*CUBO A Mathematical Journal*
*Vol.13, N$^{o}$03, (49–56). October 2011*

# On Strongly Fβp-irresolute Mappings

Ratnesh Kumar Saraf
*Department of Mathematics,*
*Government Kamla Nehru, Girls College*
*DAMOH (M.P.)-470661, India.*
and
Miguel Caldas
*Departamento de Matemática Aplicada,*
*Universidade Federal Fluminense*
*Rua Mário Santos Braga s/n$^{0}$,CEP: 24020-140, Niteroi-RJ,Brasil.*
*email:* gmamccs@vm.uff.br

**ABSTRACT**

In this paper, we introduce a new class of mappings called strongly Fβp-irresolute mappings between fuzzy topological spaces. We obtain several characterizations of this class and study its properties and investigate the relationship with the known mappings.

**RESUMEN**

En este trabajo presentamos una nueva clase de funciones llamadas funciones fuertemente Fβp-irresolute entre espacios topológicos difusos. Obtenemos varias caracterizaciones de esta clase, estudiamos sus propiedades e investigamos la relación con funciones conocidas.

**Keywords:** Fuzzy topological spaces, fuzzy β-open sets, fuzzy β-preirresolute maps, strongly fuzzy β-preirresolute maps.

**Mathematics Subject Classification:** 54C10, 54D10.

# 1 Introduction and preliminaries.

The concept fuzzy has invaded almost all branches of mathematics with the introduction of fuzzy sets by Zadeh [23] of 1965. The theory of fuzzy topological spaces was introduced and developed by Chang [6] and since then various notions in classical topology have been extended to fuzzy topological spaces. Recently Professor El-Naschie has been shown in [7] and [8] that the notion of fuzzy topology may be relevant to quantum particle physics in connection with string theory and $\varepsilon^{\infty}$ theory. Thus our motivation in this paper is to define strongly fuzzy $\beta$-preirresolute (in short St-F$\beta$p-irresolute) mappings and investigate its properties. The new defined class of mapping is stronger that M-fuzzy $\beta$-continuous mappings and is a generalization of St-F$\alpha$p-irresolute mappings.

Throughout this paper $(X,\tau),(Y,\sigma)$ and $(Z,\gamma)$ (or simply $X$, $Y$ and $Z$) represent non-empty fuzzy topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The fuzzy set $A$ of $X$ is called fuzzy $\alpha$-open (F$\alpha$-open) [5] (resp. fuzzy preopen (Fp-open) [5], fuzzy $\beta$-open (F$\beta$-open) [2]) if $A \leq Int(Cl(Int(A))$ (resp. $A \leq Int(Cl(A))$, $A \leq Cl(Int(Cl(A)))$, where $Cl(A)$ and $Int(A)$ denote the closure of $A$ and the interior of $A$ respectively. The fuzzy subset $B$ of $X$ is said to be fuzzy $\alpha$-closed (F$\alpha$-closed) (resp. fuzzy preclosed (Fp-closed), fuzzy $\beta$-closed (F$\beta$-closed)) if, its complement $B^c$ is fuzzy F$\alpha$-open (resp. Fp-open, F$\beta$-open) in $X$. By $F\alpha O(X)$, $FPO(X)$ and $F\beta O(X)$ (resp. $F\alpha C(X)$, $FPC(X)$, and $F\beta C(X)$) we denote the family of all F$\alpha$-open, Fp-open and F$\beta$-open (resp. F$\alpha$-closed, Fp-closed and F$\beta$-closed) sets of $X$. The intersection of all fuzzy $\beta$-closed sets containing $A$ is called the $\beta$-closure of $A$ and is denoted by $\beta Cl(A)$. The fuzzy $\beta$-interior [2] of $A$ denoted by $\beta$-Int(A), is defined by the union of all fuzzy $\beta$-open sets of $X$ contained in $A$.

A mapping $f : X \to Y$ is said to be:

(i) fuzzy completely weakly preirresolute [11] (resp. F$\alpha$p-irresolute [5], M-fuzzy precontinuous [3], F$\beta$p-irresolute [17]) if, $f^{-1}(V)$ is fuzzy open (resp. F$\alpha$-open), Fp-open, F$\beta$-open) in $X$ for every Fp-open set $V$ of $Y$.

(ii) strongly M-fuzzy $\beta$-continuous [16] (resp. M-fuzzy $\beta$-continuous [15], St-F$\alpha$p-irresolute [16]) if, $f^{-1}(V)$ is fuzzy open (resp. F$\beta$-open, F$\alpha$-open) in $X$ for every F$\beta$-open set $V$ of $Y$.

(iii) fuzzy strongly continuous [12] if, $f^{-1}(V)$ is fuzzy clopen in $X$ for every fuzzy subset $V$ of $Y$.

A fuzzy point in $X$ with support $x \in X$ and value $p$ $(0 < p \leq 1)$ is denoted by $x_p$. The fuzzy point $x_p$ is said to be quasi-coincident (shorty: q-coincident) with a fuzzy set $A$ of $X$ denoted by $x_p q A$ if $p + A(x) > 1$. Two fuzzy sets $A$ and $B$ are said to be quasi-coincident denoted by $AqB$, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [14] and by $\bar{q}$ we denote "is not q-coincident". It is known [14] that $A \leq B$ if and only if $A\bar{q}(1 - B)$.

Two non empty fuzzy subsets $A$ and $E$ are said to be fuzzy β-separated if there exist two fuzzy β-open subsets $G$ and $H$ such that $A \leq G$, $E \leq H$, $A\bar{q}H$ and $E\bar{q}G$. A fuzzy subset which cannot be expressed as the union of two fuzzy β-separated subsets is said to be fuzzy β-connected sets.

**Lemma 1.1.** *[22] Let $f : X \to Y$ be a mapping and $x_p$ be a fuzzy point of $X$. Then:*
*(1) $f(x_p)qB \Rightarrow x_p q f^{-1}(B)$, for every fuzzy set $B$ of $Y$.*
*(2) $x_p qA \Rightarrow f(x_p)qf(A)$, for every fuzzy set $A$ of $X$.*

## 2 St-Fβp-irresolute mappings.

**Definition 2.1.** *A mapping $f : X \to Y$ is said to be strongly fuzzy β- preirresolute (briefly St-Fβp-irresolute) if, $f^{-1}(V)$ is fuzzy preopen in $X$ for every Fβ-open set $V$ of $Y$.*

From the definitions stated, we have the following diagram:

$$\begin{array}{ccccccc} A & \to & B & \to & C & \to & D \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ E & \to & F & \to & G & \to & H \end{array}$$

Where: A = St-MFβ-continuous; B = St-Fαp-irresolute; C = St- Fβp-irresolute; D = MFβ-continuous; E = Fuzzy completely weakly preirresolute; F= Fαp-irrsesolute; G = MFp-continuous; H = Fβp-irresolute.

**Remark 2.1.** *However, converses of the above implications are not true in general, by [12, 16, 17] and the followings examples:*
*(i) Fαp-irrsesolute mapping does not imply fuzzy completely weakly preirresolute:*
*Let $X = \{a, b\}$ and $Y = \{x, y\}$. Define fuzzy sets $A(a) = 0.6$, $A(b) = 0.5$; $B(a) = 0$, $B(b) = 0.8$; $H(x) = 0.5$, $H(y) = 0.5$; $E(x) = 0.7$, $E(y) = 0.8$. Let $\tau = \{0, A, 1\}$, $\Gamma = \{0, B, 1\}$; $\sigma = \{0, H, 1\}$ and $\upsilon = \{0, E, 1\}$. The mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x$ , $f(b) = y$ is fuzzy α-preirresolute but not fuzzy completely weakly preirresolute, because $Z(x) = 0.7$, $Z(y) = 0.7$ are fuzzy preopen in $(Y, \sigma)$ but $f^{-1}(Z)$ is not fuzzy open in $X$.*
*(ii) Fuzzy completely weakly preirresolute mapping does not imply MFβ-continuous, see [[18], Example 3.2].*
*(iii) MFβ-continuous mapping does not imply MFp-continuous, see [[19], Example 3.1].*
*(iv) St- Fβp-irresolute mapping does not imply Fαp-irrsesolute, see [[19], Example 3.2].*

*(v) St- $F\alpha p$-irresolute mapping does not imply fuzzy completely weakly preirresolute, see [[16], Example 3.1].*

**Theorem 2.1.** *For a mapping $f : X \to Y$, the following are equivalent:*
*(1) $f$ is St-$F\beta p$-irresolute;*
*(2) For every fuzzy point $x_t$ in $X$ and every $F\beta$-open set $V$ of $Y$ containing $f(x_t)$, there exist a Fp-open set $U$ of $X$ containing $x_t$ such that $f(U) \leq V$;*
*(3) For every fuzzy point $x_t$ in $X$ and every $F\beta$-open set $V$ of $Y$ containing $f(x_t)$, there exist a Fp-open set $U$ of $X$ containing $x_t$ such that $x_t \in U \leq f^{-1}(V)$;*
*(4) For every fuzzy point $x_t$ in $X$, the inverse image of each $\beta$-neighbourhood of $f(x_t)$ is a preneighbourhood of $x_t$;*
*(5) For every fuzzy point $x_t$ in $X$ and each $\beta$-neighbourhood $E$ of $f(x_t)$, there exists an preneighbourhood $A$ of $x_t$ such that $f(A) \leq E$;*
*(6) $f^{-1}(V) \leq Int(Cl(f^{-1}(V)))$ for every $V \in F\beta O(Y)$;*
*(7) $f^{-1}(H) \in FPC(X)$ for every $H \in F\beta C(Y)$ ;*
*(8) $Cl(Int(f^{-1}(E))) \leq f^{-1}(\beta Cl(E))$ for every fuzzy subset $E$ of $Y$;*
*(9) $f(Cl(Int(A))) \leq \beta Cl(f(A)))$ for every fuzzy subset $A$ of $X$.*

*Proof.* $(1) \Leftrightarrow (2) \Leftrightarrow (3)$; $(4) \Rightarrow (5)$: Obvious
$(2) \Rightarrow (6)$: Let $V \in F\beta O(Y)$ and $x_t \in f^{-1}(V)$. By (2), there exists $U \in FPO(X)$ containing $x_t$ such that $f(U) \leq V$. Thus we have $x_t \in U \leq Int(Cl(U)) \leq Int(Cl(f^{-1}(V)))$ and hence $f^{-1}(V) \leq Int(Cl(f^{-1}(V)))$.
$(6) \Rightarrow (7)$: Let $H \in F\beta C(Y)$. Set $V = Y - H$, then $V \in F\beta O(Y)$. By (6) we obtain $f^{-1}(V) \leq Int(Cl(f^{-1}(V)))$ and hence $f^{-1}(H) = X - f^{-1}(Y - H) = X - f^{-1}(V) \in FPC(X)$.
$(7) \Rightarrow (8)$: Let $E$ be any fuzzy set of $Y$. Since $\beta Cl(E) \in F\beta C(Y)$, then $f^{-1}(\beta Cl(E)) \in FPC(X)$ and hence $Cl(Int(f^{-1}(\beta Cl(E)))) \leq f^{-1}(\beta Cl(E))$. Therefore we obtain
$Cl(Int(f^{-1}(E))) \leq f^{-1}(\beta Cl(E))$.
$(8) \Rightarrow (9)$: Let $A$ be any fuzzy set of $X$. by (8), we have $Cl(Int(A)) \leq Cl(Int(f^{-1}(f(A)))) \leq f^{-1}(\beta Cl(f(A)))$ and hence $f(Cl(Int(A))) \leq \beta Cl(f(A))$.
$(9) \Rightarrow (1)$: Let $V \in F\beta O(Y)$. Since $f^{-1}(Y - V) = X - f^{-1}(V)$ is a fuzzy set of $X$ and by (9), we obtain $f(Cl(Int(f^{-1}(Y - V)))) \leq \beta Cl(f(f^{-1}(Y - V))) \leq \beta Cl(Y - V) = Y - \beta Int(V) = Y - V$ and hence
$X - Int(Cl(f^{-1}(V))) = Cl(Int(X - f^{-1}(V)))) = Cl(Int(f^{-1}(Y - V)))$
$\leq f^{-1}(f(Cl(Int(f^{-1}(Y - V))))) \leq f^{-1}(Y - V) = X - f^{-1}(V)$. Therefore, we have $f^{-1}(V) \leq Int(Cl(f^{-1}(V)))$ and hence $f^{-1}(V) \in FPO(X)$. Thus, f is St-F$\beta$p-irresolute.
$(1) \Rightarrow (4)$: Let $x_t$ be a fuzzy point in $X$ and $V$ be any $\beta$-neighbourhood of $f(x_t)$, then there exists $G \in F\beta O(Y)$ such that, $f(x_t) \in G \leq V$. Now $f^{-1}(G) \in FPO(X)$ and $x_t \in f^{-1}(G) \leq f^{-1}(V)$. Thus $f^{-1}(V)$ is an preneighbourhood of $x_t$ in $X$.
$(5) \Rightarrow (2)$: Let $x_t$ be a fuzzy point in $X$ and $V \in F\beta O(Y)$ such that $f(x_t) \in V$. Then $V$ is $\beta$-neighbourhood of $f(x_t)$, so there is a preneighbourhood $A$ of $x_t$ such that $x_t \in A$, and $f(A) \leq V$. Hence there exists $U \in FPO(X)$ such that $x_t \in U \leq A$, and so $f(U) \leq f(A) \leq V$.

**Theorem 2.2.** *For a function* $f : X \to Y$*, the following are equivalent:*
*(1)* $f$ *is St-F*$\beta$*p-irresolute;*
*(2) For each fuzzy point* $x_t$ *of* $X$ *and every* $E \in F\beta O(Y)$ *such that* $f(x_t)qE$*, there exists* $A \in FPO(X)$ *such that* $x_tqA$ *and* $f(A) \leq E$*;*
*(3) For every fuzzy point* $x_t$ *of* $X$ *and every* $E \in F\beta O(Y)$ *such that* $f(x_t)qE$*, there exists* $A \in FPO(X)$ *such that* $x_tqA$ *and* $A \leq f^{-1}(E)$*.*

*Proof.* (1) $\Rightarrow$ (2) Let $x_t$ be a fuzzy point in $X$ and $E \in F\beta O(Y)$ such that $f(x_t)qE$. Then $f^{-1}(E) \in FPO(X)$, and $x_tqf^{-1}(E)$ by Lemma 1.1. If we take $A = f^{-1}(E)$ then $x_tqA$ and $f(A) = f(f^{-1}(E)) \leq E$.
(2) $\Rightarrow$ (3) Let $x_t$ be a fuzzy point in $X$ and $E \in F\beta O(Y)$ such that $f(x_t)qE$. Then by (2), there exists $A \in FPO(X)$ such that $x_tqA$ and $f(A) \leq E$. Hence we have $x_tqA$ and $A \leq f^{-1}(f(A)) \leq f^{-1}(E)$.
(3) $\Rightarrow$ (1) Let $E \in F\beta O(Y)$ and $x_t$ be a fuzzy point of $X$ such that $x_t \in f^{-1}(E)$. Then $f(x_t) \in E$. Choose the fuzzy point $x_t^c(x) = 1 - x_t(x)$. Then $f(x_t^c)qE$. And so by (3), there exists $A \in FPO(X)$ such that $x_t^cqA$ and $f(A) \leq E$. Now $x_t^cqA$ implies $x_t^c(x) + A(X) = 1 - x_t(x) + A(x) > 1$. It follows that $x_t \in A$. Thus $x_t \in A \leq f^{-1}(E)$. Hence $f^{-1}(E) \in FPO(X)$.

**Lemma 2.1.** *[1] Let* $g : X \to X \times Y$ *be the graph of a mapping* $f : X \to Y$*. If* $A$ *is a fuzzy set of* $X$ *and* $B$ *is a fuzzy of* $Y$*, then* $g^{-1}(A \times B) = A \cap f^{-1}(B)$

**Theorem 2.3.** *A mapping* $f : X \to Y$ *is St-F*$\beta$*p-irresolute if the graph mapping* $g : X \to X \times Y$*, is St-F*$\beta$*p-irresolute.*

*Proof.* Let $V$ be any Fβ-open set of $Y$, then by Lemma 2.1, $f^{-1}(V) = 1_X \cap f^{-1}(V) = g^{-1}(1_x \times V)$. Since $V$ is Fβ-open in $Y$, $1_X \times V$ is Fβ-open in $X \times Y$. Since $g$ is St-Fβp-irresolute $g^{-1}(1_x \times V) \in FpO(X)$ and hence $f^{-1}(V)$ is Fp-open in $X$ and consequently $f$ is St-Fβp-irresolute.

**Theorem 2.4.** *If* $f : X \to Y$ *is St-F*$\beta$*p-irresolute and* $g : Y \to Z$ *is M-fuzzy* $\beta$*-continuous, then* $g \circ f : X \to Z$ *is St-F*$\beta$*p-irresolute.*

*Proof.* Straightforward.

**Corollary 2.1.** *The composition of two St-F*$\beta$*p-irresolute mapping is St-F*$\beta$*p-irresolute.*

**Corollary 2.2.** *If* $f : X \to Y$ *is fuzzy strongly continuous and* $g : Y \to Z$ *is St-F*$\beta$*p-irresolute, then* $g \circ f : X \to Z$ *is St-F*$\beta$*p-irresolute.*

*Proof.* Obvious.

**Theorem 2.5.** *If* $f : X \to Y$ *is M-fuzzy* $\beta$*-continuous and* $g : Y \to Z$ *is St-F*$\beta$*p-irresolute, then* $g \circ f : X \to Z$ *is St-F*$\beta$*p-irresolute.*

**Theorem 2.6.** *Let* $\{X_i : i \in \Omega\}$ *be any family of fuzzy topological spaces. If* $f : X \to \prod X_i$ *is St-F*$\beta$*p-irresolute, then for each* $i \in \Omega$*,* $f_i : X \to X_i$ *is St-F*$\beta$*p-irresolute.*

*Proof.* Let $Pr_i$ be the projection of $\prod X_i$ onto $X_i$, we know that if a mapping is fuzzy continuous and fuzzy open, then it is M-fuzzy β-continuous [21]. So the mapping $Pr_i$ is M-fuzzy β-continuous. Now for each $i \in \Omega$, $f_i = Pr_i \circ f : X \to X_i$. It follows from Theorem 2.1 that $f_i$ is St-Fβp-irresolute since f is St-Fβp-irresolute.

# 3 Preservation of some fuzzy topological structure.

In this section preservation of some fuzzy topological structure under the St-Fβp-irresolute mapping are studied. Let us recall the definition: A space X is said to be fuzzy β-compact [4] if for every Fβ-open cover of X has a finite subcover, and X is fuzzy strongly compact [13] if for every Fp-open cover of X has a finite subcover.

**Theorem 3.1.** *Every surjective St-Fβp-irresolute image of a fuzzy strongly compact space is fuzzy β-compact.*

*Proof.* Let $f : X \to Y$ be St-Fβp-irresolute mapping of a fuzzy strongly compact space X onto a space Y. Let $\{G_i : i \in \Omega\}$ be any Fβ-open cover of Y. Then $\{f^{-1}(G_i) : i \in \Omega\}$ is a Fp-open cover of X. Since X is fuzzy strongly compact, there exist a finite subfamily $\{f^{-1}(G_{i_j}) : j = 1, 2, ..., n\}$ of $\{f^{-1}(G_i) : i \in \Omega\}$ which covers X. It follows that $\{G_{i_j} : j = 1, 2, ..., n\}$ is a finite subfamily of $\{G_i : i \in \Omega\}$ which covers Y. Hence Y is Fβ-compact.

**Theorem 3.2.** *Let $f : X \to Y$ be a St-Fβp-irresolute mapping. If A is a Fβ-connected subset of X, then f(A) is also Fβ-connected in Y.*

*Proof.* Suppose f(A) is not Fβ-connected in Y. Then there exist Fβ-separated subset G and H in Y, such that $f(A) = G \cup H$. Since G and H are Fβ-separated, there exist two Fβ-open, subset U and V such that $G \leq U$, $H \leq V$, $G\bar{q}V$ and $H\bar{q}U$. Now f being St-Fβp-irresolute so $f^{-1}(G)$ and $f^{-1}(H)$ are Fp-open in X. Thus Fβ-open in X and
$A = f^{-1}(f(A)) = f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$. It is easy to shows that $f^{-1}(G)$ and $f^{-1}(H)$ are Fβ-separated in X. Thus A is not Fβ-connected in X.

# 4 Conclusion.

Maps have always been of tremendous importance in all branches of mathematics and the whole science. On the other hand, topology plays a significant role in quantium physics, high energy physics and superstring theory [9, 10]. Thus we have obtained a new class of mappings called strongly Fβp-irresolute mappings between fuzzy topological spaces which are some generalized fuzzy continuity may have possible application in quantion physics, high energy physics and superstring theory.

Received: June 2009. Revised: August 2010.

## References

[1] Azad K.K., On fuzzy semicontinuity, fuzzy almost continuity and weakly continuity, J. Math. Anal. Appl., 82 (1981), 14-32.

[2] Balasubramanian G, Fuzzy β-open sets and fuzzy β-separation axioms, Kybernetica, 35 (1999), 215-223.

[3] Balasubramanian G, On fuzzy preseparation axioms, Bull. Cal. Math. Soc., 90 (1998), 427-434.

[4] Balasubramanian G, On fuzzy β-compact spaces and fuzzy β-extremely disconnected spaces, Kybernetica, 33 (1997), 271-278.

[5] Bin Shahna A.S., On fuzzy strong semicontinuity and fuzzy precontinuity, Fuzzy Sets and Systems, 44(1991), 303-308.

[6] Chang C.I., Fuzzy topological spaces, J. Math. Anal. Appl., 24 (1968), 182-190.

[7] El-Naschie M.S., On the uncertanity of cantorian geometry and the two-slit experiment, Chaos, Solitons and Fractals, 9(3)(1998), 517-529.

[8] El-Naschie M.S., On the certification of heterotic strings, M theory and $\varepsilon^{\infty}$ theory, Chaos, Solitons and Fractals, (2000), 2397-2408

[9] El-Naschie M.S., Wild topology, hyperbolic geometriy and fusion algebra of high energy particle physics, Chaos, Solitons and Fractals 13(2002), 1935-1945.

[10] El-Naschie M.S., Topics in the Mathematical physics of $\varepsilon^{\infty}$ theory, Chaos, Solitons and Fractals 30(3) (2006), 656-663.

[11] Hakeim K.M.A.E., Zeyada F.M and Allah M.A., Fuzzy completely weakly pre-irresolute functions, Bull. Cal. Math. Soc., 93(2) (2001), 121-128.

[12] Mukherjee M.N. and Ghosh B., Some stronger forms of fuzzy continuous mappings in fuzzy topological spaces, Fuzzy Sets and Systems, 36(1990), 375-387.

[13] Nanda S., Strongly compactness in fuzzy topological spaces, Fuzzy Sets and Systems, 42(1991), 259-268.

[14] Pu P.M. and Liu Y.M., Fuzzy topology I, neighbourhood structure of a fuzzy point and Moore Smith convergence, J. Math. Anal. Appl., 76(1989), 571-594.

[15] Roja E. and Balasubramanian G., On fuzzy $\beta$-$T_{1/2}$ spaces and its generalizations, Bull. Cal. Math. Soc., 94(6) (2002), 413-420.

[16] Saraf R.K. , Caldas M. and Navalagi G,, On strongly fuzzy α-preirresolute functions, Adv. in fuzzy Math. 3(2008), 19-25.

[17] Saraf R.K and Caldas M., On fuzzy β-preirresolute mappings, Kerala Math. Assoc. 4(2)(2007), 19-26.

[18] Saraf R.K, Caldas M. and Mishra S., Faα-irresolute mappings, Bull. Mal. Math. Sc. Soc., 24(2001), 193-199.

[19] Saraf R.K, Daleta A. and Mishra S., On Fαp-continuous mappings, Delta J. Sci. 25(2001), 31-35.

[20] Singal M.K. and Prakash N., Fuzzy preopen sets and fuzzy pre-separation axioms, Fuzzy sets And Systems, 44(1991), 273-281.

[21] Singh S., Generalization of certain fuzzy topological concepts, Ph.D. Dissertation (1999), R.D. University Jabalpur (M.P.).

[22] Yalvac T.H., Fuzzy sets and functions on fuzzy spaces, J. Math Anal. Appl., 126(1987), 409-423.

[23] Zadeh L.A., Fuzzy sets, Inform and control, 8(1965), 338-353.