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Isometric weighted composition operators on weighted Banach spaces of holomorphic functions defined on the unit ball of a complex Banach space

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ABSTRACT

Let X and Y be complex Banach spaces and B_X resp. B_Y the closed unit ball. Analytic maps $\phi : B_Y \to B_X$ and $\psi : B_X \to \mathbb{C}$ induce the weighted composition operator:

$$C_{\phi,\psi} : H(B_Y) \to H(B_X),\ f \mapsto \psi(f \circ \phi),$$

where $H(B_Y)$ resp. $H(B_X)$ denotes the collection of all analytic functions $f : B_X(\text{resp.}B_Y) \to \mathbb{C}$. We study when such operators acting between weighted spaces of analytic functions are isometric.

RESUMEN

Sea X y Y espacios de Banach complejos, B_X y B_Y las bolas unitarias cerradas correspondientes. Las aplicaciones analíticas $\phi : B_Y \to B_X$ y $\psi : B_X \to \mathbb{C}$ inducen el operador de composición con pesos:

$$C_{\phi,\psi} : H(B_Y) \to H(B_X),\ f \mapsto \psi(f \circ \phi),$$

donde $H(B_Y)$ y $H(B_X)$ denotan la colección de todas las funciones analíticas $f : B_X(\text{resp.}B_Y) \to \mathbb{C}$. Estudiamos cuándo dichos operadores que actúan entre los espacios con peso de funciones analíticas son isométricas.

Keywords and Phrases: weighted composition operators, weighted spaces of holomorphic functions on the unit ball of a complex Banach space.

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1 Introduction

Let $\mathbb{D}$ denote the open unit disk in the complex plane and $H(\mathbb{D})$ the collection of all analytic functions on $\mathbb{D}$. Then, an analytic self-map ϕ of $\mathbb{D}$ induces through composition a linear composition operator

$$C_\phi : H(\mathbb{D}) \to H(\mathbb{D}),\ f \mapsto f \circ \phi.$$

Since such operators appear naturally in a variety of problems and since they link - in the classical setting of the Hardy space H^2 (see [10] and [24]) - operator theoretical questions with classical results in complex analysis their study has a long and rich history. Now, let $\psi \in H(\mathbb{D})$. The next step is to combine the composition operator C_ϕ with a multiplication operator $M_\psi : H(\mathbb{D}) \to H(\mathbb{D}),\ f \mapsto \psi f$ to obtain the so-called *weighted composition operator*

$$C_{\phi,\psi} := M_\psi C_\phi : H(\mathbb{D}) \to H(\mathbb{D}),\ f \mapsto \psi(f \circ \phi).$$

For a bounded and continuous function (*weight*) $\nu : \mathbb{D} \to (0, \infty)$ we consider

$$H_\nu^\infty := \{f \in H(\mathbb{D});\ \|f\|_\nu := \sup_{z\in\mathbb{D}} \nu(z)|f(z)| < \infty\}.$$

Endowed with norm $\|.\|_\nu$, these spaces are Banach spaces and in the sequel we refer to them as *weighted Banach spaces of holomorphic functions*. Such spaces arise in functional analysis, partial differential equations and convolution equations as well as in distribution theory. They have been studied intensively in several articles, see e.g. [1], [2], [3], [4], [18], [19].

In [6] Bonet, Domański, Lindström and Taskinen characterized boundedness and compactness of operators

$$C_\phi : H_\nu^\infty \to H_w^\infty,\ f \mapsto f \circ \phi$$

in terms of the inducing symbol ϕ as well as the involved weights ν and w. The same properties of the weighted composition operator $C_{\phi,\psi} : H_\nu^\infty \to H_w^\infty$ were analyzed independently by Contreras and Hernández-Díaz as well as Montes-Rodríguez. In [8] we investigated under which conditions the weighted composition operator $C_{\phi,\psi}$ acting on H_ν^∞ is isometric. The work of Bonet, Domański, Lindström and Taskinen motivated Garcia, Maestre and Sevilla-Peris to study boundedness and compactness of composition operators in the following setting.

Let X be a complex Banach space, B_X its open unit ball and $H(B_X)$ the collection of all holomorphic functions $f : B_X \to \mathbb{C}$. Moreover, we consider continuous and bounded functions $\nu : B_X \to (0, \infty)$. Such a map is called a *weight*. A weight ν induces the space

$$H_\nu(B_X) := \left\{ f \in H(B_X);\ \|f\|_\nu = \sup_{x\in B_X} \nu(x)|f(x)| < \infty \right\}$$

which, endowed with the weighted sup-norm $\|.\|_\nu$ is a Banach space as in the onedimensional case. Now, an analytic map $\phi : B_Y \to B_X$ induces an operator

$$C_\phi : H(B_Y) \to H(B_X),\ f \mapsto f \circ \phi.$$

Garcia, Maestre and Sevilla-Peris, characterized when an operator

$$C_\phi : H_v(B_Y) \to H_w(B_X),\; f \mapsto f \circ \phi$$

is bounded and compact, i.e. they gave sufficient and necessary conditions in terms of the inducing map ϕ as well as of the involved weights v and w for a composition operator to be bounded resp. compact.
In this article we are interested in weighted composition operators

$$C_{\phi,\psi} : H_v(B_Y) \to H_w(B_Y),\; f \mapsto \psi(f \circ \phi).$$

Motivated by [14] we will investigate when such an operator is bounded. A full characterization when such an operator is bounded follows easily with a similar proof as given in [14]. The more interesting question (motivated by [8]) is the following: When is a bounded operator $C_{\phi,\psi}$ acting on $H_v(B_X)$ an isometry.

2 Basics on weights and weighted spaces

This section is devoted to collect some basic facts on weights and weighted spaces in the setting of a complex Banach space X and its open unit ball B_X. These can be found in [13] and [14]. We say that a set $A \subset B_X$ is B_X-*bounded* if there exists $0 < r < 1$ such that $A \subset rB_X$. We write

$$H_b(B_X) = \{f \in H(B_X);\; f \text{ bounded on the } B_X\text{-bounded sets }\}.$$

We consider

$$H_v(B_X) = \left\{ f \in H(B_X);\; \|f\|_v := \sup_{x \in B_X} v(x)|f(x)| < \infty \right\}.$$

With the norm $\|.\|_v$, the space $H_v(B_X)$ is a Banach space. A weight v is *radial* if $v(\lambda x) = v(x)$ for every $\lambda \in \mathbb{C}$ with $|\lambda| = 1$ and every $x \in B_X$.
A weight v satisfies Condition I if $\inf_{x \in rB_X} v(x) > 0$ for every $0 < r < 1$. If v satisfies Condition I, then $H_v(B_X) \subset H_b(B_X)$. If X is finite-dimensional, then all weights on B_X enjoy Condition I. In the sequel we will assume that each weight v satisfies the Condition I.
Given any weight v we consider

$$\tilde{v}(z) = \frac{1}{\sup\{|f(z)|; \|f\|_v \le 1\}}.$$

By [14] Proposition 1.1 the following hold:

(1) $0 < v \le \tilde{v}$ and $\tilde{v}$ is bounded and continuous, i.e. $\tilde{v}$ is a weight.

(2) $\tilde{v}$ is radial and decreasing whenever v is so.

(3) $\|f\|_v \le 1 \iff \|f\|_{\tilde{v}} \le 1$.

(4) For every $x \in B_X$ there is $f_x \in H_v^\infty$ with $\|f\|_v \leq 1$ such that $\tilde{v}(x) = |f_x(x)|$.

We say that a weight v is *norm-radial* if $v(x) = v(y)$ for every x, y with $\|x\| = \|y\|$. We need some extra condition on the weight -which in a sense - is an analogon to the Lusky condition $(L1)$ which appeared during his studies on the isomorphism classes of H_v^∞, see [18]. Let v be a norm-radial weight that is continuously differentiable w.r.t. x. Then we say that v satisfies condition (B) if and only if

$$(B) \quad \sup_{x \in B_X} \frac{(1 - \|x\|)|v'(x)|}{v(x)} < \infty.$$

Finally, to study isometries we need some geometric tools. The generalized pseudohyperbolic distance of two points $z, p \in B_X$ is given by

$$d(z,p) := \sup\{\rho(h(z), h(p));\ h : B_X \to \mathbb{D} \text{ holomorphic }\}$$

3 Boundedness

As we said before the following proof is very similar to the proof of Proposition 2.3 in [14]. Nevertheless we give it here for the sake of completeness.

Proposition 3.1. *Let* v, w *be two weights and* $\phi : B_X \to B_Y$ *be holomorphic. Moreover, let* $\psi \in H(B_X)$*. Then the following are equivalent:*

(a) $C_{\phi,\psi} : H_v(B_Y) \to H_w(B_X)$ *is well-defined and bounded.*

(b) $\sup_{x \in B_X} \frac{w(x)|\psi(x)|}{\tilde{v}(\phi(x))} < \infty$.

Proof. Let us first suppose that the operator is bounded. We assume to the contrary that (b) does not hold. Then we can find a sequence $(x_n)_n \subset B_X$ such that

$$\frac{w(x_n)|\psi(x_n)|}{\tilde{v}(\phi(x_n))} \geq n \text{ for every } n \in \mathbb{N}.$$

Now, for each $n \in \mathbb{N}$ we can select $f_n \in H_v(B_X)$ with $\|f_n\|_v \leq 1$ such that $|f_n(\phi(x_n))| = \frac{1}{\tilde{v}(\phi(x_n))}$. Since $C_{\phi,\psi} : H_v(B_Y) \to H_w(B_X)$ is bounded, there is $C > 0$ such that

$$C \geq \|C_{\phi,\psi} f_n\|_w \geq \frac{w(x_n)|\psi(x_n)|}{\tilde{v}(\phi(x_n))} \geq n$$

for every $n \in \mathbb{N}$, which is a contradiction. Conversely, let $f \in H_v(B_Y)$. Then we obtain for every $x \in B_X$

$$w(x)|\psi(x)||f(\phi(x))| = \frac{|\psi(x)|w(x)}{\tilde{v}(\phi(x))}\tilde{v}(\phi(x)) \leq M\|f\|_{\tilde{v}} = M\|f\|_v.$$

Thus, the claim follows. □

4 Isometries

We obtain the following lemma which was shown for the setting of the spaces H_v^∞ in [7]. However, in this setting there occur several different phenomena.

Lemma 4.1. *Let* v *be a weight on* B_X *such that* v *is norm-radial and satisfies condition (B). Moreover, let* $f \in H_v^\infty$. *Then there is a finite constant* $M > 0$ *independent of* $f \in H_v^\infty$ *such that*

$$|v(a)f(a) - v(b)f(b)| \leq M\|f\|_v d(a,b)$$

for every $a, b \in B_X$.

Proof. We fix $a, b \in B_X$ with $a \neq b$. Now, there are $n_1, n_2 \in \mathbb{N}$ such that

$$\|a\|_X < 1 - \frac{1}{n_1} \text{ and } \|b\|_X < 1 - \frac{1}{n_2}.$$

Then we can find $\varepsilon > 0$ such that

$$h : \mathbb{D} \to B_X,\ h(t) = (t-\varepsilon)b + (1-(t-\varepsilon))a.$$

Moroever $h(\varepsilon) = a$ and $h(1-\varepsilon) = b$. Now, by Cauchy's formula we obtain

$$\begin{aligned} |(f \circ h)'(\varepsilon)| &= \frac{1}{2\pi}\left|\int_{|\xi-\varepsilon|=(1-|\varepsilon|)r} \frac{(f\circ h)(\xi)}{|\xi-\varepsilon|}\, d\xi\right| \\ &\leq \frac{1}{2\pi r}\frac{1}{(1-|\varepsilon|)^2}\|f\|_v \int_{|\xi-\varepsilon|=(1-|\varepsilon|)r} \frac{|d\xi|}{v(h(\xi))}. \end{aligned}$$

Now, since $v(a) < M$ and $\|h(\xi)\|_X \leq r_0 < 1$ for every ξ with $|\xi - \varepsilon| = (1-|\varepsilon|)r$. Hence there is $C > 0$ such that

$$\frac{v(a)}{v(h(\xi))} = \frac{v(h(\varepsilon))}{v(h(\xi))} \leq C$$

for every ξ with $|\xi - \varepsilon| = (1-|\varepsilon|)r$. Thus,

$$\begin{aligned} |(f \circ h)'(\varepsilon)| &\leq \frac{C}{2\pi r^2}\frac{1}{(1-|\varepsilon|)^2}\frac{\|f\|_v}{v(h(\varepsilon))}2\pi(1-|\varepsilon|)r \\ &= \frac{C\|f\|_v}{r(1-\varepsilon)v(h(\varepsilon))}. \end{aligned}$$

Next, we consider

$$k(q) := v(q)f(q) \text{ for every } q \in B_X.$$

Then the total differential of $k \circ h$ is given by

$$d(k \circ h) = \frac{\partial(k\circ h)}{\partial t}dt + \frac{\partial(k\circ h)}{\partial \bar{t}}d\bar{t}.$$

Now, for every $t \in \mathbb{D}$ we obtain

$$\frac{\partial(k \circ h)}{\partial t} = (\nu \circ h)'(t)f(h(t)) + \nu(h(t))(\nu \circ h)'(t)$$

and

$$\frac{\partial(k \circ h)}{\partial \bar{t}} = 0$$

This yields

$$\begin{aligned} |d(k \circ h)(t)| &\leq [|(\nu \circ h)'(t)|f(h(t))| + |\nu(h(t))||(f \circ h)'(t)|] \, |dt| \\ &\leq \left[|\frac{(\nu \circ h)'(t)}{\nu(h(t))} \|f\|_\nu + \frac{C\|f\|_\nu}{r(1-|t|)\nu(h(t))} \right] |dt| \end{aligned}$$

By condition (B) we can find $C_1 > 0$ such that

$$\frac{|\nu'(h(t))|}{\nu(h(t))} \|b - a\| = \frac{|(\nu \circ h)'(t)|}{(\nu \circ h)(t)|} \leq \frac{C_1}{1 - |h(t)|}.$$

Therefore

$$|d(k \circ h)(t)| \leq \left(C_1 + \frac{C}{r} \right) \frac{\|f\|_\nu}{1 - |t|} |dt|.$$

If $d(h(p), h(q)) \leq r$, then $\rho(p, q) \leq r$ and by using

$$1 - \rho(p, q)^2 = \frac{(1 - |q|^2)(1 - |p|^2)}{|1 - \overline{p}q|^2}$$

we have that

$$\frac{|q - p|}{1 - |p|} \sim \rho(p, q).$$

Here the constants only depend on r. By integration on both sides we can find constants $C_2, C_3 > 0$ with

$$|k(h(q)) - k(h(p))| \leq C_2 \|f\|_\nu \frac{1}{1 - |p|} |q - p| \leq C_3 \|f\|_\nu \rho(p, q) \leq C_3 \|f\|_\nu d(h(p), h(q))$$

for all p, q with $d(h(p), h(q)) \leq r$. If $d(h(p), h(q)) > r$ then

$$|\nu(p)f(p) - \nu(q)f(q)| \leq 2\|f\|_\nu \leq \frac{2}{r} \|f\|_\nu d(p, q)$$

and the claim follows. □

The main ideas of the proof of the following theorem are taken from [8] but there also occur new phenomena.

Theorem 4.2. *Let* ϕ *be an analytic self-map of* B_X *and* $\psi \in H(B_X)$*. Moreover, assume that* ν *is a norm-radial weight satisfying condition* (B) *such that* ν *is continuously differentiable.*

(a) If $\sup_{x\in B_X} \frac{|\psi(x)|v(x)}{\tilde{v}(\phi(x))} \le 1$ *and*

$$(M) \quad \text{for every } a \in B_X \text{ there is } (x_n)_n \subset B_X \text{ such that } d(\phi(x_n), a) \to 0 \text{ and } \frac{|\psi(x_n)|v(x_n)}{\tilde{v}(\phi(x_n))} \to 1$$

then $C_{\phi,\psi} : H_v(B_X) \to H_v(B_X)$ *is an isometry.*

(b) Let v *be a norm-radial weight with* $v = \tilde{v}$ *such that for each* $h : B_X \to \mathbb{D}$ *holomorphic* $w(x) := \frac{v(x)}{1-|h(x)|^2)^p}$ *for every* $x \in B_X$ *is a weight for some* $0 < p < \infty$ *and* $w = \tilde{w}$*. If* $C_{\phi,\psi} : H_v(B_X) \to H_v(B_X)$ *is an isometry, then condition* (M) *holds and* $\sup_{x\in B_X} \frac{|\psi(x)|v(x)}{\tilde{v}(\phi(x))} \le 1$.

Proof. We first show (a). For every $f \in H_v(B_X)$ we have that

$$\|C_{\phi,\psi}\|_v = \sup_{z\in B_X} \frac{|\psi(x)|v(x)}{v(\phi(x))} v(\phi(x))|f(\phi(x))| \le \|f\|_v.$$

Now, let $f \in H_v(B_X)$. Then $\|f\|_v = \lim_{m\to\infty} v(a_m)|f(a_m)|$ for some sequence $(a_m)_m$. Let $m \in \mathbb{N}$ be fixed. Hence, by condition (M), there is $(x_n^m)_n \subset B_X$ such that $d(\phi(x_n^m), a_m) \to 0$ and $\frac{|\psi(x_n^m)|v(x_n^m)}{v(\phi(x_n^m))} \to 1$ when $n \to \infty$. By the previous lemma, for all m and n

$$|v(a_m)f(a_m) - v(\phi(x_n^m))f(\phi(x_n^m))| \le M\|f\|_v d(a_m, \phi(x_n^m)).$$

Hence

$$\|C_{\phi,\psi}\|_v = \sup_{x\in B_X} \frac{|\psi(x)|v(x)}{v(\phi(x))} (|f(a_m)|v(a_m) - M\|f\|_v d(\phi(x_n^m), a_m)) = v(a_m)|f(a_m)|.$$

Since this is true for all m, we have $\|C_{\phi,\psi} f\|_v \ge \|f\|_v$.
Next, we show (b). We choose $p > 0$ and fix $h : B_X \to \mathbb{D}$ holomorphic such that $w(x) = \frac{v(x)}{(1-|h(x)|^2)^p}$ is a weight on B_X with $w = \tilde{w}$. By assumption $\|C_{\phi,\psi} f\|_v = \|f\|_v$ for all $f \in H_v(B_X)$. Thus,

$$\|C_{\phi,\psi}\| = \sup_{x\in B_X} \frac{|\psi(x)|v(x)}{\tilde{v}(\phi(x))} \le 1.$$

Next, fix $a \in B_X$ and $h : B_X \to \mathbb{D}$. Then there exists $g_a \in H_w(B_X)$ with $\|g_a\|_w \le 1$ such that $g_a(a) = \tilde{w}(a)$. Put

$$f_a(z) = g_a(z) \left(\frac{(1 - |h(a)|^2)}{(1 - h(z)\overline{h(a)})^2} \right)^p.$$

Now, $\|f_a\|_v = 1$ since $|f_a(a)|v(a) = 1$. This means, that we can pick a sequence $(x_n)_n \subset B_X$ so that $|\psi(x_n)|f_a(\phi(x_n))|v(x_n) \to 1$ when $n \to \infty$. Hence

$$1 \ge \frac{|\psi(x_n)|v(x_n)}{\tilde{v}(\phi(x_n))} \ge \frac{|\psi(x_n)|v(x_n)}{\tilde{v}(\phi(x_n))} |f_a(\phi(x_n))|\tilde{v}(\phi(x_n)) = |\psi(x_n)|v(x_n)|f_a(\phi(x_n))|.$$

Finally,

$$\lim_{n\to\infty} \frac{|\psi(x_n)|v(x_n)}{\tilde{v}(\phi(x_n))} = 1.$$

Further,

$$1 \geq (1 - |\sigma_{h(a)}(h(\phi(z_n))|^2)^p = \frac{(1-|h(a)|^2)^p(1-|h(\phi(x_n))|^2)^p}{|1-h(\phi(x_n))\overline{h(a)}|^{2p}}$$
$$= \frac{|f_a(\phi(x_n))|v(\phi(x_n))(1-|h(\phi(x_n))|^2)^p}{g_a(h(\phi(x_n)))v(\phi(x_n))} \geq |f_a(\phi(x_n))|v(\phi(x_n)).$$

Since, $|f_a(\phi(x_n))|\tilde{v}(\phi(x_n)) \to 1$ when $n \to \infty$, we conclude, as $v = \tilde{v}$, that $\lim_{n\to\infty}(1 - \sigma_{h(a)}(h(\phi(x_n))|^2)^p = 1$ and $\rho(h(\phi(x_n)), h(a)) \to 0$ when $n \to \infty$. Since $h : B_X \to \mathbb{D}$ holomorphic was arbitrary the claim follows. □

Example 4.3. *Let* X *be an arbitrary complex Banach space,* $h : B_X \to \mathbb{D}$ *be holomorphic and select* $v(x) = (1 - |h(x)|^2)^p$*. For fixed* $b \in B_X$ *we put* $\phi(x) := \sigma_{h(b)}(h(x))$ *and* $\psi(x) := (\sigma_{h(b)})'(h(x))$ *for every* $x \in B_X$*. Then easy calculations show that the corresponding weighted composition operator is an isometry.*

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