CIT. Journal of Computing and Information Technology, Vol. 29, No. 3, September 2021, 181–207
doi: 10.20532/cit.2021.1005418

Business Cost Modelling of Information Processing in Supply Chain Units

Robert Bucki[1] and Petr Suchánek[2]

[1]Institute of Management and Information Technology, Bielsko-Biała, Poland
[2]Silesian University in Opava, School of Business Administration in Karvina, The Czech Republic

The paper analyzes and successively assesses the problem of information delays in a sample logistics system, in which customer inquiries are sent to adjacent or dedicated units in the logistics chain. Within such a framework time and unit costs of customer service and inquiry are analyzed. A detailed model of the logistics system is thus elaborated using a mathematical approach and its graphical representation. The accompanying case study includes the analysis of the detailed situation in which two models are compared that do not differ in the structure of logistic units but are characterized by different information flow paths. Simulation results are presented through a thorough graphic, numerical and descriptive analysis. Finally, the problem of information transmission in the logistics system is summarized.

ACM CCS (2012) Classification: Computing methodologies → Modeling and simulation → Model development and analysis → Model verification and validation

Keywords: logistic system, supply chain, modelling, simulation, optimization, customer inquiry

1. Introduction

Supply chain optimization is one of the key conditions for ensuring economic efficiency of all types of companies [1], [2]. One of the organizational and functional goals of all companies is to ensure a smooth supply of materials or input products to the production line while avoiding excessive levels of material reserves in their warehouses [3]. As follows from [4], one of the key conditions is to have correctly set up processes in the company and to use adequate methodologies and approaches within Business Continuity Management (BCM). This issue is challenging for companies engaged in the production of products of the same type and is even more challenging for companies that allow different ranges of product configuration within customer orders [5]. It turns out that servicing specific customer orders is one of the decisive factors in the efficiency of the logistics system in the production supply segment [6]. In addition, ongoing market, technological and other changes place increased emphasis on an adaptive approach to BCM [7]. The key research issue in this context is the search for specific optimized models of logistics inventory management and their distribution in direct relation to the processing of customer orders and financial costs necessary for their processing. As stated in [8], customer orders are the primary starting point for defining the production schedule which must be set so that production can be carried out smoothly without time delays. The paper focuses mainly on the issue of modelling flexible automated systems using robotic and/or cobotic systems in which the continuity of supply of production lines is the primary and necessary prerequisite [9]. This type of production systems, which we can refer to as intelligent production systems, is today a common and rapidly developing part of the digital economy and is one of the cornerstones of the Industry 4.0 initiative [10]. The subject of this paper is to focus on modelling the time delay of customer orders in relation to partial delays in the flow of information during the production process. If we take into account the fixed architecture of the production line, then the time delay within the system can be effectively modeled using a heuristic approach [11], [12] which is the case described in this paper. The reason for finding a suitable model for minimizing – or better eliminating – time delays is the fact that information delays can have a significant adverse effect on the performance of the entire system, and especially in large systems [13].

Logistics systems are characterized by a certain autonomy depending on the tasks they perform. Various logistics areas each of which is responsible for the performance of specific tasks are subject to optimization. Optimization tasks are accompanied by criteria that we can implement in a single or multi-criteria form. The criteria usually relate to the cost of the contract, its time and quality. However, little attention is paid to the issue of handling customer inquiries. This paper deals with an issue that is extremely important for modern business and logistics systems, namely the ubiquitous striving to minimize the operating costs of each enterprise. In our case, it concerns an area often neglected by decision-making operators, namely the costs of handling customer inquiries. Their minimization can unequivocally lead to the minimization of costs in the entire business area. The problem presented in this paper is a complex problem that takes into account many factors that should be taken into account when developing the algorithm of conduct within the imposed criterion of minimizing the costs of developing a client inquiry. The serial arrangement of active units in the logistics chain requires precise scheduling of information operations. The problem to be solved concerns the comparison of information flows along two different trajectories. In connection with all the above facts, the aim of the paper is to present a specific example of creating a model describing delays of the flow of order processing of individual customers in direct relation to the evaluation of the cost of providing production logistics. The measurement of the company's operating costs should be carried out in all measurable areas of its operation in accordance with all the principles of logistics cost accounting. Specifically, it is a time measurement, *i.e.* modelling of the time from order submission to final product. As an initial part of model creation, the logistical structure of the system is presented, including the time parameters. Based on this, time matrices representing delays in transits between individual logistic units and subunits are introduced. In the final phase, a specific direct link between individual delays and unit costs is presented, upon which the final value of the costs of serving a given set of orders from individual customers can be obtained.

The present paper is structured as follows. After this introductory part, Section 2 brings a review of related work, while in Section 3 the mathematical model of the logistics chain encompassing a thorough specification and detailed analysis of the respective project matters for sequential and non-sequential systems is provided. Subsequently, Section 4 presents the numerical analysis of both mentioned systems using data generated at random for a case study. The results obtained in the case study are modified in Section 5 in order to achieve a satisfactory result. Section 6 summarizes the conducted research and concludes the paper.

2. Related Work

If we focus more specifically on cost models, then these are one of the primary starting points and tools for planning and cost management, among others, in the field of industrial production and logistics systems [14]. Effective cost management ultimately reduces the scope of potential uncertainty and therefore refines the management process. Furthermore, the authors in [15] state other benefits of cost models in the form of the possibility of estimating the price of the product, supporting the elimination of errors during its design and, what is important, taking into account the customer's requirements within the individual stages of the decision-making process. Increasing the quality of logistics services usually results in an increase in their price. Following this fact, one of the goals of creating cost models is ensuring that the increase in costs does not exceed an acceptable level [16]. This paper provides an analysis of the problem of minimizing servicing costs of customers' inquiries basing on the mathematical model of the problem introduced in [17], along with the hereby considerably developed respective project. The specific output of the mentioned paper is the mathematical model created using a heuristic approach representing a proposal for the method of calculating the cost of servicing customers' inquiries. A similar methodological approach for the creation of models in the field of logistics systems was used by the authors of this paper (*e.g.*, in [18]), where the output is the design of a simulation model for optimizing the production process of a company containing geographically located workplaces. Subsequently, paper [19] presents a model for finding minimum costs of operation of a production company while, paper [20] brings a proposal for a model for organizing the company's production activities for a given set of orders, which would be effective both in terms of time and cost. The importance of the issue of cost optimization in the areas of production, logistics, supply chain and others becomes evident from a number of publications in which the authors present methodological approaches and models for solving specific optimization tasks. As a follow-up to this one, paper [21] can be cited in which the authors present a model for cost optimization based on product quality, number of deliveries and a cost margin. A model for supporting the reduction of total costs in the areas of supply, production and distribution using the methodology of integer non-linear programming is introduced in [22], further demonstrating the possible contribution of optimization models and specifically providing a reduction of total costs by approximately 20% in the given case. From the point of view of the heuristic approach methodology used, it can be stated that the possibility of using this approach for the implementation of models in the field of production, logistics and supply chain is evidenced by the publications mentioned in the Introduction, *i.e.* [11] and [12]. Additionally, a study definition of the vendor-managed inventory (VMI) and Supply Chain Management (SCM) models with respect to the size of the order, the shortest possible route and the frequency of supply is presented in [23], among aspects in relation to the cost side.

3. Mathematical Model

In order to clarify the issue of handling customers' inquiries, specific assumptions are made, which are subsequently used to create the design underlying the construction of a simulation tool enabling simulation experiments to be carried out. First of all, it is assumed that customers inquire about their orders before setting them. The units of the logistics chain are arranged in series; however, some units are backed by subunits which support their functioning. At a given moment the contents of the inquiry matrix is analyzed by the Customer Service Department (CSD). This means that customers' inquiries are subject to a thorough analysis by appropriate units in the logistics chain responsible for making products, which results in time delays. There is also a need to send inquiries between logistic units. When feedback from the logistics system to the inquiry is received by the CSD and is eventually processed, the customer is informed whether or not her/his order can be fulfilled, and what quantities and how many orders are feasible and at what date. Then the customer can either accept or decline this. After a specified time the order matrix is blocked and no more inquiries can be placed in the logistics system without additional calculations. The order inquiry matrix is transformed into the final accepted order matrix. The order inquiry matrix takes the following form:

$$Z^0 = \left[z^0_{m,n} \right], \ m = 1, \dots, M, \ n = 1, \dots, N \quad (1)$$

where $z^0_{m,n}$ is the inquiry about the possibility of making the n-th order of the m-th customer expressed in contract unit numbers at the initial state.

The vector of suppliers takes the following form:

$$D = [d_l], \ l = 1, \dots, L \quad (2)$$

where d_l is the l-th supplier for the manufacturing system.

The adjustment matrix of customers' orders to suppliers takes the following form:

$$\Omega = [\omega_{l/(m,n)}] \quad (3)$$

where $\omega_{l/(m,n)}$ is the adjustment of the l-th supplier to the n-th order of the m-th customer. At the same time $\omega_{l/(m,n)} = 1$ if the n-th order of the

m-th customer can be made from the charge material delivered by the l-th supplier, otherwise $\omega_{l/(m,\,n)} = 0$. The order inquiry matrix elements are subject to analysis to determine whether they can be made by the company or not. After the analysis, orders which can be made in the company remain in the order matrix, otherwise they are removed from it or modified. The order inquiry matrix is transformed into the order matrix after obtaining the required information as follows:

$$Z^0 = \left[z^0_{m,n} \right] \rightarrow Z^1 = \left[z^1_{m,n} \right] \tag{4}$$

At the same time, $z^0_{m,n} = z^1_{m,n}$ if the n-th product for the m-th customer can be made, otherwise $z^0_{m,n} \neq z^1_{m,n}$, *i.e.* the n-th product inquiry of m-th customer is modified (if rejected, then $z^1_{m,n} = 0$). Moreover, if $z^0_{m,n} = 0$, then CSD cannot serve the nonexistent inquiry. It is assumed that the time of processing inquiries in units depends on the number of units of a certain order which results in the need for adding an extra time for calculating the time of processing bigger customers' inquiries in units. Another assumption concerns the times of passing inquiries between units. It is assumed that problems may arise while sending larger inquiries, *e.g.* a large inquiry can be divided into a few parts depending on its volume before passing it to the corresponding units. Let us assume that it is necessary to minimize the time of dealing with customers' inquiries in logistic units, as well as the time of sending them between logistic units in case the expected acceptable values are exceeded. The matrix of amount coefficients is introduced in case of processing inquiries in units:

$$\Gamma = \gamma^{m,n}_{el},\ m = 1, \ldots, M,\ n = 1, \ldots, N \tag{5}$$

where $\gamma^{m,n}_{el} = \xi + \frac{z^0_{m,n}}{\zeta}$, ξ is the base quantitative coefficient, and ζ is the minimizing denominator for handling inquiries in logistic units. There is a need for minimizing the amount coefficient of inquiries in case it exceeds the set value while processing them in the logistic units. Let c_ζ be the maximal allowable value that cannot be exceeded in case of processing an inquiry of the m-th customer for the n-th order in each logistic unit, so:

$$\begin{gathered} \text{if} \left(\gamma^{m,n}_{el} = \xi + \frac{z^0_{m,n}}{\zeta} \right) > c_\varsigma \cdot \xi, \\ \text{then } \gamma^{m,n}_{el} = c_\varsigma \cdot \xi. \end{gathered} \tag{6}$$

The matrix of amount coefficients is introduced in case of passing inquiries in units:

$$\Gamma = \gamma^{m,n}_{pass},\ m = 1, \ldots, M,\ n = 1, \ldots, N \tag{7}$$

where $\gamma^{m,n}_{pass} = \xi + \frac{z^0_{m,n}}{\upsilon}$, ξ is the base quantitative coefficient, and υ is the minimizing denominator for information transfer between logistic units. There is here a need for minimizing the amount coefficient of inquiries in case it exceeds the set value while passing them between the logistic units. Let c_υ be the maximal allowable value that cannot be exceeded in case of passing an inquiry of the m-th customer for the n-th order to the dedicated logistic unit, so:

$$\left(\gamma^{m,n}_{pass} = \xi + \frac{z^0_{m,n}}{\upsilon} \right) > c_\upsilon \cdot \xi \tag{8}$$

E.g., if $\frac{z^0_{m,n}}{\upsilon} > 0.01 \cdot \xi$, then $\gamma^{m,n}_{pass} = 1.01 \cdot \xi$ and it is necessary to assume that $\upsilon > \zeta$.

To obtain the knowledge on whether or not an order can be made in the company, it is necessary to send an inquiry to all units responsible for all key manufacturing, as well as storing and supply operations. This inquiry can be tracked while being passed between units in sequence. The following approaches of solving the problem of servicing customers' inquiries exist:

- passing inquiries to the preceding units in sequence (see Figure 1), and
- passing inquiries to the chosen units (see Figure 2).

Let the following symbols represent logistic units which are included in the above figures:

Z: customers;

CMS: the charge material storage;

CSD: the customer service department;

REG: the regeneration unit;

MAN: the manufacturing unit;

PRS: the production support unit;

RPS: the ready product storage;

SUP: the supply center;

$d^{0}_{l/(m,n)}$: the supply zone.

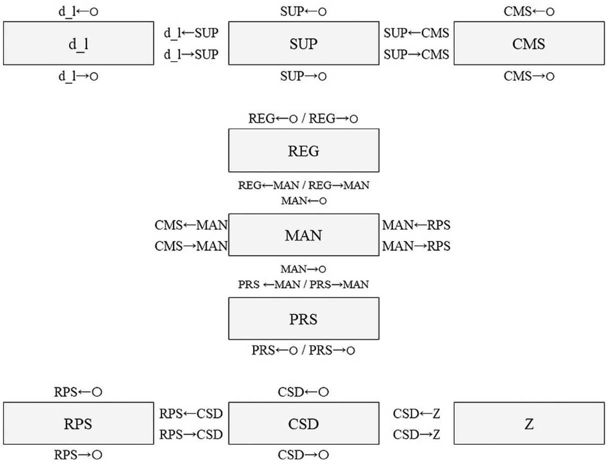


Figure 1. Passing inquiries to preceding units in sequence.

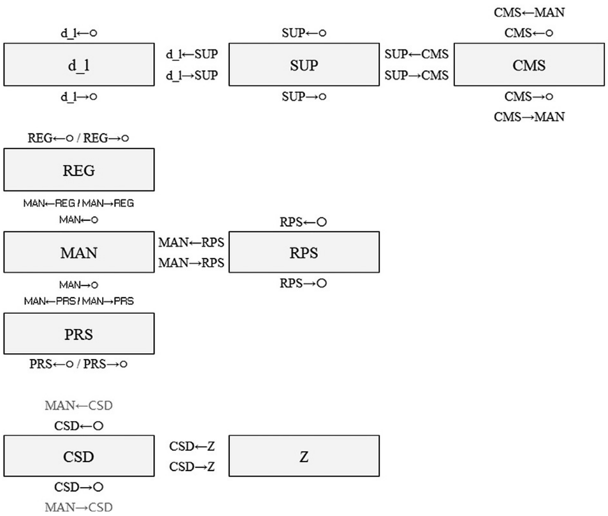


Figure 2. Passing inquiries to chosen units.

Model 1 is discussed in detail in Table 1 and Table 2, with costs of passing the inquiry between logistic units are shown in Table 1, and those of handling delays in units presented in Table 2.

Let the following assumptions be made concerning the time of inquiry delay for the n-th order of the m-th customer for both models:

$\tau^{m,n}_{CSD\leftarrow z^{0}_{m,n}}$: from the order matrix to the customer service department,

$\tau^{m,n}_{CSD\rightarrow z^{0}_{m,n}}$: from the customer service department to the order matrix,

$\tau^{m,n}_{RPS\leftarrow CSD}$: from the customer service department to the ready product storage,

$\tau^{m,n}_{RPS\rightarrow CSD}$: from the ready product storage to the customer service department,

$\tau^{m,n}_{CSD\rightarrow MAN}$: from the customer service department to the manufacturing unit,

$\tau^{m,n}_{CSD\leftarrow MAN}$: from the manufacturing unit to the customer service department,

$\tau^{m,n}_{MAN\leftarrow RPS}$: from the ready product storage to the manufacturing unit,

$\tau^{m,n}_{MAN\rightarrow RPS}$: from the manufacturing unit to the ready product storage,

$\tau^{m,n}_{REG\leftarrow MAN}$: from the manufacturing unit to the regeneration unit,

$\tau^{m,n}_{REG\rightarrow MAN}$: from the regeneration unit to the manufacturing unit,

$\tau^{m,n}_{PRS\leftarrow MAN}$: from the manufacturing unit to the production support unit,

$\tau^{m,n}_{PRS\rightarrow MAN}$: from the production support unit to the manufacturing unit,

$\tau^{m,n}_{CMS\leftarrow MAN}$: from the manufacturing unit to the charge material storage,

$\tau^{m,n}_{CMS\rightarrow MAN}$: from the charge material storage to the manufacturing unit,

$\tau^{m,n}_{SUP\leftarrow CMS}$: from the charge material storage to the supply center,

$\tau^{m,n}_{SUP\rightarrow CMS}$: from the supply center to the charge material storage,

$\tau^{m,n}_{d^{0}_{l/m,n}\leftarrow SUP}$: from the supply center to the l-th charge material subsupplier,

$\tau^{m,n}_{d^{0}_{l/m,n}\rightarrow SUP}$: from the l-th charge material subsupplier to the supply center.

It is assumed that the key logistics operating points are responsible for processing the proper inquiry from the preceding unit as well as replying to these inquiries to the subsequent unit in the logistics chain. The following formulas then represent the times needed for processing these tasks after receiving an inquiry from the preceding unit and before sending its own inquiry to the subsequent unit as shown in Table 2. The same concerns the reverse way of sending the processed inquiry.

Table 1. Costs of passing inquiries between logistic units for Model 1 and Model 2.

Times of sending the inquiry	**From**	**To**	**Unit cost of sending the inquiry**	**Cost of sending the inquiry**	**Used in model**
$\tau^{m,n}_{CSD\leftarrow z^0_{m,n}}$	$z^0_{m,n}$	CSD	$c^{unit\_m,n}_{CSD\leftarrow z^0_{m,n}}$	$c^{m,n}_{CSD\leftarrow z^0_{m,n}} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{CSD\leftarrow z^0_{m,n}} \cdot c^{unit\_m,n}_{CSD\leftarrow z^0_{m,n}}$	1, 2
$\tau^{m,n}_{CSD\rightarrow z^0_{m,n}}$	CSD	$z^0_{m,n}$	$c^{unit\_m,n}_{CSD\rightarrow z^0_{m,n}}$	$c^{m,n}_{CSD\rightarrow z^0_{m,n}} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{CSD\rightarrow z^0_{m,n}} \cdot c^{unit\_m,n}_{CSD\rightarrow z^0_{m,n}}$	1, 2
$\tau^{m,n}_{RPS\leftarrow CSD}$	CSD	RPS	$c^{unit\_m,n}_{RPS\leftarrow CSD}$	$c^{m,n}_{RPS\leftarrow CSD} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{RPS\leftarrow CSD} \cdot c^{unit\_m,n}_{RPS\leftarrow CSD}$	1
$\tau^{m,n}_{RPS\rightarrow CSD}$	RPS	CSD	$c^{unit\_m,n}_{RPS\rightarrow CSD}$	$c^{m,n}_{RPS\rightarrow CSD} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{RPS\rightarrow CSD} \cdot c^{unit\_m,n}_{RPS\rightarrow CSD}$	1
$\tau^{m,n}_{CSD\rightarrow MAN}$	CSD	MAN	$c^{unit\_m,n}_{CSD\rightarrow MAN}$	$c^{m,n}_{CSD\rightarrow MAN} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{CSD\rightarrow MAN} \cdot c^{unit\_m,n}_{CSD\rightarrow MAN}$	2
$\tau^{m,n}_{CSD\leftarrow MAN}$	MAN	CSD	$c^{unit\_m,n}_{CSD\leftarrow MAN}$	$c^{m,n}_{CSD\leftarrow MAN} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{CSD\leftarrow MAN} \cdot c^{unit\_m,n}_{CSD\leftarrow MAN}$	2
$\tau^{m,n}_{MAN\leftarrow RPS}$	RPS	MAN	$c^{unit\_m,n}_{MAN\leftarrow RPS}$	$c^{m,n}_{MAN\leftarrow RPS} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{MAN\leftarrow RPS} \cdot c^{unit\_m,n}_{MAN\leftarrow RPS}$	1, 2
$\tau^{m,n}_{MAN\rightarrow RPS}$	MAN	RPS	$c^{unit\_m,n}_{MAN\rightarrow RPS}$	$c^{m,n}_{MAN\rightarrow RPS} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{MAN\rightarrow RPS} \cdot c^{unit\_m,n}_{MAN\rightarrow RPS}$	1, 2
$\tau^{m,n}_{REG\leftarrow MAN}$	MAN	REG	$c^{unit\_m,n}_{REG\leftarrow MAN}$	$c^{m,n}_{REG\leftarrow MAN} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{REG\leftarrow MAN} \cdot c^{unit\_m,n}_{REG\leftarrow MAN}$	1, 2
$\tau^{m,n}_{REG\rightarrow MAN}$	REG	MAN	$c^{unit\_m,n}_{REG\rightarrow MAN}$	$c^{m,n}_{REG\rightarrow MAN} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{REG\rightarrow MAN} \cdot c^{unit\_m,n}_{REG\rightarrow MAN}$	1, 2
$\tau^{m,n}_{PRS\leftarrow MAN}$	MAN	PRS	$c^{unit\_m,n}_{PRS\leftarrow MAN}$	$c^{m,n}_{PRS\leftarrow MAN} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{PRS\leftarrow MAN} \cdot c^{unit\_m,n}_{PRS\leftarrow MAN}$	1, 2
$\tau^{m,n}_{PRS\rightarrow MAN}$	PRS	MAN	$c^{unit\_m,n}_{PRS\rightarrow MAN}$	$c^{m,n}_{PRS\rightarrow MAN} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{PRS\rightarrow MAN} \cdot c^{unit\_m,n}_{PRS\rightarrow MAN}$	1, 2
$\tau^{m,n}_{CMS\leftarrow MAN}$	MAN	CMS	$c^{unit\_m,n}_{CMS\leftarrow MAN}$	$c^{m,n}_{CMS\leftarrow MAN} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{CMS\leftarrow MAN} \cdot c^{unit\_m,n}_{CMS\leftarrow MAN}$	1, 2
$\tau^{m,n}_{CMS\rightarrow MAN}$	CMS	MAN	$c^{unit\_m,n}_{CMS\rightarrow MAN}$	$c^{m,n}_{CMS\rightarrow MAN} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{CMS\rightarrow MAN} \cdot c^{unit\_m,n}_{CMS\rightarrow MAN}$	1, 2
$\tau^{m,n}_{SUP\leftarrow CMS}$	CMS	SUP	$c^{unit\_m,n}_{SUP\leftarrow CMS}$	$c^{m,n}_{SUP\leftarrow CMS} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{SUP\leftarrow CMS} \cdot c^{unit\_m,n}_{SUP\leftarrow CMS}$	1, 2
$\tau^{m,n}_{SUP\rightarrow CMS}$	SUP	CMS	$c^{unit\_m,n}_{SUP\rightarrow CMS}$	$c^{m,n}_{SUP\rightarrow CMS} = \gamma^{m,n}_{pass} \cdot \tau^{m,n}_{SUP\rightarrow CMS} \cdot c^{unit\_m,n}_{SUP\rightarrow CMS}$	1, 2
$\tau^{m,n}_{d^0_{l/m,n}\leftarrow SUP}$	SUP	$d^0_{l/(m,n)}$	$c^{unit\_m,n}_{d^0_{l/m,n}\leftarrow SUP}$	$c^{m,n}_{d^0_{l/m,n}\leftarrow SUP} = \gamma^{m,n}_{pass} \cdot c^{m,n}_{d^0_{l/m,n}\leftarrow SUP} \cdot c^{unit\_m,n}_{d^0_{l/m,n}\leftarrow SUP}$	1, 2
$\tau^{m,n}_{d^0_{l/m,n}\rightarrow SUP}$	$d^0_{l/(m,n)}$	SUP	$c^{unit\_m,n}_{d^0_{l/m,n}\rightarrow SUP}$	$c^{m,n}_{d^0_{l/m,n}\rightarrow SUP} = \gamma^{m,n}_{pass} \cdot c^{m,n}_{d^0_{l/m,n}\rightarrow SUP} \cdot c^{unit\_m,n}_{d^0_{l/m,n}\rightarrow SUP}$	1, 2

Table 2. Costs of handling delays in units for Model 1 and Model 2.

Times of inquiry analysis	**Preceding unit**	**Subsequent unit**	**Unit cost of delay analysis**	**Total cost of delays**	**Used in model**
$\tau^{m,n}_{CSD\leftarrow\bullet}$	$z^0_{m,n}$	*RPS*	$c^{unit\_m,n}_{CSD\leftarrow\bullet}$	$c^{m,n}_{CSD\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{CSD\leftarrow\bullet}\cdot c^{unit\_m,n}_{CSD\leftarrow\bullet}$	1
$\tau^{m,n}_{CSD\rightarrow\bullet}$	*RPS*	$z^0_{m,n}$	$c^{unit\_m,n}_{CSD\rightarrow\bullet}$	$c^{m,n}_{CSD\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{CSD\rightarrow\bullet}\cdot c^{unit\_m,n}_{CSD\rightarrow\bullet}$	1
$\tau^{m,n}_{CSD\leftarrow\bullet}$	$z^0_{m,n}$	*MAN*	$c^{unit\_m,n}_{CSD\leftarrow\bullet}$	$c^{m,n}_{CSD\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{CSD\leftarrow\bullet}\cdot c^{unit\_m,n}_{CSD\leftarrow\bullet}$	2
$\tau^{m,n}_{CSD\rightarrow\bullet}$	*MAN*	$z^0_{m,n}$	$c^{unit\_m,n}_{CSD\rightarrow\bullet}$	$c^{m,n}_{CSD\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{CSD\rightarrow\bullet}\cdot c^{unit\_m,n}_{CSD\rightarrow\bullet}$	2
$\tau^{m,n}_{RPS\leftarrow\bullet}$	*CSD*	*MAN*	$c^{unit\_m,n}_{RPS\leftarrow\bullet}$	$c^{m,n}_{RPS\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{RPS\leftarrow\bullet}\cdot c^{unit\_m,n}_{RPS\leftarrow\bullet}$	1
$\tau^{m,n}_{RPS\rightarrow\bullet}$	*MAN*	*CSD*	$c^{unit\_m,n}_{RPS\rightarrow\bullet}$	$c^{m,n}_{RPS\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{RPS\rightarrow\bullet}\cdot c^{unit\_m,n}_{RPS\rightarrow\bullet}$	1
$\tau^{m,n}_{MAN\leftarrow\bullet}$	*MAN*	*RPS*	$c^{unit\_m,n}_{MAN\leftarrow\bullet}$	$c^{m,n}_{MAN\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{MAN\leftarrow\bullet}\cdot c^{unit\_m,n}_{MAN\leftarrow\bullet}$	1
$\tau^{m,n}_{MAN\rightarrow\bullet}$	*RPS*	*MAN*	$c^{unit\_m,n}_{MAN\rightarrow\bullet}$	$c^{m,n}_{MAN\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{MAN\rightarrow\bullet}\cdot c^{unit\_m,n}_{MAN\rightarrow\bullet}$	1
$\tau^{m,n}_{RPS\leftarrow\bullet}$	*MAN*	*MAN*	$c^{unit\_m,n}_{RPS\leftarrow\bullet}$	$c^{m,n}_{RPS\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{RPS\leftarrow\bullet}\cdot c^{unit\_m,n}_{RPS\leftarrow\bullet}$	2
$\tau^{m,n}_{RPS\rightarrow\bullet}$	*MAN*	*MAN*	$c^{unit\_m,n}_{RPS\rightarrow\bullet}$	$c^{m,n}_{RPS\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{RPS\rightarrow\bullet}\cdot c^{unit\_m,n}_{RPS\rightarrow\bullet}$	2
$\tau^{m,n}_{MAN\leftarrow\bullet}$	*MAN*	*CSD*	$c^{unit\_m,n}_{MAN\leftarrow\bullet}$	$c^{m,n}_{MAN\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{MAN\leftarrow\bullet}\cdot c^{unit\_m,n}_{MAN\leftarrow\bullet}$	2
$\tau^{m,n}_{MAN\rightarrow\bullet}$	*CSD*	*MAN*	$c^{unit\_m,n}_{MAN\rightarrow\bullet}$	$c^{m,n}_{MAN\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{MAN\rightarrow\bullet}\cdot c^{unit\_m,n}_{MAN\rightarrow\bullet}$	2
$\tau^{m,n}_{MAN\leftarrow\bullet}$	*MAN*	*REG*	$c^{unit\_m,n}_{MAN\leftarrow\bullet}$	$c^{m,n}_{MAN\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{MAN\leftarrow\bullet}\cdot c^{unit\_m,n}_{MAN\leftarrow\bullet}$	1,2
$\tau^{m,n}_{MAN\rightarrow\bullet}$	*REG*	*MAN*	$c^{unit\_m,n}_{MAN\rightarrow\bullet}$	$c^{m,n}_{MAN\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{MAN\rightarrow\bullet}\cdot c^{unit\_m,n}_{MAN\rightarrow\bullet}$	1,2
$\tau^{m,n}_{REG\leftarrow\bullet}$	*MAN*	*REG*	$c^{unit\_m,n}_{REG\leftarrow\bullet}$	$c^{m,n}_{REG\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{REG\leftarrow\bullet}\cdot c^{unit\_m,n}_{REG\leftarrow\bullet}$	1,2
$\tau^{m,n}_{REG\rightarrow\bullet}$	*REG*	*MAN*	$c^{unit\_m,n}_{REG\rightarrow\bullet}$	$c^{m,n}_{REG\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{REG\rightarrow\bullet}\cdot c^{unit\_m,n}_{REG\rightarrow\bullet}$	1,2
$\tau^{m,n}_{PRS\leftarrow\bullet}$	*MAN*	*PRS*	$c^{unit\_m,n}_{PRS\leftarrow\bullet}$	$c^{m,n}_{PRS\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{PRS\leftarrow\bullet}\cdot c^{unit\_m,n}_{PRS\leftarrow\bullet}$	1,2
$\tau^{m,n}_{PRS\rightarrow\bullet}$	*PRS*	*MAN*	$c^{unit\_m,n}_{PRS\rightarrow\bullet}$	$c^{m,n}_{PRS\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{PRS\rightarrow\bullet}\cdot c^{unit\_m,n}_{PRS\rightarrow\bullet}$	1,2
$\tau^{m,n}_{CMS\leftarrow\bullet}$	*MAN*	*SUP*	$c^{unit\_m,n}_{CMS\leftarrow\bullet}$	$c^{m,n}_{CMS\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{CMS\leftarrow\bullet}\cdot c^{unit\_m,n}_{CMS\leftarrow\bullet}$	1,2
$\tau^{m,n}_{CMS\rightarrow\bullet}$	*SUP*	*MAN*	$c^{unit\_m,n}_{CMS\rightarrow\bullet}$	$c^{m,n}_{CMS\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{CMS\rightarrow\bullet}\cdot c^{unit\_m,n}_{CMS\rightarrow\bullet}$	1,2
$\tau^{m,n}_{SUP\leftarrow\bullet}$	*CMS*	$d^0_{l/(m,n)}$	$c^{unit\_m,n}_{SUP\leftarrow\bullet}$	$c^{m,n}_{SUP\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{SUP\leftarrow\bullet}\cdot c^{unit\_m,n}_{SUP\leftarrow\bullet}$	1,2
$\tau^{m,n}_{SUP\rightarrow\bullet}$	$d^0_{l/(m,n)}$	*CMS*	$c^{unit\_m,n}_{SUP\rightarrow\bullet}$	$c^{m,n}_{SUP\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot\tau^{m,n}_{SUP\rightarrow\bullet}\cdot c^{unit\_m,n}_{SUP\rightarrow\bullet}$	1,2
$\tau^{m,n}_{d^0_{l/m,n}\leftarrow\bullet}$	*SUP*	$d^0_{l/(m,n)}$	$c^{unit\_m,n}_{d^0_{l/m,n}\leftarrow\bullet}$	$c^{m,n}_{d^0_{l/m,n}\leftarrow\bullet}=\gamma^{m,n}_{el}\cdot c^{m,n}_{d^0_{l/m,n}\leftarrow\bullet}\cdot c^{unit\_m,n}_{d^0_{l/m,n}\leftarrow\bullet}$	1,2
$\tau^{m,n}_{d^0_{l/m,n}\rightarrow\bullet}$	$d^0_{l/(m,n)}$	*SUP*	$c^{unit\_m,n}_{d^0_{l/m,n}\rightarrow\bullet}$	$c^{m,n}_{d^0_{l/m,n}\rightarrow\bullet}=\gamma^{m,n}_{el}\cdot c^{m,n}_{d^0_{l/m,n}\rightarrow\bullet}\cdot c^{unit\_m,n}_{d^0_{l/m,n}\rightarrow\bullet}$	1,2

Let us introduce the matrix of times of servicing customers' inquiries for products included in the order inquiries matrix:

$$T=[\tau^{m,n}],\ m=1,\ ...,\ M,\ n=1,\ ...,\ N \tag{9}$$

At the same time $\tau^{m,n}=\tau^{m,n}_{\leftarrow\bullet}+\tau^{m,n}_{\rightarrow\bullet}$, where in the case of Model 1 we get:

$$\begin{aligned}\tau^{m,n}_{\leftarrow\bullet}&=\tau^{m,n}_{CSD\leftarrow z^0_{m,n}}+\tau^{m,n}_{RPS\leftarrow CSD}+\tau^{m,n}_{MAN\leftarrow RPS}\\&+\tau^{m,n}_{REG\leftarrow MAN}+\tau^{m,n}_{PRS\leftarrow MAN}+\tau^{m,n}_{CMS\leftarrow MAN}\\&+\tau^{m,n}_{SUP\leftarrow CMS}+\tau^{m,n}_{d^0_{l/(m,n)}\leftarrow SUP}+\tau^{m,n}_{CSD\leftarrow\bullet}\\&+\tau^{m,n}_{RPS\leftarrow\bullet}+\tau^{m,n}_{MAN\leftarrow\bullet}+\tau^{m,n}_{REG\leftarrow\bullet}\\&+\tau^{m,n}_{PRS\leftarrow\bullet}+\tau^{m,n}_{CMS\leftarrow\bullet}+\tau^{m,n}_{SUP\leftarrow\bullet}+\tau^{m,n}_{d^0_{l/(m,n)}\leftarrow\bullet}\end{aligned}$$

$$\begin{aligned}\tau^{m,n}_{\rightarrow\bullet}&=\tau^{m,n}_{CSD\rightarrow z^0_{m,n}}+\tau^{m,n}_{RPS\rightarrow CSD}+\tau^{m,n}_{MAN\rightarrow RPS}\\&+\tau^{m,n}_{REG\rightarrow MAN}+\tau^{m,n}_{PRS\rightarrow MAN}+\tau^{m,n}_{CMS\rightarrow MAN}\\&+\tau^{m,n}_{SUP\rightarrow CMS}+\tau^{m,n}_{d^0_{l/(m,n)}\rightarrow SUP}+\tau^{m,n}_{CSD\rightarrow\bullet}\\&+\tau^{m,n}_{RPS\rightarrow\bullet}+\tau^{m,n}_{MAN\rightarrow\bullet}+\tau^{m,n}_{REG\rightarrow\bullet}\\&+\tau^{m,n}_{PRS\rightarrow\bullet}+\tau^{m,n}_{CMS\rightarrow\bullet}+\tau^{m,n}_{SUP\rightarrow\bullet}+\tau^{m,n}_{d^0_{l/(m,n)}\rightarrow\bullet}\end{aligned}$$

while in the case of Model 2 we get:

$$\begin{aligned}\tau^{m,n}_{\leftarrow\bullet}&=\tau^{m,n}_{CSD\leftarrow z^0_{m,n}}+\tau^{m,n}_{MAN\leftarrow CSD}+\tau^{m,n}_{RPS\leftarrow MAN}\\&+\tau^{m,n}_{REG\leftarrow MAN}+\tau^{m,n}_{PRS\leftarrow MAN}+\tau^{m,n}_{CMS\leftarrow MAN}\\&+\tau^{m,n}_{SUP\leftarrow CMS}+\tau^{m,n}_{d^0_{l/(m,n)}\leftarrow SUP}+\tau^{m,n}_{CSD\leftarrow\bullet}\\&+\tau^{m,n}_{MAN\leftarrow\bullet}+\tau^{m,n}_{RPS\leftarrow\bullet}+\tau^{m,n}_{REG\leftarrow\bullet}\\&+\tau^{m,n}_{PRS\leftarrow\bullet}+\tau^{m,n}_{CMS\leftarrow\bullet}+\tau^{m,n}_{SUP\leftarrow\bullet}+\tau^{m,n}_{d^0_{l/(m,n)}\leftarrow\bullet}\end{aligned}$$

$$\begin{aligned}\tau^{m,n}_{\rightarrow\bullet}&=\tau^{m,n}_{CSD\rightarrow z^0_{m,n}}+\tau^{m,n}_{MAN\rightarrow CSD}+\tau^{m,n}_{RPS\rightarrow MAN}\\&+\tau^{m,n}_{REG\rightarrow MAN}+\tau^{m,n}_{PRS\rightarrow MAN}+\tau^{m,n}_{CMS\rightarrow MAN}\\&+\tau^{m,n}_{SUP\rightarrow CMS}+\tau^{m,n}_{d^0_{l/(m,n)}\rightarrow SUP}+\tau^{m,n}_{CSD\rightarrow\bullet}\\&+\tau^{m,n}_{MAN\rightarrow\bullet}+\tau^{m,n}_{RPS\rightarrow\bullet}+\tau^{m,n}_{REG\rightarrow\bullet}\\&+\tau^{m,n}_{PRS\rightarrow\bullet}+\tau^{m,n}_{CMS\rightarrow\bullet}+\tau^{m,n}_{SUP\rightarrow\bullet}+\tau^{m,n}_{d^0_{l/(m,n)}\rightarrow\bullet}.\end{aligned}$$

Consequently, the general matrix of the total costs of responses to customers' inquiries is introduced as follows:

in case of Model 1:

$$C_{\mathrm{mod}_1}=\left[c^{m,n}_{\mathrm{mod}_1}\right],\quad m=1,\ ...,M,\ n=1,\ ...,\ N$$

in case of Model 2:

$$C_{\mathrm{mod}_2}=\left[c^{m,n}_{\mathrm{mod}_2}\right],\quad m=1,\ ...,M,\ n=1,\ ...,\ N \tag{10}$$

where $c^{m,n}_{\mathrm{mod}_1}$ is the cost of returning the m-th customer's inquiry for the n-th product in Model 1, and $c^{m,n}_{\mathrm{mod}_2}$ is the cost of returning the m-th customer's inquiry for the n-th product in Model 2. In order to generalize this process, we introduce the following matrices:

- matrix of delay times of passing the information inquiry between the preceding and subsequent units:

$$T_{\alpha\rightarrow(\alpha+1)}=\left[\tau^{m,n}_{\alpha\rightarrow(\alpha+1)}\right],\quad \alpha=1,\ ...,A,\ m=1,\ ...,M,\ n=1,\ ...,\ N \tag{11}$$

  where $\tau^{m,n}_{\alpha\rightarrow(\alpha+1)}$ is the time of the information flow between the α-th preceding logistic unit and the subsequent logistic unit $\alpha+1$;

- matrix of delay unit costs of passing the information inquiry between the preceding and subsequent units:

$$C_{\alpha\rightarrow(\alpha+1)}=\left[c^{m,n}_{\alpha\rightarrow(\alpha+1)}\right],\quad \alpha=1,\ ...,A,\ m=1,\ ...,M,\ n=1,\ ...,\ N \tag{12}$$

  where $c^{m,n}_{\alpha\rightarrow(\alpha+1)}$ is the delay unit cost of the information flow between the α-th preceding logistic unit and the subsequent logistic unit $\alpha+1$;

- matrix of the delay times of passing the information inquiry between the preceding and subsequent logistic units in the reverse mode:

$$T_{\alpha\rightarrow(\alpha+1)}=\left[\tau^{m,n}_{\alpha\rightarrow(\alpha+1)}\right],\quad \alpha=A,\ ...,\ 1,\ m=1,\ ...,M,\ n=1,\ ...,\ N \tag{13}$$

  where $\tau^{m,n}_{\alpha\rightarrow(\alpha+1)}$ is the time of the information flow between logistic units in the reverse mode;

- matrix of delay unit costs of passing the information inquiry between the preceding and subsequent logistic units in the reverse mode:

$$C_{\alpha\to(\alpha+1)} = \left[c^{m,n}_{\alpha\to(\alpha+1)}\right],\\ \alpha = A, ..., 1,\ m = 1, ..., M,\ n = 1, ..., N \tag{14}$$

where $c^{m,n}_{\alpha\to(\alpha+1)}$ is the delay unit cost of the information flow between logistic units in the reverse mode;

- matrix of servicing times of incoming customers' inquiries in logistic units:

$$T^{in}_{\alpha} = \left[\tau^{in\_m,n}_{\alpha}\right],\\ \alpha = A, ..., 1,\ m = 1, ..., M,\ n = 1, ..., N \tag{15}$$

where $\tau^{in\_m,n}_{\alpha}$ is the time of servicing the incoming inquiry for the m-th customer ordering the n-th product in the α-th logistic unit;

- matrix of delay unit costs of servicing incoming customers' inquiries in logistic units:

$$C^{in}_{\alpha} = \left[c^{in\_m,n}_{\alpha}\right],\\ \alpha = A, ..., 1,\ m = 1, ..., M,\ n = 1, ..., N \tag{16}$$

where $c^{in\_m,n}_{\alpha}$ is the delay unit cost of servicing the incoming inquiry for the m-th customer ordering the n-th product in the α-th logistic unit;

- matrix of servicing times of the outcoming inquiries of customers in logistic units:

$$T^{out}_{\alpha} = \left[\tau^{out\_m,n}_{\alpha}\right],\\ \alpha = A, ..., 1,\ m = 1, ..., M,\ n = 1, ..., N \tag{17}$$

where $\tau^{out\_m,n}_{\alpha}$ is the time of servicing the outcoming inquiry for the m-th customer ordering the n-th product in the α-th logistic unit;

- matrix of servicing times of outcoming customers' inquiries in logistic units:

$$C^{out}_{\alpha} = \left[c^{out\_m,n}_{\alpha}\right],\\ \alpha = A, ..., 1,\ m = 1, ..., M,\ n = 1, ..., N \tag{18}$$

where $c^{out\_m,n}_{\alpha}$ is the delay unit cost of servicing the outcoming inquiry for the m-th customer ordering the n-th product in the α-th logistic unit;

- matrix of delay times of passing the information inquiry between the main logistic units of the logistics chain and their subunits:

$$T_{\alpha\to\beta} = \left[\tau^{m,n}_{\alpha\to\beta}\right],\\ \alpha = 1, ..., A,\ \beta = 1, ..., B,\\ m = 1, ..., M,\ n = 1, ..., N \tag{19}$$

where $\tau^{m,n}_{\alpha\to\beta}$ is the time of the information flow between the α-th main logistic unit and its β-th logistics subunit;

- matrix of delay unit costs of passing the information inquiry between main logistic units of the logistics chain and their subunits:

$$C_{\alpha\to\beta} = \left[c^{m,n}_{\alpha\to\beta}\right],\\ \alpha = 1, ..., A,\ \beta = 1, ..., B,\\ m = 1, ..., M,\ n = 1, ..., N \tag{20}$$

where $c^{m,n}_{\alpha\to\beta}$ is the delay unit cost of the information flow between the α-th main logistic unit and its β-th logistics subunit;

- matrix of delay times of passing the information inquiry between subunits and their main units:

$$T_{\beta\to\alpha} = \left[\tau^{m,n}_{\beta\to\alpha}\right],\\ \alpha = 1, ..., A,\ \beta = 1, ..., B,\\ m = 1, ..., M,\ n = 1, ..., N \tag{21}$$

where $\tau^{m,n}_{\beta\to\alpha}$ is the time of the information flow between the β-th logistics subunit and the α-th logistic unit;

- matrix of delay unit costs of passing the information inquiry between subunits and the main units of the logistics chain:

$$C_{\beta\to\alpha} = \left[c^{m,n}_{\beta\to\alpha}\right],\\ \alpha = 1, ..., A,\ \beta = 1, ..., B,\\ m = 1, ..., M,\ n = 1, ..., N \tag{22}$$

where $c^{m,n}_{\beta\to\alpha}$ is the time of the information flow between the β-th logistics subunit and the main α-th logistic unit;

- matrix of servicing times of incoming customers' inquiries in logistics subunits:

$$T_{\beta}^{in} = \left[\tau_{\beta}^{in\_m,n}\right], \quad \beta = 1, ..., B,\ m = 1, ..., M,\ n = 1, ..., N \tag{23}$$

where $\tau_{\beta}^{in\_m,n}$ is the time of servicing the incoming inquiry for the m-th customer ordering the n-th product in the β-th logistics subunit;

- matrix of delay unit costs of servicing incoming customers' inquiries in logistics subunits:

$$C_{\beta}^{in} = \left[c_{\beta}^{in\_m,n}\right], \quad \beta = 1, ..., B,\ m = 1, ..., M,\ n = 1, ..., N \tag{24}$$

where $c_{\beta}^{in\_m,n}$ is the delay unit cost of servicing the incoming inquiry for the m-th customer ordering the n-th product in the β-th logistics subunit;

- matrix of servicing times of outcoming customers' inquiries in logistics subunits:

$$T_{\beta}^{out} = \left[\tau_{\beta}^{out\_m,n}\right], \quad \beta = 1, ..., B,\ m = 1, ..., M,\ n = 1, ..., N \tag{25}$$

where $\tau_{\beta}^{out\_m,n}$ is the time of servicing the outcoming inquiry for the m-th customer ordering the n-th product in the β-th logistics subunit;

- matrix of delay unit costs of servicing outcoming customers' inquiries in logistics subunits:

$$C_{\beta}^{out} = \left[c_{\beta}^{out\_m,n}\right], \quad \beta = 1, ..., B,\ m = 1, ..., M,\ n = 1, ..., N \tag{26}$$

where $c_{\beta}^{out\_m,n}$ is the delay unit cost of servicing the outcoming inquiry for the m-th customer ordering the n-th product in the β-th logistics subunit.

Then the logistics cost of servicing the m-th customer's inquiry ordering the n-th product is calculated as follows:

$$\begin{aligned} C_{m,n} = \left(\xi + \frac{z^0_{m,n}}{\zeta}\right)\cdot\Bigg(&\sum_{\alpha=1}^{A}\tau^{m,n}_{\alpha+1\to\alpha}\cdot c^{m,n}_{\alpha+1\to\alpha} \\ &+\sum_{\alpha=1}^{A}\tau^{m,n}_{\alpha\to\alpha+1}\cdot c^{m,n}_{\alpha\to\alpha+1} + \sum_{\alpha=1}^{A}\tau_{\alpha}^{in_{m,n}}\cdot c_{\alpha}^{in_{m,n}} \\ &+\sum_{\alpha=1}^{A}\tau_{\alpha}^{out_{m,n}}\cdot c_{\alpha}^{out_{m,n}} + \sum_{\beta=1}^{B}\sum_{\alpha=1}^{A}\tau^{m,n}_{\alpha\to\beta}\cdot c^{m,n}_{\alpha\to\beta} \\ &+\sum_{\beta=1}^{B}\sum_{\alpha=1}^{A}\tau^{m,n}_{\beta\to\alpha}\cdot c^{m,n}_{\beta\to\alpha} + \sum_{\beta=1}^{B}\sum_{\alpha=1}^{A}\tau_{\beta}^{in_{m,n}}\cdot c_{\beta}^{in_{m,n}} \\ &+\sum_{\beta=1}^{B}\sum_{\alpha=1}^{A}\tau_{\beta}^{out_{m,n}}\cdot c_{\beta}^{out_{m,n}}\Bigg) \end{aligned} \tag{27}$$

Finally, the total logistics cost of servicing delays resulting from customers inquiries is as follows:

$$\begin{aligned} C = \left(\xi + \frac{\sum_{n=1}^{N}\sum_{m=1}^{M} z^0_{m,n}}{\zeta}\right)\cdot\Bigg(&\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{\alpha=1}^{A}\tau^{m,n}_{\alpha+1\to\alpha}\cdot c^{m,n}_{\alpha+1\to\alpha} \\ &+\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{\alpha=1}^{A}\tau^{m,n}_{\alpha\to\alpha+1}\cdot c^{m,n}_{\alpha\to\alpha+1} \\ &+\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{\alpha=1}^{A}\tau_{\alpha}^{in_{m,n}}\cdot c_{\alpha}^{in_{m,n}} \\ &+\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{\alpha=1}^{A}\tau_{\alpha}^{out_{m,n}}\cdot c_{\alpha}^{out_{m,n}} \\ &+\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{\beta=1}^{B}\sum_{\alpha=1}^{A}\tau^{m,n}_{\alpha\to\beta}\cdot c^{m,n}_{\alpha\to\beta} \\ &+\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{\beta=1}^{B}\sum_{\alpha=1}^{A}\tau^{m,n}_{\beta\to\alpha}\cdot c^{m,n}_{\beta\to\alpha} \\ &+\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{\beta=1}^{B}\tau_{\beta}^{in_{m,n}}\cdot c_{\beta}^{in_{m,n}} \\ &+\sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{\beta=1}^{B}\tau_{\beta}^{out_{m,n}}\cdot c_{\beta}^{out_{m,n}}\Bigg) \end{aligned} \tag{28}$$

Let us further assume that there are two following criteria to be implemented:

- minimizing the time of responses to customers' inquiries: $Q_T \to min$, and

Table 3. Draw ranges for sequential and non-sequential flow of inquiries.

Draw range	**Model 1** sequential flow		**Model 2** non-sequential flow	
	min	**max**	**min**	**max**
Query transmission times	1	2	1	2
Unit costs of sending inquiries	3	4	3	4
Query handling times	10	50	10	50
Unit costs of handling inquiries	5	10	5	10

- minimizing the cost of responses to customers' inquiries: $Q_C \rightarrow min$.

For the case in which customers send their inquiries to the logistics system in sequence, this means that each subsequent unit is to service the incoming inquiry immediately after the preceding one sent it to the following unit, which obviously leads to forming queues that are subject to the following analysis.

Thus, let us introduce the matrix of the awaiting times of incoming customers' inquiries in units:

$$\Delta T_{\bar{\alpha}} = \left[\delta\tau_{\bar{\alpha}}^{m,n} \right], \quad \alpha = 1, \ldots, A, \; m = 1, \ldots, M, \; n = 1, \ldots, N \qquad (29)$$

where $\delta\tau_{\bar{\alpha}}^{m,n}$ is the awaiting time for servicing the incoming inquiry for the m-th customer ordering the n-th product in the α-th unit.

4. Case Study

In order to verify the specification assumptions of the presented mathematical model, the whole simulation process was carried out for the given input data. The data for the implementation of the tasks in this case study were selected using the randomization function supported by a pseudorandom code generator. Two logistics models were subject to thorough consideration in accordance with the same arrangement of units.

The difference is visible when it comes to passing the information concerning customers' inquiries. Model 1 is characterized by sending inquiries in sequence, while Model 2 is the non-sequential one. For simplicity reasons, the number of logistic units as well as the number of information messages between logistic units are identical. First, the ranges for the sequential and non-sequential flow of clients' inquiries were selected. Table 3 shows the range of input data sampling for the two analyzed models (sequential and non-sequential flow). It should be noted that the range of randomized input data for both models is identical. The differences are generated by random selection of the input data. The criterion of minimizing the cost of responses to customers' inquiries is implemented for this case study.

Subsequently, data for Model 1 (sequential flow of query information) including transfer as well as processing times and unit costs corresponding with these times were derived (see Tables 4 and 5). Likewise, the same kind of data were derived for Model 2 (see Tables 6 and 7).

Table 8 brings the number of the n-th order elements for each m-th customer drawn according to the drawing ranges. There are 12 ranges of input data draws for the number of specific orders. The ranges of the draws were selected beginning with 0 to the maximum chosen number. The maximum values were changed every 10 units. Finally, a decision was made to further implement random data in the 0-80 range.

The results of the simulation processes depending on the number of orders are shown as the minimal values of total costs obtained in 100 simulations (see Table 9).

To illustrate the problem, only the results obtained by means of drawing for the range 0 – 80 were subject to the further analysis (see Figure 3).

Table 4. Transfer and processing times for Model 1 (sequential flow of query information).

	Sending the inquiry																Elaborating the inquiry															
z(m,n)	Z→CSD	CSD→Z	CSD→RPS	RPS→CSD	MAN→RPS	RPS→MAN	MAN→REG	REG→MAN	MAN→PRS	PRS→MAN	MAN→CMS	CMS→MAN	CMS→SUP	SUP→CMS	SUP→D	D→SUP	CSD_in	CSD_out	RPS_in	RPS_out	MAN_in	MAN_out	REG_in	REG_out	PRS_in	PRS_out	CMS_in	CMS_out	SUP_in	SUP_out	D_in	D_out
(1,1)	1	2	2	1	1	2	1	2	2	1	2	2	2	1	1	2	29	46	19	17	13	22	18	39	39	10	47	31	49	28	16	22
(1,2)	1	2	2	2	1	1	2	1	1	2	1	2	2	2	1	1	38	34	23	12	11	12	42	47	23	12	11	23	20	36	11	21
(1,3)	1	1	2	1	2	1	2	1	2	2	2	1	1	2	2	1	13	28	43	22	43	40	23	34	11	23	17	23	22	27	18	31
(1,4)	2	1	1	1	1	1	2	1	1	2	1	2	2	1	1	1	37	26	23	41	13	46	34	41	38	49	43	38	11	40	28	33
(1,5)	2	1	1	1	1	2	1	1	1	1	1	1	2	1	1	2	18	36	19	14	35	33	27	14	39	12	11	38	21	12	46	36
(2,1)	2	1	1	2	2	1	1	1	2	2	2	1	2	1	1	1	35	43	40	21	29	15	18	26	28	16	25	24	27	11	35	42
(2,2)	1	2	1	1	1	2	2	2	1	2	1	2	1	1	1	1	44	19	41	46	30	31	11	34	33	40	20	14	22	41	39	14
(2,3)	1	1	2	1	1	2	2	1	1	1	1	1	1	1	2	2	14	25	25	37	17	39	25	23	49	16	12	36	44	12	35	48
(2,4)	2	2	1	2	1	1	2	2	1	1	2	1	2	1	2	1	31	24	38	39	43	46	30	10	44	21	27	40	12	25	16	11
(2,5)	1	2	2	1	2	2	2	2	2	2	2	1	2	2	2	2	13	42	42	14	46	20	37	25	41	21	40	10	49	10	38	47
(3,1)	1	2	2	1	2	1	2	1	1	1	2	2	1	2	1	1	48	44	25	17	37	26	34	34	24	32	15	23	19	36	13	28
(3,2)	1	2	1	1	1	1	2	2	2	1	2	1	1	1	1	2	27	31	43	20	37	41	21	28	44	23	12	25	32	11	34	28
(3,3)	2	2	2	2	1	1	1	2	2	1	1	1	2	1	1	1	50	16	27	29	36	22	47	25	40	37	18	33	43	37	18	43
(3,4)	2	2	2	1	2	2	2	1	1	2	2	2	2	1	1	2	45	30	44	44	31	48	36	46	30	40	37	36	20	46	35	40
(3,5)	1	1	2	1	2	2	1	1	1	2	2	2	2	1	1	2	21	39	43	24	42	24	14	36	46	23	22	16	32	48	12	35
(4,1)	2	1	1	1	2	1	1	1	2	2	1	1	1	2	1	2	29	28	11	38	38	13	46	22	19	41	12	39	21	49	13	14
(4,2)	2	1	2	2	1	2	1	1	1	2	2	1	2	2	1	1	25	41	43	37	42	38	44	47	27	19	27	41	14	45	47	20
(4,3)	2	2	2	1	1	1	1	2	2	2	2	2	1	1	2	2	32	39	15	39	22	11	15	32	48	22	12	30	50	34	27	45
(4,4)	1	1	1	1	2	1	1	2	1	2	2	2	2	2	1	2	38	24	38	41	42	34	34	35	39	18	29	45	40	48	29	31
(4,5)	1	1	1	2	2	1	1	2	2	1	1	1	2	1	1	2	32	38	11	48	30	13	37	29	32	34	27	19	17	39	43	41
(5,1)	1	1	2	2	2	1	1	1	1	2	1	1	1	1	2	2	39	46	19	43	49	40	34	41	14	13	26	26	31	34	39	35
(5,2)	2	2	2	1	2	2	2	1	2	2	2	2	1	2	2	1	28	28	22	29	30	15	21	16	12	11	32	32	29	48	24	47
(5,3)	1	2	1	1	1	1	2	2	2	1	1	2	2	2	2	1	21	33	14	40	27	43	48	34	25	36	19	31	21	18	43	26
(5,4)	2	2	2	1	1	2	2	2	2	2	2	2	1	2	2	1	29	32	48	50	34	42	48	23	17	33	23	44	21	23	17	35
(5,5)	1	2	1	1	1	1	1	1	1	2	1	1	2	1	1	2	44	37	31	13	13	20	24	27	42	29	16	33	13	48	13	39

Table 5. Unit costs for Model 1 (sequential flow of query information).

	Sending the inquiry																Elaborating the inquiry															
z(m,n)	Z→CSD	CSD→Z	CSD→RPS	RPS→CSD	MAN→RPS	RPS→MAN	MAN→REG	REG→MAN	MAN→PRS	PRS→MAN	MAN→CMS	CMS→MAN	CMS→SUP	SUP→CMS	SUP→D	D→SUP	CSD_in	CSD_out	RPS_in	RPS_out	MAN_in	MAN_out	REG_in	REG_out	PRS_in	PRS_out	CMS_in	CMS_out	SUP_in	SUP_out	D_in	D_out
(1,1)	3	3	3	3	3	4	3	3	3	3	4	4	3	3	4	3	10	6	8	10	10	9	8	6	9	8	5	6	8	9	8	9
(1,2)	3	3	3	3	4	4	4	3	4	3	4	4	3	4	3	3	5	5	6	7	6	6	9	8	10	5	5	9	9	6	7	7
(1,3)	3	4	4	3	3	3	4	3	4	3	4	3	4	4	3	4	6	7	5	10	7	6	7	5	6	6	7	9	8	8	10	6
(1,4)	4	3	4	3	4	3	4	3	3	3	3	4	3	4	4	4	10	9	9	6	6	7	9	10	10	5	10	8	6	7	7	8
(1,5)	3	4	3	4	4	3	4	3	3	4	3	4	4	3	3	3	9	10	9	9	7	7	5	5	9	8	9	7	8	6	9	9
(2,1)	3	3	3	4	3	3	3	4	3	4	3	4	4	3	4	3	6	6	5	8	10	8	8	6	6	9	7	9	9	5	7	9
(2,2)	4	4	4	3	3	4	4	4	4	3	4	4	3	3	3	3	7	8	6	5	9	8	10	6	7	9	8	8	10	6	8	8
(2,3)	3	3	4	4	4	4	3	4	3	3	3	4	3	4	4	3	6	9	5	7	9	9	9	7	5	9	9	5	6	10	9	10
(2,4)	4	3	3	3	4	3	4	3	4	3	4	4	4	4	4	3	10	9	10	9	6	10	6	6	8	7	8	6	7	7	9	7
(2,5)	4	3	3	4	4	4	4	4	3	4	4	4	4	4	4	3	10	10	6	6	6	8	7	5	10	8	9	8	6	9	10	6
(3,1)	3	3	3	4	3	3	4	4	4	4	3	4	4	4	3	4	8	7	7	7	9	7	5	8	5	8	8	8	8	8	10	9
(3,2)	3	3	4	3	3	3	3	3	4	3	3	3	3	4	3	3	9	6	8	8	5	6	9	7	6	7	7	9	6	8	9	8
(3,3)	4	4	3	4	3	3	4	4	4	3	4	4	3	3	4	4	6	6	5	5	8	8	8	8	6	6	6	10	8	9	5	10
(3,4)	4	4	4	4	4	3	4	3	4	3	3	3	4	3	3	4	10	6	6	7	8	9	9	6	7	7	9	9	7	9	9	8
(3,5)	4	4	4	4	4	3	3	3	4	3	3	3	4	3	3	4	9	10	10	10	10	5	10	10	6	9	8	8	9	6	7	6
(4,1)	3	3	4	3	3	3	4	3	3	4	4	3	3	4	4	4	7	8	9	8	5	9	9	10	7	5	8	8	6	6	8	9
(4,2)	4	4	4	3	4	4	3	4	4	4	3	3	4	4	4	3	7	6	9	9	9	8	8	10	8	10	9	6	10	9	7	9
(4,3)	3	3	4	3	4	4	4	3	4	3	3	4	4	3	4	3	6	7	9	7	5	9	6	8	7	6	8	7	7	9	6	7
(4,4)	3	3	3	3	3	3	4	4	4	4	3	3	4	4	3	4	5	8	7	7	7	10	9	6	7	5	8	6	9	7	9	10
(4,5)	3	3	4	3	4	3	3	3	3	4	3	4	4	3	4	3	7	10	5	9	7	6	10	5	6	5	8	6	9	7	9	7
(5,1)	4	3	4	4	4	3	3	4	3	4	3	4	4	4	4	4	6	7	8	7	6	7	5	6	9	7	9	7	6	6	9	9
(5,2)	4	4	4	3	4	3	4	3	3	4	4	3	3	3	3	3	7	7	6	10	9	8	6	5	8	8	5	5	7	6	9	9
(5,3)	3	4	4	3	3	3	3	3	4	3	4	3	3	4	4	3	10	7	6	5	8	10	9	8	8	8	10	7	7	7	6	10
(5,4)	3	4	4	3	3	4	3	4	3	4	3	4	3	4	3	4	10	6	10	7	9	7	10	5	8	9	10	6	8	6	6	7
(5,5)	4	3	3	4	4	3	3	4	3	4	3	4	3	4	4	4	7	7	5	8	5	6	10	5	9	10	7	6	9	9	7	6

Table 6. Transfer and processing times for Model 2 (non-sequential flow of query information).

z(m,n)	Sending the inquiry																Elaborating the inquiry															
	Z→CSD	CSD→Z	CSD→MAN	MAN→CSD	MAN→RPS	RPS→MAN	MAN→REG	REG→MAN	MAN→PRS	PRS→MAN	MAN→SUP	SUP→MAN	SUP→CMS	CMS→SUP	SUP→D	D→SUP	CSD_in	CSD_out	RPS_in	RPS_out	MAN_in	MAN_out	REG_in	REG_out	PRS_in	PRS_out	CMS_in	CMS_out	SUP_in	SUP_out	D_in	D_out
(1,1)	1	1	2	1	1	2	2	1	1	2	2	2	2	1	1	2	15	17	18	26	46	32	15	45	37	12	19	15	23	36	43	13
(1,2)	1	1	2	1	2	1	1	2	1	2	1	2	2	2	2	1	13	10	37	38	49	10	21	31	42	33	43	47	45	19	28	21
(1,3)	2	1	2	2	2	2	1	1	1	2	2	2	1	2	1	2	32	37	31	33	32	23	40	40	47	49	19	46	49	17	47	33
(1,4)	2	2	1	2	2	1	1	2	2	2	2	1	2	1	2	1	27	23	24	30	11	25	36	46	24	36	47	44	38	48	36	48
(1,5)	1	2	2	2	2	2	2	1	1	2	1	1	1	2	1	1	13	46	42	11	45	38	16	18	12	48	25	42	29	11	29	27
(2,1)	1	1	1	2	1	2	1	2	1	2	2	1	2	1	1	1	27	16	46	26	12	25	40	21	10	14	23	22	12	39	44	36
(2,2)	2	1	2	1	2	2	2	2	2	1	1	2	2	1	2	2	18	27	28	11	24	40	36	41	32	31	40	24	16	37	13	30
(2,3)	1	1	1	1	2	2	2	2	2	2	1	1	2	2	1	2	43	36	18	22	32	18	47	41	25	30	49	17	23	18	32	49
(2,4)	1	2	1	1	2	2	2	2	1	2	1	2	1	2	1	1	21	27	45	17	26	37	25	15	13	48	20	11	26	39	16	10
(2,5)	2	2	2	1	2	1	2	1	2	1	2	2	1	1	1	1	10	17	19	27	37	21	17	20	45	36	40	32	45	49	32	37
(3,1)	1	2	2	1	1	1	1	2	1	2	1	1	2	1	1	2	27	39	32	12	19	39	36	50	49	27	16	25	41	17	14	20
(3,2)	1	1	1	1	2	1	1	2	2	1	2	1	1	2	1	1	16	10	45	38	22	14	13	49	16	25	44	34	13	16	22	38
(3,3)	2	1	2	1	1	2	2	1	1	2	1	1	2	1	2	2	42	49	22	39	37	37	44	26	20	35	43	29	34	30	40	50
(3,4)	2	2	1	2	2	2	1	2	1	2	2	1	2	1	1	1	15	37	43	14	25	41	45	35	40	36	26	22	18	42	45	30
(3,5)	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	42	12	48	25	32	15	19	19	36	42	26	24	30	45	11	46
(4,1)	2	2	2	2	1	1	1	2	1	1	2	1	2	1	2	2	30	30	36	48	21	40	48	14	16	24	29	43	41	49	10	35
(4,2)	1	1	1	2	1	1	2	2	2	1	2	1	1	1	2	2	31	23	12	11	18	29	22	25	20	44	31	47	38	50	33	24
(4,3)	1	2	1	1	1	1	1	2	1	1	2	2	1	2	2	1	15	15	25	16	12	22	47	46	47	43	22	14	19	43	49	27
(4,4)	2	2	1	1	1	1	1	2	2	1	2	2	1	1	1	2	38	20	45	47	30	30	38	19	33	19	23	49	39	34	50	15
(4,5)	2	1	1	1	2	1	1	1	2	2	1	1	2	1	1	2	42	11	44	43	18	14	17	23	36	49	35	21	34	42	12	47
(5,1)	1	1	1	1	1	1	1	1	2	1	2	1	2	2	1	2	17	46	14	27	19	18	33	32	42	37	17	30	16	42	32	46
(5,2)	1	2	2	1	2	2	2	2	1	1	2	2	2	2	1	2	39	41	46	40	11	23	15	38	36	42	18	30	12	45	24	30
(5,3)	2	2	1	2	2	2	1	1	1	2	1	1	2	1	1	1	44	43	40	49	42	37	48	22	36	50	20	43	18	13	28	30
(5,4)	2	2	2	2	2	1	2	2	1	2	2	2	2	2	2	2	28	23	13	47	37	31	40	20	16	43	35	33	44	42	35	44
(5,5)	1	2	1	2	1	2	2	2	1	1	2	2	2	1	2	1	29	49	18	36	21	38	17	24	35	36	20	44	37	40	36	44

Table 7. Unit costs for Model 2 (non-sequential flow of query information).

z(m,n)	Sending the inquiry																Elaborating the inquiry															
	Z→CSD	CSD→Z	CSD→MAN	MAN→CSD	MAN→RPS	RPS→MAN	MAN→REG	REG→MAN	MAN→PRS	PRS→MAN	MAN→SUP	SUP→MAN	SUP→CMS	CMS→SUP	SUP→D	D→SUP	CSD_in	CSD_out	RPS_in	RPS_out	MAN_in	MAN_out	REG_in	REG_out	PRS_in	PRS_out	CMS_in	CMS_out	SUP_in	SUP_out	D_in	D_out
(1,1)	4	4	4	4	3	3	3	3	4	3	4	4	3	3	3	4	6	6	6	7	8	10	7	6	7	10	9	9	10	9	8	9
(1,2)	3	3	4	3	4	3	4	4	3	3	3	3	4	3	3	3	10	9	10	8	8	5	7	9	9	9	9	9	9	9	6	6
(1,3)	4	4	3	3	3	3	3	4	4	4	3	3	4	4	4	3	7	6	7	5	9	9	9	5	8	9	10	9	10	9	7	9
(1,4)	4	3	4	3	3	3	3	4	4	4	4	3	4	4	3	3	10	6	7	5	6	6	7	9	10	9	5	8	9	7	6	8
(1,5)	3	4	3	3	3	3	3	4	4	4	4	4	4	4	3	4	8	6	7	6	9	10	8	8	9	5	9	6	5	8	8	10
(2,1)	4	4	3	4	4	3	3	4	4	3	4	4	4	4	3	4	8	5	8	8	7	6	9	8	8	7	5	5	9	5	8	8
(2,2)	3	3	3	4	3	3	4	4	4	4	3	4	4	3	3	4	8	8	7	5	7	9	9	5	9	9	9	9	7	7	9	7
(2,3)	4	4	4	4	4	4	4	3	4	3	4	4	3	3	3	4	9	6	6	10	6	8	10	7	9	7	9	6	6	8	10	10
(2,4)	4	3	3	4	4	4	3	3	4	4	3	4	4	3	4	3	9	6	8	10	8	9	9	7	5	6	9	9	9	8	9	9
(2,5)	3	4	4	4	3	4	3	4	3	3	3	4	3	3	4	3	8	9	8	6	6	5	6	7	5	9	8	6	8	7	6	6
(3,1)	3	4	4	3	3	3	4	3	3	4	4	4	4	3	3	3	7	9	9	10	9	6	6	7	5	6	10	6	6	6	7	6
(3,2)	3	3	4	4	4	3	3	3	4	3	4	4	3	4	4	3	8	8	8	7	6	6	8	8	9	9	7	5	10	8	7	8
(3,3)	4	4	3	4	3	4	3	3	3	3	4	4	3	4	3	3	6	7	9	6	6	10	9	9	6	6	6	8	7	9	7	5
(3,4)	3	3	3	3	3	4	4	4	4	3	4	4	3	3	3	4	6	8	9	10	10	7	8	7	6	9	5	6	8	8	7	5
(3,5)	4	3	4	4	3	3	4	3	4	4	3	3	3	4	4	4	8	9	8	6	6	8	7	7	9	9	9	6	5	5	8	9
(4,1)	3	3	3	4	3	4	4	4	4	3	4	3	4	3	3	3	5	6	5	8	7	9	10	9	6	8	8	5	7	7	9	7
(4,2)	4	4	3	3	4	3	3	3	4	4	3	4	4	4	4	3	9	8	9	5	8	8	9	6	10	8	5	7	6	5	9	10
(4,3)	4	4	3	4	3	3	3	3	3	3	3	4	4	3	3	4	7	7	9	8	7	5	6	7	5	8	8	8	9	9	8	6
(4,4)	3	3	4	4	4	4	3	4	3	4	3	4	3	3	4	4	9	5	10	5	6	9	5	9	8	9	9	6	10	8	8	8
(4,5)	4	4	3	4	4	3	3	3	4	4	3	4	3	4	4	4	9	5	9	10	6	7	8	5	8	8	9	10	5	5	7	10
(5,1)	3	4	3	4	3	4	4	4	4	3	4	4	3	3	4	4	10	5	9	8	8	6	7	7	5	6	6	6	6	9	6	6
(5,2)	3	3	4	3	3	4	3	3	4	4	3	3	4	4	3	3	6	9	7	7	6	9	10	9	6	8	8	5	9	6	6	9
(5,3)	4	4	4	4	3	4	3	4	3	3	3	4	4	3	4	3	7	7	6	7	6	7	9	6	6	8	6	9	7	10	7	6
(5,4)	4	3	3	4	3	4	4	3	4	3	4	3	3	4	3	4	6	9	6	5	6	7	7	8	9	5	9	10	7	7	5	6
(5,5)	4	4	4	3	3	3	4	3	4	3	3	4	4	3	3	4	9	8	8	9	8	7	10	8	9	10	7	8	8	5	7	6

Table 8. Initial orders corresponding to drawing ranges.

Range: 0 - 120					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	114	100	53	40	111
$m=2$	93	11	51	110	50
$m=3$	113	19	34	69	20
$m=4$	103	117	19	85	101
$m=5$	104	16	73	102	72

Range: 0 - 110					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	69	10	15	85	20
$m=2$	93	12	58	10	9
$m=3$	74	48	30	32	38
$m=4$	84	43	10	90	98
$m=5$	43	79	66	39	8

Range: 0 - 100					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	6	51	87	61	42
$m=2$	97	31	51	46	2
$m=3$	98	29	99	27	62
$m=4$	88	3	67	23	20
$m=5$	72	86	86	36	53

Range: 0 - 90					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	3	89	28	27	19
$m=2$	71	23	3	88	29
$m=3$	30	55	66	55	33
$m=4$	4	21	68	38	37
$m=5$	12	80	34	8	84

Range: 0 - 80					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	61	21	71	78	20
$m=2$	9	58	56	2	11
$m=3$	38	70	78	19	68
$m=4$	75	38	67	37	19
$m=5$	29	4	59	61	75

Range: 0 - 70					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	32	18	19	70	37
$m=2$	67	66	11	6	33
$m=3$	68	36	66	69	8
$m=4$	21	41	37	52	3
$m=5$	43	59	70	42	32

Range: 0 - 60					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	3	32	24	49	7
$m=2$	19	51	30	49	31
$m=3$	6	30	5	41	40
$m=4$	40	41	17	42	17
$m=5$	36	5	44	49	13

Range: 0 - 50					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	16	6	2	3	5
$m=2$	20	34	39	38	26
$m=3$	25	12	14	30	3
$m=4$	27	38	45	6	23
$m=5$	11	28	18	20	39

Range: 0 - 40					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	29	7	5	21	1
$m=2$	29	25	7	6	19
$m=3$	29	34	21	25	9
$m=4$	4	33	5	32	11
$m=5$	12	29	15	32	33

Range: 0 - 30					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	7	13	7	29	2
$m=2$	16	5	15	7	30
$m=3$	13	25	7	8	19
$m=4$	18	29	7	15	17
$m=5$	27	11	22	27	7

Range: 0 - 20					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	5	10	2	5	17
$m=2$	2	18	7	15	12
$m=3$	2	18	8	10	17
$m=4$	11	17	11	19	15
$m=5$	11	5	14	10	8

Range: 0 - 10					
z	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$m=1$	6	6	4	5	1
$m=2$	4	9	9	5	9
$m=3$	2	2	7	6	8
$m=4$	10	8	4	0	9
$m=5$	6	6	4	10	3

Table 9. Simulation results depending on volume of orders.

No.	Range	Model 1	Model 2	Difference
1.	0 – 120	116942.02	118406.44	–1464.41
2.	0 – 110	116232.73	117366.76	–1134.03
3.	0 – 100	115193.00	117163.97	–1970.97
4.	0 – 90	113649.32	115111.43	–1462.11
5.	0 – 80	114415.10	115960.63	–1545.52
6.	0 – 70	114861.21	115975.94	–1114.72
7.	0 – 60	112940.99	114040.42	–1099.43
8.	0 – 50	109304.88	110286.64	–981.76
9.	0 – 40	109138.79	109872.07	–733.28
10.	0 – 30	106334.22	107105.89	–771.67
11.	0 – 20	102028.04	103010.35	–982.30
12.	0 – 10	97268.86	98464.90	–1196.04

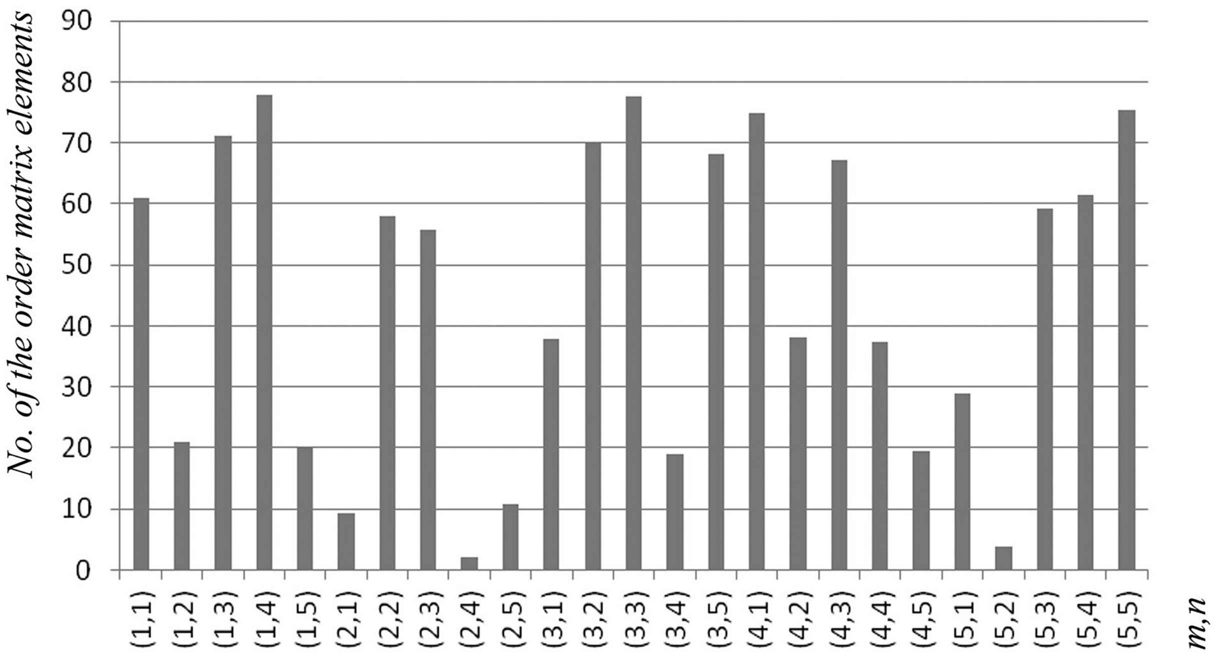


Figure 3. Initial state of order matrix for draw range 0 – 80.

In the simulation we assumed that the final values of the costs of handling customer inquiries in logistic units are the product of the minimizing coefficient for handling the given order $\gamma_{el}^{m,n}$, the unit costs of handling inquiries in the adequate units, and the expected service time. We then assumed that the final values of the transfer cost of customer inquiries between logistic units is the product of the minimizing coefficient for the transmission of inquiries for a given order $\gamma_{pass}^{m,n}$, the unit costs of sending inquiries between adequate, units and the expected time of inquiry transmission. Table 10 includes initial data necessary to calculate coefficients $\gamma_{el}^{m,n}$ as well as $\gamma_{pass}^{m,n}$. The goal of this course of action is to make the process of sending and handling inquiries more realistic, as the obtained values depend on the volume of the order: it is obvious that the larger the order is, the higher the coefficients are. They require minimizing to an acceptable value of c_ζ and c_υ (see Table 11).

Coefficients $\gamma_{el}^{m,n}$ and $\gamma_{pass}^{m,n}$ are graphically compared for both models in Figure 4 and 5.

Comparing the coefficients $\xi + \frac{z_{m,n}^{0}}{\zeta}$ and $c_\varsigma \cdot \xi$, it becomes obvious that Model 1 has an advantage in 16 cases, while in the remaining 9 ones there is an equilibrium of results taking into account the applied input data.

Comparing the coefficients $\xi + \frac{z_{m,n}^{0}}{\upsilon}$ and $c_\upsilon \cdot \xi$, it becomes again obvious that Model 1 has an advantage in 16 cases while in the remaining 9 ones there is an equilibrium of results taking into account the applied input data (see Table 12 and 13).

Based on the resulting data, it is then possible to obtain information on the total cost of handling a client inquiry in the case of:

- execution of the n-th order for the m-th customer,
- execution of the n-th order for M clients, and
- execution of N orders for the m-th customer.

Table 14 presents the total results of the simulation process for the presented cases.

The following part of the paper shows the comparison of the total cost of handling customer inquiries by orders and customers (see Figure 6). Furthermore, Figure 7 describes graphically the absolute difference between model 1 and model 2 for *C*_(1, 2, 3, 4, 5), *n* and *C*_*m*, (1, 2, 3, 4 , 5).

An important element of this study is conducting the point comparative analysis of the information flow of clients' inquiries. In order to conduct this analysis, 10 criteria, *i.e.* total costs, unit transfer costs, unit handling costs, unit transfer times, unit handling times as well as *C*_1, *n*; *C*_2, *n*; *C*_3, *n*; *C*_4, *n* and *C*_5, *n* have to be taken into account. The arithmetic mean method, the weighted average method and the percentage method were implemented to compare the sample models (see Table 15).

Based on a thorough comparative analysis of the two models of information flow in the logistics system for selected data from the ranges provided, it can be concluded that the first model is characterized by a much better result in terms of customer order handling costs. This was additionally demonstrated using the weighted average method and the percentage method, while only the arithmetic mean method shows identical results. However, the overall comparison substantiated by the scoring analysis graph proves that Model 1 has a decisive advantage in terms of cost of handling customer inquiries. As it can be seen in the automatically generated radar diagram, Model 2 could be useful for given input data if only unit transfer costs, unit handling costs and *C*_2, *n*; *C*_3, n and *C*_4, *n* are taken into account. Moreover, it should be clearly emphasized that the discussed simulation results are related to specific input data and do not constitute the basis for drawing conclusions in case of changing input data and subsequent simulations.

Table 10. Initial data to calculate coefficients $\gamma_{el}^{m,n}$ and $\gamma_{pass}^{m,n}$.

Base quantitative factor	$\xi = 1$
Minimizing denominator (handling)	$\zeta = 100$
Maximum allowable value (handling)	$c_\zeta = 1.300$
Minimizing denominator (information transfer)	$\upsilon = 1000$
Maximum allowable value (information transfer)	$c_\upsilon = 1.050$

Table 11. Minimized values of amount coefficients $\gamma_{el}^{m,n}$ and $\gamma_{pass}^{m,n}$.

$z(m, n)$	No. of orders	Real $\gamma_{el}^{m,n}$	Minimized $\gamma_{el}^{m,n}$	Real $\gamma_{pass}^{m,n}$	Minimized $\gamma_{pass}^{m,n}$
		$\gamma_{el}^{m,n} = \xi + \frac{z_{m,n}^{0}}{\zeta}$	$\gamma_{el}^{m,n} = c_{\varsigma} \cdot \xi$	$\gamma_{pass}^{m,n} = \xi + \frac{z_{m,n}^{0}}{\upsilon}$	$\gamma_{pass}^{m,n} = c_{\upsilon} \cdot \xi$
(1, 1)	61	1.611	1.300	1.061	1.050
(1, 2)	21	1.210		1.021	
(1, 3)	71	1.711	1.300	1.071	1.050
(1, 4)	78	1.778	1.300	1.078	1.050
(1, 5)	20	1.201		1.020	
(2, 1)	9	1.093		1.009	
(2, 2)	58	1.580	1.300	1.058	1.050
(2, 3)	56	1.558	1.300	1.056	1.050
(2, 4)	2	1.022		1.002	
(2, 5)	11	1.107		1.011	
(3, 1)	38	1.379	1.300	1.038	
(3, 2)	70	1.699	1.300	1.070	1.050
(3, 3)	78	1.777	1.300	1.078	1.050
(3, 4)	19	1.189		1.019	
(3, 5)	68	1.682	1.300	1.068	1.050
(4, 1)	75	1.748	1.300	1.075	1.050
(4, 2)	38	1.382	1.300	1.038	
(4, 3)	67	1.672	1.300	1.067	1.050
(4, 4)	37	1.373	1.300	1.037	
(4, 5)	19	1.194		1.019	
(5, 1)	29	1.289		1.029	
(5, 2)	4	1.037		1.004	
(5, 3)	59	1.592	1.300	1.059	1.050
(5, 4)	61	1.615	1.300	1.061	1.050
(5, 5)	75	1.754	1.300	1.075	1.050
Difference:		5.112		0.228	

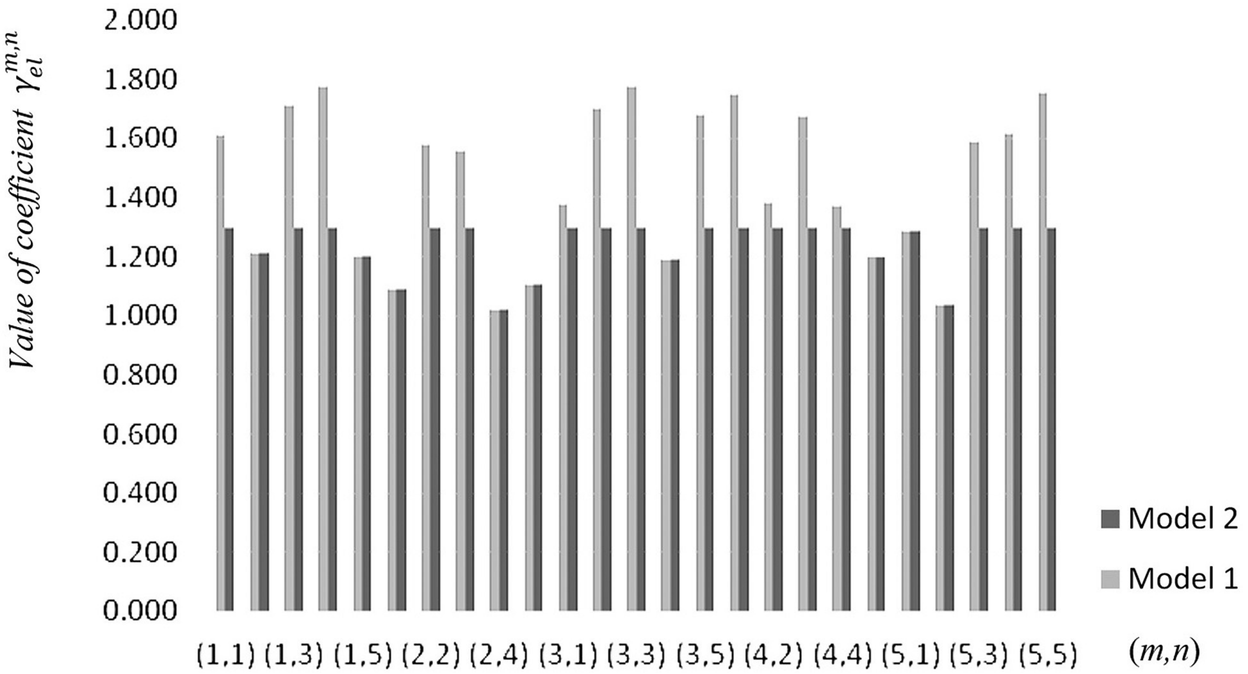


Figure 4. Comparison of $\xi + \frac{z_{m,n}^0}{\zeta}$ and $c_\varsigma \cdot \xi$.

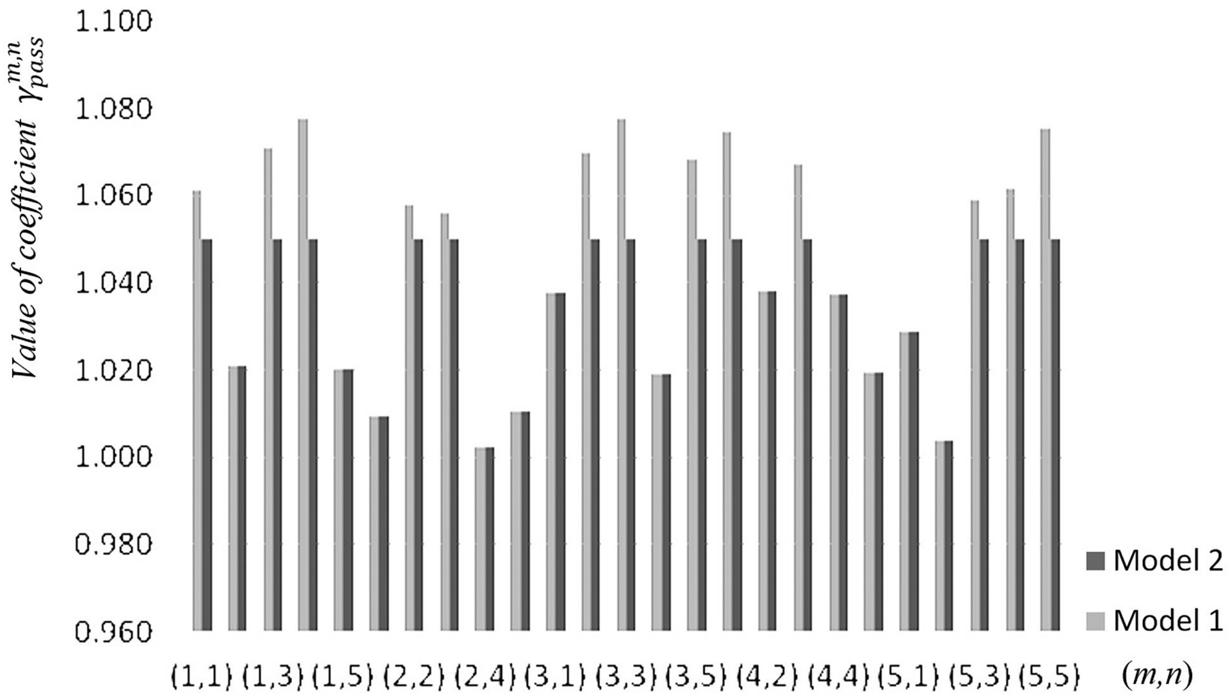


Figure 5. Comparison of $\xi + \frac{z_{m,n}^0}{\upsilon}$ and $c_\upsilon \cdot \xi$.

Table 12. Simulation results for Model 1 (for orders randomly drawn in the range 0 - 80).

	Sending the inquiry																Elaborating the inquiry															
z(m,n)	Z→CSD	CSD→Z	CSD→RPS	RPS→CSD	MAN→RPS	RPS→MAN	MAN→REG	REG→MAN	MAN→PRS	PRS→MAN	MAN→CMS	CMS→MAN	CMS→SUP	SUP→CMS	SUP→D	D→SUP	CSD_in	CSD_out	RPS_in	RPS_out	MAN_in	MAN_out	REG_in	REG_out	PRS_in	PRS_out	CMS_in	CMS_out	SUP_in	SUP_out	D_in	D_out
(1,1)	5	6	6	5	5	6	4	7	6	4	7	7	6	3	5	6	366	343	195	219	161	243	193	305	429	111	315	232	493	313	160	250
(1,2)	4	6	5	6	5	4	6	4	4	5	6	6	6	7	5	5	244	221	171	105	86	90	435	466	269	77	70	263	215	276	89	174
(1,3)	5	4	6	4	6	5	7	5	7	6	6	4	5	8	5	5	104	245	289	286	399	333	209	221	89	197	154	259	236	278	231	251
(1,4)	6	4	5	4	5	3	7	4	5	6	4	6	7	4	4	4	483	293	271	335	92	418	413	519	481	345	542	370	78	340	245	335
(1,5)	5	4	4	6	5	6	5	4	3	4	4	6	6	3	4	6	194	417	200	147	288	263	169	90	428	118	118	346	214	85	493	373
(2,1)	5	4	4	7	6	3	3	5	5	7	5	4	6	4	6	4	222	274	220	179	309	124	148	167	185	152	185	245	263	64	254	408
(2,2)	4	6	5	4	5	6	7	7	5	5	5	6	5	5	4	4	389	198	307	302	350	321	137	279	324	480	202	143	274	334	404	156
(2,3)	5	5	6	5	5	8	5	5	4	4	4	6	4	4	7	6	111	295	167	355	197	468	287	201	339	181	138	240	328	149	394	595
(2,4)	7	6	4	5	4	4	8	6	4	5	6	4	6	4	6	4	306	210	384	369	272	453	173	62	369	147	230	260	86	168	143	72
(2,5)	4	6	6	5	7	6	6	7	7	7	7	4	7	7	7	6	141	450	281	91	332	181	277	138	447	179	413	89	309	96	414	318
(3,1)	5	6	7	5	6	4	8	4	4	5	7	6	4	6	4	6	520	400	236	153	421	242	238	345	156	335	149	247	198	379	166	326
(3,2)	5	5	5	5	4	4	6	7	7	5	7	5	4	4	4	6	321	229	437	202	248	336	257	273	364	199	107	289	248	109	383	272
(3,3)	8	7	6	7	5	5	4	7	6	5	4	6	5	5	6	4	384	132	186	189	399	222	475	245	294	268	133	418	422	426	123	539
(3,4)	6	6	7	4	8	6	6	5	4	6	7	5	7	4	3	6	531	215	286	389	302	536	362	353	248	324	404	400	158	486	379	361
(3,5)	5	5	7	5	6	6	5	5	4	6	5	6	8	4	4	7	239	487	529	304	550	173	176	471	334	288	211	168	362	368	108	259
(4,1)	5	4	5	3	5	4	4	3	5	7	4	4	4	8	4	7	252	285	123	412	265	145	523	272	160	272	132	391	166	391	136	159
(4,2)	6	6	7	6	4	8	4	4	4	7	6	5	8	6	6	4	212	309	481	422	470	409	447	597	291	245	306	322	183	524	453	241
(4,3)	7	6	8	4	5	5	5	6	6	7	6	7	4	4	7	5	267	345	172	328	151	122	110	341	417	163	136	280	482	392	211	402
(4,4)	5	5	4	4	6	4	5	6	5	7	6	6	7	6	4	6	270	248	319	384	375	428	387	288	369	119	309	380	457	458	334	393
(4,5)	4	4	4	6	7	5	5	5	6	4	4	5	7	4	5	7	271	427	70	520	264	100	439	176	219	206	248	126	185	352	475	335
(5,1)	4	4	8	6	6	3	4	5	4	6	4	5	6	5	6	7	281	418	189	404	371	360	240	324	156	122	290	248	247	282	472	418
(5,2)	7	7	7	5	7	6	6	4	5	6	6	5	3	7	6	3	216	220	144	290	269	133	124	83	97	86	174	171	209	312	224	467
(5,3)	5	7	6	4	4	5	5	5	7	5	5	6	6	7	6	4	270	281	103	267	288	545	554	370	251	388	241	272	199	166	324	335
(5,4)	6	7	7	4	5	7	6	6	5	7	7	6	3	7	6	4	359	259	618	462	393	386	589	167	186	398	290	362	212	188	140	299
(5,5)	5	6	4	5	5	5	5	5	3	7	5	4	5	5	4	7	421	334	207	145	89	143	304	182	475	372	138	257	152	590	124	317

Table 13. Simulation results for Model 2 (for orders randomly drawn in the range 0 - 80).

	Sending the inquiry																Elaborating the inquiry															
z(m,n)	Z→CSD	CSD→Z	CSD→MAN	MAN→CSD	MAN→RPS	RPS→MAN	MAN→REG	REG→MAN	MAN→PRS	PRS→MAN	MAN→SUP	SUP→MAN	SUP→CMS	CMS→SUP	SUP→D	D→SUP	CSD_in	CSD_out	RPS_in	RPS_out	MAN_in	MAN_out	REG_in	REG_out	PRS_in	PRS_out	CMS_in	CMS_out	SUP_in	SUP_out	D_in	D_out
(1,1)	4	5	7	5	5	6	6	5	4	5	7	7	6	3	5	7	120	138	133	248	466	419	142	377	346	149	211	172	296	420	439	154
(1,2)	4	5	7	5	6	5	5	7	4	5	4	7	7	6	6	4	159	104	436	369	474	63	178	345	458	344	485	533	468	216	207	161
(1,3)	6	5	6	6	6	6	3	5	5	6	5	6	5	7	6	5	310	299	286	228	357	273	482	268	500	542	243	544	631	195	400	369
(1,4)	6	6	4	6	6	4	5	7	7	6	8	4	7	5	6	5	339	186	208	201	94	198	349	562	316	437	328	460	450	417	261	514
(1,5)	4	6	6	6	6	5	5	5	4	6	4	4	4	8	5	4	117	342	379	84	490	435	160	171	128	306	278	313	187	100	271	317
(2,1)	5	6	4	7	5	5	3	6	4	6	7	4	6	5	5	5	232	90	406	215	93	156	381	174	93	102	127	123	127	228	372	326
(2,2)	6	4	6	5	6	6	7	7	7	5	5	7	7	5	5	7	187	290	273	76	225	486	443	289	378	352	467	293	148	350	152	272
(2,3)	5	5	5	4	6	7	8	7	7	6	5	5	5	6	4	6	533	275	147	272	267	180	606	363	298	271	549	125	166	184	403	609
(2,4)	4	5	3	4	7	6	5	6	4	6	4	6	5	5	4	4	201	155	382	171	222	352	227	99	71	290	184	108	232	315	145	94
(2,5)	6	7	8	4	6	5	6	4	7	4	5	6	4	3	4	4	86	173	172	181	229	124	119	158	255	344	361	230	397	391	231	259
(3,1)	5	6	7	5	4	5	4	7	4	7	4	6	6	4	4	5	262	444	385	152	218	299	279	444	324	223	199	178	300	137	118	146
(3,2)	3	5	5	4	6	4	4	6	6	4	7	5	4	7	6	5	162	106	443	332	166	115	135	542	189	291	389	234	170	165	201	399
(3,3)	6	4	7	4	4	7	7	5	4	6	6	5	7	5	5	5	347	430	271	326	267	468	491	291	154	284	324	301	301	347	353	349
(3,4)	5	5	3	6	6	6	6	7	5	5	7	6	6	4	5	4	101	343	468	159	297	345	451	282	300	404	166	142	178	375	393	194
(3,5)	4	4	5	5	5	6	6	4	6	6	4	6	5	5	6	5	433	137	484	187	228	150	169	164	402	492	319	199	197	300	112	557
(4,1)	6	5	6	7	4	5	4	6	5	3	7	5	7	5	6	6	199	227	257	519	183	455	614	163	130	242	306	286	374	465	121	318
(4,2)	5	4	4	5	5	4	7	6	8	5	5	4	4	4	7	6	368	251	143	72	182	290	255	208	253	466	209	416	274	342	378	300
(4,3)	6	7	5	5	4	5	5	6	4	4	6	8	6	7	6	4	130	129	283	180	120	145	340	406	321	471	243	141	234	476	534	218
(4,4)	5	5	5	4	5	5	4	7	6	5	6	7	4	4	4	7	425	138	557	319	239	367	265	214	355	217	274	356	495	357	517	158
(4,5)	7	4	5	4	6	5	4	4	6	6	4	5	6	4	5	8	470	70	478	503	119	109	160	138	342	494	374	241	205	267	96	539
(5,1)	4	4	4	4	3	5	4	4	7	4	8	5	6	6	5	7	205	303	157	260	198	148	277	298	296	280	141	226	113	483	253	350
(5,2)	3	6	8	5	6	7	6	6	4	5	6	5	7	6	4	5	230	375	350	289	66	216	154	358	232	365	161	159	112	282	140	272
(5,3)	6	7	5	7	6	7	4	5	4	6	4	5	6	5	5	4	399	382	316	433	324	318	546	160	272	524	167	507	161	159	266	242
(5,4)	6	6	6	8	6	6	7	5	4	6	7	6	6	7	6	7	215	275	104	331	300	261	376	206	194	288	401	435	416	390	235	363
(5,5)	5	6	5	6	4	7	8	5	5	4	7	7	7	4	6	5	345	498	186	406	223	358	212	237	401	448	188	476	391	271	325	341

Table 14. Numerical results of simulation taking into account query handling costs for $C_{m,n}$, $C_{M,n}$ and $C_{m,N}$.

	Model 1				**Model 2**	
$\boldsymbol{z(m,n)}$	$\boldsymbol{C_{m,n}}$				$\boldsymbol{C_{m,n}}$	
(1,1)	3971.80		$C_{m,1}$	21561.13	4317.61	
(1,2)	4738.20		$C_{m,2}$	21686.57	5086.49	
(1,3)	5318.20		$C_{m,3}$	22693.62	6018.26	
(1,4)	4795.52	$C_{1,n}$	$C_{m,4}$	26259.41	5411.51	$C_{1,n}$
(1,5)	3618.69	21280.57	$C_{m,5}$	22214.38	4160.91	24994.77
(2,1)	2924.40		Total cost:	114415.10	3328.34	Total cost:
(2,2)	4203.40				4778.00	
(2,3)	4439.70				5339.77	
(2,4)	2893.54	$C_{2,n}$			3329.83	$C_{2,n}$
(2,5)	3450.50	20730.04			3795.34	20571.28
(3,1)	3401.98				4190.92	
(3,2)	3770.78				4120.33	
(3,3)	4528.79				5394.27	
(3,4)	4155.68	$C_{3,n}$			4685.70	$C_{3,n}$
(3,5)	3959.65	24835.93			4610.19	23001.41
(4,1)	4434.59				4944.96	
(4,2)	3789.11				4491.99	
(4,3)	4110.41				4456.96	
(4,4)	4689.50	$C_{4,n}$			5337.54	$C_{4,n}$
(4,5)	4065.21	24678.77			4687.90	23919.36
(5,1)	3479.98				4070.34	
(5,2)	3158.68				3853.84	
(5,3)	4395.28				5261.36	
(5,4)	4301.15	$C_{5,n}$			4891.33	$C_{5,n}$
(5,5)	4463.63	22889.79			5396.93	23473.81
	Total cost:	114415.10			**Total cost:**	115960.62

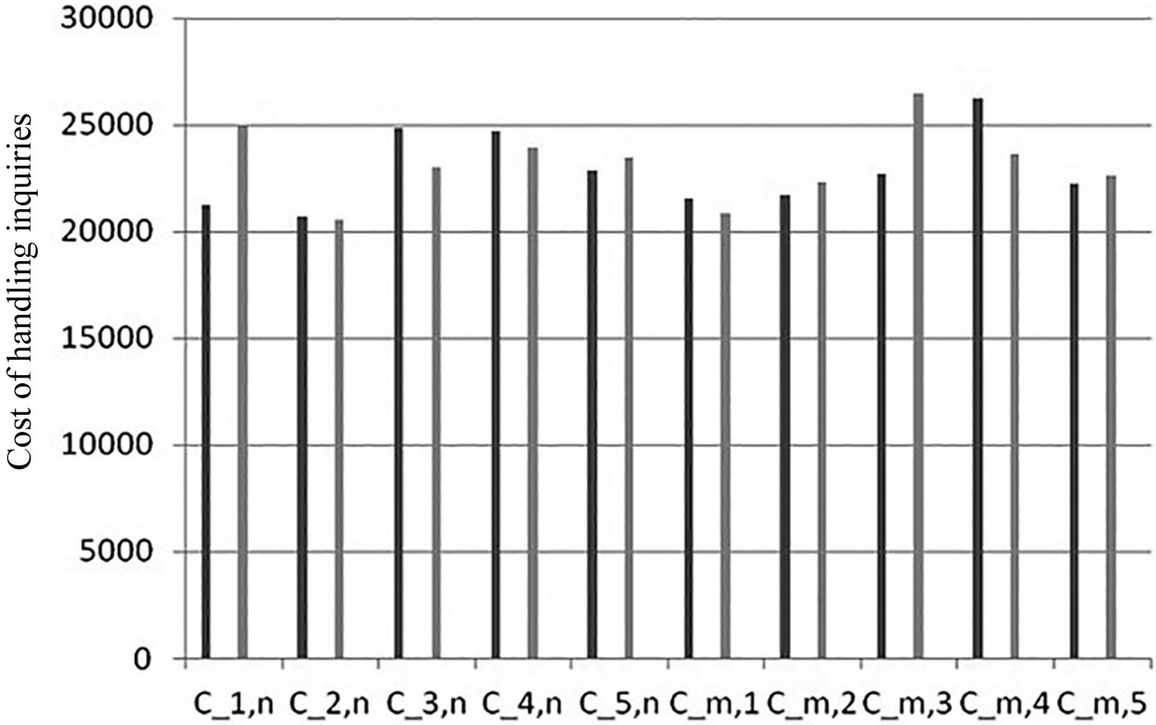


Figure 6. Comparison of total cost of handling customer inquiries by orders and customers.

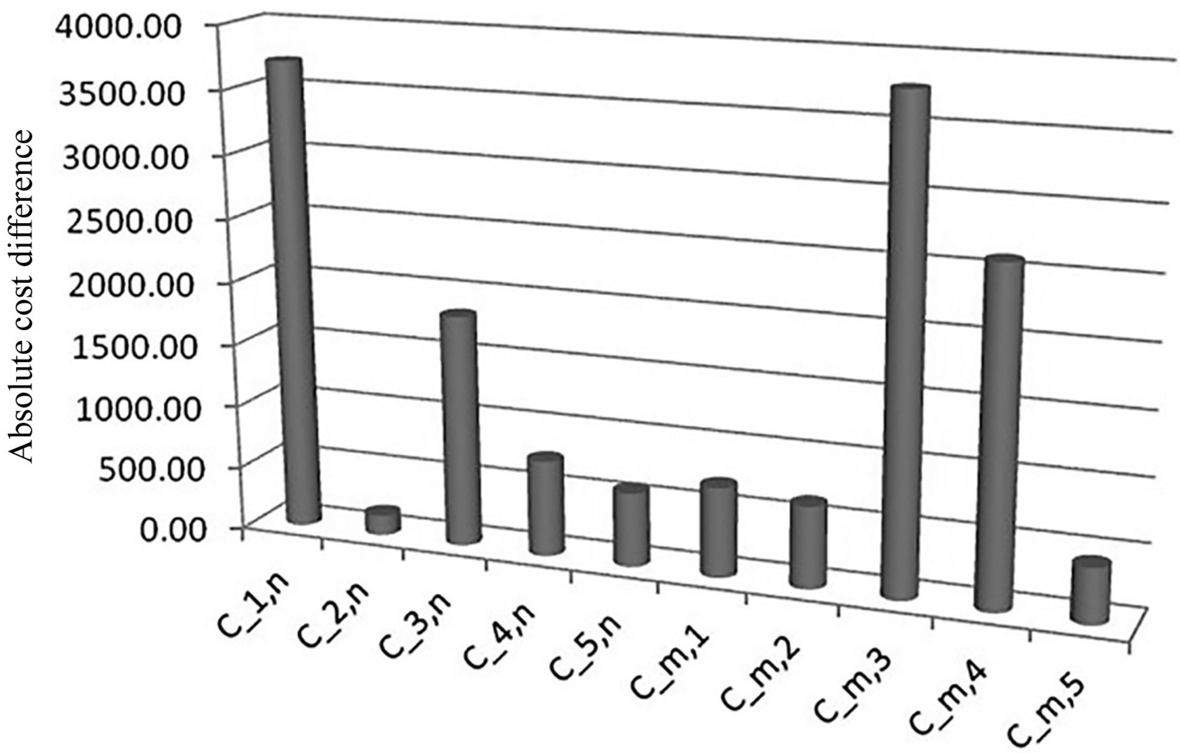
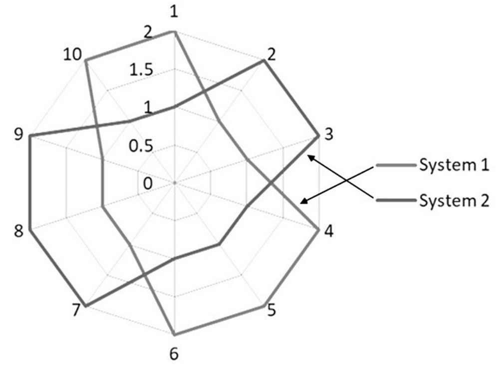


Figure 7. Absolute difference between model 1 and model 2 for *C*_(1, 2, 3, 4, 5), *n* and *C*_*m*, (1, 2, 3, 4, 5).

Table 15. Point comparative analysis of information flow of clients' inquiries.

Scale	2	1	0	0	0	0	0

No.	In game	Point analysis of the evaluation and selection of the customer inquiry service system (range 0 - 80)			
	2	Active → 1 / Inactive → 0		1	1
	Active → 1 Inactive → 0	Analized criteria	1	System	
No.			Weight	1	2
1	1	Total costs	0.35	2	1
2	1	Unit transfer costs	0.10	1	2
3	1	Unit handling costs	0.10	1	2
4	1	Unit transfer times	0.10	2	1
5	1	Unit handling times	0.10	2	1
6	1	C_1,n	0.05	2	1
7	1	C_2,n	0.05	1	2
8	1	C_3,n	0.05	1	2
9	1	C_4,n	0.05	1	2
10	1	C_5,n	0.05	2	1
	In game	Arithmetic mean method		**1.50**	**1.50**
	10	Weighted average method		**1.65**	**1.35**
		Percentage method		82.5	67.5



5. Data Modification

In order to illustrate the difference trend in total costs between Model 1 and Model 2, we decided to reduce the unit costs in Model 2 by values given in Table 16, which is graphically shown in Figure 8. The unit cost saving value was applied for each inquiry of the n-th product of the m-th customer in Model 2. As a result of this procedure, it can be noticed at which sample values of unit costs model 2 becomes more efficient for handling customer inquiries.

It should be mentioned that the value decreasing unit costs was equally applied to each unit cost cell both in the case of handling the client inquiry in each logistic unit, and to each cell representing the transmission of information related to the inquiry between units.

Subsequently, in order to illustrate the difference trend in total costs between Model 1 and Model 2, we also decided to decrease the unit times in model 2 by values given in Table 17, and graphically depicted in Figure 9. It can be noticed at what sample values decreasing the unit times Model 2 becomes more efficient for handling client queries.

First of all, draw ranges for the sequential and non-sequential flow of inquiries were introduced. Transfer and processing times, and unit costs are drawn both for the sequential and the no-sequential flow of query information for Model 1 and Model 2. The numbers of the n-th order elements for each m-th customer are consequently drawn according to 12 drawing ranges. Simulation results depending on the number of orders are shown as the minimal values of total costs obtained in 100 simulations. The results obtained by means of drawing for the range 0 – 80 were then analyzed. The matrix of amount coefficients is introduced in the case of processing inquiries in units as well as the matrix of amount coefficients, upon which expected realistic values are calculated. By comparing various aspects of the systems, the point comparative analysis of the information flow of clients' inquiries illustrates in detail the simulation result. Finally, modifications to Model 2 are introduced which lead to determining the breaking values for reducing the respective unit costs in order to minimize the total costs for this model. To conclude, the lower the number of orders, the lower the need to minimize handling and transfer information times. In our case the draw range 0 – 50 did not require minimizing these times.

Table 16. Reducing unit costs in Model 2.

Experiment number	Total costs		Cost difference between models	Unit cost saving value in Model 2
	Model 1	Model 2		
1	114415.10	112383.31	2031.80	–1.000
2	114415.10	112741.04	1674.06	–0.900
3	114415.10	113098.77	1316.33	–0.800
4	114415.10	113456.50	958.60	–0.700
5	114415.10	113814.24	600.87	–0.600
6	114415.10	114171.97	243.14	–0.500
7	114415.10	114350.83	64.27	–0.450
8	114415.10	114386.61	28.50	–0.440
9	114415.10	114404.49	10.61	–0.435
10	114415.10	114411.65	3.46	–0.433
11	114415.10	114415.22	–0.12	–0.432
12	114415.10	114422.38	–7.28	–0.430
13	114415.10	114458.15	–43.05	–0.420
14	114415.10	114493.93	–78.82	–0.410
15	114415.10	114529.70	–114.59	–0.400
16	114415.10	114887.43	–472.33	–0.300
17	114415.10	115245.16	–830.06	–0.200
18	114415.10	115602.89	–1187.79	–0.100
19	114415.10	115960.63	–1545.52	0.000

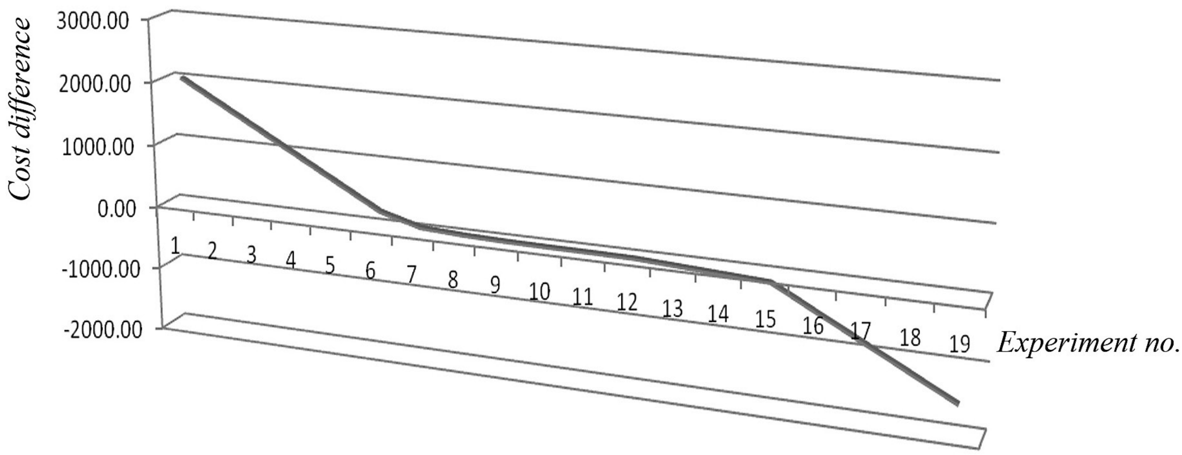


Figure 8. Total cost difference between Model 1 and Model 2 by applying unit cost saving values in 19 experiments in Model 2 according to data included in Table 16.

Table 17. Reducing unit times in Model 2.

Experiment number	**Total costs**		**Cost difference between models**	**Unit cost saving value in Model 2**
	Model 1	**Model 2**		
1	114412.10	113778.70	636.41	–0.150
2	114413.10	113924.16	490.94	–0.140
3	114414.10	114069.62	345.48	–0.130
4	114415.10	114215.08	200.02	–0.120
5	114415.10	114360.55	54.56	–0.110
6	114415.10	114404.18	10.92	–0.107
7	114415.10	114418.73	–3.63	–0.106
8	114415.10	114433.28	–18.17	–0.105
9	114415.10	114506.01	–90.90	–0.100
10	114415.10	114796.93	–381.83	–0.080
11	114415.10	115087.85	–672.75	–0.060
12	114415.10	115378.78	–963.67	–0.040
13	114415.10	115669.70	–1254.60	–0.020
14	114415.10	115960.63	–1545.52	0.000

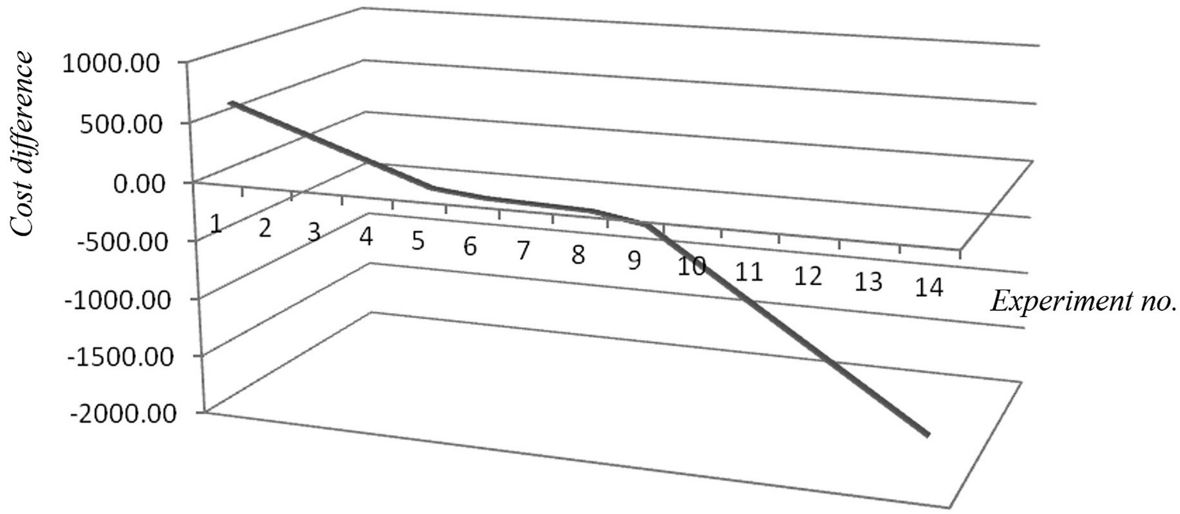


Figure 9. Total cost difference between Model 1 and Model 2 by applying unit time saving values in 14 experiments in Model 2 according to Table 17.

6. Conclusion

This paper elaborates on the problem of delays in logistics manufacturing system, with the objectives to provide a graphic and mathematical illustration of the problem of modelling delays of customers' inquiries in the system, as well as to propose a method for calculating the costs of customers' inquiries. These objectives were met, and the intended outcome achieved. The flow of inquiry information and its allocation in the logistic units were illustrated in detail, also taking into account information processing. Additionally, the comparative point analysis of the two analyzed models, taking into account 10 criteria, was also provided.

In order to make the paper an illustrative study case, we had to simplify certain assumptions. Namely, input data was generated using a pseudo-random code generator, where each new generation of input data translates into output data to be analyzed. For the set of input data chosen for simulation in the analyzed illustrative case, the sequential model turned out to be the winning model in terms of total cost of servicing the customer inquiry, although the non-sequential model proved to be useful many times for other sets of input data too, mostly due to the fact that the data is randomly generated.

The paper shows one example of passing customer inquiries between logistic units and processing them in these latter. It seems obvious that meeting the cost criterion requires minimizing either times of passing and processing inquiries or/and unit costs of these operations. Therefore, future work should focus on rearranging the routes of passing customers inquiries. Also, future research can be corroborated using process mining techniques that analyze the time-stamped events recorded in business information systems.

Acknowledgement

This paper was supported by the Ministry of Education, Youth and Sports in the Czech Republic within the Institutional Support for Long-term Development of a Research Organization in 2022.

References

[1] Q. Liu *et al.*, "Integrated Modeling and Optimization of Material Flow and Financial Flow of Supply Chain Network Considering Financial Ratios", *Numerical Algebra Control and Optimization*, vol. 9, no. 2, pp. 113–132, 2019. http://dx.doi.org/10.3934/naco.2019009

[2] W. W. Jiang *et al.*, "Efficient Assignment Algorithms to Minimize Operation Cost for Supply Chain Networks in Agile Manufacturing", *Com-*

puters & Industrial Engineering, vol. 108, pp. 225–239, 2017.
https://doi.org/10.1016/j.cie.2017.04.014

[3] K. H. Wong *et al.*, "Optimal Production Schedule in a Single-Supplier Multi-Manufacturer Supply Chain Involving Time Delays in Both Levels", *Journal of Industrial and Management Optimization*, vol. 14, no. 3, pp. 877–894, 2018.
https://doi.org/10.3934/jimo.2017080

[4] D. S. Faertes, "Reliability of Supply Chains and Business Continuity Management", in *Proceedings of the International Conference on Information Technology and Quantitative Management, ITQM 2015*, 2015, pp. 1400–1409.
https://doi.org/10.1016/j.procs.2015.07.130

[5] P. Dallasega *et al.*, "Customer-oriented Production System for Supplier Companies in CTO", *Factories of the Future in the Digital Environment*, vol. 57, pp. 533–538, 2016.
https://doi.org/10.1016/j.procir.2016.11.092

[6] I. Liberko *et al.*, "Possibilities to Optimize the Logistics Chain in the Manufacturing Plant", *Polish Journal of Management Studies*, vol. 12, no. 2, pp. 103–113, 2015.

[7] M. Labus *et al.*, "Introducing Adaptive E-Business Continuity Management", *Recent Advances in Information Systems and Technologies*, vol. 569, pp. 628–637, 2017.
https://doi.org/10.1007/978-3-319-56535-4_62

[8] R. Kampf *et al.*, "Optimization of Production Logistics", *Advances in Science and Technology-Research Journal*, vol. 12, no. 4, pp. 151–156, 2018.
https://doi.org/10.12913/22998624/100351

[9] K. I. Fattakhov and R.S. Dautov, "Optimization of Production Logistics Parameters in the Digital Economy", *Revista San Gregorio*, vol. 23, pp. 132–138, 2018.
http://dx.doi.org/10.36097/rsan.v1i23.688

[10] J. W. Strandhagen *et al.*, "The fit of Industry 4.0 applications in manufacturing logistics: a multiple case study", *Advances in Manufacturing*, vol. 5, no. 4, pp. 344–358, 2017.
https://doi.org/10.1007/s40436-017-0200-y

[11] W. Bouazza *et al.*, "Dynamic scheduling of manufacturing systems: a product-driven approach using hyper-heuristics", *International Journal of Computer Integrated Manufacturing*, vol. 34, no. 6, pp. 641–665, 2021.
https://doi.org/10.1080/0951192X.2021.1925969

[12] A. Aloui and K. Hadj-Hamou, "A heuristic approach for a scheduling problem in additive manufacturing under technological constraints", *Computers & Industrial Engineering*, vol. 154, 2021.
https://doi.org/10.1016/j.cie.2021.107115

[13] R. Caprihan *et al.*, "Evaluation of the impact of information delays on flexible manufacturing systems performance in dynamic scheduling environments", *International Journal of Advanced Manufacturing Technology*, vol. 67, no. 1–4, pp. 311–338, 2013.
https://doi.org/10.1007/s00170-013-4755-x

[14] M. Skitmore and V. Marston, "Cost Modelling", Taylor & Francis Group, London, UK, 1999.
https://doi.org/10.4324/9780203979679

[15] A. Wasim *et al.*, "An innovative cost modelling system to support lean product and process development", *Int. J. of Advanced Manufacturing Technology*, vol. 65, pp. 165–181, 2013.
https://doi.org/10.1007/s00170-012-4158-4

[16] R. Muha, "An Overview of the Problematic Issues in Logistics Cost Management", *Pomorstvo*, vol. 33, no. 1, pp. 102–109, 2019.
https://doi.org/10.31217/p.33.1.11

[17] P. Suchánek and R. Bucki, "Modelling timings of the company's response to specific customer requirements", In book Agents and Multi-Agent Systems: Technologies and Applications, pp. 209–219, 2020.
https://doi.org/10.1007/978-981-15-5764-4_19

[18] R. Bucki and P. Suchánek, "Modelling Decision-Making Processes in the Management Support of the Manufacturing Element in the Logistic Supply Chain", *Complexity*, vol. 2017, pp. 1–20, 2017.
https://doi.org/10.1155/2017/5286135

[19] R. Bucki *et al.*, "Modelling insourcing and outsourcing services for manufacturing operations", in *Proceedings of the 2nd International Conference on Decision Making for Small and Medium-Sized Enterprises (DEMSME 2019)*, 2019, pp. 52–60.

[20] R. Bucki and P. Suchánek, "Comparative Simulation Analysis of the Performance of the Logistics Manufacturing System at the Operative Level", *Complexity*, vol. 2019, pp. 1–29, 2019.
https://doi.org/10.1155/2019/7237585

[21] B. Sarkar *et al.*, "Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction", *Journal of Industrial and Management Optimization*, vol. 13, no. 2, pp. 1085–1104, 2017.
https://doi.org/10.3934/jimo.2016063

[22] M. Hajmirfattahtabrizi and H.M. Song, "Investigation of Bottlenecks in Supply Chain System for Minimizing Total Cost by Integrating Manufacturing Modelling Based on MINLP Approach", *Applied Science-Basel*, vol. 9, no. 6, pp. 1–17, 2019.
https://doi.org/10.3390/app9061185

[23] J. Sadeghi *et al.*, "Optimizing a vendor managed inventory (VMI) model considering delivering cost in a three-echelon supply chain using two tuned-parameter meta-heuristics", *Int. J. of System Assurance Engineering and Management*, vol. 4, no. 4, pp. 500–510, 2015.
https://doi.org/10.1007/s13198-014-0308-4

Received: June 2022
Revised: July 2022
Accepted: July 2022

Contact addresses:
Robert Bucki
Institute of Management and Information Technology
Bielsko-Biała
Poland
e-mail: bucki.robert@wp.pl

Petr Suchánek
Silesian University in Opava,
School of Business Administration in Karvina
The Czech Republic
e-mail: suchanek@opf.slu.cz

ROBERT BUCKI, Ph.D., is a researcher who specializes in the analysis of business-logistics systems and their successive modeling. His area of interest includes generally understood software engineering focusing on the implementation of iterative models of software development. As part of his scientific activity, he aims to create IT support systems that allow for solving optimization issues in business systems by means of simulation procedures. His publications include conference papers and scientific articles in the field of IT logistics in the number of over 80. He has been employed at the Institute of Management and Information Technology in Bielsko-Biała, Poland since 1996.

PETR SUCHÁNEK is an associate professor and experienced researcher dealing with the implementation and integration of IS/IT into business environments and the public sector. In this respect, the key solved issues are modeling, mathematical modeling and simulations of socio-economic systems such as e-business systems, logistics systems, manufacturing systems and measuring the benefits of IS/IT, especially in business and in direct relation to business intelligence applications. Since 1998, he has been employed at Silesian University in Opava, School of Business Administration in Karvina, Czech Republic. He is author or co-author of more than 90 scientific works.