

On strongly α - I -Open sets and a new mapping

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ABSTRACT

In this paper, we introduce the notion of strongly α - I -open sets in ideal topological spaces and investigate some of their properties. Further we study the continuous functions for the above set and derive the some of their properties.

RESUMEN

En este trabajo, se introduce la noción del gran conjunto α - I -abierto ideal en espacios topológicos y se investigan algunas de sus propiedades. Además se estudian las funciones continuas para el conjunto y parte de sus propiedades.

Keywords: α - I -open set, Strongly α - I -open set and B_I set.

Mathematics Subject Classification: 54A05,54D10,54F65,54G05.

1 Introduction

The notion of α -open sets was introduced and investigated by Njastad [16]. By using α -open sets. Mashhour et al. [14] defined and studied α -continuity and α -openness in topological spaces. Ideals in topological spaces have been considered since 1930. This topic has won its importance by the paper of Vaidyanathaswamy [19]. In 2002, Hatir and Noiri [6] have introduced the notion of α - I -continuous functions and used it to obtain a decomposition of continuity. The notion of B_I sets introduced by Hatir and Noiri [6] and provided a decomposition of continuity. In this paper, we introduce strongly α - I -open sets and establish a decomposition of continuity.

In 1990, Jankovic and Hamlett [9] introduced the notion of I -open sets in ideal topological spaces. An ideal is defined as a non-empty collection I of subsets of X satisfying the following two conditions. (1) If $A \in I$ and $B \subset A$, then $B \in I$. (2) If $A \in I$ and $B \in I$, then $A \cup B \in I$. An ideal topological space is a topological space (X, τ) with an ideal I on X and it is denoted by (X, τ, I) . For a subset $A \subset X$, $A^*(I) = \{x \in X : U \cap A \notin I \text{ for each neighbourhood } U \text{ of } x\}$ is called the local function of A with respect to I and τ [9]. We simply write A^* instead of $A^*(I)$ to be brief. For every ideal topological space (X, τ, I) , there exists a topology $\tau^*(I)$, finer than τ , generated by $\beta(I, \tau) = \{U - i : U \in \tau \text{ and } i \in I\}$, but in general $\beta(I, \tau)$ is not always a topology [9]. Additionally, $cl^*(A) = A \cup A^*$ defines a kuratowski closure operator for $\tau^*(I)$. Given a space (X, τ, I) and $A \subset X$, A is called I -open if $A \subset int(A^*)$ and a subset K is called I -closed if its complement is I -open [8,9].

2 Preliminaries

First we will recall some definitions used in sequel.

Definition 2.1. A subset A of an ideal topological space (X, τ, I) is said to be

1. α - I -open [6] (resp. α -open [16]) if $A \subset int(cl^*(int(A)))$ (resp. $A \subset int(cl(int(A)))$),
2. $semi$ - I -open [6] (resp. $semi$ -open [12]) if $A \subset cl^*(int(A))$ (resp. $A \subset cl(int(A))$),
3. pre - I -open [1] (resp. pre -open [13]) if $A \subset int(cl^*(A))$ (resp. $A \subset int(cl(A))$),
4. b - I -open [3] (resp. b -open [2]) if $A \subset int(cl^*(A)) \cup cl^*(int(A))$ (resp. $A \subset int(cl(A)) \cup cl(int(A))$),
5. t - I -set [6] (resp. t -set [18]) if $int(cl^*(A)) = int(A)$ (resp. $int(cl(A)) = int(A)$),
6. B_I -set [6] if $A = U \cap V$, where $U \in \tau$ and V is a t - I -set,
7. C_I -set [6] if $A = U \cap V$, where $U \in \tau$ and $int(cl^*(int(V))) = int(V)$,
8. A_I -set [11] if $A = U \cap V$, where $U \in \tau$ and $V = (int(V))^*$,

9. strongly *pre-I-open* [17] if A is *pre-I-open* as well as a C_I -set,
10. strongly *b-I-open* [4] if A is *b-I-open* as well as a C_I -set and
11. I -locally closed set [5] if $A = U \cap V$, where $U \in \tau$ and $V = V^*$.

Definition 2.2. A subset A of an ideal topological space (X, τ, I) is said to be I -nowhere dense if $\text{int}(cl^*(A)) = \phi$.

Observe that if A is rare, then a t - I -set (resp. t -set) is I -nowhere dense (resp. nowhere dense). Recall that a set A of X is rare if it has no interior points. Also notice that if A is rare, then b - I -open sets and b -open sets are *pre-I-open* and *preopen*, respectively.

Definition 2.3. A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be

1. *semi-I-continuous* [6] if for every $V \in \sigma$, $f^{-1}(V)$ is *semi-I-open*,
2. *pre-I-continuous* [5] if for every $V \in \sigma$, $f^{-1}(V)$ is *pre-I-open*,
3. *b-I-continuous* [3] if for every $V \in \sigma$, $f^{-1}(V)$ is *b-I-open*,
4. A_I -*continuous* [11] if for every $V \in \sigma$, $f^{-1}(V)$ is A_I -set,
5. B_I -*continuous* [6] if for every $V \in \sigma$, $f^{-1}(V)$ is B_I -set.
6. I -locally *continuous* [5] if for every $V \in \sigma$, $f^{-1}(V)$ is I -locally *closed*,
7. strongly *pre-I-continuous* [17] if for every $V \in \sigma$, $f^{-1}(V)$ is strongly *pre-I-open* and
8. strongly *b-I-continuous* [4] if for every $V \in \sigma$, $f^{-1}(V)$ is strongly *b-I-open*.

3 Stongly α - I -Open Sets

Definition 3.1. A subset A of an ideal space (X, τ, I) is said to be strongly α - I -open set if A is *b-I-open* as well as a B_I -set.

The family of all strongly α - I -open sets in (X, τ, I) is denoted by $S\text{-}\alpha IO(X, \tau)$ or $S\text{-}\alpha IO(X)$. For a subset A of (X, τ, I) , $\text{int}_{s\alpha}(A) = \bigcup \{U \subset A, U \in S\text{-}\alpha IO(X, \tau)\}$. Clearly $\tau \subset S\text{-}\alpha IO(X) \subset \alpha IO(X)$. The following examples 3.2 and 3.3 show that these inclusions are not reversible.

Example 3.2. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$ and $I = \{\phi, \{a\}\}$. If $A = \{c\}$, then $A^* = \{b, c, d\}$ and so $\text{int}(cl^*(\text{int}(A))) = \text{int}(cl^*(\{c\})) = \text{int}(\{b, c, d\}) = \{c\} = A$. Therefore A is α - I -open. Since X is the only open set containing A , $A = X \cap A$ is the only possibility to write A as the intersection with X . Since $\text{int}(cl^*(A)) = \text{int}(cl^*(\{c\})) = \text{int}(\{b, c, d\}) = \text{int}(A)$. This shows that A is a B_I -set and hence A is strongly α - I -open set, but A is not I -open. This shows

the existence of non trivial strongly α - I -open sets.

Example 3.3. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$ and $I = \{\phi, \{a\}\}$. If $A = \{a, b, c\}$, then $A^* = \{a, c, d\}$ and so $\text{int}(cl^*(\text{int}(A))) = \text{int}(cl^*(\{a, c\})) = \text{int}(X) = X \supset A$. Therefore A is α - I -open. Since X is the only open set containing A , $A = X \cap A$ is the only possibility to write A as the intersection with X . Since $\text{int}(cl^*(A)) = \text{int}(cl^*(\{a, b, c\})) = \text{int}(X) = X \neq \text{int}(A)$, A is not a B_I -set and hence A is not a strongly α - I -open set.

The following example shows that α - I -open sets and B_I sets are independent concepts.

Example 3.4. Consider the ideal space (X, τ, I) of Example 3.3.

(a) If $A = \{a, b, d\}$, then $\text{int}(A) = \{a\}$ and $\text{int}(cl^*(\text{int}(A))) = \text{int}(cl^*(\{a\})) = \text{int}(\{a\})$ does not contains A . Therefore A is not a α - I -open set. But $\text{int}(cl^*(A)) = \text{int}(cl^*(\{a, b, d\})) = \text{int}(\{a, b, d\}) = \{a\} = \text{int}(A)$ and $A = X \cap A$. Therefore A is a B_I -set.

(b) If $B = \{a, b, c\}$, then $\text{int}(B) = \{a, c\}$ and so $\text{int}(cl^*(\text{int}(B))) = \text{int}(cl^*(\{a, c\})) = X \supset B$. Therefore B is α - I -open set. But B is not a B_I -set, since $\text{int}(cl^*(B)) = \text{int}(X) = X \neq \text{int}(B)$.

Theorem 3.5. Every strongly α - I -open set is strongly pre - I -open.

Proof. It follows from the fact that every α - I -open set is pre - I -open and let A be a B_I set. Then $A = U \cap V$, where $U \in \tau$ and V is a t - I -set. Then $\text{int}(V) = \text{int}(cl^*(V)) \supset \text{int}(cl^*(\text{int}(V))) \supset \text{int}(V)$ and hence $\text{int}(V) = \text{int}(cl^*(\text{int}(V)))$. This shows that A is a C_I -set. Therefore A is strongly pre - I -open set.

The converse of the above theorem need not be true by the following example.

Example 3.6. Consider R , the set of all real numbers with the usual topology and the ideal I_f consisting of all finite subsets of R . If $A = Q$, the set of all rational numbers, then $A^* = R$. Since $\text{int}(cl^*(A)) = R \supset A$, A is pre - I -open. Since $A = R \cap A$ where R is open and $\text{int}(cl^*(\text{int}(A))) = \phi = \text{int}(A)$, it follows that A is strongly pre - I -open but A is not strongly α - I -open, since $\text{int}(cl^*(\text{int}(A))) = \phi$ does not contains A .

Theorem 3.7. Every strongly α - I -open set is strongly b - I -open.

Proof. It follows from Theorem 3.5. and [4, Theorem 3.7].

The converse of the above theorem need not be true by the following example.

Example 3.8. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi, \{b\}\}$. Then $A = \{a, c\}$ is strongly b - I -open, but it is not strongly α - I -open. For $\text{int}(cl^*(A)) \cup cl^*(\text{int}(A)) = \text{int}(\{a, c\}^* \cup \{a, c\}) \cup cl^*(\{a\}) = \{a, c\} \supset A$. Therefore, A is b - I -open. Since X is the only open set containing A , $A = X \cap A$ is the only possibility to write A as the intersection with X . Since $\text{int}(cl^*(\text{int}(A))) = \text{int}(cl^*(\{a\})) = \text{int}(\{a, c\}) = \text{int}(A)$ and hence A is strongly b - I -open set. Since $\text{int}(cl^*(\text{int}(A))) = \text{int}(cl^*(\{a\})) = \text{int}(\{a, c\}) = \{a\}$ is not contains A . Hence A is not

strongly α - I -open.

Proposition 3.9. Let (X, τ, I) be an ideal topological space. A subset A of X is I -locally closed set if A is both open and A_I -set.

Proof. Let A be an open and A_I -set, then $A = G \cap V$, where $G \in \tau$ and $V = (int(V))^* = V^*$. This shows that A is I -locally closed set.

Observe that if V is rare, then A is empty.

The following theorem gives a characterization of open sets in terms of strongly α - I -open sets and A_I -sets.

Theorem 3.10. If (X, τ, I) is an ideal topological space. For a subset A of X , the following conditions are equivalent.

- (a) A is open.
- (b) A is open, strongly α - I -open and A_I -set.
- (c) A is strongly α - I -open and I -locally closed set.
- (d) A is strongly pre- I -open and I -locally closed set.
- (e) A is strongly pre- I -open and A_I -set.

Proof.

(a) \Rightarrow (b) is obvious.

(b) \Rightarrow (c) It follows from Proposition 3.9.

(c) \Rightarrow (d) It follows from Theorem 3.5.

(d) \Rightarrow (e) If A is I -locally closed, then $A = G \cap A^*$ for some open set G . Since $A \subset A^*$, by [17, Lemma 2.5] $A^* = cl^*(A)$. Now $A \subset int(cl^*(A)) = int(A^*)$ and so $A^* \subset (int(A^*))^* \subset (A^*)^* \subset A^*$. Therefore $A^* = (int(A^*))^*$ which implies that A is an A_I set.

(e) \Rightarrow (a) Suppose A is strongly pre- I -open and A_I -set.

$$\begin{aligned} A &\subset int(cl^*(A)) \\ &= int(cl^*(U \cap V)) \end{aligned}$$

where U is open and $V = (int(V))^*$. By [10, Theorem 2.1.]

$$\begin{aligned} A &\subset U \cap (int(cl^*(V))) \\ &\subset U \cap int(V^*) \\ &\subset U \cap int(int(V))^* \\ &\subset U \cap int(cl^*(int(V))) \\ &\subset U \cap int(V) \\ &= int(A) \end{aligned}$$

Theorem 3.11. Let (X, τ, I) be an ideal topological space. A subset A of (X, τ, I) is pre- I -open and B_I -set if A is strongly α - I -open.

Proof. Let A be strongly α - I -open set. Since every α - I -open set is pre- I -open, then A is pre- I -open and B_I -set.

Theorem 3.12. Let (X, τ, I) be an ideal topological space. A subset A of (X, τ, I) is strongly α - I -open if and only if it is semi- I -open, pre- I -open and B_I -set.

Proof.

Necessity. It follows from the fact that every α - I -open set is semi- I -open and pre- I -open.

Sufficiency. Let A be semi- I -open, pre- I -open and B_I -set. Then, we have $A \subset \text{int}(cl^*(A)) \subset \text{int}(cl^*(cl^*(\text{int}(A)))) = \text{int}(cl^*(\text{int}(A)))$. This shows that A is α - I -open set and also A is B_I -set. Therefore A is a strongly α - I -open set.

4 Strongly α - I -Ccontinuous Maps

Definition 4.1. A mapping $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be strongly α - I -continuous if for every $V \in \sigma$, $f^{-1}(V)$ is strongly α - I -open.

Theorem 4.2. Every strongly α - I -continuous map is strongly pre- I -continuous.

Proof. It follows from Theorem 3.5.

Theorem 4.3. Every strongly α - I -continuous map is strongly b - I -continuous.

Proof. It follows from Theorem 3.7.

Theorem 4.4. Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be any mapping. Then f is I -locally continuous map if it is both continuous and A_I -continuous.

Proof. It follows from Proposition 3.9.

Theorem 4.5. Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be any mapping. Then the following conditions are equivalent.

- (a) f is continuous.
- (b) f is continuous, strongly α - I -continuous and A_I -continuous.
- (c) f is strongly α - I -continuous and I -locally continuous.
- (d) f is strongly pre- I -continuous and I -locally continuous.
- (e) f is strongly pre- I -continuous and A_I -continuous.

Proof. It follows from Theorem 3.10.

Theorem 4.6. *Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be any mapping. Then f is pre- I -continuous and B_I -continuous if f is strongly α - I -continuous.*

Proof. It follows from Theorem 3.11.

Theorem 4.7. *Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be any mapping. Then f is strongly α - I -continuous if and only if it is semi- I -continuous, pre- I -continuous and B_I -continuous.*

Proof. It follows from Theorem 3.12. **Definition 4.8.** A mapping $f : (X, \tau, I) \rightarrow (Y, \sigma, I)$ is said to be strongly α - I -irresolute if $f^{-1}(V)$ is strongly α - I -open in X for every strongly α - I -open set V of Y .

Theorem 4.9. *Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be mappings. Then the composition $g \circ f : X \rightarrow Z$ is strongly α - I -continuous if g is continuous and f is strongly α - I -continuous.*

Proof. Let W be any open subset of Z . Since g is continuous, $g^{-1}(W)$ is open in Y . Since f is strongly α - I -continuous, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is strongly α - I -open in X and hence $g \circ f$ is strongly α - I -continuous.

Theorem 4.10. *Let $f : (X, \tau, I_1) \rightarrow (Y, \sigma, I_2)$ and $g : (Y, \sigma, I_2) \rightarrow (Z, \eta, I_3)$ be mappings. Then the composition $g \circ f : X \rightarrow Z$ is strongly α - I -continuous if g is strongly α - I -continuous and f is strongly α - I -irresolute.*

Proof. Let W be any open subset of Z . Since g is strongly α - I -continuous, $g^{-1}(W)$ is strongly α - I -open in Y . Since f is strongly α - I -irresolute, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is strongly α - I -open in X and hence $g \circ f$ is strongly α - I -continuous.

Theorem 4.11. *Let $f : (X, \tau, I_1) \rightarrow (Y, \sigma, I_2)$ and $g : (Y, \sigma, I_2) \rightarrow (Z, \eta, I_3)$ be mappings. Then the composition $g \circ f : X \rightarrow Z$ is strongly α - I -irresolute if both f and g are strongly α - I -irresolute.*

Proof. Let W be any strongly α - I -open subset of Z . Since g is strongly α - I -irresolute, $g^{-1}(W)$ is strongly α - I -open in Y . Since f is strongly α - I -irresolute, then $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is strongly α - I -open in X and hence $g \circ f$ is strongly α - I -irresolute.

Definition 4.12. [15] Let A be a subset of a space (X, τ) then the set $\cap\{U \in \tau : A \subset U\}$ is called the kernel of A and denoted by $ker(A)$.

Lemma 4.13. [7] *Let A be a subset of a space (X, τ) , then*

- (a) $x \in ker(A)$ if and only if $A \cap F \neq \phi$ for any closed subset F of X with $x \in F$;
- (b) $A \subset ker(A)$ and $A = ker(A)$ if A is open in X ;
- (c) if $A \subset B$, then $ker(A) \subset ker(B)$.

Definition 4.14. Let N be a subset of a space (X, τ, I) and $x \in X$. Then N is called strongly α - I -neighbourhood of x , if there exists a strongly α - I -open set U containing x such that $U \subset N$.

Theorem 4.15. *The following statements are equivalent for a mapping $f : (X, \tau, I) \rightarrow (Y, \sigma)$.*

1. f is strongly α - I -continuous.
2. for each $x \in X$ and each open set V in Y with $f(x) \in V$, there exists a strongly α - I -open set U containing x such that $f(U) \subset V$.
3. for each $x \in X$ and each open set V in Y with $f(x) \in V$, $f^{-1}(V)$ is a strongly α - I -neighbourhood of x .

Proof.

(1) \Rightarrow (2) Let $x \in X$ and V be an open set in Y such that $f(x) \in V$. Since f is strongly α - I -continuous, $f^{-1}(V)$ is a strongly α - I -open containing x . Set $U = f^{-1}(V)$. Then we have $f(U) \subset V$.

(2) \Rightarrow (3) Let V be an open set in Y and let $f(x) \in V$. Then by (2), there exists a strongly α - I -open set U containing x such that $f(U) \subset V$. So $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is a strongly α - I -neighbourhood of x .

(3) \Rightarrow (1) Let V be an open set in Y and let $f(x) \in V$ then by (3), $f^{-1}(V)$ is a strongly α - I -neighbourhood of x . Thus for each $x \in f^{-1}(V)$ there exists a strongly α - I -open set U_x containing x such that $x \in U_x \subset f^{-1}(V)$. Hence $f^{-1}(V) \subset \cup_{x \in f^{-1}(V)} U_x$ so $f^{-1}(V) \in S\text{-}\alpha IO(X)$.

Theorem 4.16. *The following mappings are equivalent for a mapping $f : (X, \tau, I) \rightarrow (Y, \sigma)$.*

1. f is strongly α - I -continuous.
2. for every subset A of X , $f(\text{int}_{s\alpha} I(A)) \subset \ker(f(A))$.
3. for every subset B of Y , $\text{int}_{s\alpha} I(f^{-1}(B)) \subset f^{-1}(\ker(B))$.

Proof.

(1) \Rightarrow (2) Let A be any subset of X . Suppose that $y \notin \ker(f(A))$. Then by Lemma 4.13. there exists a closed subset F of Y such that $y \in F$ and $f(A) \cap F = \phi$. Thus we have $A \cap f^{-1}(F) = \phi$ and $(\text{int}_{s\alpha}(I(A))) \cap f^{-1}(F) = \phi$. Therefore, we obtain $f(\text{int}_{s\alpha}(I(A))) \cap F = \phi$ and $y \notin f(\text{int}_{s\alpha} I(A))$. This implies that $f(\text{int}_{s\alpha} I(A)) \subset \ker(A)$.

(2) \Rightarrow (3) Let B be any subset of Y by (2) and Lemma 4.13., we have $f(\text{int}_{s\alpha} I(f^{-1}(B))) \subset \ker(f(f^{-1}(B))) \subset \ker(B)$ and $\text{int}_{s\alpha} I(f^{-1}(B)) \subset f^{-1}(\ker(B))$.

(3) \Rightarrow (1) Let V be an open set of Y . Then by Lemma 4.13. and (3), we have $\text{int}_{s\alpha} I(f^{-1}(V)) \subset f^{-1}(\ker(V)) = f^{-1}(V)$ and $\text{int}_{s\alpha} I(f^{-1}(V)) = f^{-1}(V)$. This implies $f^{-1}(V)$ is strongly α - I -open.

Received: April 2009. Revised: August 2009.

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