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Decentralized H∞ Control of Interconnected Systems with Time-Varying Delays

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This paper focuses on the problem of delay dependent stability/stabilization of interconnected systems with time-varying delays. The approach is based on a new Lyapunov-Krasovskii functional. A decentralized delay-dependent stability analysis is performed to characterize linear matrix inequalities (LMIs) based on the conditions under which every local subsystem of the linear interconnected delay system is asymptotically stable. Then we design a decentralized state-feedback stabilization scheme such that the family of closed-loop feedback subsystems enjoys the delay-dependent asymptotic stability for each subsystem. The decentralized feedback gains are determined by convex optimization over LMIs. All the developed results are tested on a representative example and compared with some recent previous ones.

ACM CCS (2012) Classification: Information systems → Information systems applications → Process control systems

Theory of computation → Design and analysis of algorithms → Online algorithms

Keywords: interconnected systems, decentralized stabilization, delays-dependent stability, Lyapunov methods, time delays, linear matrix inequalities (LMIs)

1. Introduction

Time delays are habitually encountered in several practical systems such as engineering physics, neural networks, and communications networks, telecommunications, etc. The issues of stability analysis for interconnected systems have been the subject of many works and various techniques and stability criteria have been presented [1] – [12]. Indeed, the existence of time delays is often an important cause of instability and degradation of systems performances. Recently, the stability problem of interconnected systems with time delays has received considerable attention [13] – [16].

In the literatures, there are two kinds of stability analysis of interconnected systems. Delay-independent and delay-dependent. The delay dependent stability criteria, stability and performance of the system are usually ensured for delays lesser than a bound given. The Lyapunov-Krasovskii approach is widely used and it can be used to handle systems with time varying delays. Major attention has been given to problems with delay dependent stability analysis and control of time delay systems [16] – [20]. A system with time varying delay was considered in [13] and delay dependent stability criteria were suggested. Using the Jensens inequality, an improved delay dependent stability criterion for systems with time varying delays was proposed in [21]. In both cases the upper limit for the delay was defined. Problem of delay dependent stability of systems with time varying delays and structured uncertainties are given in [13], [22] and [23]. Delay dependent control of systems with time varying delays and norm bounded uncertainties are given in [24].

Our contribution is to state LMI sufficient delay-dependent conditions for the state-feedback control stabilization design, which guarantees an H∞ level. By using the Lyapunov-Krasovskii functional, the conditions for asymptotic stability are established. All these conditions are obtained in terms of Linear Matrix Inequalities (LMI's).

The rest of this paper is organized as follows: Section 2 describes the problem formulation, in which the formulation of the system and conditions of delay are given. The main results are introduced in Section 3 in which the asymptotic stability and decentralized state feedback H∞ control are presented. The numerical example is presented in Section 4, to prove the usefulness of the proposed results. Finally, the article ends with a brief conclusion.

2. Problem Formulation

Consider a class of interconnected system composed of N subsystems described by:

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + A_{di} x_i(t-\tau_i(t)) \\ &+ \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t)) + B_i u_i(t) \quad (1)\\ i, j &= 1,\dots,N\end{aligned}$$

where $x_i(t) \in \Re^{n_i}$ is the state vector, $u_i(t) \in \Re^{m_i}$ is the control vector. The system matrices A_i, A_{di}, A_{ij} and Bi are of appropriate dimensions. τ_i, η_{ij} are unknown time-delay factors satisfying the following conditions:

$$\begin{aligned}&0 \le \tau_i(t) \le \rho_i, \quad \dot{\tau}_i(t) \le \mu_i \\ &0 \le \eta_i(t) \le \rho_{ij}, \quad \dot{\eta}_{ij}(t) \le \mu_{ij}\end{aligned} \quad (2)$$

where the bounds ρ_i, ρ_{ij}, μ_i and μ_{ij} are known constants in order to guarantee smooth growth of the state trajectories.

The class of systems described by (1) subject to delay pattern (2) is frequently encountered in modelling several physical systems and engineering applications including large space structures, multi-machine power systems, transportation systems and water pollution management [3], [25] – [26].

Suppose that all the states are available; for each subsystem, the decentralized controller $u_i(t)$ can be expressed by

$$u_i(t) = k_i\, x_i(t), \quad i = 1,\dots,N \quad (3)$$

where k_i are the state feedback gains matrices of each controller laws associated with i-th subsystem.

Thus, the state representation of the interconnected closed loop system can be expressed as

$$\begin{aligned}\dot{x}_i(t) &= A_{ci} x_i(t) + A_{di} x_i(t-\tau_i(t)) \\ &+ \sum_{j=1,\, j\neq i}^{N} A_{ij} x_j(t-\eta_{ij}(t)), \quad (4)\\ i, j &= 1,\dots,N\end{aligned}$$

where $A_{ci} = A_i + B_i k_i$.

The synthesis of decentralized controller is to determine a set of the gains ensuring the stability of closed loop interconnected system (4).

We end this section with the following lemmas, which will be used for further development.

Lemma 1. (Schur complement) [28] Given three matrices Q, $R = R^T$ and $S = S^T$, the following statements are equivalent

$$\begin{bmatrix} R & Q \\ Q^T & S \end{bmatrix} \succ 0 \quad (5)$$

$$S \succ 0 \text{ and } R - QS^{-1}Q^T \succ 0. \quad (6)$$

Lemma 2. [29] For any $x, y \in \Re^n$ and positive definite matrix $P \in \Re^{n\times n}$, we have

$$2x^T y \le y^T P y + x^T P^{-1} x. \quad (7)$$

3. Main Results

In this section, the main goal is to design a decentralized controller ensuring the stability of interconnected systems described by (4).

3.1. Stability Analysis

In this subsection, we consider the problem of stability analysis for the family of subsystems (1) with $u_i(t) \equiv 0$, $i = 1, \dots, N$. Before presenting the main result, the following notations of several matrix variables are set for the sake of simplicity

$$\Upsilon_{11} = \pi_{11} + \frac{1}{N-1}\rho_i A_i^T W_i A_i \tag{8a}$$

$$\Upsilon_{33} = \pi_{33} + \left(2N-3\right) A_{ij}^T \rho_i W_i A_{ij} \tag{8b}$$

$$\pi_{11} = Z_{ji} + \frac{1}{N-1}\left(P_i A_i + A_i^T P_i + Q_i\right) \tag{8c}$$

$$\pi_{33} = -(1-\mu_{ij}) Z_{ij} \tag{8d}$$

The following theorem gives the sufficient delay-dependent conditions ensuring the asymptotic stability of the interconnected system with time-varying delays (1). Essentially, the proof is based on the construction of the Lyapunov-Krasovskii functions satisfying the Lyapunov stability theorem for a time delay system [27].

Theorem 1: Given $\rho_i \succ 0,\ \mu_i \succ 0$ and $\mu_{ij} \succ 0$, if there exist symmetric positive definite matrices P_i, Q_i, Z_{ij} and W_i, $i, j = 1, ..., N$, $i \neq j$, such that the following LMI holds:

$$\begin{bmatrix} \pi_{11} & \frac{1}{N-1}P_i A_{di} & P_i A_{ij} & 0 & \rho_i A_i^T W_i \\ * & -\frac{1-\mu_i}{N-1}Q_i & 0 & 0 & \rho_i A_{di}^T W_i \\ * & * & \pi_{33} & 0 & \left(N-1\right)\rho_i A_{ij}^T W_i \\ * & * & * & -\frac{\rho_i W_i}{N-1} & 0 \\ * & * & * & * & -\left(N-1\right)\rho_i W_i \end{bmatrix} \tag{9}$$

The system (1) with $u_i(t) \equiv 0$, $i = 1, ..., N$ is asymptotically stable for all time delays satisfying (2).

Proof. We consider the following Lyapunov-Krasovskii functional for system (1):

$$V(t) = \sum_{i=1}^{N} V_i(t) = \sum_{i=1}^{N}\left[V_{ai}(t) + V_{bi}(t) + V_{ci}(t) + V_{di}(t)\right] \tag{10}$$

where

$$V_{ai}(t) = x_i^T(t) P_i x_i(t) \tag{11a}$$

$$V_{bi}(t) = \int_{t-\tau_i(t)}^{t} x_i^T(s) Q_i x_i(s) ds \tag{11b}$$

$$V_{ci}(t) = \sum_{j=1, j\neq i}^{N} \int_{t-\eta_{ij}(t)}^{t} x_i^T(s) Z_{ij} x_j(s) ds \tag{11c}$$

$$V_{di}(t) = \int_{-\rho_i}^{0}\int_{t+s}^{t} \dot{x}_i^T(\alpha) W_i \dot{x}_i(\alpha) d\alpha ds \tag{11d}$$

where $P_i = P_i^T \succ 0,\ Q_i = Q_i^T \succ 0,\ Z_{ij} = Z_{ij}^T \succ 0,$ $W_i = W_i^T \succ 0,\ i, j = 1, ..., N,\ i \neq j,$ are the matrices of appropriate dimensions.

Differentiating (10), with respect to t, we have [13]

$$\dot{V}_{ai}(t) = 2x_i^T(t) P_i \dot{x}_i(t) = \frac{1}{\tau_i(t)} \int_{t-\tau_i(t)}^{t} 2x_i^T P_i \dot{x}_i(t) ds \tag{12}$$

$$\dot{V}_{bi}(t) \le \frac{1}{\tau_i(t)} \int_{t-\tau_i(t)}^{t} \left(x_i^T(t) Q_i x_i(t) - \left(1-\mu_i\right) x_i^T\left(t-\tau_i(t)\right) Q_i x_i\left(t-\tau_i(t)\right)\right) ds \tag{13}$$

$$\begin{aligned} \dot{V}_{ci}(t) &= \sum_{j=1, j\neq i}^{N}\left[x_j^T(t) Z_{ij} x_j(t) - (1-\dot{\eta}_{ij}(t)) x_j^T(t-\eta_{ij}(t)) Z_{ij} x_j(t-\eta_{ij}(t))\right] \\ &\le \sum_{j=1, j\neq i}^{N}\left[x_j^T(t) Z_{ij} x_j(t) - (1-\mu_{ij}) x_j^T(t-\eta_{ij}(t)) Z_{ij} x_j(t-\eta_{ij}(t))\right] \end{aligned} \tag{14}$$

$$\begin{aligned}\dot{V}_{di}(t) &= \int_{-\rho_i}^{0}\left[\dot{x}_i^T(t)W_i\dot{x}_i(t)-\dot{x}_i^T(t+s)W_i\dot{x}_i(t+s)\right]ds \le \rho_i\dot{x}_i^T(t)W_i\dot{x}_i(t)-\int_{t-\tau_i(t)}^{t}\dot{x}_i^T(s)W_i\dot{x}_i(s)ds \\ &= \frac{1}{\tau_i(t)}\int_{t-\tau_i(t)}^{t}\left[\rho_i\dot{x}_i^T(t)W_i\dot{x}_i(t)-\tau_1(t)\dot{x}_i^T(s)W_i\dot{x}_i(s)\right]ds\end{aligned} \tag{15}$$

we obtain

$$\begin{aligned}\sum_{i=1}^{N}\dot{V}_i(t) &= \sum_{i=1}^{N}\left[\dot{V}_{ai}(t)+\dot{V}_{bi}(t)+\dot{V}_{ci}(t)+\dot{V}_{di}(t)\right] \\ &\le \sum_{i=1}^{N}\left\{\frac{1}{\tau_i(t)}\int_{t-\tau_i(t)}^{t}\left[2x_i^T(t)P_i\left(A_ix_i(t)+A_{di}x_i\left(t-\tau_i(t)\right)+\sum_{j=1,j\neq i}^{N}A_{ij}x_j\left(t-\eta_{ij}(t)\right)\right)\right.\right. \\ &\quad + x_i^T(t)Q_i\,x_i(t)-\left(1-\mu_i\right)x_i^T\left(t-\tau_i(t)\right)\cdot Q_i\,x_i\left(t-\tau_i(t)\right)+\rho_i\,\dot{x}_i^T(t)W_i\,\dot{x}_i(t) \\ &\quad -\tau_i(t)\dot{x}_i^T(s)W_i\,\dot{x}_i(s)+\sum_{i=1,j\neq i}^{N}x_j^T(t)Z_{ij}x_j(t) \\ &\quad \left.\left.-\sum_{j=1,j\neq i}^{N}\left(\left(1-\mu_{ij}\right)x_j^T\left(t-\eta_{ij}(t)\right)\cdot Z_{ij}x_j\left(t-\eta_{ij}(t)\right)\right)\right]ds\right\}\end{aligned} \tag{16}$$

where

$$\begin{aligned}\dot{x}_i^T(t)W_i\dot{x}_i(t) &= \left(x_i^T(t)A_i^T+x_i^T\left(t-\tau_i(t)\right)A_{di}^T+\sum_{j=1,j\neq i}^{N}x_j^T(t-\eta_{ij}(t))A_{ij}^T\right)W_i \\ &\quad \times\left(A_ix_i(t)+A_{di}x_i\left(t-\tau_i(t)\right)+\sum_{j=1,j\neq i}^{N}A_{ij}x_j\left(t-\eta_{ij}(t)\right)\right) \\ &= x_i^T(t)A_i^TW_i\;A_ix_i(t)+x_i^T(t)A_i^TW_i\;A_{di}x_i\left(t-\tau_i(t)\right) \\ &\quad + x_i^T(t)A_i^TW_i\sum_{j=1,j\neq i}^{N}A_{ij}x_j\left(t-\eta_{ij}(t)\right)+x_i^T\left(t-\tau_i(t)\right)A_{di}^TW_i\;A_ix_i(t) \\ &\quad + x_i^T\left(t-\tau_i(t)\right)A_{di}^TW_i\;A_{di}x_i\left(t-\tau_i(t)\right)+x_i^T(t-\tau_i(t))A_{di}^TW_i\sum_{j=1,j\neq i}^{N}A_{ij}x_j(t-\eta_{ij}(t)) \\ &\quad + \sum_{j=1,j\neq i}^{N}x_j^T\left(t-\eta_{ij}(t)\right)A_{ij}^TW_i\;A_ix_i(t)+\sum_{j=1,j\neq i}^{N}x_j^T\left(t-\eta_{ij}(t)\right)A_{ij}^TW_i\;A_{di}x_i\left(t-\tau_i(t)\right) \\ &\quad + \sum_{j=1,j\neq i}^{N}x_j^T\left(t-\eta_{ij}(t)\right)A_{ij}^TW_i\cdot\sum_{j=1,j\neq i}^{N}A_{ij}x_j\left(t-\eta_{ij}(t)\right)\end{aligned} \tag{17}$$

Note that the following structural identity holds:

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}x_j^T(t)Z_{ij}x_j(t)=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}x_j^T(t)Z_{ij}x_j(t) \tag{18}$$

Also, the following inequality is verified (see Appendix):

$$\sum_{j=1, j\neq i}^{N} x_j^T\left(t-\eta_{ij}(t)\right)A_{ij}^T \sum_{j=1, j\neq i}^{N} A_{ij}x_j\left(t-\eta_{ij}(t)\right) \leq \left(2N-3\right)\sum_{j=1, j\neq i}^{N} x_j^T\left(t-\eta_{ij}(t)\right)A_{ij}^T A_{ij}x_j\left(t-\eta_{ij}(t)\right) \quad (19)$$

Then the inequality (16) can be expressed as follows:

$$\sum_{i=1}^{N}\dot{V}_i(t) \leq \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \zeta_i^T(t)\Phi_i\zeta_i(t) \qquad (20)$$

where

$$\zeta_i^T(t) = \left[x_i^T(t) \;\; x_i^T\left(t-\tau_i(t)\right) \;\; x_i^T\left(t-\eta_{ij}(t)\right) \;\; \dot{x}_i^T(s)\right] \qquad (21)$$

$$\Phi_i = \begin{bmatrix} \Upsilon_{11} & \frac{1}{N-1}\left(\rho_i A_i^T W_i A_{di} + P_i A_{di}\right) & P_i A_{ij} + \rho_i A_i^T W_i A_{ij} & 0 \\ * & \frac{1}{N-1}\left(\rho_i A_{di}^T W_i A_{di} - (1-\mu_i)Q_i\right) & \rho_i A_{di}^T W_i A_{ij} & 0 \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} \end{bmatrix} \qquad (22)$$

The condition $\dot{V}(t) \prec 0$ is satisfied if

$$\Phi_i = \begin{bmatrix} \Upsilon_{11} & \frac{1}{N-1}\left(\rho_i A_i^T W_i A_{di} + P_i A_{di}\right) & P_i A_{ij} + \rho_i A_i^T W_i A_{ij} & 0 \\ * & \frac{1}{N-1}\left(\rho_i A_{di}^T W_i A_{di} - (1-\mu_i)Q_i\right) & \rho_i A_{di}^T W_i A_{ij} & 0 \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} \end{bmatrix} \qquad (23)$$

which can be written as

$$\Pi_i + \begin{bmatrix} \rho_i A_i^T W_i \\ \rho_i A_{di}^T W_i \\ (N-1)\rho_i A_{ij}^T W_i \\ 0 \end{bmatrix} \left((N-1)\rho_i W_i\right)^{-1} \times \left[\rho_i W_i A_i \;\; \rho_i W_i A_{di} \;\; (N-1)\rho_i W_i A_{ij} \;\; 0\right] \prec 0 \qquad (24)$$

where

$$\Pi_i = \begin{bmatrix} \pi_{11} & \frac{1}{N-1}P_i A_{di} & P_i A_{ij} & 0 \\ * & -(1-\mu_i)Q_i & 0 & 0 \\ * & * & \pi_{33} & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} \end{bmatrix} \qquad (25)$$

Using Lemma 1 (Schur complements), inequality (25) is equivalent to the LMI (9).

This establishes the internal asymptotic stability.

Remark 1. Theorem 1 presents a new stability criterion for system (1) with time-varying delay. It is worth noting that condition (9) is an LMI, which can be readily checked by using the standard numerical software.

In the light of the result of Theorem 1, we are able to present our result on decentralized stabilization via linear state feedback for interconnected system of (1).

Theorem 2. Given $\rho_i \succ 0$, $\mu_i \succ 0$ and $\mu_{ij} \succ 0$, if there exist symmetric positive definite matrices P_i, Q_i, Z_{ij} and W_i, $i, j = 1, ..., N$, $i \neq j$, such that the following LMI holds:

$$\begin{bmatrix} \upsilon_{11} & \frac{1}{N-1}A_{di}X_i & A_{ij} & 0 & \rho_i\left(X_iA_i^T + y_i^TB_i^T\right) \\ * & -\frac{1-\mu_i}{N-1}R_i & 0 & 0 & \rho_iX_iA_{di}^T \\ * & * & \pi_{33} & 0 & (N-1)\rho_iA_{ij}^T \\ * & * & * & -\frac{\rho_iW_i}{N-1} & 0 \\ * & * & * & * & -(N-1)\rho_iS_i \end{bmatrix} \prec 0 \tag{26}$$

where

$$\upsilon_{11} = R_{ji} + \frac{1}{N-1}\left(A_iX_i + X_iA_i^T + B_iy_i + y_i^TB_i^T + R_i\right)$$
$$\pi_{33} = -\left(1-\mu_{ij}\right)Z_{ij},$$

The system (1) is decentralized stabilizable for any time delays $\tau_i(t)$, $\eta_{ij}(t)$ satisfying (2).

Moreover, the decentralized state feedback gain matrix is given by:

$$k_i = y_iX_i^{-1}. \tag{27}$$

Proof. By applying Theorem 1 to the closed-loop system of (4) we obtain

$$\Omega = \begin{bmatrix} \pi_{11} & \frac{1}{N-1}P_iA_{di} & P_iA_{ij} & 0 & \rho_iA_{ci}^TW_i \\ * & -\frac{1-\mu_i}{N-1}Q_i & 0 & 0 & \rho_iA_{di}^TW_i \\ * & * & \pi_{33} & 0 & (N-1)\rho_iA_{ij}^TW_i \\ * & * & * & -\frac{\rho_iW_i}{N-1} & 0 \\ * & * & * & * & -(N-1)\rho_iW_i \end{bmatrix} \prec 0 \tag{28}$$

where

$$\pi_{11c} = Z_{ji} + \frac{1}{N-1}\left(P_iA_{ci} + y_{ci}^TP_i + Q_i\right)$$
$$\pi_{33} = -\left(1-\mu_{ij}\right)Z_{ij}.$$

Pre-multiplying Ω by $\text{diag}\left[P_i^{-T}, P_i^{-T}, I, I, W_i^{-T}\right]$ and $\text{diag}\left[P_i^{-1}, P_i^{-1}, I, I, W_i^{-1}\right]$ and let $X_i = P_i^{-1}$, $S_i = W_i^{-1}$, $R_{ji} = X_iZ_{ji}X_i$, $R_i = X_iQ_iX_i$, we obtain

$$\begin{bmatrix} \vartheta_{11} & \frac{1}{N-1}A_{di}X_i & A_{ij} & 0 & \rho_iX_iA_{ci}^T \\ * & -\frac{1-\mu_i}{N-1}R_i & 0 & 0 & \rho_iX_iA_{di}^T \\ * & * & \pi_{33} & 0 & (N-1)\rho_iA_{ij}^T \\ * & * & * & -\frac{\rho_iW_i}{N-1} & 0 \\ * & * & * & * & -(N-1)\rho_iS_i \end{bmatrix} \prec 0 \tag{29}$$

where $\vartheta_{11} = R_{ji} + \frac{1}{N-1}\left(A_{ci}X_i + X_iA_{ci}^T + R_i\right)$.

Defining $y_i = k_iX_i$, we obtain the LMI conditions of Theorem 2.

Remark 2. The interconnections of the subsystems influence stabilization of the system as a whole. In order to guarantee the performance of local controllers, we propose to minimize the effect of interconnections between the subsystems by means of a H∞ criterion.

3.2. Decentralized State Feedback H∞ Control

In this section, we consider the problem of decentralized H∞ control of interconnected closed-loop system described by (4) using the Lyapunov method with LMI. Our objective is to provide a method of synthesis of decentralized control laws which allows both the asymptotic stability of each subsystem (4) and reduce the effect of interconnections between subsystems. We rely on a criterion H∞ to minimize the influence of j-th subsystem ($j = 1, ..., N$, and $j \neq i$) on the i-th subsystem. This criterion is given as follows:

For $i = 1, ..., N$,

$$\int_0^\infty x_i^T(t)x_i(t)dt \prec \gamma_1^2 \int_0^\infty \varphi_i^T(t)\varphi_i(t)dt \qquad (30)$$

where

$$\varphi_1(t) = \sum_{j=1, j\neq i}^{N} A_{ij}x_j\left(t-\eta_{ij}(t)\right) \qquad (31)$$

$\gamma_i \succ 0$ are the H∞ performances levels.

Note that the vector $\varphi_i(t)$ reflects the influence of a j-th subsystem on i-th and, in this case, it is appropriate to minimize the performance rate H∞.

The approach ensuring the simultaneous stabilization of the system (4), via the decentralized controller $u_i(t) = k_i x_i(t)$ and reducing the effect of interconnections between subsystems is summarized in the following theorem.

Theorem 3. Given $\rho_i \succ 0$, $\mu_i \succ 0$ and $\mu_{ij} \succ 0$, the decentrelized H∞ control problem for the system (4) is solvable if there exist symmetric positive definite matrices P_i, Q_i, Z_{ij} and W_i, $i, j = 1, ..., N$, $i \neq j$, and positive scalars γ_i, for any time delays $\tau_i(t)$, $\eta_{ij}(t)$ satisfying (2), which satisfy the following conditions LMI.

Minimizing γ_i such as

$$\begin{bmatrix} \vartheta_{11} & \frac{1}{N-1}A_{di}X_i & A_{ij} & 0 & \rho_i\left(X_iA_i^T+y_i^TB_i^T\right) & X_i \\ * & \frac{1-\mu_i}{N-1}R_i & 0 & 0 & \rho_iX_iA_{di}^T & 0 \\ * & * & \Theta_{33} & 0 & (N-1)\rho_iA_{ij}^T & 0 \\ * & * & * & -\frac{\rho_iW_i}{N-1} & 0 & 0 \\ * & * & * & * & -(N-1)\rho_iS_i & 0 \\ * & * & * & * & * & -(N-1)I \end{bmatrix} \prec 0 \qquad (32)$$

where

$$\vartheta_{11} = R_{ji} + \frac{1}{N-1}\left(A_iX_i + X_iA_i^T + B_iy_i + y_i^TB_i^T + R_i\right)$$

$$\Theta_{33} = -\gamma_i^2\left(2N-3\right)A_{ij}^TA_{ij} - \left(1-\mu_{ij}\right)Z_{ij}.$$

Moreover, the decentralized state feedback gain matrix is given by:

$$k_i = y_iX_i^{-1}, \quad i = 1, ..., N. \qquad (33)$$

Proof. The closed loop system (4) is stable under the criterion H∞ if:

$$\sum_{i=1}^{N}\left[\dot{V}_i(t) + x_i^T(t)x_i(t) - \gamma_i^2\left(\sum_{j=1,j\neq i}^{N} A_{ij}x_j\left(t-\eta_{ij}(t)\right)\right)^T \cdot \sum_{j=1,j\neq i}^{N} A_{ij}x_j\left(t-\eta_{ij}(t)\right)\right] \prec 0 \qquad (34)$$

The above inequality (34) may be increased by (see Appendix):

$$\sum_{i=1}^{N}\left[\dot{V}_i(t) + x_i^T(t)x_i(t) - \gamma_i^2\left(2N-3\right) \cdot \sum_{j=1,j\neq i}^{N}\left(x_j^T(t-\eta_{ij}(t))A_{ij}^TA_{ij}x_j(t-\eta_{ij}(t))\right)\right] \qquad (35)$$

with

$$\sum_{i=1}^{N}\dot{V}_i(t) \le \sum_{i=1}^{N}\left\{\frac{1}{\tau_i(t)}\int_{t-\tau_i(t)}^{t}\left[2x_i^T(t)P_i\left(A_{ci}x_i(t)+A_{di}x_i\left(t-\tau_i(t)\right)+\sum_{j=1,j\neq i}^{N}A_{ij}x_j\left(t-\eta_{ij}(t)\right)\right)\right.\right.$$
$$+\, x_i^T(t)Q_ix_i(t)-\left(1-\mu_i\right)x_i^T\left(t-\tau_i(t)\right)Q_ix_i\left(t-\tau_i(t)\right)+\rho_i\dot{x}_i^T(t)W_i\dot{x}_i(t)-\tau_i(t)\dot{x}_i^T(s)W_i\dot{x}_i(s)$$
$$\left.\left.+\sum_{i=1,j\neq i}^{N}x_i^T(t)Z_{ji}x_i(t)-\sum_{i=1,j\neq i}^{N}\left(1-\mu_{ij}\right)x_j^T\left(t-\eta_{ij}(t)\right)Z_{ij}x_j\left(t-\eta_{ij}(t)\right)\right]ds\right\} \tag{36}$$

Let

$$\zeta_i^T(t)=\left[x_i^T(t)\;\; x_i^T\left(t-\tau_i(t)\right)\;\; x_i^T\left(t-\eta_{ij}(t)\right)\;\; \dot{x}_i^T(s)\right] \tag{37}$$

we obtain

$$\sum_{i=1}^{N}\left[\dot{V}_i(t)+x_i^T(t)x_i(t)-\gamma_i^2\left(2N-3\right)\cdot\sum_{j=1,j\neq i}^{N}\left(x_j^T(t-\eta_{ij}(t))A_{ij}^TA_{ij}x_j(t-\eta_{ij}(t))\right)\right]\le\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\zeta_i^T(t)\Xi_i\zeta_i(t) \tag{38}$$

with

$$\Xi_i=\begin{bmatrix}\Lambda_{11} & \frac{1}{N-1}\left(\rho_iA_{ci}^TW_iA_{di}+P_iA_{di}\right) & P_iA_{ij}+\rho_iA_{ci}^TW_iA_{ij} & 0\\ * & \frac{1}{N-1}\left(\rho_iA_{di}^TW_iA_{di}-\left(1-\mu_i\right)Q_i\right) & \rho_iA_{di}^TW_iA_{ij} & 0\\ * & * & \Lambda_{33} & 0\\ * & * & * & -\frac{\rho_iW_i}{N-1}\end{bmatrix}\prec 0 \tag{39a}$$

$$\Lambda_{11}=Z_{ji}+\frac{1}{N-1}\left(P_iA_{ci}+A_{ci}^TP_i+\rho_iA_{ci}^TW_iA_{ci}+Q_i+I\right) \tag{39b}$$

$$\Lambda_{33}=-\gamma_i^2\left(2N-3\right)A_{ij}^TA_{ij}-\left(1-\mu_{ij}\right)Z_{ij}+\left(N-1\right)A_{ij}^T\rho_iW_iA_{ij}. \tag{39c}$$

That is to say, for all $i, j = 1, ..., N$ and $j \neq i$

$$\begin{bmatrix}\Lambda_{11} & \frac{1}{N-1}\left(\rho_iA_{ci}^TW_iA_{di}+P_iA_{di}\right) & P_iA_{ij}+\rho_iA_{ci}^TW_iA_{ij} & 0\\ * & \frac{\rho_iA_{di}^TW_iA_{di}}{N-1}-\left(1-\mu_i\right)Q_i & \rho_iA_{di}^TW_iA_{ij} & 0\\ * & * & \Lambda_{33} & 0\\ * & * & * & -\frac{\rho_iW_i}{N-1}\end{bmatrix}\prec 0 \tag{40}$$

Inequality (40) can be rewritten as:

$$\Pi_i + \begin{bmatrix} \rho_i A_{ci}^T W_i \\ \rho_i A_{di}^T W_i \\ (N-1)\rho_i A_{ij}^T W_i \\ 0 \end{bmatrix} \left((N-1)\rho_i W_i\right)^{-1} \times \begin{bmatrix} \rho_i W_i A_{ci} & \rho_i W_i A_{di} & (N-1)\rho_i W_i A_{ij} & 0 \end{bmatrix} \prec 0 \tag{41}$$

where

$$\Pi_i = \begin{bmatrix} \Theta_{11} & \frac{1}{N-1} P_i A_{di} & P_i A_{ij} & 0 \\ * & -(1-\mu_i) Q_i & 0 & 0 \\ * & * & \Theta_{33} & 0 \\ * & * & * & -\frac{\rho_i W_i}{N-1} \end{bmatrix} \tag{42a}$$

$$\Theta_{11} = Z_{ji} + \frac{1}{N-1}\left(P_i A_{ci} + A_{ci}^T P_i + Q_i + I \right) \tag{42b}$$

$$\Theta_{33} = -\gamma_i^2 (2N-3) A_{ij}^T A_{ij} - (1-\mu_{ij}) Z_{ij} \tag{42c}$$

Applying the Schur complement we obtain

$$\begin{bmatrix} \Theta_{11} & \frac{1}{N-1} P_i A_{di} & P_i A_{ij} & 0 & \rho_i A_{ci}^T W_i \\ * & -\frac{1-\mu_i}{N-1} Q_i & 0 & 0 & \rho_i A_{di}^T W_i \\ * & * & \Theta_{33} & 0 & (N-1)\rho_i A_{ij}^T W_i \\ * & * & * & -\frac{\rho_i W_i}{N-1} & 0 \\ * & * & * & * & -(N-1)\rho_i W_i \end{bmatrix} \prec 0 \tag{43}$$

(43) is satisfied if:

$$\psi = \begin{bmatrix} \psi_{11} & \frac{1}{N-1} P_i A_{di} & P_i A_{ij} & 0 & \rho_i A_{ci}^T W_i & I \\ * & -\frac{1-\mu_i}{N-1} Q_i & 0 & 0 & \rho_i A_{di}^T W_i & 0 \\ * & * & \Theta_{33} & 0 & (N-1)\rho_i A_{ij}^T W_i & 0 \\ * & * & * & \frac{\rho_i W_i}{N-1} & 0 & 0 \\ * & * & * & * & -(N-1)\rho_i W_i & 0 \\ * & * & * & * & * & -(N-1) I \end{bmatrix} \prec 0 \tag{44}$$

where

$$\psi_{11} = Z_{ji} + \frac{1}{N-1}\left(P_i A_{ci} + A_{ci}^T P_i + Q_i\right).$$

Multiplying ψ left and right respectively by diag$[P_i^{-T}, P_i^{-T}, I, I, W_i^{-T}, I]$ and diag$[P_i^{-1}, P_i^{-1}, I, I, W_i^{-1}, I]$, and let $X_i = P_i^{-1}$, $y_i = k_i X_i$, $S_i = W_i^{-1}$, $R_{ji} = X_i Z_{ji} X_i$, $R_i = X_i Q_i X_i$.

Finally, we obtain the LMI condition (32) of Theorem 3. This completes the proof.

4. Numerical Example

In this section, a numerical example is supplied to show the advantage of the proposed approach. Consider the interconnected system [13] given by

$$A_1 = \begin{bmatrix} -2 & 0 \\ -2 & -1 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0 \\ -2 & -2 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}, A_{21} = \begin{bmatrix} -1 & -2 \\ 3 & 6 \end{bmatrix},\ A_{23} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}, A_3 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix},\ A_{d3} = \begin{bmatrix} 0 & 0 \\ -1 & -2 \end{bmatrix},$$

$$A_{31} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix},\ A_{32} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix},\ B_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

The interconnections of the subsystems influence the stabilization of the system as a whole. In order to guarantee performance of the local controllers, we propose to minimize the effect of the interconnections between the subsystems by means of a H∞ criterion.

Applying Theorem 3, for:

$$\rho_1 = 2;\ \mu_1 = 0.6;\ \rho_2 = 2;$$

$$A_{31} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix},\ A_{32} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix},\ B_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\mu_{12} = 0.5;\ \mu_{13} = 0.7;\ \mu_{23} = 0.2;$$

$$\mu_{21} = 0.9;\ \mu_{31} = 0.6;\ \mu_{32} = 0.3,$$

we obtain

$$P_1 = \begin{bmatrix} 1.4664 & -0.1928 \\ -0.1928 & 0.1505 \end{bmatrix};$$

$$Q_1 = \begin{bmatrix} 28.4358 & -4.0533 \\ -4.0533 & 0.7780 \end{bmatrix};$$

$$W_1 = \begin{bmatrix} 0.1334 & -0.0020 \\ -0.0020 & 0.1343 \end{bmatrix}; P_2 = \begin{bmatrix} 2.6855 & 1.0542 \\ 1.0542 & 0.5788 \end{bmatrix};$$

$$Q_2 = \begin{bmatrix} 116.5229 & 48.1746 \\ 48.1746 & 20.2471 \end{bmatrix}; W_2 = \begin{bmatrix} 0.1313 & 0.0116 \\ 0.0116 & 0.1018 \end{bmatrix};$$

$$P_3 = \begin{bmatrix} 0.7291 & 1.6097 \\ 1.6097 & 4.9738 \end{bmatrix}; Q_3 = \begin{bmatrix} 46.8429 & 137.7018 \\ 137.7018 & 409.9437 \end{bmatrix};$$

$$W_3 = \begin{bmatrix} 0.1942 & -0.0143 \\ -0.0143 & 0.1883 \end{bmatrix},$$

since P_i, Q_i, $W_i \succ 0$, $i = 1, 2, 3$.

Then the conditions provided by Theorem 3 are satisfied.

The results obtained with our approach, as well as those in the literature, are summarized in Tables 1 and 2.

Table 1 compares minimum H∞ performances for the three subsystems. We note that the gamma obtained by applying theorem 3 is smaller than that given by [13], whereas Theorem 3 is less conservative than that of [13]. This shows the effectiveness of our approach.

Table 2 shows the gain matrices obtained for the three subsystems. It can be seen that the three local controllers are capable of stabilizing the three subsystems.

Also, from Tables 1 and 2 follows that Theorem 3 in our approach yields less conservativeness than the previous result. This shows that our stabilization conditions give better results than [13].

Table 1. Minimum H∞ performance $\gamma_{\min}$ of interconnected system with time-varying delays.

Methods	$\gamma_{\min}$		
	First subsystem	Second subsystem	Third subsystem
By [13]	$\gamma_1 = 1.8011$	$\gamma_2 = 13.4931$	$\gamma_3 = 1.4746$
By Theorem 3	$\gamma_1 = 1.6725$	$\gamma_2 = 0.5162$	$\gamma_3 = 0.9170$

Table 2. Decentralized control laws k_i.

Methods	γ_{min}		
	First subsystem	Second subsystem	Third subsystem
By [13]	$k_1 = [1.1187 \;\; 0.8962]$	$k_2 = \begin{bmatrix} -8.8441 & 1.3237 \\ -3.9501 & -3.3300 \end{bmatrix}$	$k_3 = [-0.6289 \;\; -0.2550]$
By Theorem 3	$k_1 = [1.1374 \;\; 0.4022]$	$k_2 = \begin{bmatrix} -0.3479 & -1.1906 \\ 0.5539 & 1.2342 \end{bmatrix}$	$k_3 = [-0.0237 \;\; -0.6346]$

5. Conclusion

This paper focuses on delay-dependent stability/stabilization and the H∞ control problem of interconnected system with time-varying delays. Based on the Lyapunov- Krasovskii functional, a new decentralized delay-dependent stabilization and H∞ control conditions are established in terms of linear matrix inequalities (LMIs). The behavior and efficiency of the control design approach has been illustrated by means of an example and compared with some recent approaches by other authors.

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Appendix

A. List of Symbol Used

$[a_{ij}]$	Matrix with a coefficient of the *i*-th line and *j*-th collone is a_{ij}
M^T	Transpose of the matrix M
M^{-1}	Inverse of the matrix M
$M \succ 0$	Positive definite matrices
$M \prec 0$	Negative definite matrices
$\|G(s)\|_\infty$	Norm of a transfer matrix $G(s)$
N	Set of natural numbers
$\Re^n$	Set of real vectors of dimension n
$\Re^{n\times m}$	Set of real matrices of size $n \times m$
I	Identity matrix of appropriate dimension
*	Symmetric term of a square symmetric matrix

B. Decentralized State Feedback H∞ Control

In this Appendix, we verify inequality (19) used in Section 3, with:

$$\varphi_i = \sum_{j=1, j\neq i}^{N} A_{ij} x_j \left(t - \eta_{ij}(t)\right) \tag{A1}$$

$$
\begin{aligned}
\varphi_i^T \varphi_i &= \left(\sum_{j=1, j\neq i}^{N} A_{ij} x_j \left(t-\eta_{ij}(t)\right) \right)^T \cdot \sum_{j=1, j\neq i}^{N} A_{ij} x_j \left(t-\eta_{ij}(t)\right) \\
&= \sum_{j=1, j\neq i}^{N} \sum_{\substack{l=1 \\ l\neq i, l\neq j}}^{N} \left[x_j^T \left(t-\eta_{ij}(t)\right) A_{ij}^T A_{il} x_l \left(t-\eta_{ij}(t)\right) + x_l^T \left(t-\eta_{ij}(t)\right) A_{il}^T A_{ij} x_j \left(t-\eta_{ij}(t)\right) \right] \\
&= \sum_{j=1, j\neq i}^{N} x_j^T \left(t-\eta_{ij}(t)\right) A_{ij}^T A_{ij} x_j \left(t-\eta_{ij}(t)\right) + \sum_{\substack{l=1 \\ l\neq i, l\neq j}}^{N} \left[x_j^T \left(t-\eta_{ij}(t)\right) A_{ij}^T A_{il} x_l \left(t-\eta_{ij}(t)\right) \right. \\
&\quad \left. + x_l^T \left(t-\eta_{ij}(t)\right) A_{il}^T A_{ij} x_j \left(t-\eta_{ij}(t)\right) \right]
\end{aligned} \tag{A2}
$$

Applying the lemma of the square matrix, we obtain:

$$
\begin{aligned}
& \sum_{\substack{l=1 \\ l\neq i, l\neq j}}^{N} \left[x_j^T \left(t-\eta_{ij}(t)\right) A_{ij}^T A_{il} x_l \left(t-\eta_{ij}(t)\right) + x_l^T \left(t-\eta_{ij}(t)\right) A_{il}^T A_{ij} x_j \left(t-\eta_{ij}(t)\right) \right] \\
& \leq \sum_{\substack{l=1 \\ l\neq i, l\neq j}}^{N} \left[A_{ij} x_j \left(t-\eta_{ij}(t)\right) \right]^T \cdot \left[A_{ij} x_j \left(t-\eta_{ij}(t)\right) \right] + \left[A_{il} x_l \left(t-\eta_{ij}(t)\right) \right]^T \left[A_{il} x_l \left(t-\eta_{ij}(t)\right) \right] \\
& = (N-2) \left[A_{ij} x_j \left(t-\eta_{ij}(t)\right) \right]^T \cdot \left[A_{ij} x_j \left(t-\eta_{ij}(t)\right) \right] + \sum_{\substack{l=1 \\ l\neq i, l\neq j}}^{N} \left[A_{il} x_l \left(t-\eta_{ij}(t)\right) \right]^T \cdot \left[A_{il} x_l \left(t-\eta_{ij}(t)\right) \right]
\end{aligned} \tag{A3}
$$

Then

$$
\varphi_i^T \varphi_i \leq \sum_{j=1, j\neq i}^{N} \left[\left(N-1\right) x_j^T (t-\eta_{ij}(t)) \cdot A_{ij}^T A_{ij} x_j (t-\eta_{ij}(t)) + \sum_{\substack{l=1 \\ l\neq i, l\neq j}}^{N} x_l^T (t-\eta_{ij}(t)) A_{il}^T A_{il} x_l \left(t-\eta_{ij}(t)\right) \right] \tag{A4}
$$

Since

$$
\sum_{j=1, j\neq i}^{N} \left(\psi_{ij} + \sum_{\substack{l=1 \\ l\neq i, l\neq j}}^{N} \psi_{il} \right) = \left(N-1\right) \sum_{j=1, j\neq i}^{N} \psi_{ij} \tag{A5}
$$

inequality (A.4) can be rewritten as follows:

$$
\varphi_i^T \varphi_i \leq \left(2N-3\right) \cdot \sum_{j=1, j\neq i}^{N} x_j^T (t-\eta_{ij}(t)) A_{ij}^T A_{ij} x_j \left(t-\eta_{ij}(t)\right) \tag{A6}
$$

Finally, inequality (35) is verified.

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