similar interpretation to VaR .

For the financial position X and $\lambda \in (0,1)$, ES at level λ is defined as:

$$ES_\lambda(X) := E[VaR_\alpha(X) | \alpha \leq \lambda]. \quad (8)$$

For a portfolio $\mathbf{p} = (p_1, \dots, p_N)$ of financial N dimensional vector r_t , representations of VaR and ES at the time t are defined as follows:

$$VaR_\lambda(\mathbf{p}) := \inf\{r | P[\mathbf{p} \cdot r_{t+1} + r < 0 | \Omega_t] \leq \lambda\}, \quad (9)$$

and

$$ES_\lambda(\mathbf{p}) := E[VaR_\alpha(\mathbf{p}) | \alpha \leq \lambda], \quad (10)$$

where Ω_t denotes the information set up to the time t .

Since the conditional distribution G in the VAR(1)-DCC-GARCH(1,1) model described in the Section 1.1 is assumed to be t-Student, VaR and ES are estimated using Monte Carlo method.

1.3. Portfolio Optimization

The portfolio optimization problem is the procedure providing us with the portfolio with the minimal risk in the class of portfolios with a given expected return. In this article, we construct three different optimal portfolios: the portfolio with the minimal VaR , the portfolio with the minimal ES , both under the VAR(1)-DCC-GARCH(1,1) model assumption, and classical Markowitz portfolio.

Let the N -dimensional stochastic vector process $\{r_t\}$ be represented by the model presented in the Section 2.1. The portfolio \mathbf{p}_t^{VaR} is obtained as the result of the following optimization problem:

$$\begin{aligned} \sum_{i=1}^N p_{i,t}^{VaR} &= 1, \\ E(\sum_{i=1}^N p_{i,t}^{VaR} \cdot r_{i,(t+1)|t}) &= r, \end{aligned} \quad (11)$$

$$VaR_{\lambda}(\sum_{i=1}^N p_{i,t}^{VaR} \cdot r_{i,(t+1)|t}) \rightarrow \min!,$$

where $p_{i,t}^{VaR}$ are coordinates of the vector \mathbf{p}_t^{VaR} , and $r_{i,(t+1)|t}$ denotes the conditional distribution of the i -th asset return, for $i = 1, \dots, N$. The \mathbf{p}_t^{VaR} vector is the N -dimensional time series, which represents the investment with the expected return r at the time $t + 1$ and the minimal VaR .

Analogously, the portfolio \mathbf{p}_t^{ES} is built as the result of the following optimization problem:

$$\begin{aligned} \sum_{i=1}^N p_{i,t}^{ES} &= 1, \\ E(\sum_{i=1}^N p_{i,t}^{ES} \cdot r_{i,(t+1)|t}) &= r, \\ ES_{\lambda}(\sum_{i=1}^N p_{i,t}^{ES} \cdot r_{i,(t+1)|t}) &\rightarrow \min!, \end{aligned} \quad (12)$$

where $p_{i,t}^{ES}$ are coordinates of the vector \mathbf{p}_t^{ES} . The \mathbf{p}_t^{ES} vector is the N -dimensional time series, which represents the investment at the time t with the expected return r at the time $t + 1$, and the minimal ES .

The third constructed optimal portfolio – \mathbf{p}_t^M is obtained using the standard Markowitz's mean-variance model:

$$\begin{aligned} \sum_{i=1}^5 p_{i,t}^M &= 1, \\ E(\sum_{i=1}^5 p_{i,t}^M \cdot \hat{r}_{i,(t+1)|t}) &= r, \\ \text{var}(\sum_{i=1}^5 p_{i,t}^M \cdot \hat{r}_{i,(t+1)|t}) &\rightarrow \min!, \end{aligned} \quad (13)$$

where $p_{i,t}^M$ are coordinates of the vector \mathbf{p}_t^M . The \mathbf{p}_t^M vector is the N -dimensional time series, which represents the investment at the time t with the expected return r at the time $t + 1$, and the minimal variance. It is assumed that the vector $\hat{\mathbf{r}}_{(t+1)|t}$ with coordinates $\hat{r}_{i,(t+1)|t}$ is normally distributed with the mean vector and the covariance matrix obtained from the preceding observations.

2. Empirical Study

2.1. Data

The investigation covers five global indices constructed using market indices from all over the world. A random sample, diversified enough to capture all specific properties in terms of both geographical and economical dimensions are selected to study. These markets are combined into five regions: (i) North America, (ii) Japan (iii) Pacific Asia, (iv) Western Europe and (v) Eastern Europe. The global index of Northern American includes the

USA (DJIA, NASDAQ) and Canadian (TSE300) indices. The global Asian index includes indices of India (BSE), Hong Kong (HSI), Indonesia (JCI), Malaysia (KLCI), South Korea (KOSPI), China (SHBS) and Singapore (STI). The global index of Western Europe includes indices of Germany (DAX), the Netherlands (AEX), Austria (ATX), France (CAC40), the UK (FTSE), Switzerland (SMI) and Spain (IBEX). The index of Eastern Europe includes indices of Poland (WIG), Czech Republic (PX), Hungary (BUX) and Turkey (XU100). The Japanese market is represented by the NIKKEI index. The data set used in the study has been taken from the stooq database (<http://www.stooq.com>).

All considered indices have been denominated in the US dollar. Daily returns come from the period from October 2002 to April 2012. To deal with the missing data in the sample, the linear approximation has been performed. Daily returns are computed as the difference between the logarithm of the closing price on the day t and the logarithm of the closing price on the day $t - 1$. The global indices' returns are constructed as the mean of component indices' returns weighted by the corresponding market capitalization. Therefore, the return i_t of a global index at the time t is given by:

$$i_t = \sum_{j=1}^d w_{j,t} i_{j,t}, \quad (14)$$

where $i_{j,t}$ is the return of the j -th component index at the time t , and the weight factor $w_{j,t}$, for $j = 1, \dots, d$, is defined by:

$$w_{j,t} = \frac{c_{j,t}}{\sum_{k=1}^d c_{k,t}}, \quad (15)$$

where $c_{j,t}$ is the capitalization of the market corresponding to the j -th index at the time t . In this article we have used the annual data of capitalization. Therefore, every weight factor is constant in the period of a particular year.

Table 1 presents summary statistic for the five discussed percent (multiplied by 100) logarithmic returns of the global indices.

Table 1. Descriptive statistics of the data (October 2002 to April 2012)

	Average	Median	Std dev.	Skewness	Kurtosis	p-value of Engle test	p-value of Ljung-Box test
Northern American	0.0629	0.1774	1.8628	-0.5225	7.8354	0.0000	0.0000
Western Europe	0.0163	0.0463	1.6036	-0.0715	9.9381	0.0000	0.8772
Eastern Europe	0.0279	0.0776	1.2675	-0.1971	11.2966	0.0000	0.0012
Asian	0.0565	0.1082	1.2586	-0.3713	7.4732	0.0000	0.0256
Japan	0.0207	0.0842	1.5479	-0.7544	10.0213	0.0000	0.0000

Note: presented values are calculated for corresponding global indices defined in (14)

Averages of returns are close to zero. An asymmetry is suggested by the higher value of the median than the average, for all global indices. This is confirmed by the negative skewness for all series under study. However, this skewness seems to be relatively small so it is not included in the model. The kurtosis is high, taking values from 7.47 to 11.30, which suggests the fat-tailedness of analyzed time series. In order to examine the properties of the time series, especially, autocorrelation and heteroscedasticity, the Ljung-Box and Engle tests were performed. The test results indicated existence of autocorrelation for all indices, except for Western Europe, and the GARCH effect for all considered returns. The presented properties of the 5-dimensional time series justify the use of the multivariate DCC-GARCH model with the vector autoregressive mean described in Section 1.1 by formulas (1)–(6).

2.2. Estimation Results

For model estimating purposes, the time zone difference between studied regions has been taken into account. The non-synchronous trading can cause a bias in the estimation. Several transformations of the indices returns were considered to omit this problem. Lagging American index return or accelerating the Asian seems adequate. However, there is the period during a day, when American and European stock exchanges are working at the same time. Therefore, in the analysis we have used only the accelerated Asian index return.

In the preliminary step, the structure of the multivariate model has been investigated. The five dimensional VAR(1)-DCC-GARCH(1,1) model with the Student's-*t* conditional distribution and dynamic conditional correlations (DCC) has been considered to describe the returns' process. In order to determine whether the chosen lags in the model are proper, Ljung-Box and Engle tests on the obtained residual were conducted. The results of the tests showed clearly that there is no autocorrelation, nor GARCH effect in the residual series.

Table 2. Vector autoregressive parameters

	Eastern Europe	Western Europe	Northern America	Asia	Japan
Eastern Europe	-0.0621**	0.0193	0.3062***	0.5204***	0.1173***
Western Europe	-0.0031	-0.2267***	0.2442***	0.4951***	0.0857***
Northern America	-0.0165	0.0294	-0.1637***	0.3344***	-0.0304
Asia	0.0188	-0.0287	0.1103***	0.0679**	-0.1026***
Japan	0.0059	-0.0225	0.1102**	0.2430***	-0.2338***

Note: the significance levels are used: *** for 0.001, ** for 0.01 and * for 0.05.

Table 2 presents the estimation result of the VAR(1) part of the considered model. Table 2 presents elements of the matrix \mathbf{A} from equation (1) calculated for the entire sample.

Estimates presented in Table 2 show that the current returns of the index data are strongly affected by lagged returns of themselves. The Eastern European region is significantly dependent on the others, apart from the Western Europe region. On the other hand, the Western European region is also dependent on the others and it is not dependent only on the preceding state of markets of Eastern Europe. Northern American markets are affected only by the Asian region. Pacific Asian and Japanese regions are dependent on Northern America and on each other. It can be noticed that Asian and Northern American markets affect all the other regions strongly and positively. The Japanese market impacts European markets also positively but weaker than American or Asian markets. The only negative impact is observed in the case of Japan market influencing the Asian one. It indicates that a growth on Japanese markets results in decreases on Asian markets. Note that, described relations are irrespective to correlations among the series itself. In particular, both European regions are strongly correlated, however, ones current value does not depend on the preceding value of the other.

Furthermore, considered regions (all except the Pacific Asian) tend to correct daily returns, because of significant negative values on the diagonal of the autoregressive matrix.

Table 3. The DCC model parameters

	Eastern Europe	Western Europe	Northern America	Asia	Japan	joint
ω_i	0.0796**	0.0101	0.0122	0.0098	0.0577*	α 0.0096*
α_i	0.0981**	0.0923***	0.0795**	0.0664**	0.0989***	β 0.9853***
β_i	0.8710***	0.9063***	0.9103***	0.9280***	0.8748***	df 11.786

Note: the significance levels are used: *** for 0.001, ** for 0.01 and * for 0.05.

Table 3 summarize results of the DCC model estimation. The α_i and the β_i parameters are similarly framed in each series, about 90% of the standard deviation value comes from the previous value and about 10% comes from the realization of the previous error term. The conditional correlation matrix is more persistent.

Figure 1 presents the estimated correlations between considered global indices. There are presented the correlation between Eastern Europe to other discussed regions (1a), the correlation between Western Europe and the others (1b), the correlations between Northern America and the others (1c), the correlation between Asian index and the others (1d) and the correlations of Japan (1e).



Figure 1a. The estimated realized correlations between Eastern Europe and the others



Figure 1b. The estimated realized correlations between Western Europe and the others

Figure 1 shows that the global index of the Eastern Europe region is the most correlated with the global index of Western Europe region, similar to Western Europe, it is the least correlated with the global index of Pacific Asian region. On the other hand, the Western Europe region is the most correlated with Northern American, except for the beginning of the financial crisis (October 2007 to July 2008), when it was the most correlated with Eastern Europe. The Northern American region is the most correlated with

Western Europe markets, partially with the Japanese market and it is the least correlated with Eastern Europe (up to the middle of the year 2005) and the Pacific Asian region (after the year 2006). The noted region (Pacific Asian) is the most correlated with Japan. The intensity of the relationships between region global indices are time varying. For example, until September 2006, Eastern European was correlated with Northern America, Pacific



Figure 1c. The estimated realized correlations between Northern America and the others



Figure 1d. The estimated realized correlations between Pacific Asia and the others



Figure 1e. The estimated realized correlations between Japan and the others

Asia and Japan at similar levels, after that it was correlated more strongly with Japan and Northern America. The American region appeared to be correlated at the same level with Japan and Western Europe until March 2009, after that it was correlated more strongly to Western Europe. Until the turn of the years 2009/2010 America was the least correlated to Eastern Europe, from the beginning of 2010 to March 2011 it was the least correlated to Pacific Asia, after that it was correlated to Pacific Asia and Eastern Europe at similar levels.

2.3. Properties of Optimal Portfolios

Dynamic dependences which derive from the analysis presented in the paragraph 2.2 should be taken into consideration in the process of building optimal portfolios. That is why two dynamic portfolios are constructed: the portfolio of the minimal *VaR* and portfolio of the minimal *ES*. To show properties of these procedures, results of the Markowitz portfolio optimization is presented.

For this purpose, at every time, starting from January 2009, parameters of VAR(1)-DCC-GARCH(1,1) model, described in Section 2.1 have been estimated using preceding 7 years (1601 observations). The estimation has been conducted using the data of the same length at every time point and reestimated every 50 observations. It provides us with the 27-month series (863 moments) in the *ex post* analysis. For each t , the distribution of the random vector of returns $\mathbf{r}_{(t+1)|t}$, with coordinates $r_{i,(t+1)|t}$, for $i = 1, \dots, 5$, is approximated using Monte Carlo method. We simulate 100,000 realiza-

tions at each time point. Technically, the analysis has been performed as if we have not known the future and been doing it for 27 months.

The estimates in Tables 2 and 3 have been presented in order to visualize additional properties of the series. These results differ from the estimates obtained during the procedures. In the analysis, we reestimate the model repeatedly, therefore we do not omit parameters which occurred to be insignificant in Tables 2 and 3. It would be difficult to control significance of parameters at every step and would not change results of the analysis substantially.

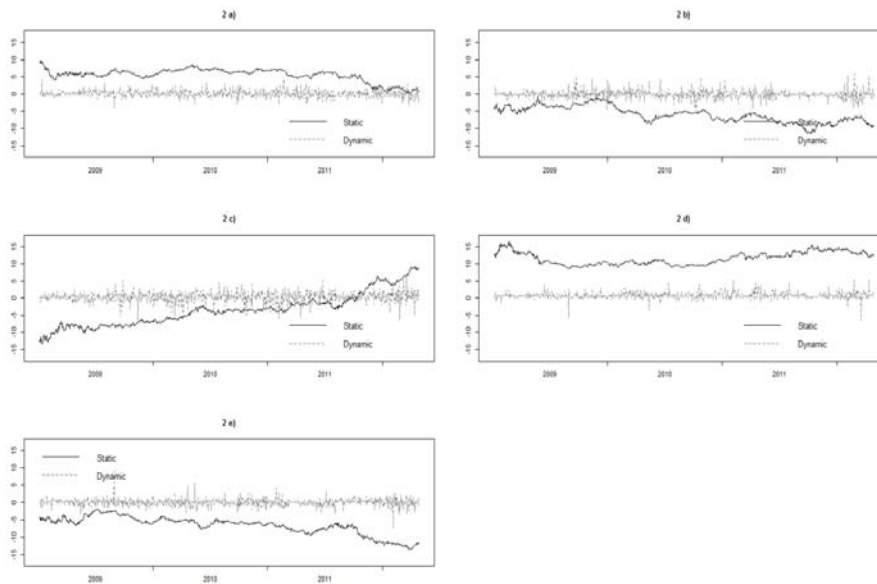


Figure 2. The portfolio shares obtained by the *ES* (dynamic) and the Markowitz (static) models, of: 2a) Eastern Europe 2b) Western Europe 2c) Northern America index 2d) Pacific 2e) Japan

The results are discussed for $\lambda = 0.1$, the similar results are obtained by taking $\lambda = 0.05$, which are standard levels for *VaR*. In the regulations such as the Basel regulations, *VaR* levels are 0.01 or even 0.005. The presented procedures have no relation to these regulations. According to these regulations, an investor is obliged to keep the *VaR* at the level 0.01 (or 0.005) below some value. It does not mean, that one do not fulfill this condition using presented method. Moreover, conducting the same procedure for $\lambda = 0.01$ does not assure investor that he fulfills the regulations. For lower levels, such as $\lambda = 0.01$, the analysis is not stable. We fixed the expected interest

rate on the level 1%, which is much higher than the average of any analyzed variable (see Table 1).

The realized interest rates of the constructed portfolios are computed and analyzed. The realizations in the $(t + 1)$ time point of portfolios constructed in the time t are considered. The compositions of both dynamic portfolios (*VaR* and *ES*) are similar, therefore, corresponding portfolios have similar interest rate. Thus, we present only returns of the portfolio obtained by minimizing *ES* under the dynamic VAR(1)-DCC-GARCH(1,1) model assumption (dynamic portfolio) and portfolio under the Markowitz's mean-variance model (static portfolio) in Figure 1.

Figure 2 presents shares of the static and the dynamic portfolios. Firstly, we have noticed the differences in the scale of the portfolio shares. Large absolute values of shares in the Markowitz portfolios are caused by the height of the assumed expected return. If the expected return had been assumed to be lower, the absolute values of shares in the Markowitz portfolio would have decreased, however, it would not have changed the preferences essentially. Despite the assumption about height of the expected return, the shares of the *ES* and *VaR* optimal portfolios remain relatively small. Additionally, we have noticed differences of biases of estimation of the expected interest rate. It turns out that the average of the realized returns of the Markowitz portfolios is equal to 0.605%, which is far from the assumed 1%, whereas the return average of the *ES* and the *VaR* optimal portfolios are very close to 1% (0.98% and 1.14%, respectively). Secondly, vivid differences in shares of the static and dynamic portfolios can be observed. Comparing the two portfolios we notice that Markowitz portfolio gives clear verification of the attractiveness of regions while shares obtained by minimizing the *VaR* or the *ES* portfolios under the dynamic model assumption are highly variable and it is hard to notice any tendencies. In the case of Markowitz model, it can easily be noticed that the Pacific Asian region is the most attractive for investors during the period under study. At the beginning of the period the Eastern Europe region was also attractive but in the last year its attractiveness decreased radically. The regions of Western Europe and Japan are unattractive for investors during the whole period, while the American region after an initial phase of selling tendency proved to be quite attractive for investors.

Figure 3 illustrates that returns of the static portfolio are much more volatile than returns of the dynamic portfolio. Without a doubt, Markowitz procedure provides us with the much more risky portfolio than the two other proposed.

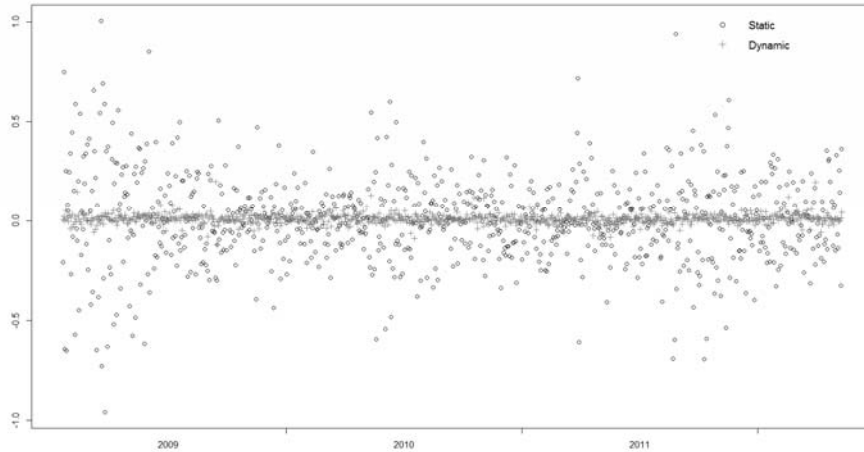


Figure 3. The realized optimal portfolio returns of the *ES* (dynamic) and the Markowitz (static) portfolios

In order to illustrate investment regions preferred by the dynamic portfolio, the Figure 4 shows the monthly moving average shares for the *ES* strategy.

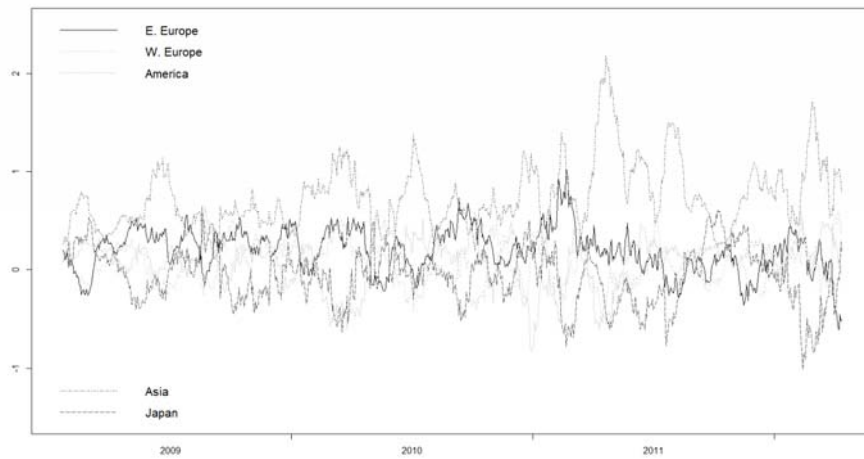


Figure 4. The moving average for the dynamic strategy

The optimal investment strategy strongly suggests buying Pacific Asian assets most of the time. The Eastern Europe region is also regarded to be attractive for investors but it is less preferred than the Pacific Asian region.

Optimal shares of Northern American, Western Europe and Japan assets are mostly negative. The meaning of these findings is not apparent. The attractiveness read from the Figure 4 should be interpreted correctly. It does not mean that optimal portfolios had been containing a positive value of Pacific Asian assets during the analyzed period, it means that it had been containing a positive value of the assets in average. However, the change of the composition of the optimal portfolio is intense, an investor has to rebuild the portfolio completely every day in order to keep it optimal.

Conclusions

We have constructed, for the region attractiveness study purposes, global indices which represent five regions: Northern America, Pacific Asia, Japan, Eastern Europe and Western Europe. The study covers the period of January 2009 to April 2012. The period of October 2002 to January 2009 has been used only for the model estimation. The five-dimensional time series' mean has been modeled by the vector autoregressive term and the variance's dynamics have been described by the generalized autoregressive conditional heteroscedastic model with dynamic conditional correlation. The estimated correlations between considered regions have been computed due to the VAR(1)-DCC-GARCH(1,1) model, they illustrate relations among the investigated regions. The analysis confirms the claim that dependences between financial markets are higher in a period of crisis than during a prosperity time. Dynamic dependences were included in the construction of the optimal portfolio. That is why two dynamic optimal portfolios have been built: the portfolio of the minimal *VaR* and the portfolio of the minimal *ES*. In order to show the properties of these portfolios, the result of the implementation of the Markowitz model, still very commonly used in practice, has been presented. The share of the index corresponding with a given region in the optimal portfolio have determined the region's daily attractiveness.

The Markowitz portfolio composition gives clear results of the attractiveness of a region, however, properties of this portfolio have been inferior to properties of the dynamic portfolio. The Markowitz procedure estimates the mean of a portfolio incorrectly. Moreover, the risk of the portfolio obtained using this method is much higher than the risk of the portfolio obtained under the dynamic model assumption. The optimal portfolio obtained using the dynamic model have attained the assumed return with relatively small risk.

The diversification of the optimal dynamic portfolio is highly volatile and it has hardly any connection to a global financial situation. The cost of

such an investment strategy is a daily rebuilding of the portfolio. It requires almost complete decomposition of the portfolio every day. Therefore it is not possible to summarize the profitability of the regions in general. The daily attractiveness is dynamical and it should be taken into account by international investors. As a illustration of the result, monthly moving averages of the daily attractiveness (measured by the share of assets from the region in the optimal portfolio) have shown Pacific Asian region as the most attractive during the period under study. Eastern European markets also have appeared to be profitable. Western European, American and especially Japanese markets have appeared to be unattractive.

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Badanie zyskowności wybranych regionów świata w międzynarodowej dynamicznej strategii inwestycyjnej

Zarys treści. Artykuł przedstawia empiryczną weryfikację przydatności wybranych regionów świata do inwestowania na ich rynkach finansowych. Do badania wybrano pięć regionów świata: region Ameryki Północnej, region Europy Zachodniej, region Europy Wschodniej, region Azji i Pacyfiku oraz Japonię. Jako reprezentant nastrojów finansowych danego regionu zdefiniowano globalne wskaźniki będące ważoną kapitalizacją stopą zwrotu głównych indeksów rynków wchodzących w skład rozważanych regionów. Do modelowania zależności pomiędzy tak utworzonymi globalnymi wskaźnikami zastosowano wielowymiarowy model VAR(1)-DCC-GARCH(1,1) z warunkowym wielowymiarowym rozkładem t-Studenta. Atrakcyjność regionu jest definiowana jako udział globalnego wskaźnika związanego z danym regionem w optymalnym portfelu. Rozważono trzy typy konstrukcji optymalnych portfeli. Własności optymalnych portfeli opartych na minimalizacji Value at Risk oraz na minimalizacji Expected Shortfall zostały porównane do własności standardowego portfela Markowitza.

Słowa kluczowe: portfel optymalny, wielowymiarowe modele dynamiczne, miary ryzyka.

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