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Determination of the Time of Contagion in Capital Markets Based on the Switching Model

A b s t r a c t. This article attempts to compare conclusions made about market contagion based on the periods indicated by using the Markov-switching model and based on a range for unconditional correlations as well as on arbitrary arrangements. DCC-model was used to control for correlation change over time. Determination of extremely high correlations by using a range for unconditional correlations and the *MS(3)* switching model yields similar results regarding conclusions about the occurrence of the process of contagion in a market. Conclusions about contagion are, however, made at a higher significance level in the case of the switching model.

K e y w o r d s: switching model, DCC-GARCH model, contagion.

J E L Classification: G01, G15, C24.

Introduction

Current economical and financial crises in general have international – character. Propagation mechanisms across countries and markets are called the transmissions for fundamental linkages. In literature contagion term is applied only to the financial markets, however it should not be identified only with the financial linkages – it can also concern the markets which are not significantly financially connected. Many authors claim that increase in financial integration intensifies contagion effects. On the subject of interde-

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pendence between markets, contagion effects and transmission channels treat, among others, the works of: Eichengreen et al. (1995, 1996), Goldstein (1998), Masson (1998), Kaminsky and Reinhart (2000, 2002), Forbes and Rigobon (2002), Pericoli and Sbracia (2003), Pesaran and Pick (2004), Dungey et al. (2005).

The most restrictive definition provided by the World Bank assumes that market contagion occurs when the correlation between markets in a time of crisis is significantly higher than during the period of tranquillity[1]. It is possible to control for correlation change over time by using, for example, a dynamic conditional correlation model. Researchers often also adopt an additional definition of contagion that would "suit the purpose of a given research method". If volatility models are used, then contagion is identified with the spread of uncertainty across financial markets.

The assessment of the significance of a contagion process requires dividing a sample into observations from the time of tranquillity and from the time of crisis in financial markets. A period of tranquillity is a benchmark period for determining connections between markets, which is a point of reference for changes observed during a crisis. The transition from a tranquillity period to the period of crisis is usually established based on events which may change the behaviour of certain indicators. The results of research studies depend on the division which has been made and the time of crisis often covers both high and low correlations between researched markets. Establishing a potential time of market contagion by using the Markov-switching model makes it possible to make an assumption about the differences in a stochastic process that determines correlations in particular regimes. The main hypothesis refers to the possibility of using the one-dimensional Markov switching model to determine the time of contagion in capital markets. Results were compared with conclusions made about market contagion based on a range for unconditional correlations as well as on arbitrary arrangements. The consequences of adopting particular divisions are, in fact, important information for researchers.

Research results presented in this paper concern the *assessment of the significance of contagion in certain capital markets in the years 2007–2009.* Selected stock exchange indices represent the situation in securities markets[2]. In empirical studies that are described later, the concept of market

[1] *Contagion of Financial Crises*, World Bank, http://www.worldbank.org/economic-policy/managing%20volatility/contagion/definitions.htm (14 May 2012). This definition is cited based on Forbes and Rigobon's paper (2002).

[2] Capital market crisis is identified with sharp decline in stock prices, maintaining for an extended period of time. Role of stock market indexes is broadly described by Jajuga (2006).

contagion means a contagion spreading from an index representing the U.S. market to an index representing the studied market[3].

Section 1 presents the DCC-GARCH model and section 2 describes the Markov-switching model which has been used in the research. Section 3 contains information on the tested stock exchange indices as well as the criteria for an arbitrary division of the set of observations into those relating to the time of crisis and those relating to the period of tranquillity in securities markets. The obtained research results are presented in section 4.

1. A dynamic Conditional Correlation Model

Let us assume that an n-dimensional vector of rates of return $\mathbf{s}_t$ ($t = 1,\ldots,T$) can be decomposed into the following form:

$$\mathbf{s}_t = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t, \tag{1}$$

$$\boldsymbol{\varepsilon}_t = \mathbf{H}^{1/2}\boldsymbol{\xi}_t, \tag{2}$$

where $\boldsymbol{\mu}_t$ is the vector of conditional expected values of vector $\mathbf{s}_t$ based on model VAR(p). In empirical research it is usually assumed that p =1[4]. The Student's t-distribution was used because of an increased kurtosis for process $\boldsymbol{\xi}_t$.

The dynamic conditional correlation (DCC) model can be formulated as (Engle, 2002):

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t, \tag{3}$$

$$\mathbf{D}_t = diag(\sqrt{h_{11,t}},\ldots,\sqrt{h_{NN,t}}), \tag{4}$$

[3] Causality tests are sometimes used for establishing the direction of contagion (Cheung, Ng, 1996; Coporale, Pittis and Spagnolo, 2002). Their usefulness, however, is limited. This is because these tests are based on Granger's concept related to analysing correlations between studied processes and the consequences of events. It is often a researcher who decides whether to test a particular causal relationship exists based on his or her knowledge and experience (Osińska, 2006; Fiszeder, 2009).

[4] A vector autoregression model also controls for the mutual interdependence between markets through connections between the delayed values of endogenous variables. Empirical studies described in literature have found that linear relationship between stock returns are low significant. Some researchers suggest that it is better to resign from expected value model than include incorrectly specified model, especially in the case of total model for expected values and variances (Doman, Doman, 2009).

$$h_{i,t} = c_i + \sum_{j=1}^{q} (\alpha_{ij}\varepsilon_{i,t-j}^2 + \gamma_{ij} I(\varepsilon_{i,t-j} < 0)\varepsilon_{i,t-j}^2) + \sum_{j=1}^{p} \beta_{ij} h_{i,t-j}, \qquad i = 1,...N, \tag{5}$$

$$\mathbf{R}_t = (diag(\mathbf{Q_t}))^{-1/2} \mathbf{Q}_t (diag(\mathbf{Q}_t))^{-1/2}, \tag{6}$$

$$\mathbf{Q}_t = \left(1 - \sum_{k=1}^{K} \alpha_k - \sum_{l=1}^{L} \beta_l \right) \overline{\mathbf{Q}}_t + \sum_{k=1}^{K} \alpha_k \boldsymbol{\xi}_{t-k} \boldsymbol{\xi}_{t-k}^{'} + \sum_{l=1}^{L} \beta_l \mathbf{Q}_{t-l}, \tag{7}$$

Matrix $\mathbf{R}_t$ is a positively defined symmetric matrix with ones along the main diagonal; vector $\boldsymbol{\xi}_t = \mathbf{D}_t^{-1}\boldsymbol{\varepsilon}_t$ in this case denotes the vector of standardised residuals from model VAR(1). Matrix $\mathbf{D}_t$ was estimated based on the one-dimensional GJR-GARCH(1,1) model (Glosen, Jagannathan, Runkle, 1993). In equation (5) $I(\cdot)$ is an indicator function and it assumes the value of 1 for $\varepsilon_{i,t-j} < 0$ and the value of 0 for $\varepsilon_{i,t-j} \geq 0$ ($i = 1,..., N$). Positive values of parameter γ_{ij} which are significantly different from zero prove that the leverage effect occurs[5].

Covariance stationarity and thus a finite variance in equation (5) is ensured by satisfying these conditions:

$$c_i > 0, \alpha_{ij}, \beta_{ij} \geq 0 \text{ and } \sum_{j=1}^{q} (\alpha_{ij} + \gamma_{ij} / 2) + \sum_{j=1}^{p} \beta_{ij} < 1. \tag{8}$$

In equation (6) $\overline{\mathbf{Q}}_t$ denotes a square matrix of unconditional covariances of the vector $\boldsymbol{\xi}_t$ variables. In addition, it is required that $\alpha_k, \beta_l \geq 0$, $\sum_{k=1}^{K} \alpha_k + \sum_{l=1}^{L} \beta_l < 1$.

The model's parameters are estimated in two stages. The logarithmic likelihood function is the sum of likelihood functions for a volatility model and likelihood functions for the parameters of dynamic correlations (Engle, 2002).

[5] The leverage effect results from an asymmetric response of rates of return to positive and negative information reaching the financial market. It is a consequence of a negative correlation between securities prices and the volatility of rates of return. The higher the value of parametr γ_{ij}, the stronger the leverage effect (the additional impact of negative information).

2. The Markov-Switching Model

In Markov-switching models it is assumed that a switch between the behaviours of rates of return in regimes (periods), and thus the process of contagion in a market, depend on certain hidden factors which are not directly observable. One can only observe the external symptoms of regime change by observing, for example, the mutual correlations between rates of return. Theoretically, for a time series of dynamic conditional correlations ρ_t, a one-dimensional switching model can be used, in which switches occur as a result of changes in the expected value μ, variance σ^2 or the expected value and variance of the studied correlations (Hamilton, 1989; Davidson, 2013).

If a switching model is only constructed for the purpose of classifying the already obtained theoretical values of correlations, it can be assumed that, in each regime, values are generated by independent processes with a different constant expected value and constant variance:

$$\rho_{it} = \mu(r_t = i) + \varepsilon_{it} \qquad \varepsilon_{it} \sim N(0, \sigma^2(r_t = i), i = 0,1,2, \tag{9}$$

where $\mu(r_t = i)$, $\sigma^2(r_t = i)$ denote the expected value and variance of conditional correlations, respectively, in the *i*-regime. Such an approach allows one to use a one-dimensional model, in which it is assumed that three regimes will be analysed, i.e. $r_t = i$ ($i = 0, 1, 2$), which are related to an extremely low, average and extremely high correlation between rates of return. The proposed sequential procedure entails estimating the model of dynamic conditional correlations and the switching model separately, which makes it possible to avoid many problems related to estimating multidimensional models[6].

The series of random variables r_t in the subsequent moments in time t (t=1,..., T) has the Markov property, i.e. its value at the time moment t+1, i.e. r_{t+1}, depends only on the regime at the t moment, rather than on all the preceding regimes, which is formally formulated as:

[6] In practice, the use of multidimensional *switching models* is associated with many problems because of the number of estimated parameters which grows exponentially, as in the multidimensional *VechGARCH* model (Billio, Lo Duca, Pelizzon, 2005). If one assumes that only two states (of high and low volatility) can occur in each of two studied markets, than one already allows for the occurrence of four regimes, and the matrix of conditional probabilities has dimensions [4 x 4]. The final number of parameters depends on the assumptions regarding the differences between processes determining the behaviour of rates of return in particular regimes.

$$P(r_{t+1} = j \mid r_t = i, r_{t-1} = k, ...) = P(r_{t+1} = j \mid r_t = i) = p_{ij}, \quad i, j = 0,1,2. \tag{10}$$

Probabilities p_{ij} denote the probability of transition of the dependence between rates of return from regime i to regime j.
If at the $t - 1$ moment the process was under the $r_{t-1} = i$ regime, then the conditional density function of the explained variable ρ_t can be represented as $f(s_t \mid r_t = i, I_{t-1})$, where I_{t-1} denotes the history of the process until the $t - 1$ moment. Any suppositions on the i regime may be made by means of a *conditional probability*:

$$P(r_t = i \mid I_t) = \frac{f(s_t \mid r_t = i, I_{t-1}) \cdot P(r_t = i \mid I_{t-1})}{\sum_{j=0}^{2} f(s_t \mid r_t = j, I_{t-1}) \cdot P(r_t = j \mid I_{t-1})}, \tag{11}$$

where

$$P(r_t = i \mid I_{t-1}) = \sum_{j=0}^{2} p_{ij} P(r_{t-1} = j \mid I_{t-1}). \tag{12}$$

The model's parameters are estimated by using the maximum likelihood method (Davidson, 2013)[7].

3. The Statistical Material and an Arbitrary Division of the Sample

In the empirical research, daily continuously compounded rates of return on six indices representing the situation on stock exchanges during the period from August 17, 2005 to July 31, 2009 were used (1022 observations for each stock exchange):

$$s_{it} = 100 \cdot (\ln(P_{i,t}) - \ln(P_{i,t-1})).$$

Two indices from strong EU economies – DAX and CAC, representing the situation on the German and French stock exchanges, as well as two indices from weaker economies from the "old" European Union – the Spanish IBEX and the Greek ATEX, and two indices from the countries of Central and Eastern Europe – the Hungarian BUX and the Polish WIG20 were selected for the purpose of the analysis (source: the Stooq database). The Dow Jones

[7] The relevant likelihood function is presented as part of the description of the TSM (Time Series Modelling) program. It is not easy to estimate the model's parameters. Numerical problems result from the occurrence of local extrema of the logarithmic likelihood function. This is why normally two, up to three, regimes under which a process may be are distinguished.

Industrial Average (DJIA) index represented the situation on the U.S. stock exchanges. Gaps in the data were filled by using the linear interpolation method. Due to the different quotation times, data were smoothed by using a two-period moving average (Dungey et al., 2007).

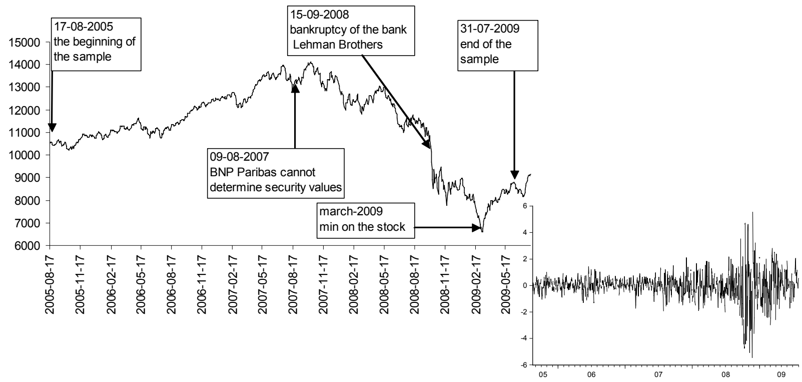

Figure 1. The Dow Jones Industrial Average index during the period from August 17, 2005 to July 31, 2009 (lower figure – daily rates of returns from DJIA index)

The behaviour of the Dow Jones Industrial index during the studied period is shown in Figure 1. An arbitrary division of the observation set into two subsets related to the period of crisis (high volatility of rates of return) and the period of tranquillity (low volatility) in securities markets should be made in such a way that the time of tranquillity immediately precedes the time of crisis. This is because it constitutes a benchmark for comparisons. Quotations preceding, for example, the collapse of Lehman Brothers, hardly represented a time of tranquillity as the Dow Jones Industrial Average had already been declining for some time and the rate of return on the index had been characterised by increased volatility. Therefore, it was decided that the information about the difficulties related to evaluating assets which was announced by the French BNP Paribas would be used when dividing the set of observations. On 9 August 2007 this bank suspended payments from three funds investing in the market of bonds secured by subprime mortgages. The period of crisis was extended beyond the time when securities were trading at the lowest level because of the increased volatility of rates of return which persisted until the end of July 2009. There were 511 observations for the time of crisis determined in this way. In order to ensure comparability of results, 511 former observations were analysed for events contributing to financial turmoil. It was a time of rising asset prices, with minor adjustments, which is why it has been assumed to be a period of tranquillity on the stock exchange.

4. Research Results

The estimation of a model of dynamic conditional correlations should be preceded by a test justifying their use. The results of such tests depend on the adopted specification of volatility models. Therefore, two tests were used in the research, i.e. the Tse test (2000) as well as Engle and Sheppard test (2001) in two versions with delays p=5 and p=10. For all indices at least one test indicated the reasonableness of constructing a dynamic conditional correlation model[8].

A significant increase of correlations in the time of crisis confirms the occurrence of the process of market contagion. Forbes and Rigobon (2002) propose using Fisher's transformation of correlation coefficients while testing the significance of change in correlation between rates of return.
After Fisher's transformation, the sample correlation coefficient can be treated as the realization of a random variable with a normal distribution with the expected value of $E(\rho)=\frac{1}{2}\ln\frac{1+\hat{\rho}}{1-\hat{\rho}}$, where $\hat{\rho}$ is the estimated correlation coefficient. The variance of this variable is $Var(\rho)=\frac{1}{T-3}$. The null hypothesis $H_0:\rho^K \le \rho^S$ is tested against an alternative hypothesis, i.e. $H_1:\rho^K > \rho^S$ (index S means the tranquility period, K - the time of potential market contagion).
The empirical statistic in the test for two expected values is as follows:

$$FR=\frac{0,5\ln\left(\frac{1+\hat{\rho}^K}{1-\hat{\rho}^K}\right)-0,5\ln\left(\frac{1+\hat{\rho}^S}{1-\hat{\rho}^S}\right)}{\sqrt{\frac{1}{T_K-3}+\frac{1}{T_S-3}}}. \tag{13}$$

The FR-statistic have a normal distribution N(0,1) and even if the sample is small it allows one to use the critical values of a standard normal distribution.

[8] Calculations were carried out by using the OxMetrics 6.10 program.

The parameters of the GJR-GARCH(1,1) model are presented in the upper part of Table 1. Let us remember that the models were estimated on the basis of residuals from model VAR(1). Therefore, a slightly different model for the DJIA index was connected with each index. In all the volatility models for the DJIA index, the alfa parameter which describes the impact of positive residual impulses was insignificant. In the models for the European indices, the alfa parameter was only significant for the Greek (ATH), the Hungarian (BUX) and the Polish (WIG20) indices. The parameters that were significant were beta and gamma which describe the impact of past variance as well as the leverage effect (an additional impact of negative information reaching the market). The model's assumptions require that the alfa parameter be significantly greater than zero. Thus, in order to standardise the residuals from model VAR(1), it was finally the GARCH(1,1) models that were used[9]:

$$h_{ij,t} = c_i + \sum_{j=1}^{q} \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{ij} h_{ii,t-j}, \quad i = 1,...N, \tag{14}$$

where $\sum_{j=1}^{q} \alpha_{ij} + \sum_{j=1}^{p} \beta_{ij} < 1$, $c_i > 0, \alpha_{ij} \geq 0, \beta_{ij} \geq 0.$ This time the obtained parameter estimates meet the models' assumptions; the arch effect is significant in all of the models (the lower part of Table 1). This means that the impact of negative information on many markets was considerably stronger than the impact of positive information. Such markets could be identified by estimating the GJR-GARCH model at the first stage of the research.

Estimates of parameter β exceed the value of 0.8, which confirms the volatility clustering phenomenon. Both in the GARCH(1,1) model and in the conditional correlation model DCC-GARCH(1,1) the requirement of covariance stationarity is satisfied. Also the conditions for variance (non-negative, significant model parameters) are met. The sum of parameters $(\alpha + \beta)$ in the GARCH model is close to one, which means the occurrence of persistence (long-term dependencies) and suggest test of integrated model (IGARCH, FIGARCH) in further studies. The highest unconditional correla-

[9] The same volatility model was adopted for two time series due to program limitations. In the research GARCH (1,2), GARCH (2,1) and GARCH (2,2) models were also tested – only in GARCH (1,1) model all parameters were significant and was the lowest information criterion value (AIC). Conditional correlations from two models (DCC-GJR-GARCH (1,1) and DCC-GARCH) were slightly different.

tion with the DJI index was recorded for the CAC and the DAX indices and the lowest for the BUX and the WIG20 indices.

Table 1. The DCC-GARCH model's parameters

GJR–GARCH(1,1)								
Index	for index i				for index DJIA			
	const(i)	alfa(i)	beta(i)	gamma(i)	const	alfa	beta	gamma
ATH	0.017***	0.069**	0.842***	0.146***	0.006***	–0.003	0.894***	0.211***
CAC	0.010***	0.022	0.900***	0.123***	0.006***	0.002	0.891***	0.201***
DAX	0.013***	0.032	0.868***	0.162***	0.006***	–0.005	0.892***	0.214***
IBEX	0.015**	0.043	0.853***	0.164***	0.007***	0.006	0.880***	0.212***
BUX	0.037**	0.113***	0.798***	0.135***	0.006***	0.005	0.881***	0.209***
WIG20	0.027**	0.059**	0.889***	0.064**	0.006***	–0.003	0.889***	0.221***

GARCH(1,1)						
Index	for index i			for index DJIA		
	const(i)	alfa(i)	beta(i)	const	alfa	beta
ATH	0.016**	0.165***	0.823***	0.005**	0.111***	0.882***
CAC	0.009**	0.107***	0.883***	0.005**	0.116***	0.877***
DAX	0.011**	0.129***	0.861***	0.005**	0.112***	0.881***
IBEX	0.015**	0.159***	0.824***	0.006**	0.118***	0.875***
BUX	0.034**	0.172***	0.806***	0.004**	0.116***	0.880***
WIG20	0.026*	0.098***	0.883***	0.005**	0.114***	0.879***

DCC–GARCH(1,1)					
Index	Alfa	beta	df	unconditional correlations	Log–likelihood
ATH	0.022***	0.958***	16.960***	0.384***	–2060.58
CAC	0.016*	0.962***	12.636***	0.641***	–1820.90
DAX	0.027**	0.951***	10.960***	0.649***	–1801.13
IBEX	0.016**	0.971***	11.360***	0.590***	–1845.05
BUX	0.075	0.829***	16.020***	0.319***	–2292.27
WIG20	0.018**	0.967***	16.971***	0.378***	–2346.02

Note: a parameter's significance for $\alpha = 0.01$ is marked with three asterisks, for $\alpha = 0.05$ with two asterisks and for $\alpha = 0.1$ with one asterisk.

The parameters of the MS(3) switching model are provided in Table 2. Regime 2 is related to an extremely high correlation (the shaded area in Figure 2).

As for the two indices – CAC and DAX, regime 2 covered both significantly high and significantly low correlation between markets. This probably resulted from a very high volatility of extreme correlations. A correct classification was obtained by simplifying the process to a model in which only the expected value would change.

For the remaining four indices, the variance of extremely high and extremely low correlations was significantly higher than the variance of conditional correlations in the time of tranquillity, and the model made it possible to make a correct classification[10].

Table 2. The estimates of the MS(3) switching model's parameters

Switches occur as a result of changes →		the expected value and variance				the expected value	
	Regime	ATH	IBEX	BUX	WIG20	CAC	DAX
Number of observations	0	283	423	178	312	223	222
	1	379	387	405	336	614	461
	2	360	212	439	374	185	339
Expected value	0	0.264	0.511	0.103	0.255	0.567	0,520
	1	0.372	0.598	0.271	0.372	0.638	0,632
	2	0.482	0.672	0.440	0.468	0.691	0,710
FR-statistic		1,820**	1.449*	2.811***	1.545*	1.132	1.989**
Variance	0	0.060	0.031	0.085	0.047	x	x
	1	0.027	0.017	0.052	0.029	x	x
	2	0.049	0.029	0.078	0.038	x	x
H-statistic		601.6***	589.2***	854.1***	596.2***	492.4***	650.4***
D-statistic	0	15.89***	12.90***	16.69***	13.77***	10.83***	11.61***
	1	24.53***	23.10***	29.23***	23.78***	19.97***	22.72***
Probability of transition	p_{0/0}	0.977	0.983	0.880	0.987	0.978	0,961
	p_{0/1}	0.023	0.017	0.120	0.013	0.000	0,039
	p_{0/2}	0.000	0.000	0.000	0.000	0.022	0,000
	p_{1/0}	0.017	0.018	0.052	0.012	0.008	0,018
	p_{1/1}	0.962	0.970	0.880	0.964	0.984	0,965
	p_{1/2}	0.021	0.012	0.068	0.024	0.008	0,017
	p_{2/0}	0.000	0.000	0.061	0.019	0.000	0,000
	p_{2/1}	0.022	0.021	0.000	0.000	0.046	0,021
	p_{2/2}	0.978	0.979	0.939	0.981	0.954	0,979
Log-likelihood		1708,9	2257.9	1063.5	1824.5	2307.3	1797.5
AIC		–3,325	–4.399	–2.062	–3.551	–4.499	–3.502

Note: the *FR*-statistic refers to the difference in correlation between regimes 2 and 1; the *D*-statistic refers to the difference distributions between regime 2 and regime 0 or 1.

The Jarque-Berra test rejects at conventional significance level the normality of correlation in three regimes (not reported). It is the reason of the use of nonparametric variance analysis (Kruskal and Wallis-test) to evaluate the quality of classification (division of the sample into observations from the time of tranquillity and from the time of potential market contagion). In the first research stage, *H*-statistic indicates the diversification of distribution at least in two regimes. In the second stage, *D*-statistics indicates the

[10] High variance in 0 regime (extremely low correlations) indicates, that differentiation of the sign of returns on two markets causes the increase of uncertainty among investors.

diversification of distribution in all regimes for all studied indices. Relevant statistics are provided in Table 2. Results confirms the legitimacy of the use of one-dimensional switching model. Simple model can give satisfactory results.

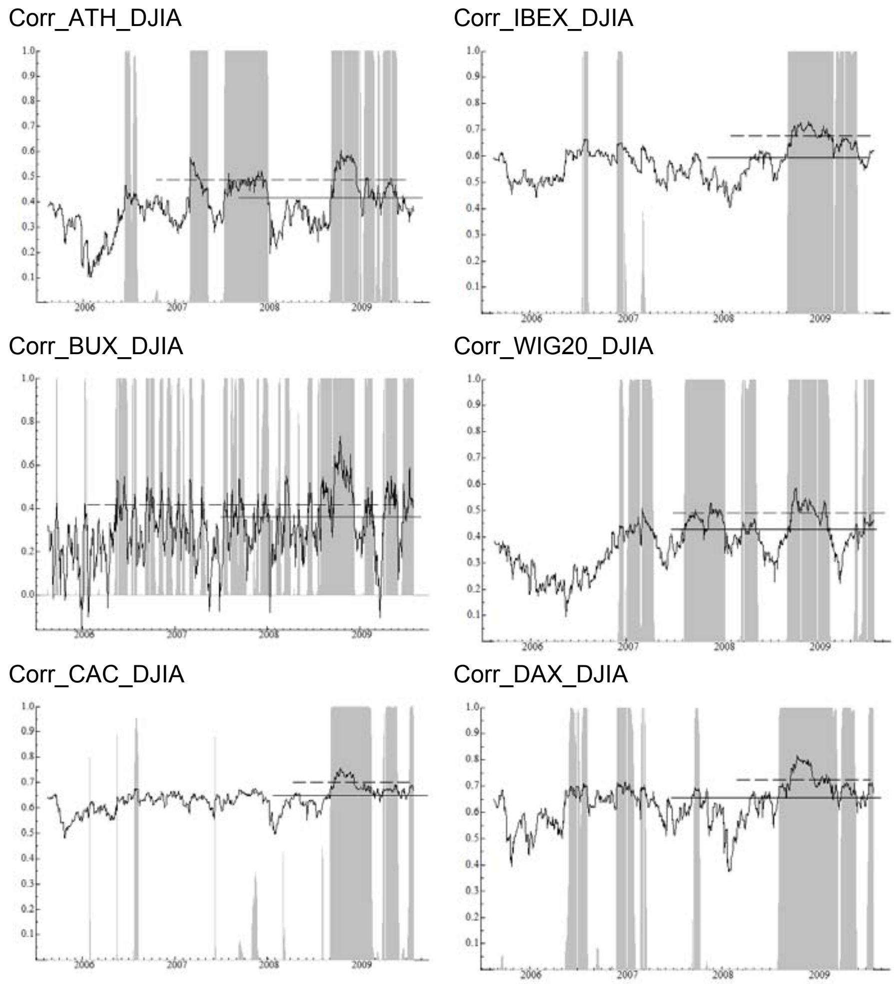

Figure 2. Potential periods of market contagion as determined based on conditional correlations from the DCC-GARCH(1.1) model

Note: the shaded area: the time of extremely high correlations (regime 2); —— the average value of conditional correlations in the time of crisis (August 9, 2007 to July 31, 2009); - - - the upper limit of the range: unconditional correlation + 2 error.

The occurrence of extreme correlations under regime 2 only means the time of potential market contagion. Only the rejection of the null hypothesis in the test for two expected values means that the process of market contagion has occurred. The expected value of correlation in regime 2 is significantly higher than the expected value of correlation in regime 1 (the time of tranquillity in the market) in all securities markets except for the French market (CAC). The significance level that allows one to reject the null hypothesis is, however, varied, which is highlighted in the table. The *FR*-statistics assumes the highest value for the Hungarian market. It can be assumed that the value of statistics reflects the effects of contagion. The higher the value of statistics, the more severe the effects of market contagion.

The probability of remaining under each of the regimes, which is provided in Table 2, is high, which means that the highlighted regimes are persistent. The analysis of charts in Figure 2 allows one to compare the frequency of an index being under regime 2 and the time of remaining there. The additional, horizontal and dashed line makes it possible to relate indicated periods to the time of potential market contagion as identified based on a range for unconditional correlations. In this case, those correlations which fall outside the upper and the lower limits determined by a double estimation error are assumed to be extremely high and extremely low correlations, respectively. The time of tranquillity in a market is represented by correlations from a range determined in this way. As for arbitrary arrangements, it should be remembered that the sample was only divided into two subsets.

Total duration times of the potential market contagion period are provided in Table 3. The longest time is for an arbitrary division and the shortest for the range for unconditional correlations. The switching model indicated that the period of potential contagion in the Hungarian market was the longest.

Table 3. The number of observations during the potential period of market contagion

Metod	ATH	IBEX	BUX	WIG20	CAC	DAX
Arbitrary arrangements	511	511	511	511	511	511
Switching model – regime 2	360	212	439	374	185	339
Range for unconditional correlations	161	89	243	101	57	81

A comparison of the results of tests of the significance of occurrence of contagion spreading from the U.S. market to a given market is presented in Table 4.

Table 4. A comparison of the results of testing the significance of contagion in a market

Method	Index	ATH	IBEX	BUX	WIG20	CAC	DAX
Arbitrary arrangements	Contagion	0.416	0.594	0.360	0.427	0.650	0.654
	Tranquility	0.349	0.562	0.267	0.315	0.614	0.613
	FR-statistic	1.309*	0.770	1.641*	2.072**	0.966	1.096
Switching model	Contagion	0.482	0.672	0.440	0.468	0.691	0.710
	Tranquility	0.372	0.598	0.271	0.372	0.638	0.632
	FR-statistic	1.820**	1.449*	2.811***	1.545*	1.132	1.989**
Range for unconditional correlations	Contagion	0.535	0.702	0.492	0.574	0.731	0.771
	Tranquility	0.384	0.584	0.321	0.405	0.639	0.653
	FR-statistic	2.019**	1.788**	2.647***	1.572*	1.237	2.030**

Note: "*contagion*" means the time of potential market contagion.

For five indices the values of the *FR*-statistics recorded in the case of the switching model are lower than the corresponding statistics in the analysis of unconditional correlations, which has an effect on conclusions about contagion. The occurrence of the contagion process is registered more often (for lower significance levels) in the analysis of a range for unconditional correlation. The opposite is true only for the BUX index, which probably results from small differences between the duration times of market contagion that are determined by using different methods. The lowest values of the *FR*-statistics are usually recorded for an arbitrary division. Detailed results and significance levels are provided in Table 4.

Conclusions

It is relatively difficult to date a crisis in financial markets. During periods determined based on events that change the behaviour of rates of return, both high and low correlation between markets can be observed.

This paper proposes indicating the periods of potential market contagion on the basis of a one-dimensional switching model. Tests made confirm the legitimacy of the use of simple switching model to determine potential market contagion periods. Further studies should be conducted – it is important to compare obtained results with the results from multidimensional model, where switches are determined based on the changes of expected value, variance, and covariance. In the paper such comparisons were not made because of the lack of appropriate software. Occurrence of persistence suggest, that further studies should also include inference based on the integrated model.

Results confirm the conclusions made by the author on the subject of contagion on the basis of logit model for panel data (Burzała, 2012). Significant contagion effects were observed on German market, less significant –

on Polish, and the lack of significant contagion effects were observed on French market.

Determination of extremely high correlations by using a range for unconditional correlations and the MS(3) switching model yields similar results regarding conclusions about the occurrence of the process of contagion in a market. Conclusions about contagion are, however, made at a higher significance level in the case of the switching model. It is worth emphasising that it is necessary that the appropriate tests be conducted which would confirm the significance of the increase of correlation between markets. Also, the time of potential market contagion determined on the basis of a regime with an extremely high correlation (the switching model) is longer. In further studies more attention should be paid to the issue of determining the direction of contagion as well as extremely low correlations which may be a harbinger of a crisis.

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Wyznaczanie czasu zarażania rynków kapitałowych na podstawie modelu przełącznikowego

Z a r y s t r e ś c i. W artykule podjęto próbę porównania wnioskowania o zarażaniu rynków na podstawie okresów wskazanych przez model przełącznikowy Markowa z wnioskowaniem opartym na przedziale dla korelacji bezwarunkowych i ustaleniach arbitralnych. W celu kontrolowania zmieniających się w czasie korelacji wykorzystano model DCC. Ustalenie ekstremalnie wysokich korelacji przy wykorzystaniu przedziału dla korelacji bezwarunkowych lub modelu przełącznikowego *MS(3)* prowadzi do podobnych rezultatów w zakresie wnioskowania o wystąpieniu procesu zarażania rynku. Wnioski o zarażaniu są jednak stawiane przy wyższym poziomie istotności w przypadku modelu przełącznikowego.

S ł o w a k l u c z o w e: model przełącznikowy, model DCC, zarażanie.