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On Weak concircular Symmetries of Lorentzian Concircular Structure Manifolds

Dhruwa Narain
D.D.U.Gorakhpur University,
Department of Mathematics & Statistics,
Gorakhpur, India.
profdndubey@yahoo.co.in

Sunil Yadav
Alwar Institute of Engineering & Technology,
Department of Mathematics,
Matsya Industrial Area, Alwar-301030,
Rajasthan, India.
prof_sky16@yahoo.com

ABSTRACT

The object of the present paper is to study weakly concircular symmetric, weakly concircular Ricci symmetric and special weakly concircular Ricci symmetric Lorentzian concircular structure manifolds.

RESUMEN

El objetivo del presente artículo es estudiar las variedades de estructura simétricas concirculares débiles, las simétricas Ricci concirculares débiles y concirculares Lorentzianas simétricas Ricci concirculares débiles especiales.

Keywords and Phrases: Weakly concircular symmetric manifold, weakly concircular Ricci symmetric manifold, concircular Ricci tensor, special weakly concircular Ricci symmetric and Lorentzian concircular structure manifold.

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1 Introduction

The notion of weakly symmetric manifolds was introduced by Tamassy and Binh [8].A non-flat Riemannian manifold (M^n, g) $(n > 2)$ is called *weakly symmetric* manifold if its curvature tensor R of type $(0, 4)$ satisfies the condition

$$\begin{aligned}(\nabla_X R)(Y,Z,U,V) &= A(X)R(Y,Z,U,V) + B(Y)R(X,Z,U,V)\\ &+H(Z)R(Y,X,U,V) + D(U)R(Y,Z,X,V) + E(V)R(Y,Z,U,X)\end{aligned}, \tag{1.1}$$

for all vector fields $X, Y, Z, U, V \in \chi(M^n)$, $\chi(M)$ being the Lie-algebra of the smooth vector fields of M, where A, B, H, D and E are $1-$forms (not simultaneously zero) and ∇ denote the operator of the covariant differentiations with respect to Riemannian metric g. The $1-$forms are called the associated $1-$forms of the manifold and $n-$dimensional manifold of this kind is denoted by $(WS)_n$.

In 1999, De and Bandyopadhyay [2] studied a $(WS)_n$ and prove that in such a manifold the associated 1–forms $B = H$ and $D = E$. Hence from (1.1) reduces to the following:

$$\begin{aligned}(\nabla_X R)(Y,Z,U,V) &= A(X)R(Y,Z,U,V) + B(Y)R(X,Z,U,V)\\ &+B(Z)R(Y,X,U,V) + D(U)R(Y,Z,X,V) + D(V)R(Y,Z,U,X).\end{aligned} \tag{1.2}$$

A transformation of n–dimensional Riemannian manifold M, which transform every geodesic circle of M into a geodesic circle, is called a *concircular transformation* [11].The intersecting invariant of a concircular transformation is the concircular curvature tensor $\tilde{C}$ which is defined by [11].

$$\tilde{C}\,(Y,Z,U,V) = R\,(Y,Z,U,V) - \frac{k}{n\,(n-1)}\,[\,g\,(Z,U)g(Y,V) - g(Y,U)\,g(Z,V)]\,, \tag{1.3}$$

where k is the scalar curvature of the manifold.

Recently Shaikh and Hui [5] introduced the notion of weakly concircular symmetric manifolds. A Riemannian manifold is called *weakly concircular symmetric* manifold if its concircular curvature tensor $\tilde{C}$ of type $(0, 4)$ is not identically zero and satisfies the condition

$$\begin{aligned}\left(\nabla_X \tilde{C}\right)(Y,Z,U,V) &= A\,(X)\,\tilde{C}(Y,Z,U,V) + B\,(Y)\,\tilde{C}(X,Z,U,V)\\ &+H(Z)\tilde{C}(Y,X,U,V) + D\,(U)\,\tilde{C}(Y,Z,X,V) + E\,(V)\,\tilde{C}(Y,Z,U,X)\end{aligned}, \tag{1.4}$$

for all vector fields $X, Y, Z, U, V \in \chi(M^n)$ where A, B, H, D and E are 1–form (not simultaneously zero) an n–dimensional manifold of this kind is denoted by $\left(W\,\tilde{C}\,S\right)_n$ .Also it is known that [5], in a $\left(W\tilde{C}S\right)_n$ the associated 1–forms $B = H$ and $D = E$, and hence the defining the condition (1.4) of a $\left(W, \tilde{C}S\right)_n$ reduces to the following form:

$$\begin{aligned}\left(\nabla_X \tilde{C}\right)(Y,Z,U,V) &= A\,(X)\,\tilde{C}(Y,Z,U,V) + B\,(Y)\,\tilde{C}(X,Z,U,V)\\ &+B\,(Z)\,\tilde{C}(Y,X,U,V) + D\,(U)\,\tilde{C}(Y,Z,X,V) + D\,(V)\,\tilde{C}(Y,Z,U,X)\end{aligned}, \tag{1.5}$$

where A, B and D are 1-forms (not simultaneously zero).

Again Tamassy and Binh [9] introduced the notion of weakly Ricci symmetric manifolds. A Riemannian manifold $(M^n, g), (n > 2)$ is called *weakly Ricci symmetric* manifold if its Ricci tensor S of type$(0, 2)$ is not identically zero and satisfies the condition:

$$(\nabla_X S)(Y, Z) = A(X)S(Y, Z) + B(Y)S(X, Z) + D(Z)S(Y, X), \tag{1.6}$$

where A, B and D are three non-zero 1–forms called the associate 1-forms of the manifold, and ∇ is the operator of covariant differentiation with respect to metric g. Such n–dimensional manifold is denoted by $(WRS)_n$. If $A = B = D$ then is called *pseudo Ricci symmetric.*

Let$\{e_i : i = 1, 2, ...n\}$ be an orthonormal basis of the tangent space at each point of the manifold and let

$$\tilde{S}(Y, V) = \sum_{i=1}^{n} \tilde{C}(Y, e_i, e_i, Y)$$

Then from (1.3), we have

$$\tilde{S}(Y, V) = S(Y, V) - \frac{k}{n} g(Y, V), \tag{1.7}$$

The tensor $\tilde{S}$ is called the *concircular Ricci symmetric tensor* which is symmetric tensor of type $(0, 2)$. In [1] De and Ghose introduced the notion of weakly concircular Ricci symmetric manifolds. A Riemannian manifold $(M^n, g), (n > 2)$ is called *weakly concircular Ricci symmetric* manifolds [1] if its concircular Ricci tensor $\tilde{S}$ of type $(0, 2)$ is not identically zero satisfies the condition:

$$\left(\nabla_X \tilde{S}\right)(Y, Z) = A(X)\tilde{S}(Y, Z) + B(Y)\tilde{S}(X, Z) + D(Z)\tilde{S}(Y, X), \tag{1.8}$$

where$A$, B and D are three 1-form (not simultaneously zero).If $A = B = D$ then M^n is called *pseudo concircular Ricci symmetric.* A Riemannian manifold is called *special weakly Ricci symmetric* manifold if

$$(\nabla_X S)(Y, Z) = 2A(X)S(Y, Z) + A(Y)S(X, Z) + A(Z)S(Y, X), \tag{1.9}$$

where A is a 1–form and is defined by

$$A(X) = g(X, \rho). \tag{1.10}$$

where ρ is the associated vector field.

Motivated by above studied we define and syudy special weakly concircular Ricci symmetric manifold. An n–dimensional Riemannian manifold is called *special weakly concircular Ricci symmetric* manifolds. If

$$\left(\nabla_X \tilde{S}\right)(Y, Z) = 2A(X)\tilde{S}(Y, Z) + A(Y)\tilde{S}(X, Z) + A(Z)\tilde{S}(Y, X). \tag{1.11}$$

where A is a 1–form and is defined by (1.10).

An $(2n+1)$-dimensional Lorentzian manifold M is smooth connected Para contact Hausdorff manifold with Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to R$ is a non degenerate inner product of signature $(-,+,.....+)$where $T_pM$denotes the tangent space of M at p and R is the real number space. In a Lorentzian manifold (M,g) a vector field ρ defined by

$$g(X,\rho) = A(X)$$

for any vector field $X \in \chi(M)$ is said to be *concircular vector field* [5] if

$$(\nabla_X A)(Y) = \alpha\,[g(X,Y) + \omega(X)A(Y)]$$

where α is a non zero scalar function, A is a 1-form and ω is a closed 1-form.

Let M be a Lorentzian manifold admitting a unit time like concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi,\xi) = -1, \tag{1.12}$$

Since ξ is the unit concircular vector field, there exist a non zero 1-form η such that

$$g(X,\xi) = \eta(X), \tag{1.13}$$

the equation (1.13) of the following form holds:

$$(\nabla_X \eta)(Y) = \alpha\,[g(X,Y) + \eta(X)\eta(Y)] \qquad (\alpha \neq 0), \tag{1.14}$$

for all vector field X,Y, where ∇denotes the operator of covariant differentiation with respect to Lorentzian metric g and α is a non zero scalar function satisfying

$$(\nabla_X \alpha) = (X\alpha) = \rho\eta(X), \tag{1.15}$$

where ρ being a scalar function. If we put

$$\phi X = \frac{1}{\alpha}\nabla_X \xi, \tag{1.16}$$

Then from (1.14) and (1.16), we have

$$\phi^2 X = X + \eta(X)\xi, \tag{1.17}$$

from which it follows that ϕ is a symmetric $(1,1)$-tensor. Thus the Lorentzian manifold M together with unit time like concircular vector field ξ, it's associate 1-form η and $(1,1)$–tensor field ϕ is said to be *Lorentzian concircular structure manifolds* (briefly $(LCS)_{2n+1}$-manifold) [6]. In particular if $\alpha = 1$, then the manifold becomes LP-Sasakian structure of Matsumoto [3].

2 Lorentzian Concircular Structure manifolds

A differentiable manifold M of dimension $(2n+1)$ is called $(LCS)_{2n+1}$-manifold if it admits a $(1,1)$ -tensor ϕ, a contravarient vector field ξ, a covariant vector field η and a Lorentzian metric g which satisfy the following

$$\eta(\xi) = -1, \tag{2.1}$$

$$\phi^2 = I + \eta \otimes \xi, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X, \xi) = \eta(X), \tag{2.4}$$

$$\phi\xi = 0 \quad , \eta(\phi X) = 0, \tag{2.5}$$

for all $X, Y \in TM$. Also in a $(LCS)_{2n+1}$-manifold the following relations are satisfied [7].

$$\eta(R(X, Y)Z) = (\alpha^2 - \rho)\left[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)\right], \tag{2.6}$$

$$R(X, Y)\xi = (\alpha^2 - \rho)\left[\eta(Y)X - \eta(X)Y\right], \tag{2.7}$$

$$R(\xi, X)Y = (\alpha^2 - \rho)\left[g(X, Y)\xi - \eta(Y)X\right], \tag{2.8}$$

$$R(\xi, X)\xi = (\alpha^2 - \rho)\left[\eta(X)\xi + X\right], \tag{2.9}$$

$$(\nabla_X \phi)(Y) = \alpha\left[g(X, Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\right], \tag{2.10}$$

$$S(X, \xi) = 2n(\alpha^2 - \rho)\eta(X), \tag{2.11}$$

$$S(\phi X, \phi Y) = S(X, Y) + 2n(\alpha^2 - \rho)\eta(X)\eta(Y), \tag{2.12}$$

Definition 2.1 *A Lorentzian concircular structure manifold is said to be η–Einstein if the Ricci operator Q satisfies*

$$Q = a Id \ + \ b\eta \otimes \xi,$$

where a and b are smooth functions on the manifolds, In particular if $b = 0$, then M is an Einstein manifold.

3 Main Results

Definition 3.1 *A Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *is said to be weakly concircular symmetric if its concircular curvature tensor* $\tilde{C}$ *of type* $(0, 4)$ *satisfies (1.5)*

Substituting $Y = V = e_i$ in (1.5) and taking summation over i, $1 \leq i \leq 2n + 1$, we get

$$\begin{aligned}&(\nabla_X S)(Z, U) - \tfrac{d\kappa(X)}{n} g(Z, U) = A(X)\left[S(Z, U) - \tfrac{\kappa}{n} g(Z, U)\right] + B(Z)\,[S(X, U)\\&- \tfrac{\kappa}{n} g(X, U) + D(U)\left[S(X, Z) - \tfrac{\kappa}{n} g(X, Z)\right] + B(R(X, Z)U) + D(R(X, U)Z)\\&- \tfrac{\kappa}{n(n-1)}\,[\{B(X) + D(X)\; g(Z, U) - B(Z)g(X, U) - D(U)g(Z, X)]\end{aligned} \tag{3.1}$$

Again setting $X = Z = U = \xi$ in (3.1) and using (2.7)and (2.11), we have

$$A(\xi) + B(\xi) + D(\xi) = \frac{dk(\xi)}{k - 2n^2(\alpha^2 - \rho)} \tag{3.2}$$

This leads to the following result.

Theorem 3.1. *In a weakly concircular symmetric Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *the relation (3.2) holds.*

Corollary 3.1 *In a weakly concircular symmetric Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *the sum of 1-forms* A, B *and* D *is zero everywhere if and only if the scalar curvature* κ *of the manifold is constant.*

Next, putting X and Z by ξ in (3.1) and using (2.4), (2.7)and (2.11) we obtain

$$\begin{aligned}&(\nabla_X S)(\xi, U) - \tfrac{d\kappa(\xi)}{n}\eta(U) = [A(\xi) + B(\xi)]\left\{2n(\alpha^2 - \rho) - \tfrac{\kappa}{n}\right\}\eta(U) +\\&\left[\tfrac{\kappa}{n} - 2n(\alpha^2 - \rho) - \tfrac{\kappa}{n(n-1)} + 1\right] D(U) \; + D(\xi)\left[(\alpha^2 - \rho) - \tfrac{\kappa}{n(n-1)}\right]\eta(U).\end{aligned} \tag{3.3}$$

Also from (2.11), we have

$$(\nabla_\xi S)(\xi, U) = 0. \tag{3.4}$$

In view of (3.2) and (3.4), equation (3.3) reduces to

$$D(U) = \left[\frac{k + (n-1)\left\{k - 2n^2(\alpha^2 - \rho)\right\}}{-k + (n-1)\{k - 2n^2(\alpha^2 - \rho)\}}\right] D(\xi)\,\eta(U). \tag{3.5}$$

Next setting $X = U = \xi$ in (3.1) and proceeding in the similar manner as above, we have

$$B(Z) = \left[\frac{k - (n-1)\left\{k - 2n^2(\alpha^2 - \rho)\right\}}{-k + (n-1)\{k - 2n^2(\alpha^2 - \rho)\}}\right] B(\xi)\,\eta(Z), \tag{3.6}$$

Again, substituting $Z = U = \xi$, (3.1), we obtain

$$\begin{aligned}(\nabla_X S)(\xi,\xi) + \frac{d\kappa(X)}{n} &= A(X)\left[\frac{\kappa}{n} - 2n(\alpha^2-\rho)\right] + \left[\frac{\kappa}{n(n-1)} - (\alpha^2-\rho)\right] \\ &\{B(X)+D(X)+(B(\xi)+D(\xi))\eta(X)\} + \left[2n(\alpha^2-\rho) - \frac{\kappa}{n}\right]\{B(\xi)+D(\xi)\}\eta(X)\end{aligned} \tag{3.7}$$

On the other hand we have

$$(\nabla_\xi S)(\xi,\xi) = \nabla_X S(\xi,\xi) - 2S(\nabla_X\xi,\xi),$$

which yield by using (1.16) and (2.1) that.

$$(\nabla_\xi S)(\xi,\xi) = -2n(\alpha^2-\rho)\xi, \tag{3.8}$$

In view of (3.7) and (3.8), we get

$$\begin{aligned}A(X) &= \left[\frac{dk(X) - 2n^2(\alpha^2-\rho)\xi}{k-2n^2(\alpha^2-\rho)}\right] - \left[\frac{k}{(n-1)\{(k-2n^2(\alpha^2-\rho)\}}\right] \\ &\{B(X)+D(X)\} - \{D(\xi)+B(\xi)\}\eta(X) - \left\{\frac{dk(\xi)}{k-2n^2(\alpha^2-\rho)} - A(\xi)\right\}\eta(X)\end{aligned} \tag{3.9}$$

This leads to the following result.

Theorem 3.2. I*n a weakly concircular symmetric Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *the associated 1-forms* D, B *and* A *are given by (3.5) (3.6) and (3.9) respectively.*

Definition 3.2 A *Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *is said to be weakly concircular Ricci symmetric if its concircular Ricci tensor* $\mathbb{S}$ *of type* $(0, 2)$ *satisfies (1.8).*

In view of (1.8) and (1.9) yield

$$\begin{aligned}(\nabla_X S)(Y,Z) - \frac{dk(X)}{n}g(Y,Z) &= A(X)\left[S(Y,Z) - \frac{k}{n}g(Y,Z)\right] + B(Y) \\ &\left[S(X,Z) - \frac{k}{n}g(X,Z)\right] + D(Z)\left[S(X,Y) - \frac{k}{n}g(X,Y)\right]\end{aligned} \tag{3.10}$$

Setting $X = Y = Z = \xi$ in above we get the relation (3.2). Hence we can state the following

Theorem 3.3. *In a weakly concircular Ricci symmetric Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *the relations (3.2) holds*

Corollary 3.2. *In a weakly concircular Ricci symmetric Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *the sum of 1-forms* A, B *and* D *is zero everywhere and only if the scalar curvature k of the manifold is constant.*

Now, taking X and Y by ξ in (3.10), we have

$$(\nabla_\xi S)(\xi, Z) - \frac{d\kappa(X)}{n}\eta(Z) = \{A(\xi) + B(\xi)\} + D(Z)\left[S(\xi,\xi) + \frac{\kappa}{n}\right] \tag{3.11}$$

In view of (3.2) and (3.4), equation (3.11) yields.

$$D(Z) = \frac{-dk(\xi)}{k - 2n^2(\alpha^2-\rho)}\eta(Z) + \left[\frac{dk(\xi)}{k-2n^2(\alpha^2-\rho)} - D(\xi)\right]\eta(Z), \tag{3.12}$$

Again putting $X = Z = \xi$ in (3.11) and proceeding in a similar manner as above, we get

$$B(Y) = \frac{-dk(\xi)}{k - 2n^2(\alpha^2-\rho)}\eta(Y) + \left[\frac{dk(\xi)}{k-2n^2(\alpha^2-\rho)} - B(\xi)\right]\eta(Y), \tag{3.13}$$

$$A(X) = \frac{-2n^2(\alpha^2-\rho)\xi\eta(x)}{k-2n^2(\alpha^2-\rho)} + \frac{dk(X)}{k-2n^2(\alpha^2-\rho)} + \left[\frac{dk(\xi)}{k-2n^2(\alpha^2-\rho)} - A(\xi)\right]\eta(X), \tag{3.14}$$

This leads to the following result.

Theorem 3.4. *In a weakly concircular Ricci symmetric Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *the associated 1-form* D, B *and* A *are given by (3.12) (3.13) and (3.14) respectively*

Adding equations (3.12) (3. 13) and (3.14), using (3.3) we obtain

$$A(X) + B(X) + D(X) = \frac{dk(X) - 2n^2(\alpha^2-\rho)\xi}{k-2n^2(\alpha^2-\rho)} \tag{3.15}$$

for any vector field X .

This leads to the following result.

Theorem 3.5. *In a weakly concircular Ricci symmetric Lorentzian concircular structure manifold* (M^{2n+1}, g) $(n > 1)$ *the sum of the associated 1-form* A, B *and* D *is given by (3.15)*

Corollary 3.3 *There is no weakly concircular Ricci symmetric Lorentzian concircular structure manifold* $(M^{2n+1}, g)(n > 1)$ *unless the sum of the 1-forms is everywhere zero if* $dk(X) = 2n^2(\alpha^2 - \rho)\xi$.

Also taking cyclic sum of (1.11), we get

$$\left(\nabla_X\tilde{S}\right)(Y,Z) + \left(\nabla_Y\tilde{S}\right)(Z,Y) + \left(\nabla_Z\tilde{S}\right)(X,Y) = 4\,A(X)\,\tilde{S}(Y,Z) + A(Y)\,\tilde{S}(X,Z) + A(Z)\,\tilde{S}(Y,X), \tag{3.16}$$

Let M^{2n+1} admits a cyclic Ricci tensor. Then (3.16) reduces to

$$A\,(X)\,\tilde{S}(Y,Z)+A\,(Y)\,\tilde{S}(X,Z)+A\,(Z)\,\tilde{S}(Y,X)=0.$$

Taking $Z=\xi$, in above and then using (1.7), (1.10) and (2.11), we obtain

$$\left[2n^2(\alpha^2-\rho)-\frac{\kappa}{n}\right]\{A(X)\eta(Y)+A(Y)\eta(X)\}+\eta(\rho)\mathbb{S}(X,Y)=0. \tag{3.17}$$

Again taking $Z=\xi$, in (3.17), we get

$$2\eta(\rho)\eta(X)=A(X) \tag{3.18}$$

Taking $X=\xi$, in (3.18)and using (1.7), we yields

$$\eta(\rho)=0. \tag{3.19}$$

In view of (3.18) and (3.19), we get $A(X)=0,\ \forall X$.

This leads to the following result.

Theorem 3.6.*If a special weakly concircular Ricci symmetric Lorentzian concircular structure manifold* (M^{2n+1},g) $(n>1)$ *admits Cyclic Ricci tensor then the 1-form* A *must vanishes.*

Finally for Einstein manifold $(\nabla_X S)\,(Y,Z)=0$ and $S(Y,Z)=ag(Y,Z)$. Then (1.7) and (1.11), we get

$$\begin{aligned}-\frac{d\kappa(X)}{n}g(Y,Z)=2A(X)\left[\left(a-\frac{\kappa}{n}\right)g(Y,Z)\right]+A(Y)\left[\left(a-\frac{\kappa}{n}\right)g(X,Z)\right]\\+A(Z)\left[\left(a-\frac{\kappa}{n}\right)g(X,Y)\right],\end{aligned} \tag{3.20}$$

Plugging $Z=X=Y=\xi$, in (3.20), we obtain that

$$4\eta\;(\rho)\;(an-\kappa)=d\kappa\,(\xi)$$

which implies that if κ is constant then $\eta(\rho)=0$,that is $A(Y)=0,\forall Y$. Therefore we state the results

Theorem 3.7. *A special weakly concircular Ricci symmetric Lorentzian concircular structure manifold* (M^{2n+1},g) $(n>1)$*can not Einstein manifold if the scalar curvature of the manifold is constant.*

Corollary 3.4.*In a special weakly concircular Ricci symmetric Lorentzian concircular structure manifold* (M^{2n+1},g) $(n>1)$ *the 1-form* A *is given by*

$$\left[A\,(\xi)=\frac{d\,k\,(\xi)-2n^2\,(\alpha^2-\rho)}{2\,\{k-2n^2\,(\alpha^2-\rho)+n\}}.\right]$$

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