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A DYNAMIC MODEL OF A NONLIFE INSURANCE COMPANY

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J E L Classification: G22, C61.

Abstract: A Dynamics model of a nonlife insurance company is developed. Goodwill representing the awareness of the company by the public and the perceived quality of its services, and the technical capability representing the ability of the company to calculate the risk premium of the risks it considers to accept, are two state variables. The level of investment in advertising and quality improvement, and investments in technical capability are determined optimally to maximize the discounted profits of the company over an infinite horizon. The technical capability elasticity of number of customers and the claim ratio are shown to be determining parameters affecting the optimal paths of investments. The stability of equilibrium points are also shown to be dependent on these parameters.

Introduction

Nonlife insurance sector is an important part of financial sector which fuels economic growth. It grew by 2.9% in 2014 when the advanced economies grew by less than this rate (Sigma 2015). Liberalization of the sector has resulted in intense price competition (Borscherd, Haueter 2012). Therefore, the companies in that sector have to compete by investing in other factors such as quality

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of service, advertising, and technical capability to better evaluate risks to decrease the claims. Nair and Narasimhan (2006) have shown that quality is an important determinant of goodwill. Another study has shown that perceived quality of the service provided and name familiarity are important factors which affect the choice company by the customers (Arora and Stoner 1996). Hanson (2001) states that "quality is seen in the context of the essential transformation problem which may exists between the suppliers and the customers". Promotional activities, image of the company, customer convenience, and procedures are shown to be important factors that determine the company choice by life insurance customers (Sunega, Sharma 2008). The factors that determine customer choice are not same in all countries. Computerization, online production of policies, connection to the banks, speed and efficiency of transactions, and clear communication with the customers are considered as important in a city in India (Mathur, Tripathi 2014). In another study by Gangwar (2011) procedural efficiency, accessibility, advertising, redressal of complaints, and efficiency of claim settlement are shown to be important for life insurance customers. Chowdhury, Raahman, and Afra (2007) have studied the same problem in Pakistan and shown that foreign ownership, quality of service, reputation of the company, and quality of personnel are the most important factors determining customer choice of insurance company. Yet in another similar study, Akan (2013) has shown that the confidence in the company that it will pay claims, quality of claim service, and price are most important competition variables. Price was not even mentioned in another study (İDE 2012:56). Other studies conducted in Turkey have also similar conclusions (Karaali ve Özçelik 2008 and Kaya et. all 2008).

However, goodwill is built up slowly. It is built up by slowly offering a matrix of very good services (advertising for name familiarity, computerization, speed of policy production and distribution, claim payment procedures, good communication with the customers etc.) over time and it is costly to do so. Hence, in optimal control theory terminology, goodwill is a state variable and it is denoted as G in this paper.

Pricing risks correctly (determining the risk premium) and underwriting these risks is a core function of an insurance company and this function is often called underwriting or technical department. A high technical capability will result in lower incurred claims. A high technical capability will also help increase the number of customers since the customers will be more aware of the risks that they face and hence they will insure them. However, underwriting or technical capability, like goodwill, can only be built up in time by investing in it. Thus it is also a state variable and it will be denoted as T in this paper.

A nonlife insurance company tries to insure many homogenous risks by correctly calculating their risk premium (formation of underwriting portfolio) and price, collecting the premiums from the sellers of policies, paying commissions, claims, general expenses out of its financial portfolio, and optimally investing the remaining funds in financial portfolio. Therefore there are two actual portfolios (underwriting and financial) and one important but immeasurable portfolio (goodwill) that it has to manage optimally. However how exactly goodwill affects the company is not clear. The effect of technical capability on the portfolio of the company is evident but the exact functional form is not known. The effect of financial portfolio is also evident and is not taken into account here since it is assumed to be optimally managed.

The objective of this paper is to develop and solve a dynamic model of an insurance company to maximize its profits over an infinite horizon by optimally deciding on investments in goodwill and technical capability portfolio.

THE RESEARCH METHODOLOGY AND THE COURSE OF THE RESEARCH PROCESS

Model

Revenues of a nonlife insurance company are composed of premiums and financial income. Expenses are composed of claims, commissions, and general expenses. Financial income is neglected in this paper since the financial portfolio is assumed to be optimally managed. General expenses are also neglected since they are mostly of fixed character over significantly large time periods and output levels. Commissions are disregarded since they are generally a fixed percentage of premiums.

The instantaneous profit of the company at time t is expressed as:

$$\Pi\,(\mathrm{t}) = pN(G(t),T(t))(1-H(T(t)))-q(t)-w(a(t))$$

where;

- p: price (premium) of the policy, a constant since the insurance sector is assumed to perfectly competitive.
- N(G(t),T(t)): Number of policies sold at time t is assumed to be a function of goodwill G(t) and the level of technical capability T(t). It has the following characteristics.

$$N_G \geq 0,\ \mathrm{N}_T \geq 0,\ \mathrm{N}_{GT} \geq 0, N_{GG} \leq 0, N_{TT} \leq 0$$

- H(T(t)): Claim ratio (claims as a portion of premiums) as a function of technical capability of the firm with the following characteristics;

$$H_T \leq 0, H_{TT} \geq 0$$

a(t) is the investment in goodwill with w(a) as the cost of investing in goodwill, q(t) is the level of investment in technical capability. Cost of investing in technical capability is assumed to be linear function of q(t) with a unit cost of 1.

Hence better the technical capabilities of an insurance company lower will be its loss ratio since risk premium (expected loss of the company for accepting a certain risk) is a large part of the premium paid by customers. However, improving technical capability is possible only by properly investing in it. Employment of qualified technical underwriting personnel, education of such personnel, acquisition of hardware and software to analyze relevant data to measure risks are all important and are expensive. The issue of reinsurance is not included in the model since, in the long run, reinsurance is a partnership.

Mathematically dynamics of technical capability is expressed as:

$$dT / dt = T'(t) = q(t) - \delta T(t) \tag{1}$$

Here;

- T(t): denotes the technical capability at time t.
- δ: rate of obsolescence of technical capability (attrition of personnel, obsolescence of methodologies used to measure risks, etc.).

Equation (1) states that the technical capability increases by investment in it (q) and decreases exponentially at a rate δ due to obsolescence and depreciation of this capability (loss of experienced underwriters, obsolescence of both software and hardware used by underwriters, etc.).

Another important portfolio which affects the company is the goodwill portfolio. Akan (2013), Karaali and Özçelik (2008) and Kaya, Akın and Nalan (2008) have shown that confidence in a company and the quality of service are very important factors in the choice of insurance company by the customers. Hence the perception of customers about an insurance company is important. It is of paramount importance for company to have optimal product quality, service quality, company awareness (all elements of goodwill) and to be able to keep these at optimal levels. However, the development of this portfolio is very difficult since the investments in all elements of goodwill are expensive (cost of investment in goodwill is assumed to be a convex function).

The dynamics of this portfolio is expressed as:

$$dG / dt = G'(t) = a(t) - \eta G(t) \tag{2}$$

Here;

- a(t): the level of expenditures made to increase goodwill (advertising, timely claim payments, advisory functions, education of the personnel, computer hardware and software for faster issuance of policies ,etc.).
- η: the rate of decrease in the level of goodwill due to bad experience of customers with the company, advertising by competitors, better performance of other companies, etc. This rate is assumed to be constant.

The objective of the firm is assumed to be to maximize the present value of the firm over an infinite horizon by optimally choosing the time path of its expenditures a(t) and q(t) and at the same time meet the constraints represented by equation(1) and (2). Mathematically;

$$\max_{a,q\geq 0} \int_0^\infty e^{-rt}[pN(G(t),T(t))(1-H(T))-q(t)-w(a(t))]dt$$

$$T'(t) = q(t) - \delta T(t) \qquad \mathrm{T(0)=T_0}$$

$$G'(t) = a(t) - \eta G(t) \qquad \mathrm{G(0)=G_0}$$

The term e^{-rt} in the integral represents the present value factor. The discount rate r is assumed to be constant since time-dependent discount rate would complicate the solution of the model even though it is more realistic.

Solution

Optimal Control Theory will be employed to solve this problem (Kamien and Schwartz 2012, L.S, Pontrayagin 1962 among others).

The current Value Hamiltonian is:

$$H = pN(G,T)(1-H(T)) - q - w(a) + \lambda_1(q - \delta T) + \lambda_2(a - \eta G)$$

The necessary conditions for optimality are:

$$H_q = 0 = -1 + \lambda_1$$
$$\mathrm{H}_a = 0 = -w'(a) + \lambda_2$$
$$\lambda'_1 = \lambda_1(r+\delta) - pN_T(G,T)(1-H(T)) + pN(G,T)H'(T)$$
$$\lambda'_2 = \lambda_2(r+\eta) - pN_G(G,T)\ (1\text{-H(T)})$$

with equation (1) and equation (2).

The Hamiltonian is assumed to be jointly concave in the variables a, q, G and T. Therefore, the necessary conditions are also sufficient for optimality. The solution is possible to determine the optimal solution even with unknown forms of functions N, H, and w. However, we will assume specific forms for these functions for better exposition of the solution as follows:

$$H(T) = (k + b / T^{\alpha})$$
$$w(a) = Ba^{\beta}$$
$$N(G,T) = AG^{\gamma}T^{\varepsilon}$$

The parameters $\alpha, \varepsilon, \gamma$ are assumed to be between zero and one to assure the concavity of the Hamiltonian. A and B are parameters of scale. The parameter k, 0 < k < 1, represents the lowest value of claim ratio than can be obtained since there will always be claims no matter what the level of technical capability is. The function is assumed to be convex, i.e. β>1. Then the necessary conditions can be rewritten as:

$$H_q = 0 = -1 + \lambda_1 \tag{3}$$

$$\mathrm{H}_a = 0 = -B\beta a^{\beta-1} + \lambda_2 \tag{4}$$

$$\lambda'_1 = \lambda_1(r+\delta) - pAG^{\gamma}\ \mathrm{T}^{\varepsilon-1}(\varepsilon(1-k) + \mathrm{b}(\alpha\text{-}\varepsilon)/\mathrm{T}^{\alpha}) \tag{5}$$

$$\lambda'_2 = \lambda_2(r+\eta) - pA\gamma G^{\gamma-1}\mathrm{T}^{\varepsilon}(1\text{-k-bT}^{-\alpha}) \tag{6}$$

with equations (1) and (2).

From equation (3),

$$\lambda'_1 = 0 \tag{7}$$

Using this equation with equation (3) in equation (5), we have;

$$r + \delta = pAG^{\gamma}\,\mathrm{T}^{\varepsilon-1}(\varepsilon(1-k) + \mathrm{b}(\alpha-\varepsilon)/T^{\alpha}) \tag{8}$$

Using equation (8);

$$\begin{gathered} G^{\gamma} = (\mathrm{r}+\delta)\mathrm{T}/(\mathrm{pAT}^{\varepsilon}(\varepsilon(1-\mathrm{k}) + \mathrm{b}(\alpha-\varepsilon)/\mathrm{T}^{\alpha})) \\ \text{or} \\ \mathrm{G} = ((\mathrm{r}+\delta)\mathrm{T}^{1+\alpha-\varepsilon}/\,pA(\varepsilon(1-k)T^{\alpha} + b(\alpha-\varepsilon)))^{1/\gamma} \end{gathered} \tag{9}$$

Then, using equation (9), we can express T as a function of G, as

$$\mathrm{T} = \mathrm{M(G)} \tag{10}$$

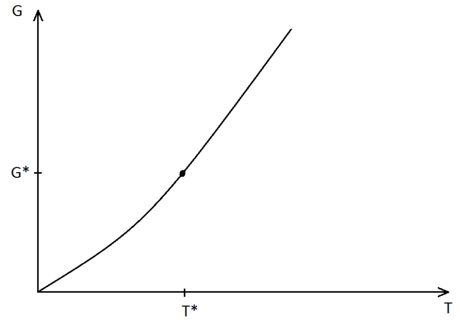
Using equations (4) and (9) in equation (6), we have

$$\begin{gathered} \lambda'_2 = w''(a)a' = w'(a)(r+\eta) - pAG^{\gamma-1}\gamma T^{\varepsilon}(1-k-bT^{-\alpha}) \text{ or,} \\ \lambda'_2 = B\beta(\beta-1)a^{\beta-2}a' = B\beta a^{\beta-1}(r+\eta) - pAG^{\gamma-1}\gamma M(G)^{\varepsilon}(1-k-bM(G)^{-\alpha}) \end{gathered} \tag{11}$$

Equations (11) and (2) represents a homogeneous, first order differential equation system in (G, a) space. It is not possible to solve the system since M(G) and (a) are not linear. Phase Diagrammatic analysis will be conducted to characterize the optimal solution (Kaplan 1958). However before this analysis, equation (9) has to be analyzed. The parameters will be important since their relative size will affect the solution. There are two cases:

A. If $\alpha > \varepsilon$ (technical capability elasticity of claim ratio is greater than the technical capability elasticity of number of customers)
   In this case equation (9) is represented in Figure 1 below.

Figure 1. Graphic representation of Equation (9)



S o u r c e : developed by the author.

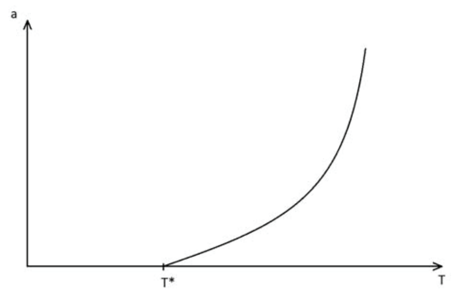
First the loci of points where both a' = 0 and G' = 0 needs to be determined (equations 2 and 11) in (G, a) space.

a'=0 curve, using equation(11) can be written as:

$$B\beta a^{\beta-1}(r+\eta)=p\gamma\,\mathrm{A}((\mathrm{r}+\delta)\mathrm{T}^{1+\alpha-\varepsilon}/(\varepsilon(1-\mathrm{k})\mathrm{T}^{\alpha}+b(\alpha-\varepsilon)))^{(\gamma-1)/\gamma}((1-k)T^{\varepsilon}-bT^{\varepsilon-\alpha}) \quad (12)$$

This relationship between a and T is represented in Figure 2 below.

Figure 2. Graphic representation of Equation (12)

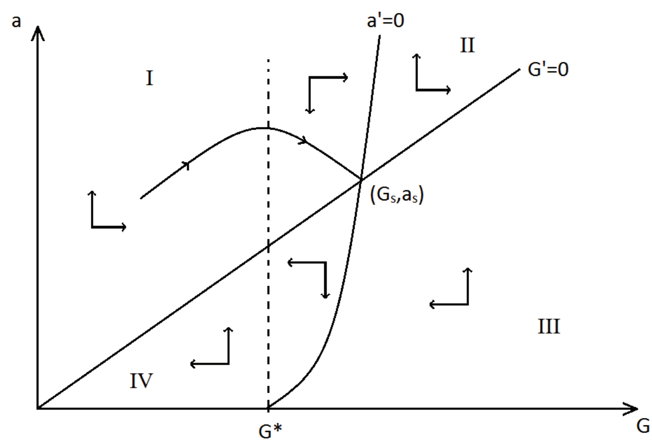


S o u r c e : developed by the author.

This curve, however, is in (T, a) space. We need to translate it to (G, a) space using equation (9) and Figure (1).

$G^{*}$is the value of G in equation (9) when $T^{*}=(b/1-k)^{1/\alpha}$ in equation (9). Therefore, a'=0 locus can be represented in (G, a) space as in Figure 3 using Figures 1, 2, and equation (9). It can be shown that above the curve represented by equation (12) which is a'=0 locus, a' > 0, and below it a' < 0. This dynamic procedure is represented by the directional arrows in Figure 1. Notice that a < 0 in equation (9) when T < $(b/(1-k))^{1/\alpha}$ which corresponds to G^* in equation (9). So this section of this curve is omitted. Notice also that a'>0 to the left of G^* due to equation (11) because to the left of T^* the last term in equation (11) is negative making a'>0. Using equation (2), it is shown that above G'=0, G'>0, and below G'=0, G'<0. The intersection of these loci represents equilibrium point (G_s,a_s).

Figure 3. Phase Diagram-Equations (2) and (11)



Source: developed by the author.

It is possible to reach the equilibrium point (G_s,a_s) from quadrants I and III. New or companies with low goodwill must begin with increasing the level of technical capability to the level implied by equation (9) by a jump in that state variable and continue with very high levels of expenditures (advertising, expenditures to increase quality, and technical capability) at an increasing rate first to increase goodwill to G^*, technical capability to $T^*=b/(1-k))^{1/\alpha}$ (where the claim ratio is one) implied by equation (9), then keep investing in goodwill and technical capability to further decrease the claim ratio to T_s and increase G to G_s which are the desired levels of goodwill and technical capability (Quadrant I). This strategy is represented by the curve which starts in the first quadrant with arrows on it. For companies with high level of beginning goodwill, the optimal strategy will be to keep investing in both goodwill and quality as to bring the level of goodwill to the level desired level in the long term (Quadrant III).

Strategies starting in quadrants II and IV will not lead to equilibrium. Stability of the equilibrium point is studied in the appendix. It is shown that this equilibrium may or may not be stable. It is stable only if the relationship between goodwill G and the technical capability T is not very strong, i.e. M'(G) is small.

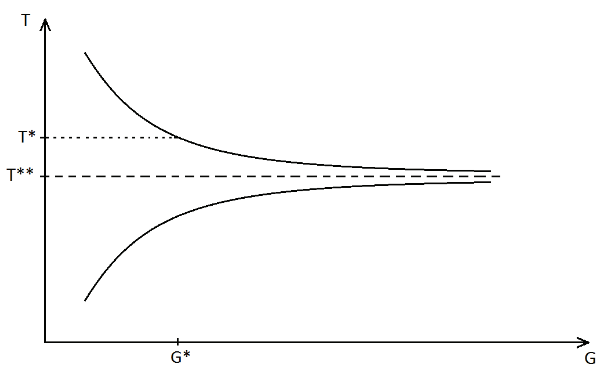
B. If $\alpha < \varepsilon$ (technical capability elasticity of claims ratio is less than the technical capability elasticity of number of claims)

There will be no change in G'=0 loci (a=). The loci a'=0, as was done before (using equations (9) and (11)) can be rewritten as:

$$B\beta a^{\beta-1}(r+\eta) = p\gamma \mathrm{A}((\mathrm{r}+\delta)\mathrm{T}^{1+\alpha-\varepsilon} / (\varepsilon(1-\mathrm{k})\mathrm{T}^{\alpha} + b(\alpha-\varepsilon)))^{(\gamma-1)/\gamma}((1-k)T^{\varepsilon} - bT^{\varepsilon-\alpha}) \quad (13)$$

However, this relationship is in (T, a) space. Equation (9) will be employed to represent this relationship (a'=0 locus) in (G, a) space to carry out the phase plane analysis.

Figure 4. Graphic representation of Equation (9)

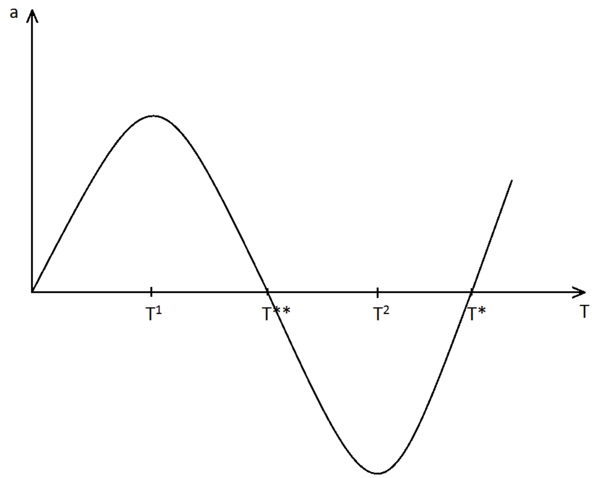


Source: developed by the author.

T^{**} is the value of T that makes the denominator in equation (9) zero. G becomes infinite at this value of T. This relationship and its graph will be employed to translate equation (13) in (G, a) space. T^{*} is same as defined previously. Notice that T^{*} is greater than T^{**}. The values of T less than T^{*} will be disregarded since a<0 in that case due to equation (13).

Equation (13) is graphed is also in Figure 5 for better exposition with the full knowledge that it is not a sinusoidal curve.

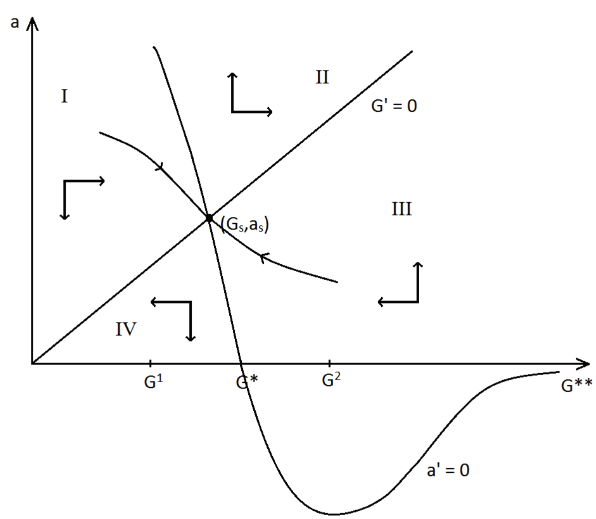
Figure 5. Graphic representation of Equation (13)



Source: developed by the author.

Using Figures (4) and (5), a'=0 locus can be defined in (G, a) space. These loci, a'=0 and G'=0, are represented in Figure (6). The part of the graph where a<0 should be omitted.

Figure 6. Phase Diagram-Equations (2) and (11)



Source: developed by the author.

It can be easily shown that the directional arrows are as shown in Figure (6). (G_S,a_S) are the values of a and G at the equilibrium point. It is shown in the appendix that this equilibrium point is a saddle point.

Following conclusions can be drawn from the diagram above:

- Companies with low level of beginning goodwill (Quadrant I) must begin with increasing the level of technical capability to the level implied by equation (9) by a jump in that state variable and continue with high levels of advertising expenditures to increase their goodwill and adjust technical capability T in accordance with equation (9) until equilibrium levels are reached. In practice, this strategy implies that the companies with low goodwill should first invest heavily in technical capability to improve claim ratio to improve profitability then invest heavily in advertising and quality to increase the number of customers.
- For companies with very high levels of goodwill (Quadrant III) optimal behavior will be to gradually decrease the goodwill level to the level desired in the long run (G_s), and adjusts technical capability level T in accordance with equation (9) to the long term technical capability (T_s).
- Starting in other Quadrants (II and IV) will not lead to equilibrium points.

The outcome of the research process and conclusions

Findings

When the impact of increasing the technical capability (T) on claim ratio is greater than its impact on number of customers ($\alpha > \varepsilon$) or revenues, the optimal strategy is to first to decrease the claim ratio to one as quickly as possible (a jump),then to keep investing in it until the equilibrium level (T_s) implied by equation (9) is reached. A similar strategy should be followed with respect to goodwill (G). Goodwill should be aggressively increased first until the claim ratio becomes one and then to continue investing until the long term desired level (G_s) is reached.

However, if the impact of increasing technical capability on claim ratio is less than its impact on the number of customers ($\varepsilon > \alpha$), the strategy should be to increase the goodwill to increase the number of customers, and technical capability in accordance with equation (9).

In any case, the equilibrium level of goodwill (G_s) and the technical capability level (T_s) are greater in the first case than they are in the second case (Figures 3 and 6). This implies high spending levels for a company in such an environment which in turn may require high capitalization at the beginning discouraging small companies to enter into this sector.

It is clear that the parameters α and ε are very important in determining the optimal strategy for an insurance company. Therefore an insurance company must make an analysis to determine these parameters before determining a general strategy.

Discussion

The model developed above is a strategic planning model in terms of the state variables Goodwill G(t) and Technical Capability T(t), control variables, advertising and quality improvement a(t) and technical investment q(t). It cannot be used for short term profit maximization. The parameters in the model are assumed to be simple to be able to carry out an indicative analysis. For example $\beta=2$ while $\gamma=0.5$.

The assumed forms of number of customers N(t) and the claim ratio H(t) are arbitrary but somehow reflective of their true forms. All cost items (rents, water, electricity, personnel, etc.) other than investments in goodwill and technical capability are assumed to be fixed and hence are not taken into account.

The major strength of the model is that it is dynamic and its solution gives an insight about the optimal strategic performance to the management of nonlife insurance companies. The major weakness of the model is the assumption made on the functional forms related to claim ratio, cost of advertising, and number of customers even though the author believes that these functions reflect the reality. Another weakness is the assumption that all factors affecting the company will remain the same during the life of the company even though this assumption is widely used in optimal control theory models.

Suggestions for further research

All cost items related to volume of business can be added to the model as a fraction of total revenues as defined in the nonlife insurance sector (expense ratio). However, this will have no impact on the general solution. Proportional insurance can easily be introduced into the model. However non-proportional reinsurance function will make the model very difficult.

Appendix: Stability Analysis of Equilibrium Points

The Taylor's expansion of the nonlinear system represented by equations (2) and (11) around the equilibrium point (G_s,a_s) is analyzed. The signs of the roots of the linear system determine the stability of the system.

Then the system of nonlinear differential equations rewritten below will be expanded around the equilibrium point.

$$B\beta(\beta-1)a^{\beta-2}a' = B\beta a^{\beta-1}(r+\eta) - pAG^{\gamma-1}\gamma M(G)^{\varepsilon}(1-k-bM(G)^{-\alpha})$$

and

$$G'(t) = a(t) - \eta G(t)$$

Rewriting this system and simplifying;

$$a' = a\,c_1 - c_2 G^{\gamma-1} M(G)^{\varepsilon}(1-k-bM(G)^{-\alpha})\,a^{2-\beta} = ac_1 - c_2 N(G)a^{2-\beta}$$

$$G'(t) = a(t) - \eta G(t)$$

where $N(G) = G^{\gamma-1}M(G)^{\varepsilon}(1-k-bM(G)^{-\alpha})$ for later use and;

$c_1 = (r+\eta)/(\beta-1)$ and

$c_2 = pA\gamma / B\beta(\beta-1)$

Which are all positive constants since β is assumed to be greater than one. The Taylor's expansion of the system is written as:

$$a' = (c_1 - N(G)(2-\beta)a^{1-\beta})(a-a_s) - c_2(a^{2-\beta}dN(G)/dG)(G-G_s)$$
$$G' = (a-a_s) - \eta(G-G_s)$$

Rewriting and neglecting the constants, we have;

$$a' = Da - EG$$
$$G' = a - \eta G$$

where

$$D = (c_1 - N(G)(2-\beta)a^{1-\beta})$$

and

$E = c_2 a^{2-\beta} dN(G)/dG$ evaluated at equilibrium point.

The roots of this system are calculated as:

$$r_1, r_2 = ((\mathrm{D}-\eta) \pm \sqrt{(\mathrm{D}-\eta)^2 - 4(\mathrm{E}-D\eta)})/2$$

It is clear that the stability depends on the term (D-η) and the term (E-Dη). The term D is assumed to be positive.

It can be shown that (the details are not included here) the term dN(G)/dG is negative if α < ε. This implies that the term E is negative making the term in the square root above is positive which in turn implies that one root is positive when the other is negative. Thus the equilibrium point is a saddle point if α<ε i.e. when the impact of increase of technical capability on claim ratio is less than its impact on number of customers the equilibrium reached will be a saddle point. The stability analysis in the case of α>ε, we do not have a definite result on the sign of E thus the equilibrium can be of any type.

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