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RESEARCH ARTICLE

# Prediction the Groundwater Depth using Kriging Method and Bayesian Kalman Filter Approach in Erbil Governorate

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## ABSTRACT

The aim of this research is using the kriging method as one of geostatistics interpolation methods on the measured value of the specific part and Bayesian Kalman filter to identifying the depth of Groundwater in Erbil. Geostatistics is a tool which is developed for statistical analysis of any continuous data that can be measured at any location in the space. The Kalman filter is the Bayesian optimum solution to the problem of estimating the unknown state of a dynamic system from noisy data and is more efficient than computing the estimate directly from the entire past observed data. The main goal of this work is to predict anew value at the unmeasured location by kriging method and Bayesian Kalman filter and compare these two methods. The dataset is the observed values of the (295) wells that had been taken from a known specific place which called Shaqlawa – in Erbil Governorate. The comparison was done by calculating mean absolute error (MAE) and root mean square error (RMSE) for the value of the depth of groundwater in the eara of the study. The values of (MAE and RMSE) of each models are compared and the smaller values of them are the better interpolation as it shown in analyzing to evaluate the precision of the prediction.

**Keywords:** Bayesian estimation, covariance function, Gaussian random field, groundwater-surface interpolation, Kalman filter, kriging interpolation method

## INTRODUCTION

The Kriging is a spatial interpolation Geostatistical method used for the first time in meteorology, geology, environmental sciences, agriculture, and others fields. This method is used to find the best estimator under the assumption of the second order stationarity. Geostatistics is a set of tools and models that are developed for statistical analysis of any continuous data that can be measured at any location in the space. Verify three data feature in statistical continuous data analysis: Dependency, stationery, and distribution. With these features, you can proceed to the modeling of the geostatistical data analysis like simple kriging. In addition, the goal of this work is to predict a new value at the unmeasured location by Gaussian Semivariogram function (model) and compare the results of this model based on the simple kriging method and understanding their spatial variability with another approach called Bayesian Kalman Filter.

Kalman filter is an optimal linear estimator which provides the estimation of signals in noise. Kalman used the state transition models for the dynamic system in the estimation process.

## GROUNDWATER-SURFACE INTERPOLATION

In general, spatial statistics and geographic information system (GIS) rely on each other in many ways. Arc GIS is software which can be used to create covariates for inclusion in all statistical models and to bring out the results from statistical models. The work in this study might be very important to evaluate the results from different models in simple kriging interpolation approaches. This kind of comparison presents a relevant meaning for the variability of a physical model which used as a reference to validate the interpolation results.

Groundwater depth of the wells of any sample point data for generates surface need to be evaluated and preprocessed before interpolation. The locations and values of sample point data will impact the interpolation result. First, all the collected data should come from the same type of wells in the same aquifer. The well information should be carefully evaluated to make sure the data reflect the dynamics of groundwater in the target aquifer, not other aquifers. Second, the spatial distribution of sample point data should be carefully considered. The clustered data and sparse data in one area

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will cause different interpolation results. After collecting suitable data for interpolation, raster calculator in GIS is used to calculate the elevation of groundwater table in each well by dividing the surface elevation (digital elevation model) with the groundwater depth.[1]

## GAUSSIAN RANDOM FIELD

Used Gaussian Random Field $Z(s)$ to identify the spatial correlation structure, any field of spatial is a set of random variables which parameterized by some set ($D\subset R^d$). The simplest stochastic process form is as follows:

$$\{Z(s): s \in D \subset R^d\} \quad (1)$$

Where:
$Z(s)$: Field of random spatial.
$s$: Coordinates spatial random variable.
$D$: Domain of spatial random variable.
$R^d$: d-dimensional Euclidean space.

Any finite collection$\{Z(s_1), Z(s_2).,Z(s_k).,\}$ is multivariate normal:

$$\begin{bmatrix} Z(s_1) \\ \cdot \\ \cdot \\ Z(s_k) \end{bmatrix} \sim N\left(\begin{bmatrix} \mu(s_1) \\ \cdot \\ \cdot \\ \mu(s_k) \end{bmatrix}, \left[Cov(Z(s_i), Z(s_j))\right]\right)$$

If the following assumptions hold, then a spatial random field is called second-order stationary: [2]

$$E(Z(s)) = \mu \quad \forall s \in D \quad (2)$$

$$Cov\left(Z(s_1), Z(s_2)\right) = E\left[\left(Z(s_1) - \mu_{s1}\right)\left(Z(s_2) - \mu_{s2}\right)\right] = C(s_1 - s_2) \quad s_1, s_2 \in D \quad (3)$$

Where:
$E$: Expected value. $Cov$: Covariance function of two locations.
$h=s_2-s_1$: Vector distance between $Z(s_1)$ and $Z(s_2)$.

However, we know that the covariance function can be expressed as follows:

$$\begin{aligned} Cov\left(Z(s_1), Z(s_2)\right) &= Cov\left[\left(Z(s_1), Z(s_1+h)\right)\right] = C(h) \\ &= E\left[\left(Z(s_1) - \mu_{s1}\right)\left(Z(s_1+h) - \mu_{s1+h}\right)\right] \\ &= E\left[Z(s_1)Z(s_1+h)\right] - \mu^2 \end{aligned} \quad (4)$$

When the expected value is equal to zero.

$$\mu = E(Z(s)) = 0, \quad \forall s \in D \quad (5)$$

Then,

$$\begin{aligned} Cov\left(Z(s_1), Z(s_2)\right) &= Cov\left[\left(Z(s_1), Z(s_1+h)\right)\right] \\ &= C(h) = E\left[Z(s_1)Z(s_1+h)\right] \end{aligned} \quad (6)$$

When the mean of a second-order stationary of spatial random field is equal to zero over the $D$ and the covariance function of the locations does not depend on $s_1$ and $s_2$ but the vector $h$.

And for $Cov(h=0) = Cov\left[\left(Z(s), Z(s+h)\right)\right] = V\left[Z(s)\right]$

If variance or covariance function does not exist, the intrinsic hypothesis and the spatial random field are called (intrinsic stationery) if the following assumption holds as:

$$\begin{aligned} & E(Z(s)) = \mu \text{ or } E\left[Z(s_1) - Z(s_2)\right] = 0 \;\; \forall s \in D \\ & V\left[Z(s_1) - Z(s_2)\right] = 2\gamma(s_1 - s_2) \;\; \forall s_1, s_2 \in D \end{aligned} \quad (7)$$

Where,
$V$: Variance, γ: Semivariogram, 2γ: Variogram

$$\begin{aligned} 2\gamma &= V\left[Z(s_1) - Z(s_2)\right] = V\left[Z(s_1) - Z(s_1+h)\right] \\ &= E\left(\left[Z(s_1+h) - Z(s_1)\right]^2\right) - \left(E\left[Z(s_1+h) - Z(s_1)\right]\right)^2 \\ &= E\left(\left[Z(s_1+h) - Z(s_1)\right]^2\right) \end{aligned} \quad (8)$$

In addition, if the covariance function $C(s_1-s_2)=C(h)$ or semivariogram $\gamma(s_1-s_2)=\gamma(h)$ depends only on separation distance between $s_1$ and $s_2$, $h=||s_1-s_2||$, then the spatial random field is called isotropic.[3]

### The Kriging

Kriging is a spatial interpolation geostatistical method used for the 1st time in meteorology, geology, environmental sciences, agriculture, and other fields. This method is used to find the best estimator under the assumption of the second-order stationarity. A geological process may not be stationary in reality. In case, where the process is non-stationary, we could use non-linear functions.[4] Here, the classifying Geostatistical techniques are as follows [Table 1].

### Assumptions

In some of the interpolation methods especially Geostatistical methods, they have their own assumptions as follow:[5]

1. Stationarity
2. Intrinsic hypothesis
3. Isotropy and anisotropy
4. Unbiased.

### Semivariogram Function and Covariance Function

The semivariogram function is a structure of intrinsically stationary spatial random field which describes a broader class of Erath phenomena. In the case of the second-order stationary spatial random processes, there is an equivalence between covariance function and semivariogram function as follows:

**Table 1:** Classification of geostatistical techniques

| Model | Stationary | Nonstationary |
|---|---|---|
| Linear | Ordinary/simple kriging | Universal kriging; kriging using IRF-K |
| Nonlinear | Disjunctive kriging simulation | Simulation of IRF-K |

IRF-k: Intrinsic random functions of order k

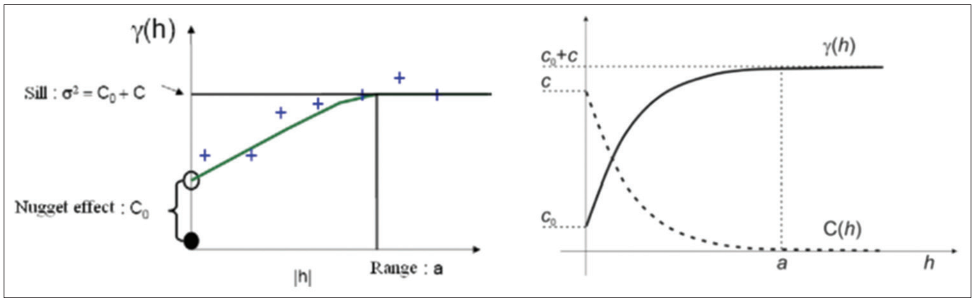


**Figure 1:** The relationship between the $\gamma(h)$ and $C(h)$ for the second-order stationary of a spatial random variable

$$\begin{aligned} V[Z(s+h)-Z(s)] &= V[Z(s+h)]+V[Z(s)] \\ &-2Cov[Z(s+h),Z(s)] \\ &= 2V[Z(s)]-2Cov[Z(s+h),Z(s)] \\ &= 2[C(0)-C(h)] = 2\gamma(h) \div 2 \\ &\gamma(h) = C(0)-C(h) \end{aligned} \quad (10)$$

The semivariogram function is a measure of dissimilarity between pairs of locations or observed value $Z(s+h)$ and $Z(s)$. There are three parameters of semivariogram function for the spatial second-order stationary processes: Nugget effect ($c_0$), range ($a$), and partial sill ($c$) which are shown in Figure 1. The sum ($c_0+c$) is called the sill (Marcin and Marek, 2010), (Sluiter, 2009).

## The Gaussian Semivariogram Model

The Gaussian model is commonly used to represent events with a small scale spatial structure,[6] the equation for this model is similar to the normal cumulative distribution function, and it is given by [Figure 2]:

$$\gamma_z(h) = C_0 + C\left[1-\exp\left(\frac{-h^2}{a_0^2}\right)\right], h>0 \quad (11)$$

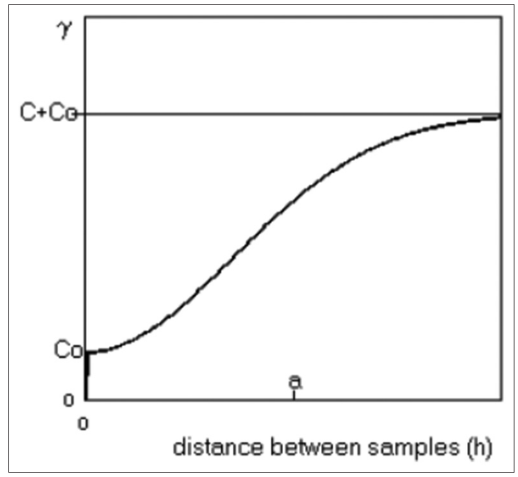


**Figure 2:** The Gaussian semivariogram model with parameters

## The Simple Kriging Interpolation Method

Simple kriging can deliver the value at any unmeasured location ($s_0$) using a linear estimator $\lambda_i$ for the measured values at locations ($s_1,s_2,.,s_n$):

$$Z(s_0) = \sum_{i=1}^{n} \lambda_i Z(s_i), \sum_{i=1}^{n} \lambda_i = 1 \quad (12)$$

As the sum of all linear estimators equals one, the unbiased prediction is ensured. Using the intrinsic hypothesis, the estimation variance is calculated by the formula:

$$\begin{aligned} \sigma_e^2(s_0) &= Var(Z(s_0) = E\left((\hat{Z}(s_0)-Z(s_0))^2\right) \\ &= 2\sum_{i=1}^{n} \lambda_i \gamma(s_i - s_0) - \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda_j \gamma(s_i - s_j) \end{aligned} \quad (13)$$

The semivariance ($s_j-s_0$) and($s_i-s_0$) are taken from the variogram model. The goal is to minimize $\sigma^2(s)$ under the unbiased conditions and to find the corresponding weight.

The weights are chosen to fulfill: $E(\hat{Z}(s_0)-Z(s_0))=0$ according to the assumption that $E[Z(s_1)-Z(s_2)]=0$ which called intrinsic stationery and to solve this the Lagrange multiplier ($\mu$) is introduced, not easy solving this model by any other ways:

$$Z(s_o) = \sum_{i=1}^{n} \lambda_i \gamma(s_i - s_j) + \mu(s_0) = \gamma(s_j - s_0) \quad (14)$$

The solutions to this linear equation system are the values of the linear estimators. This system is called kriging system.[7] The kriging variance is determined by:

$$\sigma_k^2 = \sum_{i=1}^{n} \lambda_i \gamma(s_i - s_0) + \mu(s_0) \quad (15)$$

# KALMAN FILTERING

The Kalman filter gives unbiased, linear, and minimum variance recursive estimate to the state of a dynamic system from noisy data taken in separate real-time.[10] KF used in the formulation of a dynamic model for linear dynamical systems provides a recursive solution to the problem of the optimal linear filter. The recursive solution in every state update estimate is calculated from previous estimates and data entry; only the previous estimate requires storage. KF has been widely used in the fields of signal processing and modern control and airborne surveillance systems, radar signal processing and control, and adaptive controls.

Here, we will provide only the equations needed to develop discrete recursion KF, discrete state equations are given as linear dynamic for signal $\theta_t$ and observation $Y_t$.[11] We have two equations:

1. Observation equation: $Y_t = F_t\theta_t + \upsilon_t$

2. System equation: $\theta_t = G_t\theta_{t-1} + \omega_t$

The initial state vector has a mean $\theta_0$ and a covariance matrix $P_0$ that is:

$E[\theta_0] = \hat{\theta}_0$ and $Var[\theta_0] = P_0$

$Y_t$ is the vector of noisy observation at time $t$, of dimension m×1.

$F_t$ is the known matrix at time $t$, of dimension m×$p$ relates the state vector to the observation vector $Y_t$.

υt is the vector of observation error (stochastic error term) at time $t$, having normal distribution with zero mean and covariance $V_t$, of dimension $m$×θt is the vector of unknown state parameters (signal) at time $t$, of dimension $p$×1, system states.

$G_t$ is the matrix of time-varying state transition matrix dimension pxp.

ωt is the system error (stochastic error term) at time $t$, having normal distribution with zero mean and covariance $W_t$, of dimension $p$×1.

Where, $V_t$ and $W_t$ are variance-covariance measurement and process noise matrices, respectively. The noise vectors $\omega_t$ and $\upsilon_t$ process and measurement noise, respectively, are uncorrected white noises (have zero means) they are independent of each other and have normal probability distributions.

The disturbances $\upsilon_t$ and $\omega_t$ uncorrected with the initial state that as:

$E(\upsilon_t\theta_0^T) = 0$ and $E(\omega_t\theta_0^T) = 0$ for $t$=1, 2, 3,…,$T$

## Bayesian Estimation of Dynamic Linear Models

Kalman filter is used for estimating a state of a dynamic system which gives the best-unbiased estimator using previous measurement knowledge. Previous algorithms stand from the use of all the previous information to estimate the state of the system at the time of the next step.

This dynamic linear model is the simple model represented as:

Observation equation $Y_t=\mu_t+\varepsilon_t$

System equation $\mu_t=\mu_{t-1+}\sigma_{\mu 2}$

Where the errors $\varepsilon_t$ and $\omega_t$ are mutually independent and independent of initial information probability $(\mu_0|D_0)$

Initial information probability $(\mu_0|D_0)$ represents forecaster's probabilistic information of about the $\mu_0$ at time $t$=0. The mean $m_0$ and the variance $c_0$ are a point estimate and the associated uncertainty of $\mu_0$. $D_t$ consists all the information available up until time $t$, including $D_0$, the values variances values $\{\sigma_\varepsilon^2, \sigma_\mu^2 : t > 0\}$, and the the observations Yt,$Y_{t-1}$,…,$Y_1$ or, the only new information becomes available at any time $t$ is the observed value $Y_t$, where $D_t=\{Y_t,D_{t-1}\}$.

Here, we start expressing initial information concerning the parameter $\mu_0$ was described in the form of a normal probability distribution with mean $m_0$ and variance $c_0$ is:

$$\mu_0 \sim N(m_0, c_0)$$

Using a mathematical process and Bayes' theorem at the time $(t-1)$ of the parameter $\mu_{t-1}$ is:

$$(\mu_{t-1} \mid D_{t-1}) \sim N(m_{t-1t}, c_{t-1})$$

Where, $D_{t-1}$ is $D_{t-1} = \{Y^{t-1}, \sigma_\varepsilon^2, \sigma_\mu^2\}$.

We aim now to find a final distribution for the parameter $\mu_t$ at last time is the time appointed by our knowledge of all available data, or the time that we want to filter. We recompense an extremely presence of the information is:

$$D_t = \{Y_t, D_{t-1}\}.$$

Then Bayes' theorem is

Posterior ∝ Observed likelihood×Prior

$$P(\mu_t \mid D_t) \alpha P(Y_t \mid \mu_t) * P(\mu_t \mid D_{t-1}) \quad (16)$$

Where

$P(Y_t|\mu_t)$ represents the likelihood distribution function at time $t$

$P(\mu t|D_{t-1})$ represents the prior distribution at time $t$

Both of them are distributed normally

Now to find all of the weighting function $P(\mu_t|D_{t-1})$ and the prior probability before observation $Y_t$, using dynamic linear model and probability distribution as:

$$\begin{aligned} E(Y_t \mid \mu_t) &= E(\mu_t + \varepsilon_t \mid \mu_t) \\ &= E(\mu_t \mid \mu_t) + E(\varepsilon_t \mid \mu_t) \\ &= \mu_t \end{aligned}$$

And

$$\begin{aligned} V(Y_t \mid \mu_t) &= V(\mu_t + \varepsilon_t \mid \mu_t) \\ &= V(\mu_t \mid \mu_t) + V(\varepsilon_t \mid \mu_t) \\ &= \sigma_\varepsilon^2 \end{aligned}$$

Or

$$(Y_t \mid \mu_t) \sim N(\mu_t, \sigma_\varepsilon^2)$$

And from the system equation:

$$\begin{aligned} E(\mu_t \mid D_{t-1}) &= E(\mu_{t-1} + \sigma_{\mu_t}) \\ &= E(\mu_t \mid D_{t-1}) + E(\sigma_{\mu_t} \mid D_{t-1}) \\ &= m_{t-1} \end{aligned}$$

And

$$\begin{aligned} V(\mu_t \mid D_{t-1}) &= V(\mu_{t-1} + \sigma_{\mu_t}) \\ &= V(\mu_t \mid D_{t-1}) + V(\sigma_{\mu_t} \mid D_{t-1}) \\ &= c_{t-1} + \sigma_{\mu_t}^2 = R_t \end{aligned}$$

Or

$$P(\mu_t \mid D_{t-1}) \sim N(m_{t-1}, R_t)$$

Then by substituting each of (Yt|$\mu$t), $P(\mu t|D_{t-1})$ in Equation (2.10.1) we get

$$P(\mu_t \mid D_t) - \alpha e \frac{1}{2}[\frac{1}{\sigma_\varepsilon^2}(Y_t - \mu_t) + \frac{1}{R_t}(\mu_t - m_{t-1})] \quad (17)$$

We can find that:

$$P(\mu_t \mid D_t) \alpha e - \frac{1}{2}[\frac{1}{c_t}(\mu_t - m_t)] \quad (18)$$

Now we get mean $m_t$ and variance ct of final distribution, respectively, are:

$$m_t = \frac{\frac{1}{\sigma_\varepsilon^2} Y + \frac{1}{R_t} m_{t-1}}{\frac{1}{\sigma_\varepsilon^2} + \frac{1}{R_t}}, \text{ and } c_t = \frac{\sigma_\varepsilon^2 + R_t}{\sigma_\varepsilon^2 + R_t}$$

Then, the final distribution for parameter $\mu$t at time $t$ is

$$P(\mu_t \mid D_t) \sim N(m_t, c_t)$$

We can write the mean and variance of posterior probability distribution after simplifying them as:

$$m_t = m_{t-1} + K_t(Y_t - m_{t-1}) \quad (19)$$

$$c_t = K_t * \sigma_\varepsilon^2 \quad (20)$$

Where

$$K_t = R_t(R_t + \sigma_\varepsilon^2)^{-1} \quad (21)$$

Where the Equation (19) called Kalman filter, sometimes called for two Equations (19), (20) together Kalman filter, and Equation (21) is Kalman gain.

## Forecasting Dynamic Linear Models

We mean by forecasting finding the $h$-step ahead for each distribution given the data dynamic linear model y1,y2.,yt. For a dynamic linear model, the $h$-step-ahead forecasting distributions, for states and observations, are obtained as a product of the Kalman filter.[12,13] From the observations $y_{1:t}$ to $Y_t$+h we noticed that the data $Y_{1:t}$ provide information about θt, which, in turn, gives information about the future state evolution up to $\theta_{t+k}$ and consequently on $Y_{t+h}$.[13,14]

Then, the general dynamic linear model for h step is:

Observation equation:

$$Y_{t+h} = F_{t+h}\theta_{t+h} + \upsilon_{t+h} \quad \upsilon_t \sim N[0, V_t] \quad (22)$$

System equation:

$$\theta_{t+h} = G_{t+h}\theta_{t+h-1} + \omega_{t+h} \quad \omega_t \sim N[0, W_t] \quad (23)$$

Let a0=mt, R0=Ct, then for $h \geq 1$, for forecasting at time $t$, the forecaster requires the $h$-step ahead marginal distributions, ($Y_{t+h}$|Dt) and ($\theta_t$+h|Dt), predictions start from the posterior estimate of the state, obtained from the Kalman filter, on the time ($t$) on which the forecast is made.

The mean and variance for the observation equation are:

$$\begin{aligned}\hat{Y}_{t+h} &= E(Y_{t+h} \mid D_t) \\ &= E(F_{t+1}\theta_{t+1} + \upsilon_{t+1} \mid D_t) \\ &= FG^k m_t\end{aligned} \quad (24)$$

$$\begin{aligned}Q_t(t) &= Var(Y_{t+h} \mid D_t) \\ &= FG^h C_t (G^k)^T F^T + \sum_{i=0}^{h-1} FG^i W (G^i)^T F^T + V\end{aligned} \quad (25)$$

# APPLICATION, RESULTS AND DISCUSSION

## The Study Area and Data Collection

Spatial variability of any regional variable is a result of complex processes which is working at the same time and over long periods of time. Variation of a regional variable has never been an easy task or work. Many regional random variables vary not only horizontally but also with depth such as wells of water, oil, and gas. The spatial attribute includes approximately all wells at fields that exist which spatially distributed across the study area to represent the fluctuations of levels from place to another place in whole area.

The source of dataset is the observed values of the 295 wells that had been taken from the known specific place which called Shaqlawa in Erbil Governorate. These observations are collected by GPS by the ministry of agriculture, Figure 3 expresses the location of each well, and each point of the dataset has its name and properties. Furthermore, each point has its goal for drilling the well, some of them for drinking or irrigation and agriculture.

Figure 3 shows the geographical location of the data points of Shaqlawa or all wells. They are shown as a surface which can be represented by the most probably prediction map and also by estimated prediction when we are applying a model of the simple kriging interpolation. The figure shows cycles that are many points closer together tend to be more alike than things that are farther apart (quantified here as spatial autocorrelation).

## Results of Groundwater-Surface Interpolation

Figure 4 can be described as continuous data and represented the random field. It shows a histogram of the observed values and how they distributed in the region. It represents the curve of the observations values, and the results after taken the suitable transformation that has a small standard deviation equal to 0.37924, the skewness of dataset is equal to −0.61614, near to zero and the kurtosis of the dataset is equal to 4.5715 not near to three. These indicated that the distributions of depth in sample data points were approximate to normal distribution.

## Results of Gaussian Semivariogram Simple Kriging Surface Interpolation

Spatial dependency can be detected in this dataset using several tools available in geostatistical analysis exploratory spatial data analysis and geostatistical wizard in GIS software.

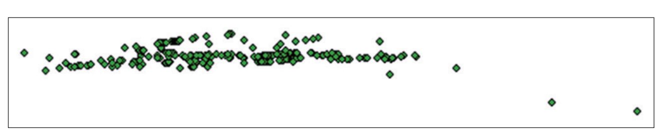

**Figure 3:** Geographical location of the dataset

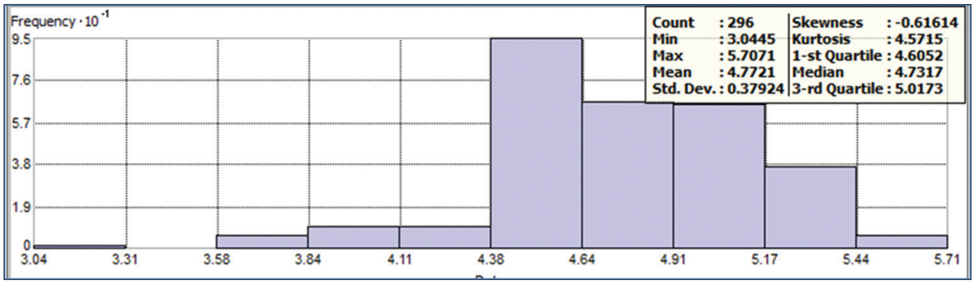


**Figure 4:** The Histogram of the dataset

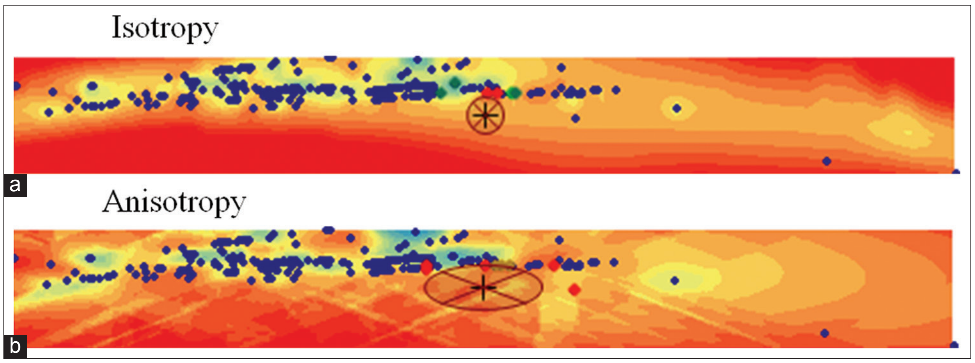


**Figure 5:** (a and b) Surfaces of the Gaussian semivariogram model

In geostatistics semivariogram is called spatial modeling (called structural analysis or variography).

Table 2 shows the model called the Gaussian semivariogram. This gives a quantitative description of variability about the same regional variation. The important part of the variogram is the rang which describes the distance. It shows the isotropy of the Gaussian semivariogram model for (295) dataset of groundwater of wells in Shaqlawa, which all points have equal directional of variability as being north-south and east-west. Furthermore, the anisotropy of Gaussian semivariogram model expresses the same data (554) that change with the direction which described an ellipsoid, and this ellipsoid specified by the length of two orthogonal axis (Major and Minor) with its orientation Angle θ.

Figure 5 shows Gaussian semivariogram model for isotropy and anisotropy surfaces, these used to predict of random field (spatial field) of Groundwater well $Z(s_0)$ unknown value in the same area, which depends on only maximum (5) neighbors value of the measured values. Table 2 contains all cross-validation of isotropy and anisotropy of simple kriging Gaussian semivariogram model and the prediction value of the depth of unknown new value of well.

Table 3 shows all information about simple kriging by Guassian semivariogram model to predict a new location of groundwater in the same area for both of isotropy and anisotropy. The prediction value of unknown is measured also depending on (5) maximum and (2) minimum value of neighbors from measured value for the depth of new well with longitude and latitude. In anisotropy Gaussian semivariogram model is equal to 173.2582 and its greater than the depth of the isotropy exponential semivariogram model which equal to 132.3405.

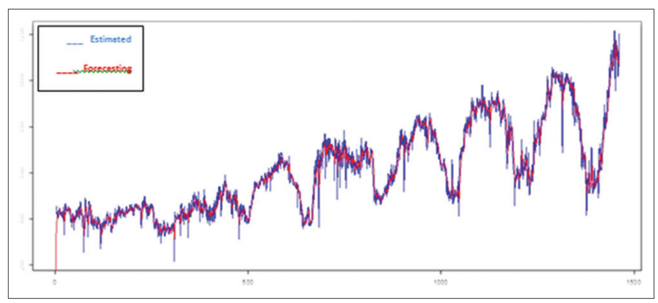


**Figure 6:** Difference between estimate values and forecast values for groundwater depth

## Estimation the Depth of Groundwater using Kalman Filter

The estimation process of parameters is made using the initial values of the depth of groundwater mean and variances are being of an estimate of it is values. Equations (19), (20), and (21) are used to estimate parameters, Error Covariance, and Kalman Gain for univariate simple dynamic linear model for each observation of the groundwater depth.

Table 4 gives the actual value, estimated groundwater depth parameter, Error Covariance, and Kalman Gain. Examining the table reveals the following: The Error Covariance is convergence at a time point ($t$=18) with the value (802.1407). Kalman Gain is convergence at a time point ($t$=16) with the value (0.383983). There is two convergences time and the last time convergence is the best estimate for the same time.

A compression between groundwater depth resulting from the estimate and forecasting groundwater depth values for all observations is shown in Figure 6. The results show how

**Table 2:** Results of gaussian semivariogram model

| Simple kriging exponential semivariogram | Isotropy | Anisotropy |
|---|---|---|
| Nugget ($C_0$) | 0.0714 | 0 |
| Rang (α) | 2 | 2 |
| Major rang | 2554.326 | 4998.32 |
| Minor rang | 2554.326 | 1949.16 |
| Partial sill ($C$) | 0.081173 | 0.093287 |
| Lag size ($h$) | 416.5267 | 416.5267 |
| Number of lag | 12 | 12 |

**Table 3:** Accuracy criteria for each approach

| Methods | MAE | RMSE |
|---|---|---|
| Kalman filter | 13.2975 | 18.078 |
| Simple kriging/gaussian model | 29.7364 | 43.719 |

MAE: Mean absolute error, RMSE: Root mean square error

closely the estimated model matches the forecasting electricity groundwater depth. The blue line is estimate values and the red line is forecasting values.

## Evaluating Accuracy (or Error) Measures for Estimation

For selections, the best approach (method) for prediction the groundwater depth two accuracy criteria were used, which are (mean absolute error [MAE] and root mean square error [RMSE]). They are based on the error estimate, which is the difference between the estimated and forecast groundwater depth values. The results are shown in Table 5.

Table 3 represents that Kalman Filter is the better approach based on the two criteria, where each of them has a small value than simple kriging using Gaussian semivariogram model for both of isotropy and anisotropy.

## CONCLUSIONS

The major results of performing the analysis of two approaches (Simple Kriging and Kalman Filter), the following main conclusions have been achieved:

1. The data follow approximately to normal distribution after taken log transformation for variability and second-order stationary to remove the trend.
2. The variability of the depth of groundwater wells elevation in the region changes from north to south or from west-north to south-east, this is due to the spatial dependency in the area which is one of the reasons of the depth of well.
3. For unmeasured value used simple kriging with Gaussian semivariogram model, in this model the predicted value by isotropy semivariogram model is better than the anisotropy semivariogram model depending on the value of the depth of groundwater [Table 2].
4. When applied the Kalman filter for estimating and forecasting the depth of groundwater, we found that the KF is suitable for estimation and yield good results, because in the presence of white Gaussian noise because when the normality condition holds, the KF will give optimum and sufficient results.
5. The result shows that the Bayesian Kalman filter approach has the best estimation and forecasting results compared to simple kriging using the accuracy criteria (MAE and RMSE).

**Table 4:** Results of simple kriging gaussian semivariogram model

| Simple kriging exponential semivariogram | Isotropy | Anisotropy |
|---|---|---|
| Maximum neighbors | 5 | 5 |
| Minimum neighbors | 2 | 2 |
| Predicted value | | |
| X | 400,739 | 400,739 |
| Y | 421,531 | 421,531 |
| Depth (m) | 132.3405 | 173.2582 |

**Table 5:** Estimated parameters when ( $\hat{\theta}_0$ =126.02, $\hat{P}_0$ =2089.088, and W=500)

| Convergence time | Actual values | Estimate | Error covariance | Kalman gain |
|---|---|---|---|---|
| 1 | 189 | 126.02 | 2089.088 | 0.73123 |
| 2 | 201 | 167.5177 | 1156.157 | 0.55345 |
| 3 | 91 | 133.6806 | 923.7829 | 0.442213 |
| 4 | 142 | 137.0526 | 846.7026 | 0.405315 |
| 5 | 100 | 122.529 | 818.8316 | 0.391973 |
| 6 | 130 | 125.4203 | 808.4435 | 0.387 |
| 7 | 160 | 138.7378 | 804.528 | 0.385126 |
| 8 | 140 | 139.223 | 803.046 | 0.384416 |
| 9 | 50 | 104.9482 | 802.4841 | 0.384147 |
| 10 | 151 | 122.6342 | 802.271 | 0.384045 |
| 11 | 183 | 145.8151 | 802.1901 | 0.384007 |
| 12 | 100 | 128.2224 | 802.1594 | 0.383992 |
| 13 | 100 | 117.3854 | 802.1478 | 0.383986 |
| 14 | 220 | 156.7878 | 802.1434 | 0.383984 |
| 15 | 193 | 170.6927 | 802.1417 | 0.383984 |
| 16 | 180 | 174.2665 | 802.1411 | 0.383983 |
| 17 | 200 | 184.1478 | 802.1408 | 0.383983 |
| 18 | 147 | 169.8836 | 802.1407 | 0.383983 |
| 19 | 202 | 182.2158 | 802.1407 | 0.383983 |
| 20 | 151 | 170.2295 | 802.1407 | 0.383983 |
| . | . | . | . | . |
| . | . | . | . | . |
| 291 | 239 | 164.2836 | 802.1407 | 0.383983 |
| 292 | 211 | 182.2219 | 802.1407 | 0.383983 |
| 293 | 73 | 140.2825 | 802.1407 | 0.383983 |
| 294 | 91 | 121.3589 | 802.1407 | 0.383983 |
| 295 | 82 | 106.2469 | 802.1407 | 0.383983 |

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